A few applications of model section on GW astrophysics

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The rough idea

- Given some data and two (or more) alternative models that could explain it, how to pick the "best" model ?
- Use Bayesian model selection!
  - Advantages
    - Occam's razor is built in
    - Can always update evidence as more data gets in
  - Caveats
    - Can be computationally expensive
    - You get a relative ranking. I.e. which of the model fits best the data, w/o guarantees that any of them is actually correct!

Bayesian model selection

- Given two models A and B and GW data d, build odds ratio

$$\frac{p(A|dI)}{p(B|dI)} = \frac{p(d|AI)}{p(d|BI)}\frac{p(A|I)}{p(B|I)} \equiv \mathcal{O}_B^A \frac{p(A|I)}{p(B|I)}$$

- If many detections are available and data is independent

Cumulative Bayes factor

$$\mathcal{O}_B^A(\vec{d}) = \prod_j \frac{p(d_j|AI)}{p(d_j|BI)}$$

Evidence

- Each Bayes factor is the ratio of the evidence for the data given each of the two models

Evidence for model A, event j

$$Z_A^j \equiv p(d_j|AI)$$

$$\mathcal{O}_B^A(\vec{d}\,) = \prod_j \frac{Z_A^j}{Z_B^j}$$

LALInference

- The evidence is the marginalized posterior of the data given the model and the model's parameters

$$Z_A^j \equiv p(d_j|AI) = \int \mathrm{d}\vec{\theta}\, p(d_j|\vec{\theta}AI)p(\vec{\theta}|AI)$$

- In the LVC, we very often use LALInference (Veitch+ 2014) to calculate the evidence
  - Nested sampling or MCMC

LALInference

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Likelihood — $p(d_j|\vec{\theta}AI)$

Prior — $p(\vec{\theta}|AI)$

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TIGER

- GW have allowed us for the first time to access in a direct way the dynamical regime of GR.
  - Is GR still a good description of gravity at those energies?
- Test Infrastructure for GEneral Relativity (Li+ 2011, Agathos+ 2013)
- The idea:
  - Use model section to verify if detected gravitational waves are consistent with what expected within GR
  - Does not require knowledge of alternative theory
  - Tested for binary neutron stars (BNS)

TIGER – 2

- One of the two models we test is, of course, GR
- How is the "GR is wrong" model implemented?

- In post-Newtonian theory, the phase coefficients are known functions of the two component masses and spins
- One can leave these coeffs free to vary from their GR values

$$\varphi_i \to \varphi_i \left(1 + d\varphi_i\right)$$

TIGER -3

- Introduce auxiliary hypotheses
  - $H_{i_1 i_2 \ldots i_k}$: the PN coeffs $i_1, i_2, \ldots, i_k$ are potentially different from GR, while the others are fixed at their GR values
- The *modified GR* (modGR) can be then written as a sum of mutually exclusive hypotheses

$$\mathcal{H}_{\mathrm{modGR}} = \bigvee_{i_1 < i_2 < \ldots < i_k} H_{i_1 i_2 \ldots i_k}$$

- We want the odds

$$O_{\mathrm{GR}}^{\mathrm{modGR}} \equiv \frac{P(\mathcal{H}_{\mathrm{modGR}} | d, \mathrm{I})}{P(\mathcal{H}_{\mathrm{GR}} | d, \mathrm{I})}$$

TIGER -4

- Since the aux models were defined as mutex, the numerator is just the sum of the posteriors of the aux modes.
- Finally one has

$$O_{\mathrm{GR}}^{\mathrm{modGR}} = \sum_{i_1 < i_2 < \ldots < i_k} \frac{P(H_{i_1 i_2 \ldots i_k} | \mathrm{I})}{P(\mathcal{H}_{\mathrm{GR}} | \mathrm{I})} B_{\mathrm{GR}}^{i_1 i_2 \ldots i_k}$$

