## PHYS222 Uncertainty Notes for The Thin Lenses Experiment

The uncertainty for the **lens equation** (i.e.,  $\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$ ) is given by the relationship

$$\delta f = f \sqrt{\left(\frac{i}{o(o+i)}\delta o\right)^2 + \left(\frac{o}{i(o+i)}\delta i\right)^2}$$

where  $\delta o$  and  $\delta i$  are the uncertainties in object and image distances respectively.

We can assume that the uncertainty in object distance  $\delta o$  (i.e., the distance from the lens to the object or light source) is small compared to the uncertainty of the image distance  $\delta i$  (i.e., the distance from the lens to the image). This is a good approximation if we fix both the light source (i.e., the object) and lens and vary only the screen location while focusing.

This yields an object uncertainty  $\delta o$  of approximately  $\sqrt{\left(\frac{1}{2}mm\right)^2 + \left(\frac{1}{2}mm\right)^2} = 0.707 mm = 0.07 cm$  which we can ignore. This value is an order of magnitude smaller than the typical error  $\delta i$  in image distance.

Thus, we can use the following approximation for the uncertainty of the lens equation

$$\delta f \approx f \left( \frac{o}{i(o+i)} \delta i \right)$$
 Eq-1)

where

$$\delta i = \frac{fuzzy_{left} - fuzzy_{right}}{2} = \frac{\Delta fuzzy}{2}.$$

Since we make 5 measurements we will use the average of the 5 uncertainties in our final result to go along with the average of the 5 focal lengths. See table below for  $\delta i$ .

The uncertainty for the conjugate foci equation (i.e.,  $f = \frac{L^2 - d^2}{4L}$ ) is given by

$$\delta f = f \sqrt{\left[ \left( \frac{L^2 + d^2}{\left( L^2 - d^2 \right) L} \delta L \right)^2 + \left( \frac{2d}{L^2 - d^2} \delta d \right)^2 \right]}$$

For the same reasons given above we can ignore the uncertainty between light and screen (i.e.,  $\delta L \approx 0$ ) and use

$$\delta f = f\left(\frac{2d}{L^2 - d^2}\delta d\right)$$
 Eq-2) where 
$$\delta d = \sqrt{\left[\frac{\Delta Fuzzy_{small\ image}}{2}\right]^2 - \left[\frac{\Delta Fuzzy_{large\ image}}{2}\right]^2}$$