Four ways of doing cosmography with gravitational waves

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Cosmography recap

- Hubble parameter:

$$
\begin{aligned}
H^2(a) &\equiv \left(\frac{\dot{a}}{a}\right)^2 \\
&= H_0^2\left[\Omega_{\rm M} a^{-3} + \Omega_{\rm R} a^{-4} + \Omega_k a^{-2} + \Omega_{\rm DE}\exp\left(3\int_0^a \frac{da'}{a'}\,[1+w(a')]\right)\right]
\end{aligned}
$$

- Fractional densities of matter, radiation, spatial curvature, dark energy:

$$
\Omega_{\rm M} = \frac{8\pi}{3H_0^2}\rho_{M,0}, \quad \Omega_{\rm R} = \frac{8\pi}{3H_0^2}\rho_{\rm R,0}, \quad \Omega_k = -\frac{k}{H_0^2}, \quad \Omega_{\rm DE} = \frac{8\pi}{3H_0^2}\rho_{\rm DE,0}
$$

Dark energy equation of state: $P(t) = w(t)\,\rho(t)$ with $w(t) < 0$

- Luminosity distance as a function of redshift ($1/a = 1 + z$):

$$
D_{\rm L}(z) = c\,(1+z)\begin{cases} |k|^{-1/2}\sin\left[|k|^{1/2}\int_0^z \frac{dz'}{H(z')}\right] & \text{for } \Omega_k < 0 \\ \int_0^z \frac{dz'}{H(z')} & \text{for } \Omega_k = 0 \\ |k|^{-1/2}\sinh\left[|k|^{1/2}\int_0^z \frac{dz'}{H(z')}\right] & \text{for } \Omega_k > 0 \end{cases}
$$

Cosmography: determining $\vec{\Omega} \equiv (H_0, \Omega_{\rm M}, \Omega_{\rm R}, \Omega_k, \Omega_{\rm DE}, w(t))$ by fitting $D_{\rm L}(z)$

Cosmography recap

- Luminosity distance as a function of redshift:

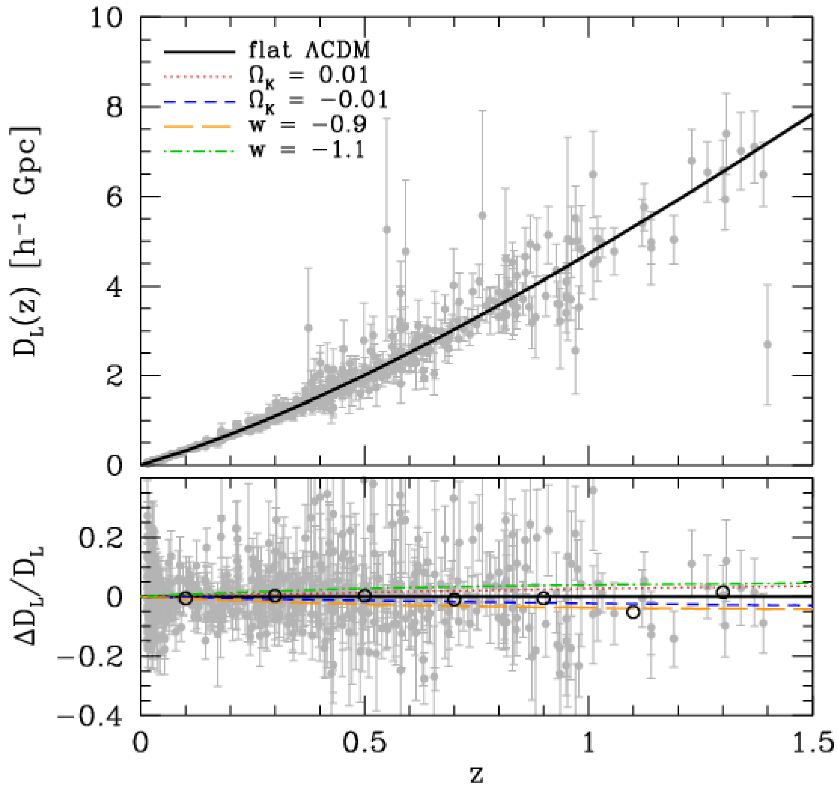
$$D_{\rm L}(z) = c\,(1+z) \begin{cases} |k|^{-1/2} \sin\left[|k|^{1/2} \int_0^z \frac{dz'}{H(z')}\right] & \text{for } \Omega_k < 0 \\ \int_0^z \frac{dz'}{H(z')} & \text{for } \Omega_k = 0 \\ |k|^{-1/2} \sinh\left[|k|^{1/2} \int_0^z \frac{dz'}{H(z')}\right] & \text{for } \Omega_k > 0 \end{cases}$$