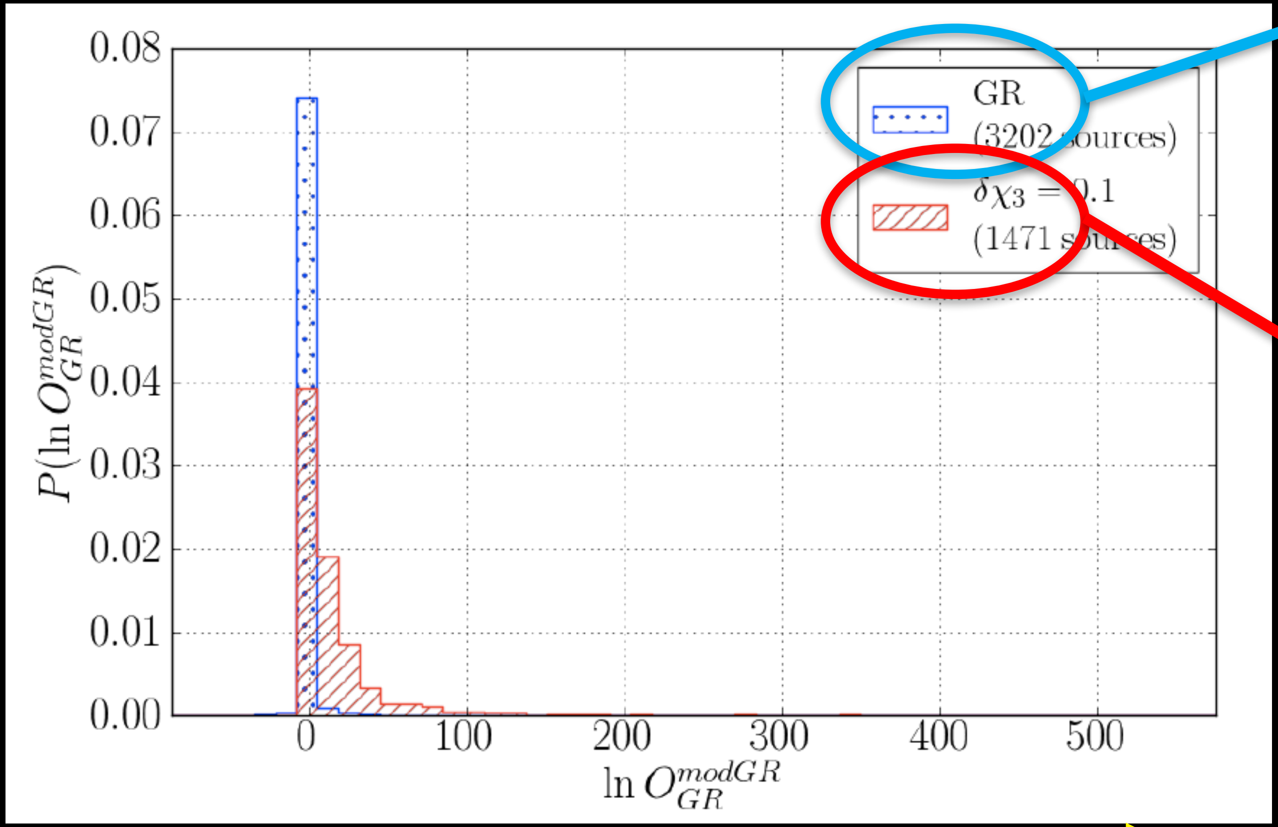
- Trivial to extend to more than one source

The need for a background

- If our understand if the data were perfect, getting a large odds in favor of modGR would be enough
- In practice, while we assume the noise is Gaussian, it is really not
- Run TIGER on (simulated) GR signals to build a background of odds ratio in **absence** of deviations from GR
- RUN TIGER on (simulated) modGR signals and check how the odds compare with the background

