

Experiment 5: Newton's Second Law



Figure 5.1: Newton's Second Law Setup

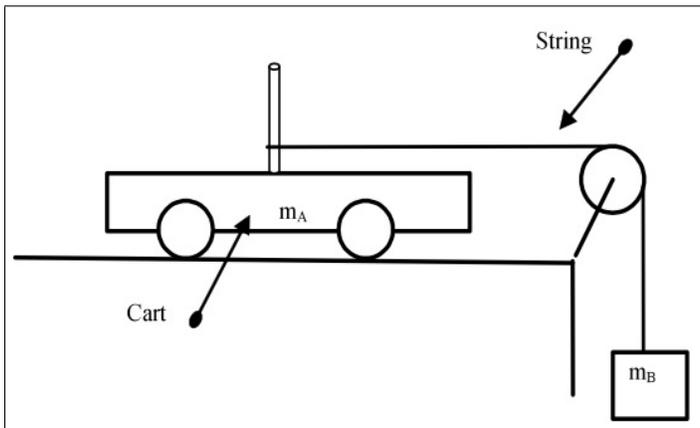


Figure 5.2: Note: String from cart to pulley must be horizontal and aligned with the pulley.

EQUIPMENT

- Low-Friction Cart
- Pulley and String
- Triple-Beam Balance
- Digital Balance
- Stopwatch
- Meter Stick
- Mass Hanger
- (1) 10 g Mass
- (2) 20 g Masses
- (1) 50 g Mass
- Paper Clips (small masses)
- Plumb Bob
- Wastebasket

FREE-BODY DIAGRAM SOLUTION METHOD: INSTRUCTIONS

Step 1: Sketch the problem/situation and specify the coordinate system for each object in your system.

Step 2: Draw all forces (arrows that represent these vectors) acting on *each object* in the system you are investigating. All forces should extend *away from* the object in the direction of the force. Remember that the length of the arrow is an indication of the magnitude of the force.

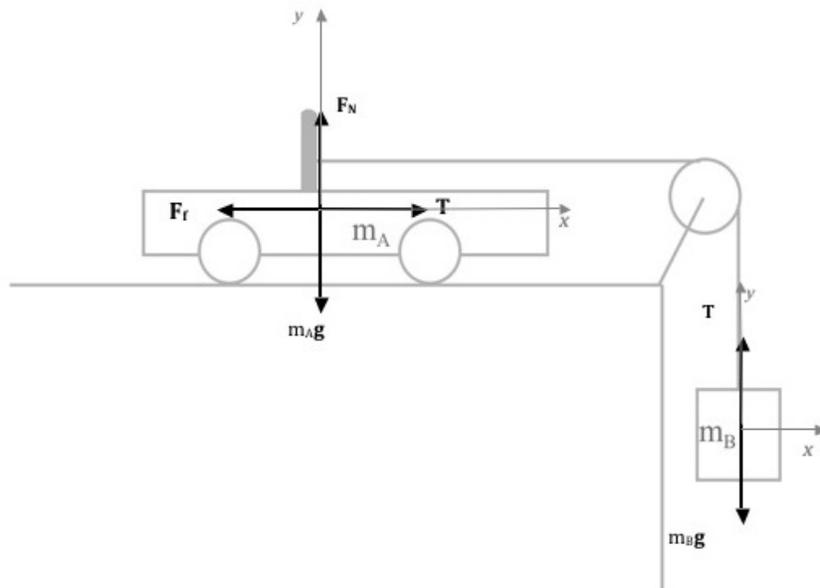


Figure 5.3: Free-Body Diagram of Modified Atwood's Machine (Example for \vec{v} constant, $\Sigma \vec{F} = m\vec{a} = 0.0 \text{ N}$).

Step 3: Write Newton's 2nd Law ($\Sigma \vec{F} = m\vec{a}$) in component form ($\Sigma F_x = ma_x$ and $\Sigma F_y = ma_y$) for *each* object in the system. For this example, m_A and m_B :

$$m_A: \begin{aligned} \Sigma F_{A_x} &= m_A a_{A_x} \\ \Sigma F_{A_y} &= m_A a_{A_y} \end{aligned}$$

$$m_B: \begin{aligned} \Sigma F_{B_x} &= m_B a_{B_x} \\ \Sigma F_{B_y} &= m_B a_{B_y} \end{aligned}$$

Step 4:

Refer to your diagram to sum the forces for each object as instructed by Sir Isaac Newton. For this example:

$$\Sigma F_{A_x} = m_A a_{A_x} = T - F_f$$

$$\Sigma F_{A_y} = m_A a_{A_y} = F_N - m_A g = 0.0 \text{ N}$$

(a_{A_y} : Constant velocity means this acceleration equals 0.0 m/s^2)

$$\Sigma F_{B_x} = m_B a_{B_x} = 0.0 \text{ N}$$

(m_B : No forces acting in the horizontal direction.)

$$\Sigma F_{B_y} = m_B a_{B_y} = T - m_B g$$

(Note that since m_A and m_B are connected and moving together, T is the same for each object, and $a_{A_x} = -a_{B_y}$.)

