Experiment 20: Exponentials and Oscilloscopes

Figure 20.1: Part 2: RC Circuit

**EQUIPMENT**
- Universal Circuit Board
- (1) 680 kΩ Resistor
- (4) Jumpers
- (1) 47 µF Capacitor
- (4) Wire Leads
- Digital Multi-Meter (DMM)
- Power Supply
- Stopwatch

Figure 20.2: Exponential growth general equation is $N = N_0 e^{\lambda t}$

Figure 20.2: Exponential growth general equation is $N = N_0 e^{\lambda t}$

Exponential decay general equation is $N = N_0 e^{-\lambda t}$
Advance Reading (Exponentials)

Text: Exponential decay, RC circuit

Lab Manual: Appendix B

Objective
The objective of Part 2 is to investigate an exponential curve by analysis of an RC circuit. Our RC circuit will be a resistor \( R \) and a capacitor \( C \) connected in parallel.

Theory
When the rate of change of a quantity is proportional to the initial quantity, there is an exponential relationship.

An exponential equation is one in which \( e \) is raised to a power. \( e \) is the irrational number 2.7182818\ldots

An exponential equation has the form:

\[
A(t) = A_0 e^{Bt} + C
\]  

(20.1)

where \( A \) is the amount or number after a time \( t \) and \( A_0 \) is the initial amount or number.

Exponential rates are found everywhere in nature. Some examples include exponential growth (e.g., population of Earth) and exponential decay (e.g., decay of radioactive elements), as well as heating and cooling rates.

Analysis of exponential curves often involves the doubling-time or half-life (\( T_{1/2} \)). Doubling-time is the time required for an initial amount to double in quantity (what is \( t \) when \( A = 2A_0 \)?). The half-life is the time required for \( 1/2 \) of the initial amount to be gone (what is \( t \) when \( A = \frac{1}{2}A_0 \)?).

Analyzing these curves is often simple. For doubling-time or half-life, begin with the appropriate value on the \( y \)-axis. Then draw a horizontal line that intersects the plot. Next, drop a perpendicular line to the \( x \)-axis. Read the value of the time directly from the \( x \)-axis.

The data for our exponential curve will be obtained by measuring the potential difference across the resistor of an RC circuit as a function of time.

A capacitor is connected in parallel with a resistor, then charged to some initial voltage. When the power supply is disconnected, the potential difference across the capacitor will decrease exponentially. The voltage, \( V \), across the capacitor as it discharges is given by:

\[
V_t = V_0 e^{-t/RC}
\]  

(20.2)

where \( V_0 \) is the initial potential difference across the capacitor at time \( t_0 \), \( R \) is resistance, and \( C \) is capacitance.

Consider Eq. 20.2. When \( t = RC \),

\[
V_t = V_0 e^{-1} = V_0 \frac{1}{e}
\]  

(20.3)

We define \( \tau \) (Greek letter, tau) to be the time it takes for the voltage to drop to \((1/e) \cdot (V_0)\) its original value (about 37\%). This is the value when \( \tau = RC \).

For our discharging capacitor:

A graph of Voltage vs. Time is an exponential decay curve. Analysis of this curve provides the time constant by locating the point at which \( V \) has dropped to \(1/e\) its original value of \( V_0 \).
Part 1  EQUIPMENT

Circuit Gear Digital Oscilloscope
Oscillator
BNC Cable
DMM
BCN - Coaxial Adapter
Advance Reading (Oscilloscopes)

Text: Wavelength ($\lambda$), frequency ($f$), and period ($T$), oscilloscope, EKG.

Objective

The objective of Part 1 is to become familiar with oscilloscopes by measuring various quantities using an analog oscilloscope and a digital oscilloscope.

Theory

An oscilloscope is a measuring device that plots Voltage vs. Time. When a signal is received by the oscilloscope, a trace (line) appears on the screen. The signal could be generated from an oscillator, a tuning fork, or a heart monitor at the doctor’s office.

The screen of an analog oscilloscope has a grid, or gradecule, superimposed on it (Fig. 20.4). The gradecule is used as a reference to read information from the screen of an oscilloscope. The controls of an oscilloscope will adjust the view of an incoming signal; they will not adjust the signal itself.

Voltage

We will generate a periodic function (a sine wave; AC voltage) for Part 1 using an oscillator. Note that a sine wave has an amplitude of equal magnitude both above and below zero, resulting in an average value of zero. To obtain a meaningful value for this AC voltage, we calculate the root-mean-square, $rms$, of the voltage.

Refer to Fig. 20.5. We will determine, from the trace, the number of DIVISIONS for both the peak-to-peak voltage ($y$-axis) and the time required for one wavelength ($x$-axis).

$V_{\text{max}}$ is the amplitude of the wave (half of the peak-to-peak voltage).