Cosmography: determining $D_{\rm L}(z)$

$$\vec{\Omega} \equiv (H_0, \Omega_{\rm M}, \Omega_{\rm R}, \Omega_k, \Omega_{\rm DE}, w(t))$$

by fitting $D_{\rm L}(z)$

- Example using Type Ia supernovae:
  - Intrinsic luminosity believed known $\rightarrow$ $D_{\rm L}$ from observed luminosity
  - Redshift z from spectrum



Weinberg et al., Phys. Rept. **530**, 87 (2013)

Cosmography recap

- Luminosity distance as a function of redshift:

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Cosmography: determining

$$\vec{\Omega} \equiv (H_0, \Omega_{\rm M}, \Omega_{\rm R}, \Omega_k, \Omega_{\rm DE}, w(t))$$

by fitting $D_{\rm L}(z)$

- In most of this presentation: $\Omega_{\rm R} \simeq 0, \quad \Omega_{\rm M} + \Omega_k + \Omega_{\rm DE} = 1$

$$\rightarrow \quad H(z) = H_0\, \left[\Omega_{\rm M}(1+z)^3 + \Omega_k(1+z)^2 + (1 - \Omega_{\rm M} - \Omega_k)\, E(z)\right]^{1/2}$$

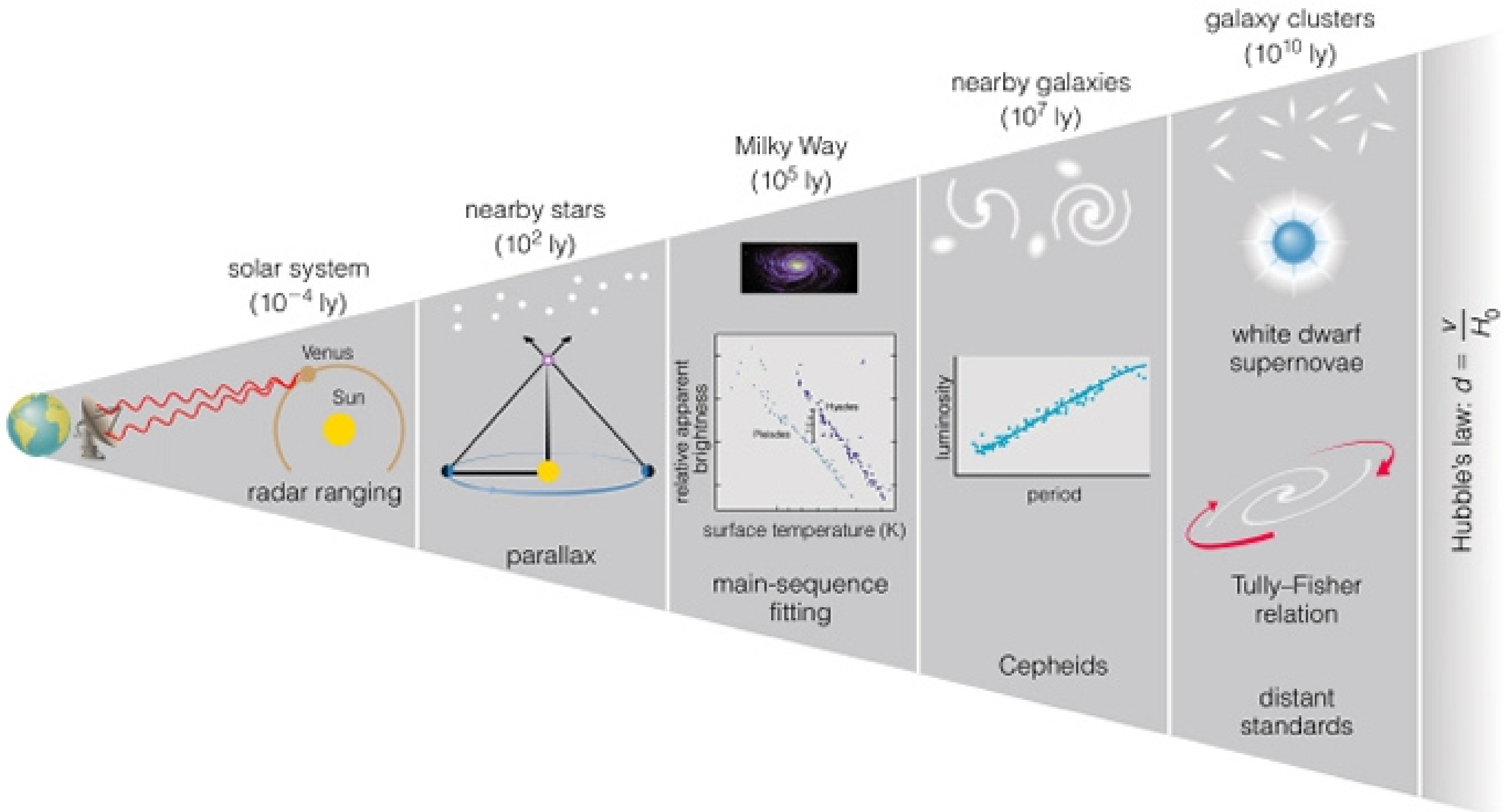
Dark energy equation of state:

