Experiment 20: Exponentials and RC Circuits

**EQUIPMENT**

Universal Circuit Board
(1) 680 kΩ Resistor
(4) Jumpers
(1) 47 µF Capacitor
(4) Wire Leads
Digital Multi-Meter (DMM)
Power Supply
Stopwatch

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Figure 20.1: *Part 1: RC Circuit*

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**Figure 20.2: Exponential growth general equation** is 

\[ N = N_0 e^{\lambda t} \]

**Figure 20.2: Exponential decay general equation** is 

\[ N = N_0 e^{-\lambda t} \]

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Exponential decay general equation is \( N = N_0 e^{-\lambda t} \)
**Advance Reading (Exponentials)**

*Text:* Exponential decay, RC circuit

*Lab Manual: Appendix C*

**Objective**

The objective of Part 1 is to investigate an exponential curve by analysis of an RC circuit. Our RC circuit will be a resistor $R$ and a capacitor $C$ connected in parallel.

**Theory**

When the rate of change of a quantity is proportional to the initial quantity, there is an exponential relationship.

An exponential equation is one in which $e$ is raised to a power. $e$ is the irrational number 2.7182818...

An exponential equation has the form:

$$A(t) = A_0e^{Bt} + C$$  \hspace{1cm} (20.1)

where $A$ is the amount or number after a time $t$ and $A_0$ is the initial amount or number.

Exponential rates are found everywhere in nature. Some examples include exponential growth (e.g., population of Earth) and exponential decay (e.g., decay of radioactive elements), as well as heating and cooling rates.

Analysis of exponential curves often involves the *doubling-time* or *half-life* ($T_{1/2}$). Doubling-time is the time required for an initial amount to double in quantity (what is $t$ when $A = 2A_0$?). The half-life is the time required for 1/2 of the initial amount to be gone (what is $t$ when $A = \frac{1}{2}A_0$?).

Analyzing these curves is often simple. For doubling-time or half-life, begin with the appropriate value on the *y-axis*. Then draw a horizontal line that intersects the plot. Next, drop a perpendicular line to the *x-axis*. Read the value of the time directly from the *x-axis.*

The data for our exponential curve will be obtained by measuring the potential difference across the resistor of an RC circuit as a function of time.

A capacitor is connected in parallel with a resistor, then charged to some initial voltage. When the power supply is disconnected, the potential difference across the capacitor will decrease exponentially. The voltage, $V$, across the capacitor as it discharges is given by:

$$V_t = V_0e^{-t/RC}$$  \hspace{1cm} (20.2)

where $V_0$ is the initial potential difference across the capacitor at time $t_0$, $R$ is resistance, and $C$ is capacitance.

Consider Eq. 20.2. When $t = RC$,

$$V_t = V_0e^{-1} = V_0\frac{1}{e}$$  \hspace{1cm} (20.3)

We define $\tau$ (Greek letter, tau) to be the time it takes for the voltage to drop to $(1/e) \cdot (V_0)$ its original value (about 37%). This is the value when $\tau = RC$.

For our discharging capacitor:

A graph of *Voltage vs. Time* is an exponential decay curve. Analysis of this curve provides the time constant by locating the point at which $V$ has dropped to $1/e$ its original value of $V_0$. 

Exponential Curve

1. Refer to Fig. 20.2. Consider the equations:

\[ y = Ae^{(C-x)} + B \]
\[ N = N_0e^{(\lambda t)} \]

What is \( N_0 \) [i.e. \( N_{t=0} \)]? (You may read the information directly from the auto-fit box.) (10 pts)

2. Refer to Fig. 20.3, below. The doubling-time is the time it takes for the initial number or amount to double. What is the doubling-time \( (N = 2N_0) \)? (Find the value on the y-axis, then move horizontally until you intersect the plot. Drop a perpendicular line to the x-axis and read the time.) (20 pts)

Figure 20.3: Exponential Growth
Radioactive Decay - Carbon Dating (30 pts)

Assume Fig. 20.4 represents 100% of the carbon found in all living matter. Carbon 14 ($^{14}$C) has a half-life ($T_{1/2}$) of 5,730 years. The square represents a sample of $^{14}$C. $^{14}$C emits $\beta^-$ particles.

3. Divide the square in half with a vertical line and write “5,700 yrs” (rounded for simplification) on the left side, to represent the amount of $^{14}$C decayed after its half-life of 5,700 years ($T_{1/2} = 5,700$ yrs).

4. Now divide the right side in half with a horizontal line and write “11,400 yrs” to represent the amount of $^{14}$C decayed after an additional half-life.

5. Continue to divide the remaining sample in this manner to show the amount of $^{14}$C decayed after 17,100 yrs; 22,800 yrs; 28,500 yrs; 34,900 yrs; and 45,600 yrs.

Figure 20.4: Carbon Dating Square
**PROCEDURE**

**PART 1: RC Circuit**

The capacitors are electrolytic. They must be connected with the negative end of the capacitor on the negative side of the power supply; failure to do so will damage the capacitor.

Refer to Fig. 20.5

1. Determine the nominal resistance of $R_1$. Then measure $R_1$ with the DMM. Record these in the table provided.

2. Measure the capacitance, $C$, of the capacitor using the DMM at the TA’s table. Take care not to plug it in backwards - this will damage the capacitor.

3. Construct the circuit shown in Fig. 20.5. Connect, but do not plug in, the power supply.

4. Measuring voltage with a voltmeter changes the equivalent resistance of the circuit. To account for this, calculate $R_{eq}$ of the resistor in parallel with the 10 MΩ voltmeter. Use the measured value of $R_1$.

5. Calculate the theoretical value of the time constant $\tau$ using $R_{eq}$ and $C$.

6. Ask your TA to approve the circuit. Plug in and turn on the power supply. Charge the capacitor by applying 10.0 V across the circuit.

7. Disconnect one power supply lead from the circuit. Take voltage measurements every five seconds for three minutes. [The voltage should drop to the mV range, requiring you to adjust the voltmeter scale twice. If voltage drops to zero, check your circuit.]

8. Disassemble the circuit. Turn off the DMM.

**PART 2: Graphing**

9. Graph $V$ vs. $t$ using *Graphical Analysis*. Analyze the curve using a natural exponent fit. Determine the time constant from your curve fit. Print the graph.

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**QUESTIONS**

1. Show that the product of $RC$ has units of seconds.

2. If an RC circuit has $\tau = 15$ seconds, how long would it take for the circuit to discharge to $1/e^7$ its original value?

3. Calculate $\tau$ for an RC circuit consisting of a 3 μF capacitor and a 1.5 MΩ resistor.