Results

- Odds distribution for individual sources



background

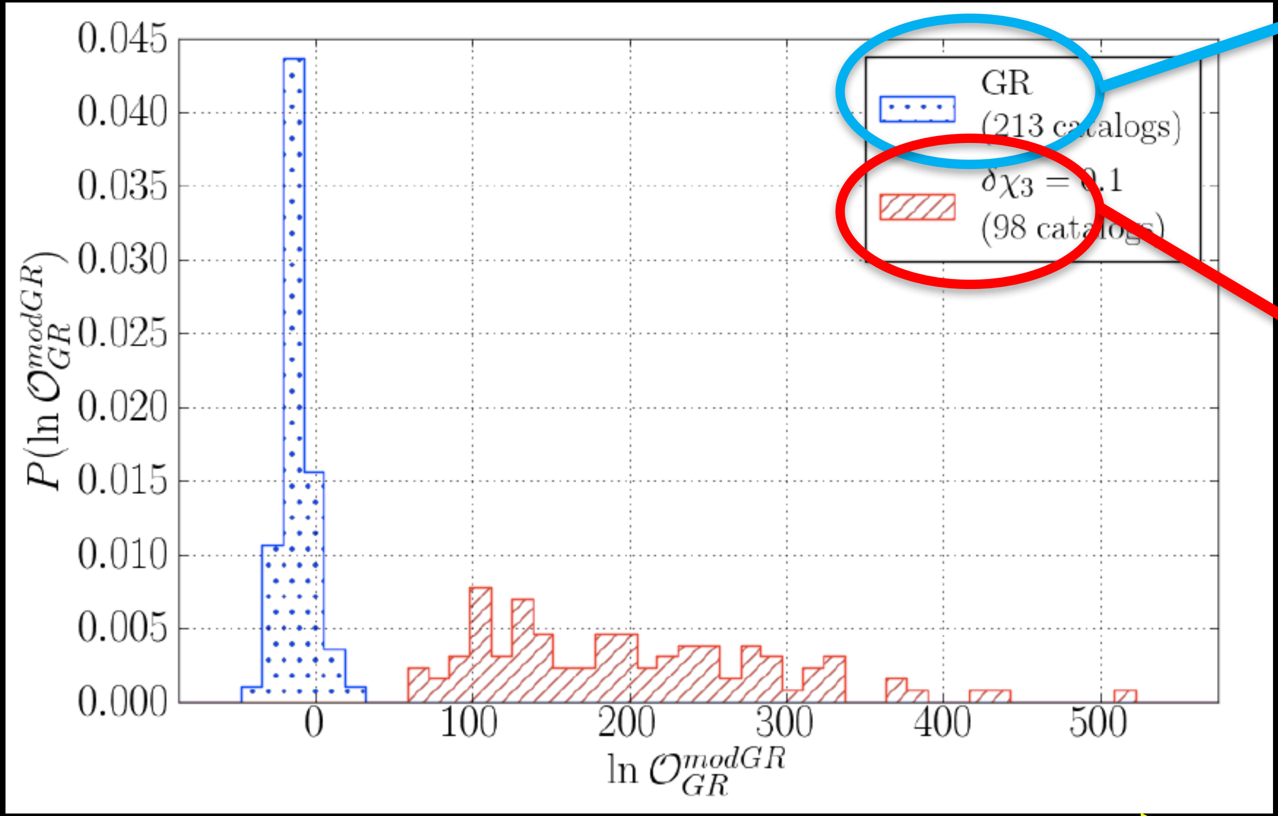
modGR signals (10% added to the 1.5PN)

Favors GR

Favors modGR

Results

- Cumulative odds distribution for catalogs of 15 sources



background

modGR signals (10% added to the 1.5PN)

Favors GR → Favors modGR

TIGER - caveats

- Odds in favor of modGR **not** necessarily equivalent to “GR is wrong”
- Could be that waveform model is inappropriate to start with
- Something weird with the data or calibration
- Unaccounted (GR) physics
  - E.g. non-linear NS tides (Essick+ 2016)
- Priors on GR parameters (?)
- Most of these effects shown to be under control in Agathos+ 2013

Neutron star equation of state (EOS)

- One of the key open questions in astrophysics, with potentially large impact on nuclear physics.
- Impossible to study in a lab
- Hard to study with electromagnetic radiation
  - Need very precise measurement of NS radius
  - NICER lunched soon, but only 1 sure target