$$\begin{aligned} w(z) &= P_{\rm DE}/\rho_{\rm DE} = w_0 + w_a(1-a) + \mathcal{O}\left[(1-a)^2\right] \\ &\simeq w_0 + w_a \frac{z}{1+z}, \end{aligned}$$

Cosmic distance ladder

- Commonly used distance markers (like Type Ia supernovae) need calibration

  → *Cosmic distance ladder*



Compact binary inspirals as "standard sirens"

- Gravitational wave amplitude during inspiral:

$$\mathcal{A}(t) = \frac{1}{D_{\rm L}} \mathcal{M}^{5/3} g(\theta, \phi, \iota, \psi)\, \omega^{2/3}(t)$$

- From the gravitational wave phase $\Phi(t)$ one separately obtains
  - Chirp mass $\mathcal{M} = (m_1 m_2)^{3/5}/(m_1 + m_2)^{1/5}$
  - Instantaneous frequency $\omega(t) = \dot{\Phi}(t)$
- The function $g(\theta, \phi, \iota, \psi)$ depends on sky position and orientation
  - Sky position (θ, ϕ) : EM counterpart, or multiple detectors
  - Orientation (ι, ψ) : multiple detectors

→ Possibility of extracting distance $D_{\rm L}$ from the signal itself

→ No need for cosmic distance ladder

But: Fitting of $D_{\rm L}(z)$ *also requires separate determination of redshift* z

1.

Redshift

from

electromagnetic counterparts

1. Redshift from EM counterparts

- Short-hard gamma ray bursts (GRBs) are assumed to result from NS-NS or NS-BH mergers
  - If localizable on the sky: (θ, ϕ), and host galaxy identification would provide z
  - With multiple detectors: (ι, ψ)
    - Beaming of the GRB: $\iota \lesssim 20^\circ$

- At redshifts accessible to 2nd generation detectors: $D_\mathrm{L} \simeq \frac{cz}{H_0}$
  - If redshift can be determined with negligible error then with a *single* source, uncertainty is $\frac{\Delta H_0}{H_0} = \frac{\Delta D_\mathrm{L}}{D_\mathrm{L}}$
  - With *multiple* sources, accuracy improves roughly as $\sim \sqrt{N}$
    - With 50 NS-NS events: $\Delta H_0 / H_0 \sim 5\%$

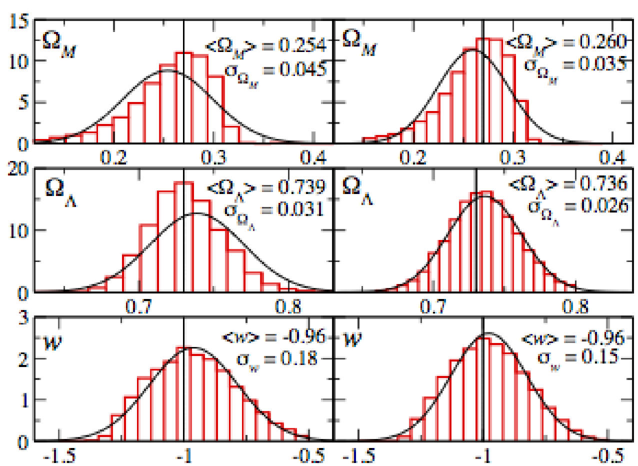
Nissanke et al., Astrophys. J. **725**, 496 (2010)