Step 5:

Write the known quantities. Write the question.

Always write equations algebraically first, then insert the known values.

Now do the math! That's all there is to it.

Advance Reading

Text: Newton's Second Law, acceleration, velocity, displacement, vectors.

Lab Manual: Appendix C

Objective

The objective of this lab is to explore and analyze the relationship between force, mass, and acceleration.

Theory

According to Newton's Second Law, the acceleration, \vec{a} , of a body is directly proportional to the vector sum of the forces, $\Sigma\vec{F}$, applied to the body:

$$\Sigma\vec{F} = m\vec{a} \quad (5.1)$$

where m is the mass of the body.

The experimental configuration for this experiment is a variation of Atwood's machine (Fig. 5.2, Fig. C.6).

A force T (tension) will be applied to the cart, m_A , by means of a string with an attached mass, m_B . If one can ignore the force of friction acting on the cart, then Eq. 5.1 in the direction of motion simplifies to:

$$\Sigma F_{Ax} = m_A a_{Ax} = T \quad (5.2)$$

Thus, if the mass of the cart is doubled while T is held constant, the acceleration of the cart is halved (*Part 1*). Correspondingly, if T is doubled while m_A is held constant, the acceleration of the cart is doubled (*Part 2*).

This analysis assumes a frictionless environment. For simplicity, F_f will be counterbalanced by a small mass, m_f , hanged from one end of the system. When the weight of m_f is equal to the force of friction ($m_f g = F_f$), the system will be in equilibrium.

$$\Sigma F = 0 \text{ N} \quad a = 0 \text{ m/s}^2$$

In equilibrium, the cart set in motion continues moving with constant velocity. A new mass m_f must be found each time the cart's mass is adjusted in *Part 1*, as F_f will have changed.

Once F_f has been counteracted, any additional mass will be directly related to the acceleration of the system as in Eq. 5.1. The added mass, m_B , exerts a force equal to its weight on the cart/mass system.

When the cart is accelerating, deriving T and a of the cart is more involved. The relevant equations for this experiment are provided:

$$a = \frac{m_B \cdot g}{m_A + m_B + m_f} \quad (5.3)$$

$$T = \frac{m_A \cdot m_B \cdot g}{m_A + m_B + m_f} + m_f \cdot g \quad (5.4)$$

We will compare the acceleration from Eq. 5.3 to the acceleration obtained from the *kinematic equations for constant acceleration* (Page 22, *Experiment 4*).

In this experiment, acceleration will be found experimentally tracking the cart's motion across a set distance. The time of travel will be carefully measured using a stopwatch.

Note that since the cart and m_B are connected, their acceleration, velocity, and distance traveled are equal at all times. Thus, the horizontal distance the cart travels, Δx , is equal to the vertical drop of the attached mass, Δy .

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \quad (5.5)$$

We now define calculated and measured accelerations as $a_{Theo.}$ and $a_{Meas.}$:

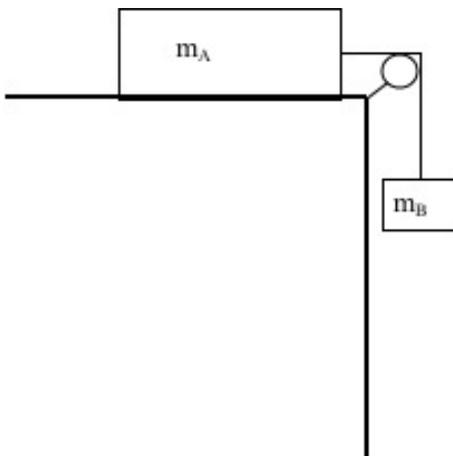
$a_{Theo.}$ is determined by Eq. 5.3;

$a_{Meas.}$ is determined by Eq. 5.5.