- (At least conceptually) Simpler with GWs

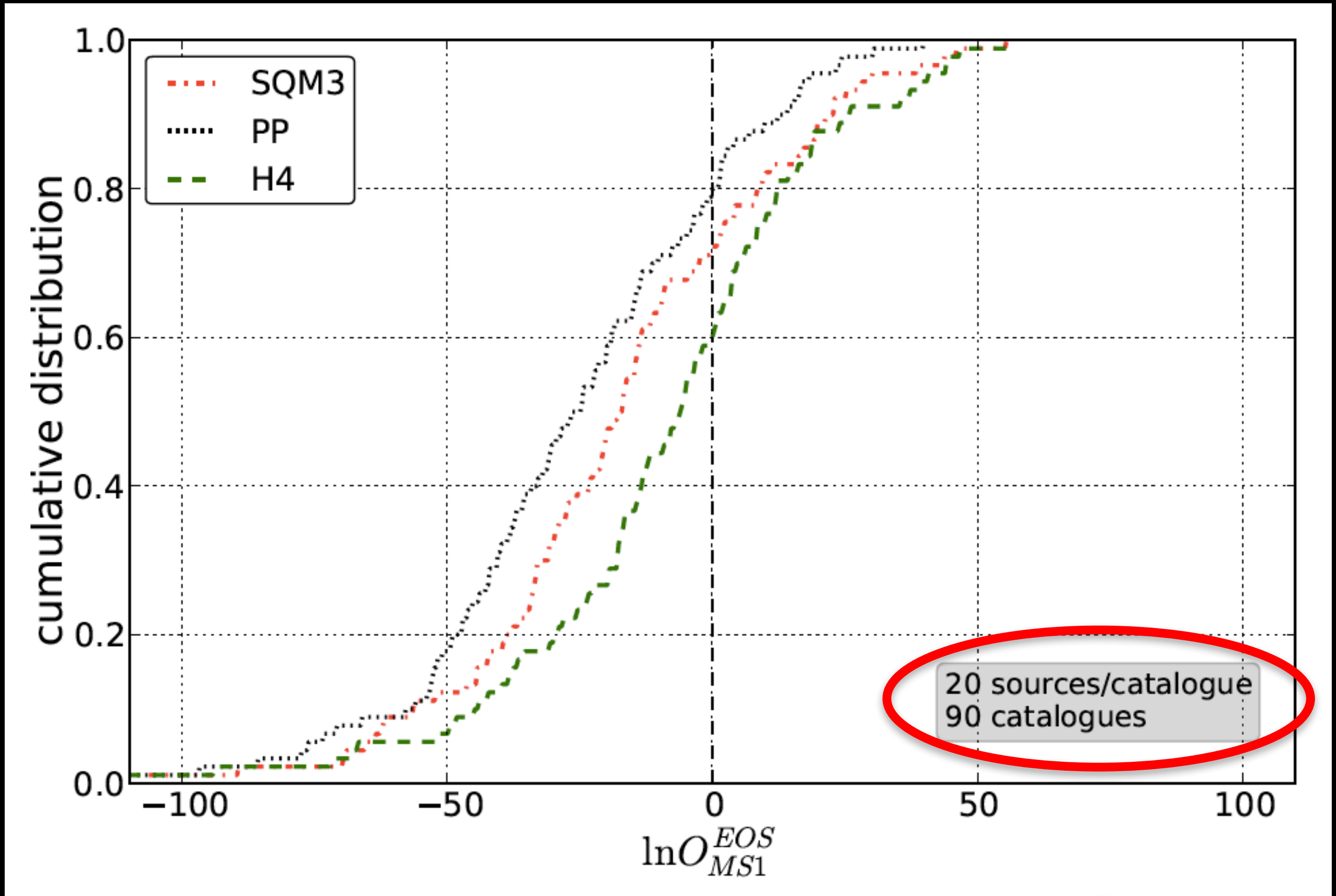
Linear tides

- Linear tides will affect the waveforms in a way that depends on the EOS
- Late inspiral, high PN order (>6PN), high frequency effect
  - Very hard to measure for any one event
- Model selection approach (Del Pozzo+, Agathos+ 2014)
  - Assume all NS have same EOS
  - Introduce a model for each proposed EOS
  - Rank the models given N BNS detectios

Method

- Simulate BNS sources with a given EOS (e.g. MS1)
- Consider 3 other models: PP (point particle), SQM3, H4
- Build cumulative odds of alternative EOS vs MS1