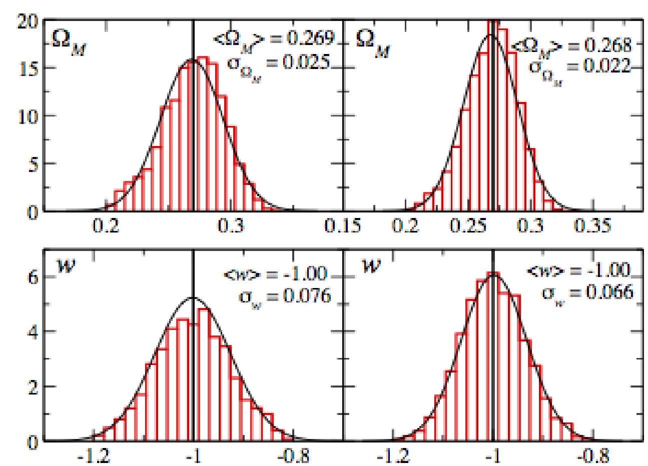
1. Redshift from EM counterparts

- 3rd generation detectors: in principle access to the entire set

$$(H_0, \Omega_{\rm M}, \Omega_{\rm DE}, w_0, w_a)$$

- Einstein Telescope:
  - Over a period of O(10) years, assume 1000 NS-NS out to z = 2, localizable through EM counterparts, and redshifts available
  - Assume constraint on inclination angle: $\iota \lesssim 20^\circ$
- Measuring subsets of parameters assuming other parameters known:





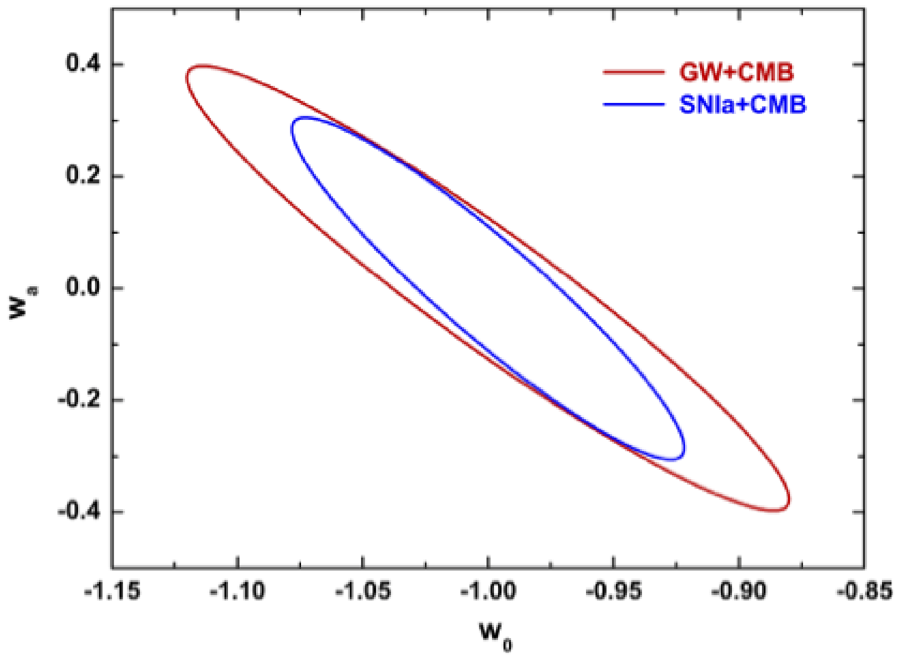
Sathyaprakash, Schutz, Van Den Broeck, Class. Quantum Grav. **27**, 215006 (2010)

1. Redshift from EM counterparts

- Time evolution of dark energy equation of state:

$$w(z) \simeq w_0 + w_a \frac{z}{1+z}$$

- Assuming $(H_0, \Omega_{\rm M}, \Omega_{\rm DE})$ known and using prior from CMB for (w_0, w_a):



- Similar constraints on dark energy equation of state as with supernova surveys on similar timescale
- But, no dependence on cosmic distance ladder

Zhao, Van Den Broeck, Baskaran, Li, Phys. Rev. D **83**, 023005 (2011)

2.