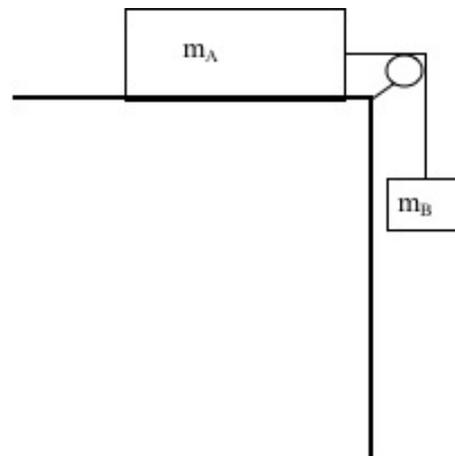
Name: _____

1. State Newton's First and Second Laws (qualitative explanations). (20 pts)
2. How is tension applied to the cart in this experiment? (15 pts)
3. How is friction compensated for in this experiment? (15 pts)
4. Should the acceleration determined in *Step 5* (or any part of this experiment) be greater than, less than, or equal to the acceleration of gravity? Explain. Assume friction is negligible; refer to Eq. 5.3. (20 pts)
5. Complete the free-body diagrams for the two situations shown below. Draw to scale (*i.e.*, your diagrams should delineate between $\Sigma \vec{F} = m\vec{a} = 0.0 \text{ N}$ and $\Sigma \vec{F} = m\vec{a} > 0.0 \text{ N}$). (Refer to the lab manual *Theory*) (30 pts)

Situation 1: $\Sigma \vec{F} = m\vec{a} = 0.0 \text{ N}$



Situation 2: $\Sigma \vec{F} = m\vec{a} > 0.0 \text{ N}$



PROCEDURE**PART 1: Vary the Mass of the Cart, m_a**

Trial #1

1. Measure the mass of the cart, m_A , using the triple beam balance. Record it in the table provided.
2. Measure the frictional force acting on the cart:
Add small masses to the string until the cart maintains a constant velocity when tapped. The weight of m_f is equal to F_f . Record m_f in the table provided.
3. Attach m_B (0.05 kg) to the end of the string. [Measure its actual mass and record it]
4. Use a stopwatch to measure the time it takes for the cart to travel a distance Δx (or for m_B to fall the same distance Δy). Your data will be more accurate if Δx is as large as possible. Do not allow the cart to hit the pulley.
5. Determine theoretical and measured acceleration for the cart.

Trial #2

6. Remove m_B from the system. Add 0.5 kg to the cart and measure its total mass.
7. Determine the new force of friction on the cart and adjust m_f .
8. Replace m_B and measure time and distance of the cart's motion.
9. Determine acceleration for this heavier cart.

Trial #3

10. Remove m_B and add 0.5 kg to the cart. Measure the force of friction and adjust m_f for this case.
11. Replace m_B and determine acceleration for this case.

PART 2: Vary the Applied Force

12. Keep the weight of the cart (approx. 1.5 kg) and keep the same m_f as in the previous step.
13. Trial #1:
Increase m_B to 0.07 kg, find $a_{Theo.}$ and $a_{Meas.}$
14. Trial #2:
Increase m_B to 0.09 kg, find $a_{Theo.}$ and $a_{Meas.}$
15. Trial #3:
Increase m_B to 0.11 kg, find $a_{Theo.}$ and $a_{Meas.}$

PART 3: Graphing

The entire cart/hanging mass system follows the same law, $\Sigma F = ma$. This means that plotting force vs. acceleration yields a linear relationship (of the form $y = mx$).

16. Open Graphical Analysis. Graph force ($m_B g$) vs. acceleration ($a_{Meas.}$) for the *Varying Force* trials. Apply a linear fit to the three data points. Print this graph.
17. Create a similar graph of $m_B g$ vs. $a_{Theo.}$ and print it.

QUESTIONS

Reminder: Use the method specified on Page 5 for Question 2

1. Assume a modified Atwood's Machine arrangement similar to today's experiment and a frictionless cart that continues moving after the hanging mass has reached the floor, thus no longer exerting a force on the cart. Qualitatively sketch v vs. t for this arrangement, starting from rest at $t = 0.0$ s, and include times after the hanging mass has reached the floor (long table!).
2. Draw a free-body diagram for the cart/hanging mass system shown in Fig. 5.2, *ignoring friction*. Use this diagram to derive an equation of force, T , that has only masses and acceleration due to gravity, g (*i.e.*, an equation similar to Eq. 5.4).