The rms voltage, $V_{\text{rms}}$, is given by:

$$V_{\text{rms}} = \sqrt{\frac{(V_{\text{max}})^2}{2}} = 0.707 V_{\text{max}} \quad (20.4)$$

Recall that once the period $T$ of a cyclic function is known, the frequency $f$ is given by:

$$T = \frac{1}{f} \quad (20.5)$$

![Figure 20.4: Gradecule](image1)

![Figure 20.5: Gradecule Sample](image2)
Exponential Curve

1. Refer to Fig. 20.2. Consider the equations:

\[ y = Ae^{(C \cdot x)} + B \]
\[ N = N_0e^{(\lambda t)} \]

What is \(N_0\) [i.e. \(N_{(t=0)}\)]? (You may read the information directly from the auto-fit box.) (10 pts)

2. Refer to Fig. 20.6, below. The doubling-time is the time it takes for the initial number or amount to double. What is the doubling-time \((N = 2N_0)\)? (Find the value on the y-axis, then move horizontally until you intersect the plot. Drop a perpendicular line to the x-axis and read the time.) (20 pts)
Radioactive Decay - Carbon Dating (30 pts)

Assume Fig. 20.7 represents 100% of the carbon found in all living matter. Carbon 14 ($^{14}$C) has a half-life ($T_{1/2}$) of 5,730 years. The square represents a sample of $^{14}$C. $^{14}$C emits $\beta^-$ particles.

3. Divide the square in half with a vertical line and write “5,700 yrs” (rounded for simplification) on the left side, to represent the amount of $^{14}$C decayed after its half-life of 5,700 years ($T_{1/2} = 5,700$ yrs).

4. Now divide the right side in half with a horizontal line and write “11,400 yrs” to represent the amount of $^{14}$C decayed after an additional half-life.

5. Continue to divide the remaining sample in this manner to show the amount of $^{14}$C decayed after 17,100 yrs; 22,800 yrs; 28,500 yrs; 34,900 yrs; and 45,600 yrs.

Figure 20.7: Carbon Dating Square
6. Define frequency, wavelength, and period. (10 pts)

7. What does an oscilloscope plot? (10 pts)

8. Determine the period and peak-to-peak voltage of Fig. 20.8, below (refer to Fig. 20.4). (20 pts)
   Assume each major box is $\frac{1}{2}$ V on the vertical scale and $\frac{1}{4}$ ms on the horizontal scale.

![Figure 20.8](image)
PROCEDURE

PART 1: Oscilloscopes

1. Connect the oscilloscope to the computer. Open the Syscomp Circuitgear software on the desktop.

2. Plug in the oscillator and connect the output to the oscilloscope, as in Fig. 20.3. The signal has been set for you. You may shift the waveform by dragging the A, B, or X cursors at the edge of the gradecule. The horizontal and vertical scales can be adjusted by using the Timebase Controls and Vertical Controls, respectively.

3. Measure the peak-to-peak voltage of your trace (refer to Fig. 20.9). Note the VOLTS/DIV scale on-screen under Vertical Controls. You can use the measurement tool under View ⇒ Toggle Channel A Cursors to facilitate more accurate measurement. Record this value in the table provided.

4. Calculate $V_{\text{max}}$ (do not measure).

5. Calculate $V_{\text{rms}}$ (Eq. 20.4).

6. Measure the period, $T$, of your trace. Note the SEC/DIV scale onscreen under Timebase Controls. You can use the measurement tool under View ⇒ Toggle Time Cursors to facilitate measurement.

7. Use your measurement of period to calculate the frequency of the signal (Eq. 20.5).

8. Connect the oscillator to the DMM using a BNC - Coaxial adapter. Set the DMM to 20V ACV (instead of DCV) to measure root-mean-square voltage.

9. Record the frequency, $f$, of the signal by reading the oscillator display.

10. Reconnect the BNC cable to the oscilloscope. Turn off the oscilloscope, oscillator, and DMM.
**PART 2: RC Circuit**

The capacitors are electrolytic. They must be connected with the negative end of the capacitor on the negative side of the power supply; failure to do so will damage the capacitor. Refer to Fig. 20.11.

11. Determine the nominal resistance of \( R_1 \). Then measure \( R_1 \) with the DMM. Record these in the table provided.

12. Measure the capacitance, \( C \), of the capacitor using the DMM at the TA’s table. Take care not to plug it in backwards - this will damage the capacitor.

13. Construct the circuit shown in Fig. 20.11. Connect, but do not plug in, the power supply.

14. Measuring voltage with a voltmeter changes the equivalent resistance of the circuit. To account for this, calculate \( R_{eq} \) of the resistor in parallel with the 10 M\( \Omega \) voltmeter. Use the measured value of \( R_1 \).

15. Calculate the theoretical value of the time constant \( \tau \) using \( R_{eq} \) and \( C \).

16. Ask your TA to approve the circuit. Plug in and turn on the power supply. Charge the capacitor by applying 10.0 V across the circuit.

17. Disconnect one power supply lead from the circuit. Take voltage measurements every five seconds for three minutes. [The voltage should drop to the mV range, requiring you to adjust the voltmeter scale twice. If voltage drops to zero, check your circuit.]

18. Disassemble the circuit. Turn off the DMM.

**PART 3: Graphing**

19. Graph \( V \) vs. \( t \) using Graphical Analysis. Analyze the curve using a natural exponent fit. Determine the time constant from your curve fit. Print the graph.

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**QUESTIONS**

**Part 1**

1. Compare the two \( f \) values (oscilloscope and oscilator).

2. Compare the two \( V_{rms} \) values (oscilloscope and DMM).

3. Given \( V_{peak\rightarrow peak} = 18.54 \text{ V} \), calculate \( V_{rms} \).

**Part 2**

4. Show that the product of \( RC \) has units of seconds.

5. If an RC circuit has \( \tau = 15 \text{ seconds} \), how long would it take for the circuit to discharge to \( 1/e^7 \) its original value?

6. Calculate \( \tau \) for an RC circuit consisting of a 3 \( \mu \text{F} \) capacitor and a 1.5 M\( \Omega \) resistor.