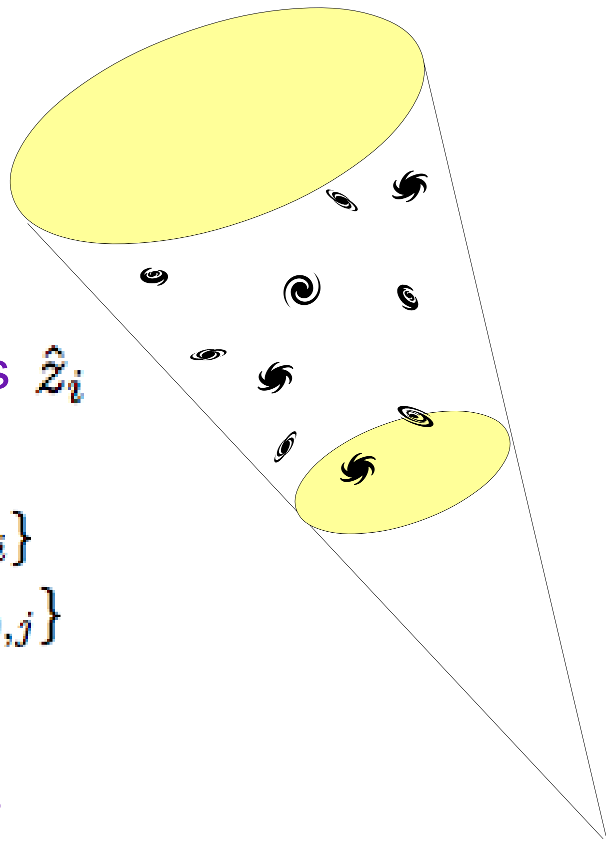
Cross-correlating

with

galaxy catalogs

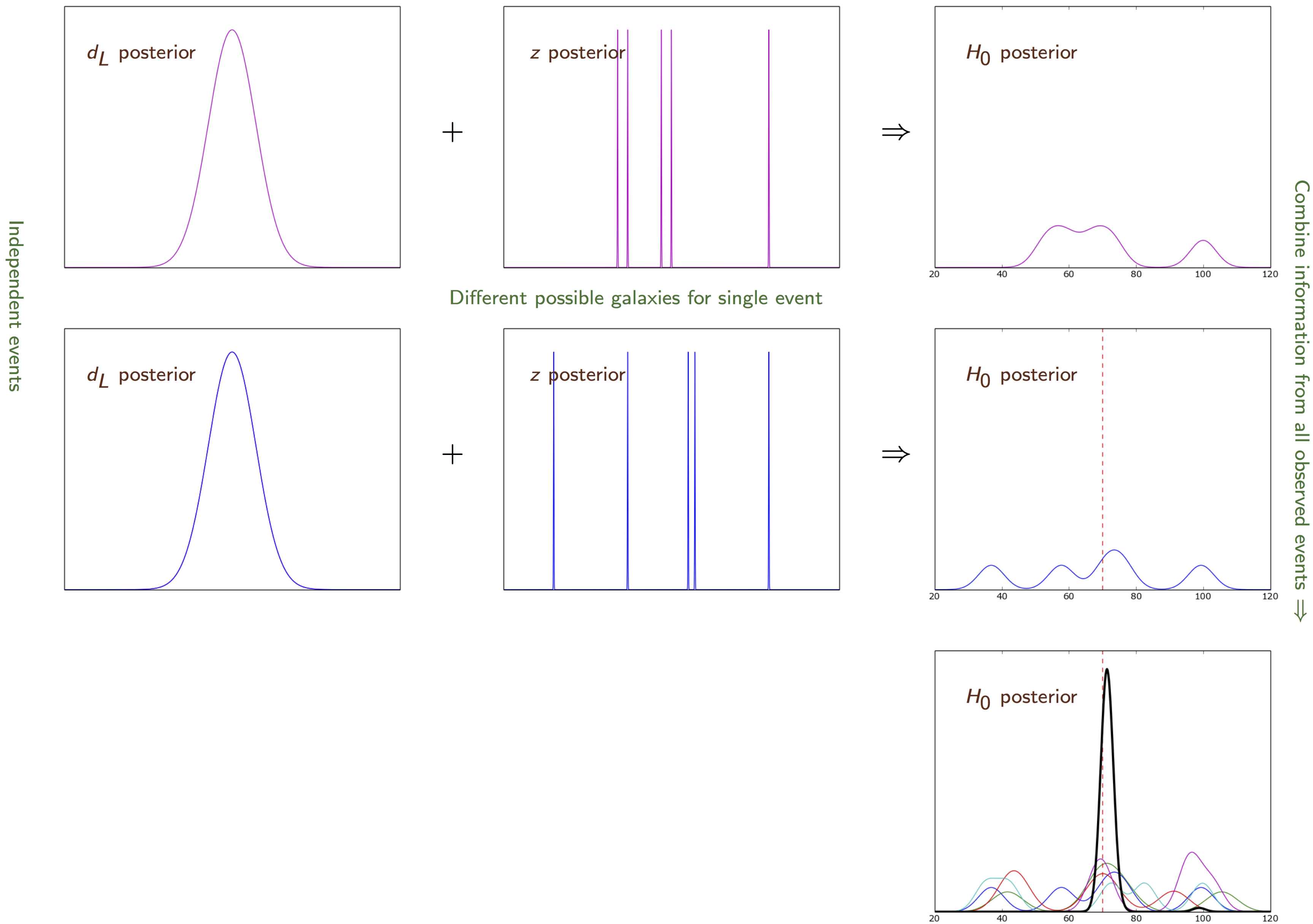
2. Cross-correlating with galaxy catalogs

- Basic strategy:
  - Given one detection:
    - Measurement of distance D_L
    - Volume error box
    - Galaxies inside box have redshifts $\hat{z}_i$
    - From $D_\mathrm{L} = cz/H_0$
      obtain list of possible values $\{H_{0,i}\}$
  - Next detection gives *different* list $\{H_{0,j}\}$
  - ...
  - Eventually correct value H_0 emerges