Results



No spins

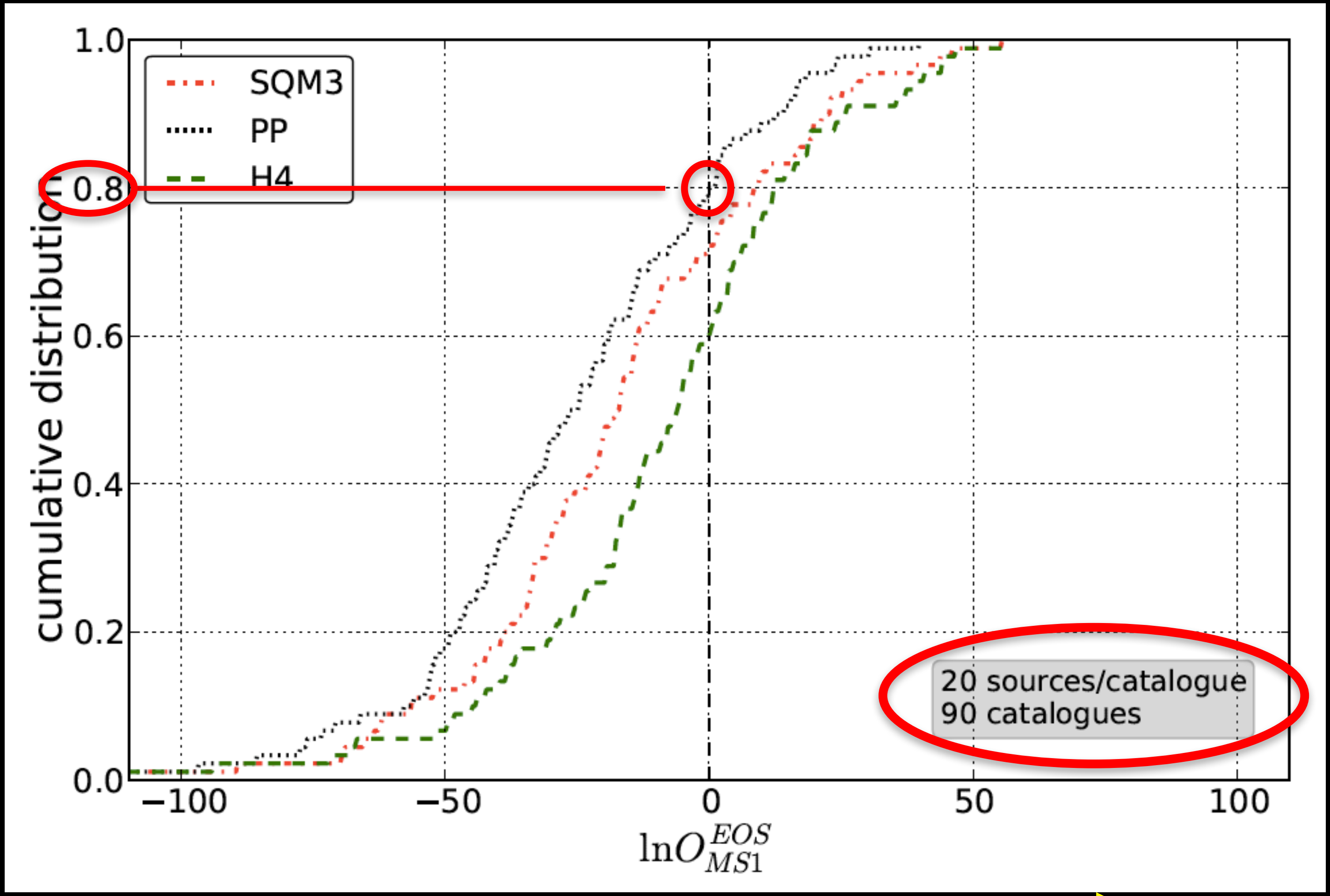
Favors MS1

Favors alternative EOS

Results

For 80% of the catalogs the (correct) MS1 model is preferred over PP

Alternative models raked by increasing hardness

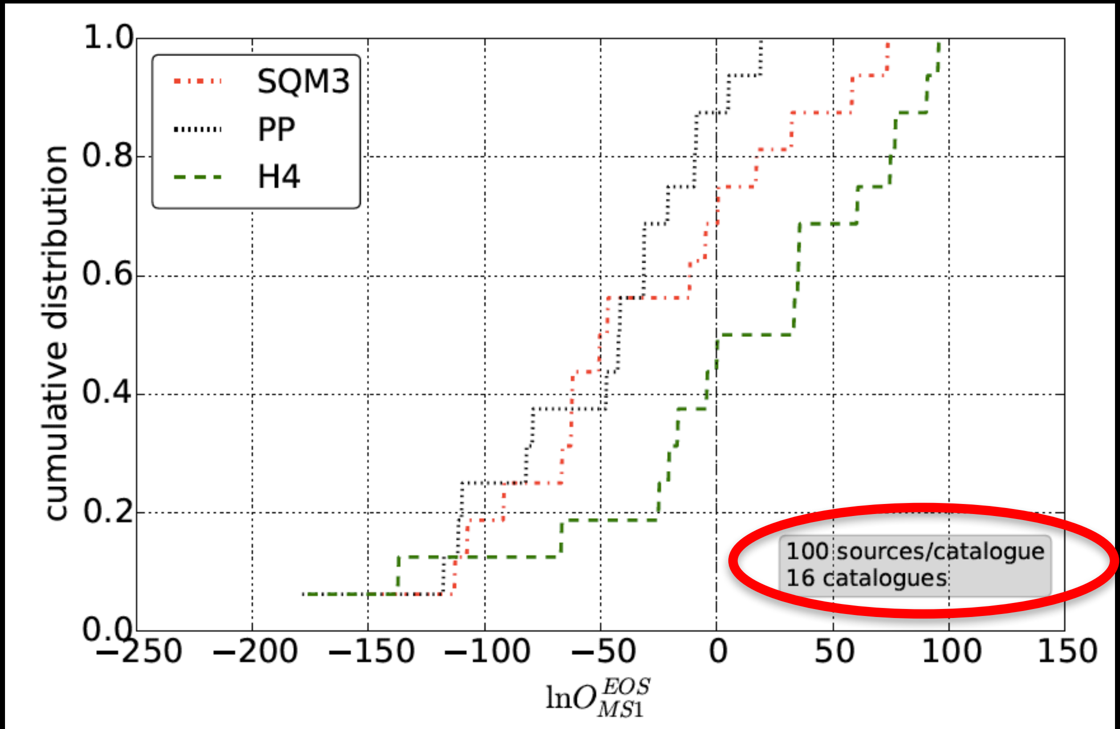


No spins

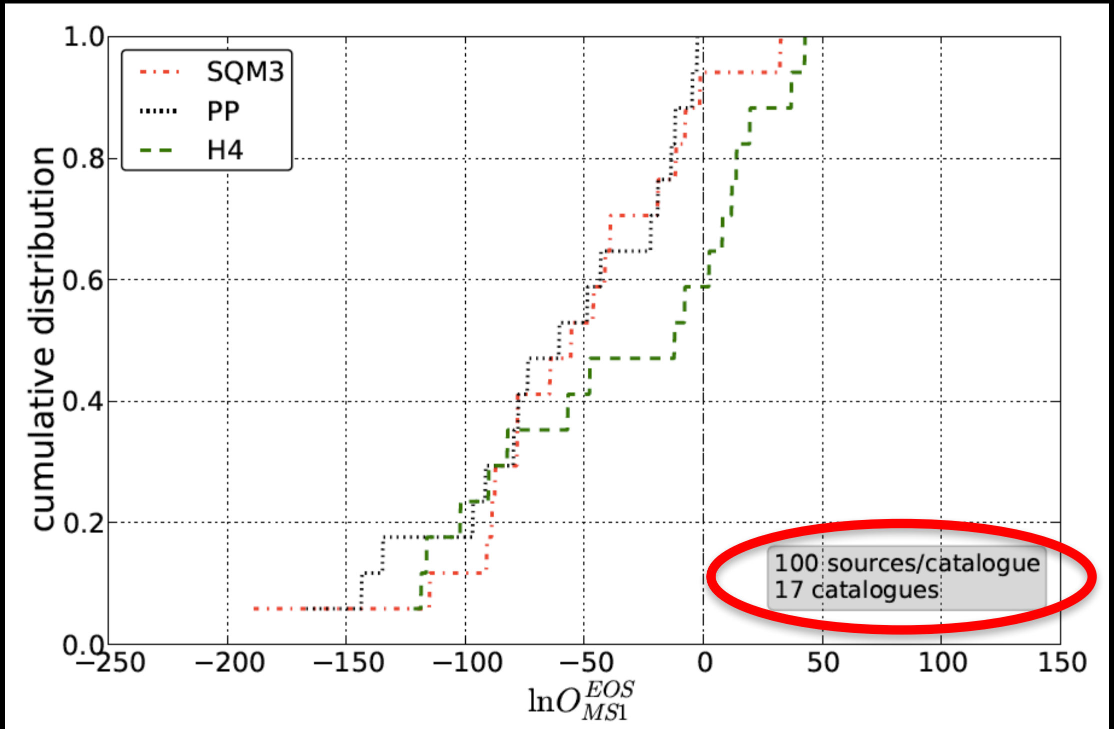
Favors MS1 Favors alternative EOS

Results

- Checked effect of prior mismatch



Injection masses from peaked gaussian (sigma=0.05M) while using flat prior ([1,2]M)



Injection spins from peaked gaussian (sigma=0.02) while using flat prior ([-0.1,0.1])

Non-linear tides

- Weinberg (XXX) suggested non-linear tides could play a role
  - Resonance of children modes could make the effect enter the waveform **very early** in the inspiral

Plot here

CBC formation channels

A binary model selection approach

- Assume only two formation channels exist
  – Common envelope (galactic fields)
  – Random encounters (globular clusters)
- Will distinguish them using the (expected, hoped?) resulting spins orientation distributions:
  – Common envelope -> spin ~(anti) aligned with L
  – Random encounters -> random spin distribution