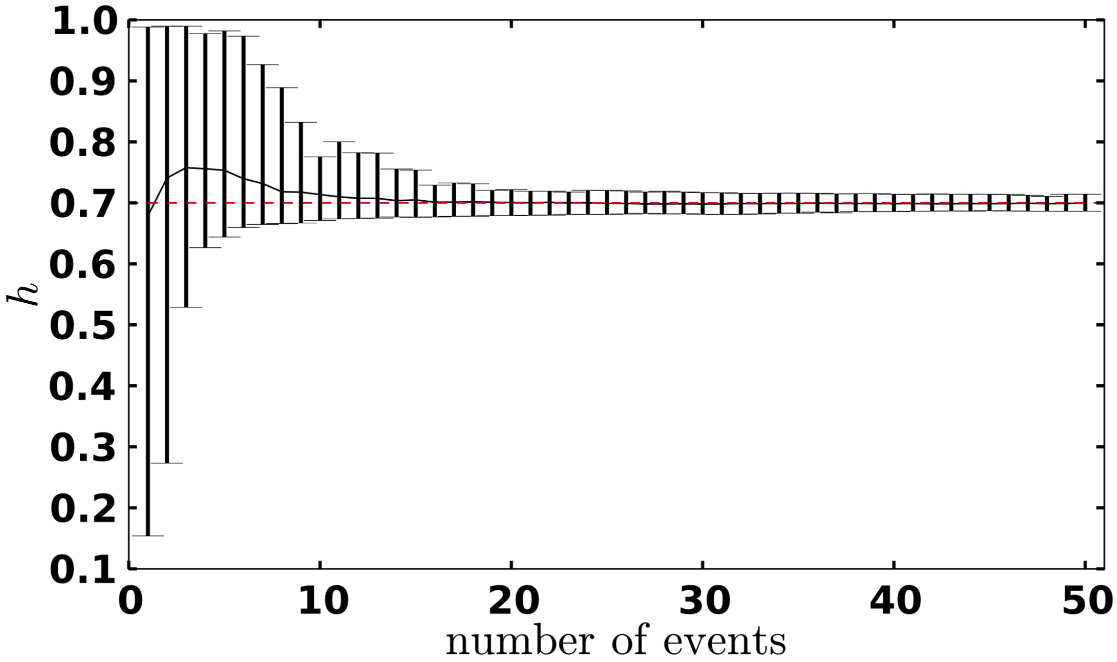
Del Pozzo, Phys. Rev. D **86**, 043011 (2012)

2. Cross-correlating with galaxy catalogs



2. Cross-correlating with galaxy catalogs

- All binary coalescences, also BBH, can be used!
  - Will need to correct for galaxy catalog incompleteness for z > 0.1
- 2nd generation detector network at design sensitivity: few percent inaccuracy after O(10) sources



Del Pozzo, Phys. Rev. D **86**, 043011 (2012)

- Possibility of having only *one* galaxy in 3D error box?
  See talk by Nishizawa

Nishizawa, arXiv:1612.06060

3.

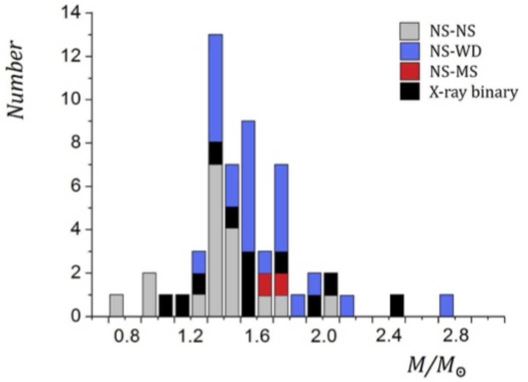
Redshift

from

the neutron star mass distribution

3. Redshift from the neutron star mass distribution

- Masses of neutron stars in binaries are in a relatively tight distribution:



Kiziltan et al., arXiv:1011.4291
Valentim et al., arXiv:1101.4872

- Observed masses differ from physical ones by redshift factor:

$$m_{\rm obs} = (1+z)\, m_{\rm phys}$$

- Advanced detector network
  - 100 NS-NS detections: $\Delta H_0/H_0 \sim 10\%$

Taylor, Gair, Mandel, Phys. Rev. D **85**, 023535 (2012)

- Einstein Telescope:
  - 10^5 NS-NS detections: again similar determination of (w_0, w_a) as forecast for future CMB+BAO+SNIa surveys

Taylor and Gair, Phys. Rev. D **86**, 023502 (2012)

4.

Redshift

using

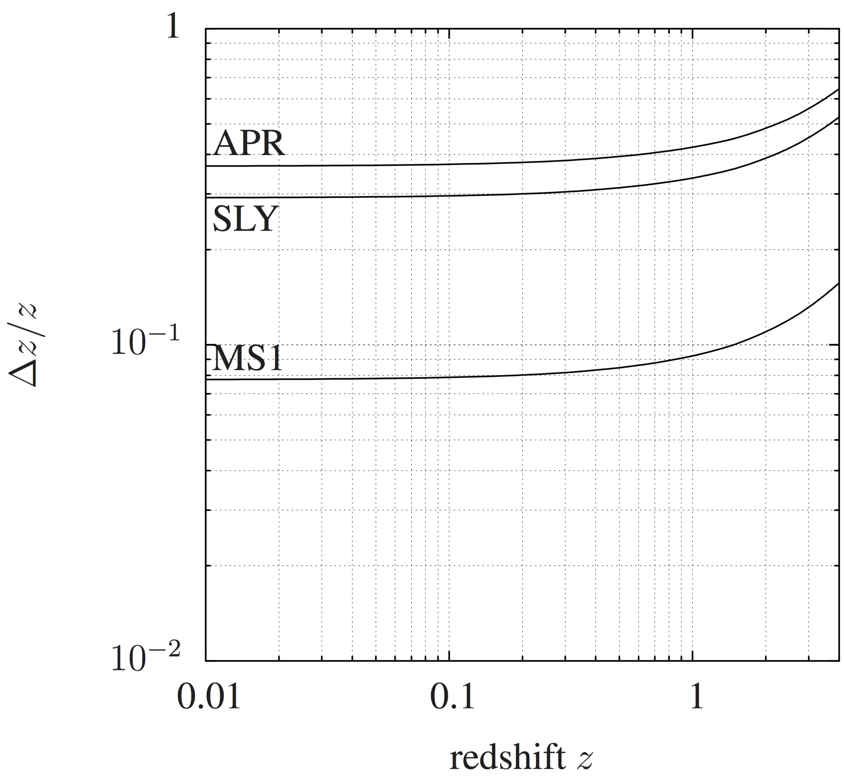
neutron star equation of state

4. Redshift using the neutron star equation of state

- Equation of state of neutron stars enters late inspiral through

$$\frac{\lambda(m)}{m^5} \propto \left(\frac{R(m)}{m}\right)^5$$

  - $\lambda(m)$ the tidal deformability,
  - $R(m)$ the radius
  - Both set by equation of state $P(\rho)$
- *This quantity breaks mass-redshift degeneracy*
  - Obtain $m_{\rm obs} = (1+z)\, m_{\rm phys}$ from low-order PN contributions
  - Obtain $m_{\rm phys}$ from tidal contribution
  - Extract redshift



Messenger and Read, Phys. Rev. Lett. **108**, 091101 (2012)

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  - $\lambda(m)$ the tidal deformability,
  - $R(m)$ the radius
  - Both set by equation of state $P(\rho)$

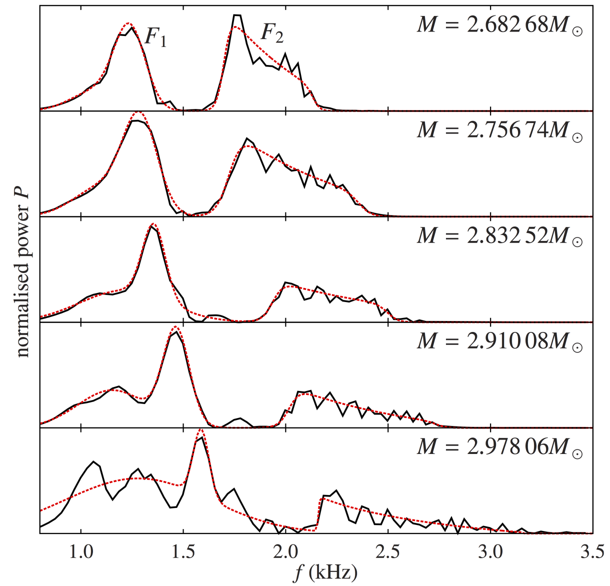
- Dark energy equation of state by combining information from multiple sources:

Model \ N	10^3	10^4	10^5	10^6	10^7
Δw_0					
General FRW+DE	0.8×10^0	2.5×10^{-1}	0.8×10^{-1}	2.5×10^{-2}	0.8×10^{-2}
Δw_1					
General FRW+DE	0.9×10^0	2.9×10^{-1}	0.9×10^{-1}	2.9×10^{-2}	0.9×10^{-2}