Implementation

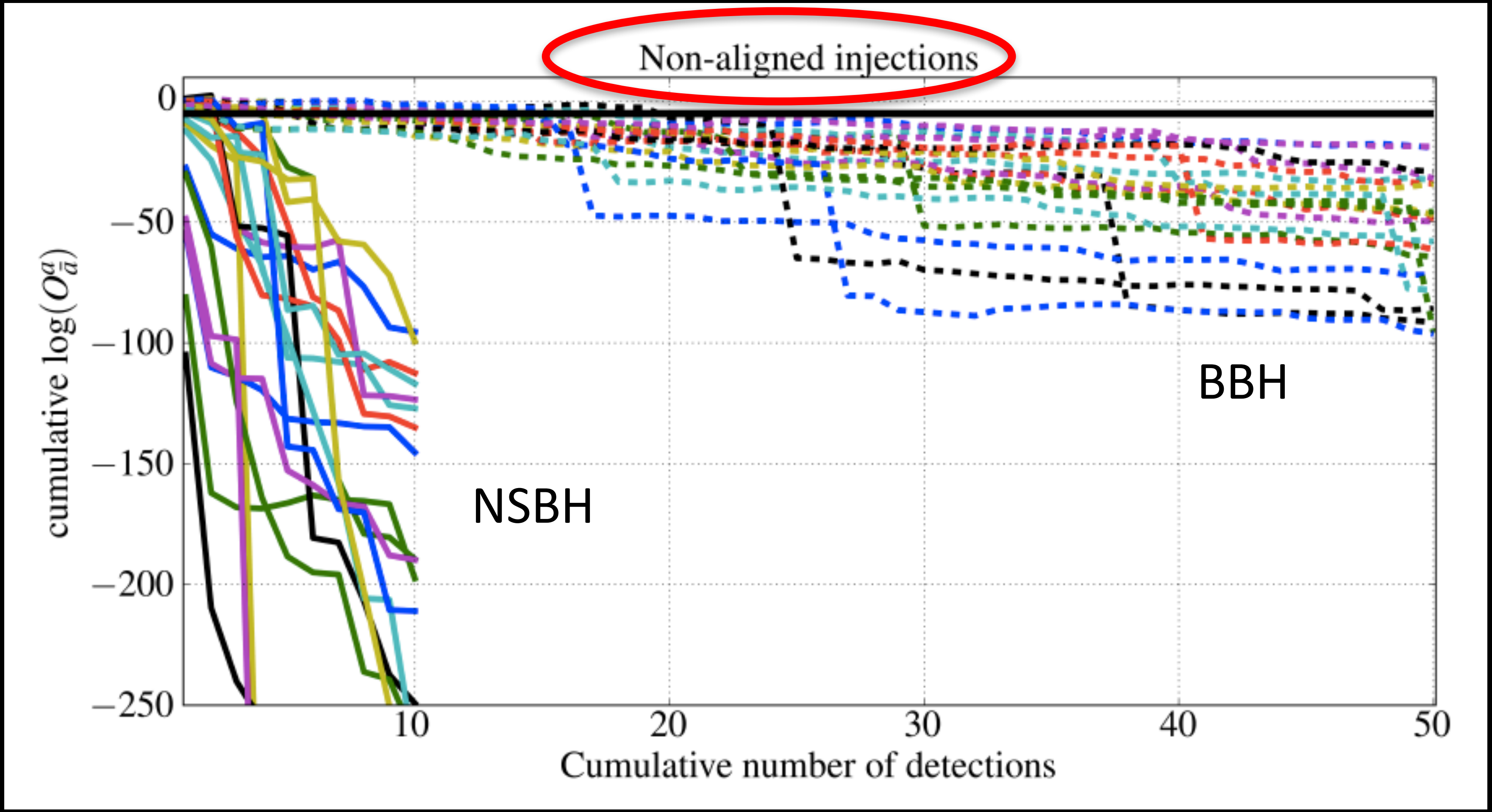
- “Aligned” model: **both** tilt angles smaller than 10degs
- “Non-Aligned” model: isotropic tilt angles (excluding the region where aligned is true)
- Enforced through different prior ranges in the evidence calculation.