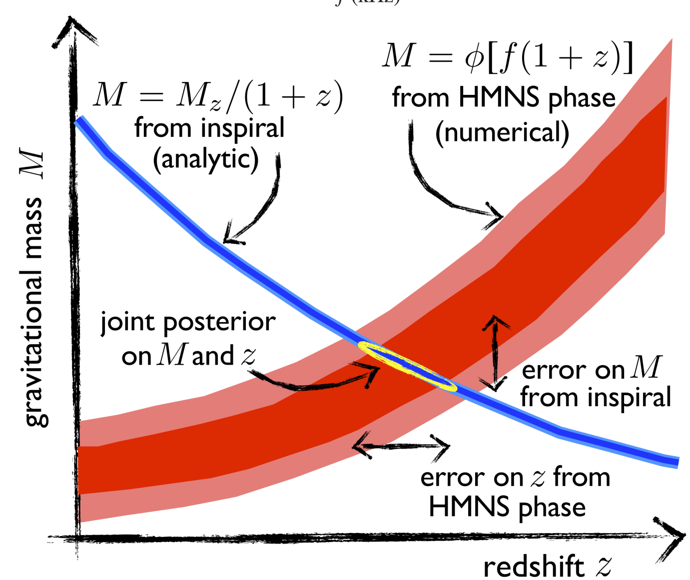
Del Pozzo, Li, Messenger, Phys. Rev. D **95**, 043502 (2017)

4. Redshift using the neutron star equation of state

- More information if one uses both
  - Inspiral
  - Post-merger:
    high-mass NS oscillations with chacteristic frequencies



- Single BNS events, Einstein Telescope: measure redshift with ~20% for z < 0.04



Messenger et al., Phys. Rev. X **4**, 041004 (2014)

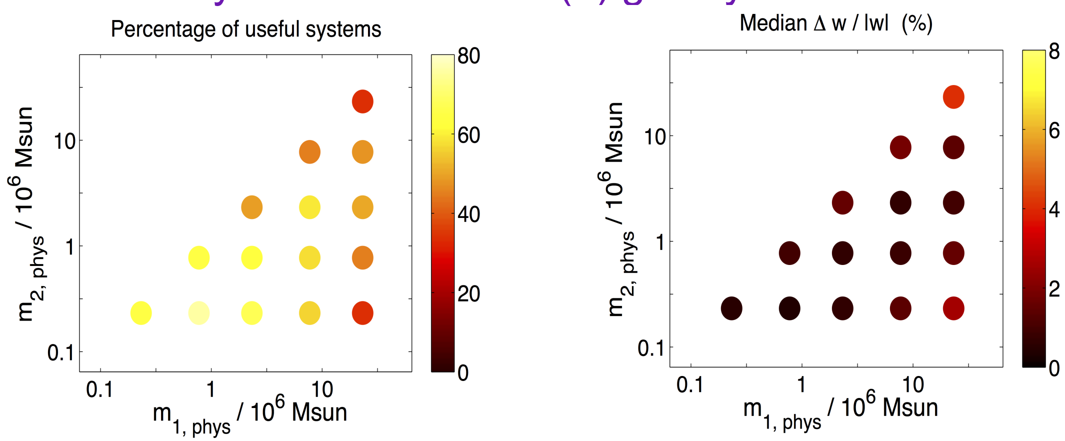
How about space-based?

Cosmography with LISA

- 3D error box can be small
    - Depending on binary, can have < 10^{-4} sterad sky localization, < 1% in distance
- Measuring H_0 with extreme mass ratio inspirals
    - Use galaxy clustering to get statistical redshift information
    - Few percent accuracy after O(20) sources out to z ~ 0.5?

McLeod & Hogan, Phys. Rev. D **77**, 043512 (2008)

- Measuring w_0 with supermassive binary black holes (SMBBH)
    - Consider system "useful" if O(1) galaxy clusters in 3D error box



Van Den Broeck, Trias, Sathyaprakash, Sintes, Phys. Rev. D **81**, 124031 (2010)

- SMBBH with EM counterparts: **see talk by Tamanini**

Summary

- 2nd generation detectors: Hubble constant to few percent accuracy
  - Redshift through EM counterpart, NS mass distribution, galaxy catalog
  - Need few tens of events
- 3rd generation ground-based: $(H_0, \Omega_M, \Omega_{DE}, w_0, w_a)$
  - Redshift through EM counterpart, NS mass distribution, NS equation of state
  - Benefit from large number of detections
  - Similar accuracies to non-GW methods, but completely different systematics
- Space-based:
  - Hubble constant from EMRIs?
  - Dark energy EOS from supermassive BBH

Bottom line: *Cosmography with GW will (probably) not do better than other methods on the same timescales - but complementary method, different systematics*