- Simulate signals from either type and build cumulative odds ratio (Vitale+, 2017)

Results

Favors aligned

Favors non-aligned



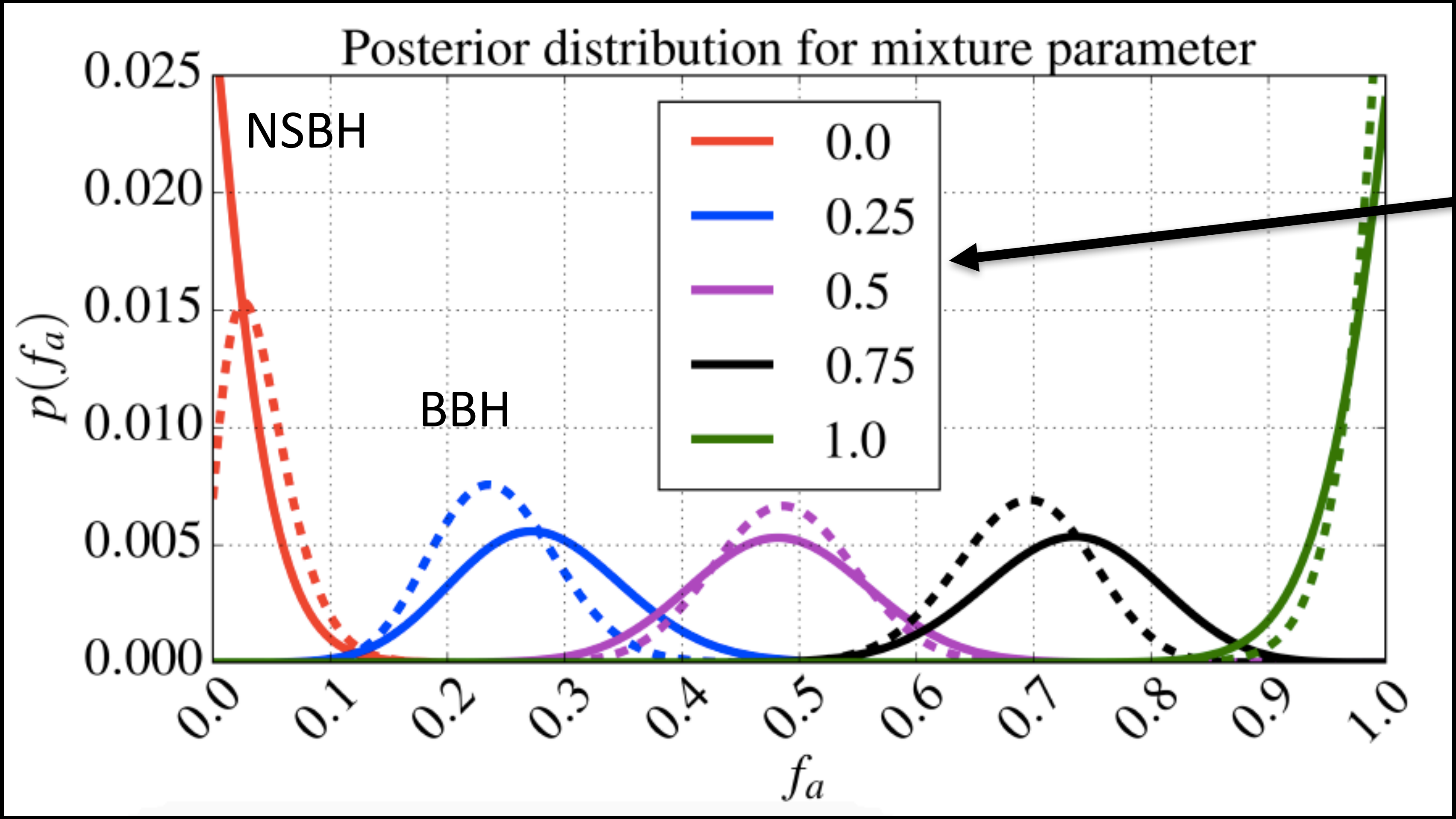
Caveats

- Can only build cumulative odds if one believes only one channels is possible
  - In reality both are probably possible
- Can modify method to measure the posterior fraction of aligned systems

$$\log p(f_a|\vec{d}) \;=\; \sum_{k=1}^{N}\left(\log Z_k^a + \log\left[f_a + (1-f_a)\frac{Z_k^{\bar{a}}}{Z_k^a}\right]\right)$$

Measuring the mixture fraction

100 NSBH
200 BBH



True underlying fraction of aligned sources

Caveats – To dos

- Assumed what I called "aligned" is what the universe calls aligned – should include possible prior mismatch
- Can extend the model so that they also take into account mass ratios, eccentricity, or anything else that might be useful to distinguish
- Can include more than 2 models

Beyond CBCs

- Model selection currently used in other LIGO groups
- Core collapse supernovae: infer explosion mechanism (Gossan+ 2016)
- Look for extra GR polarizations
- As a detection statistic (Lynch+ 2015, Kanner+ 2016)