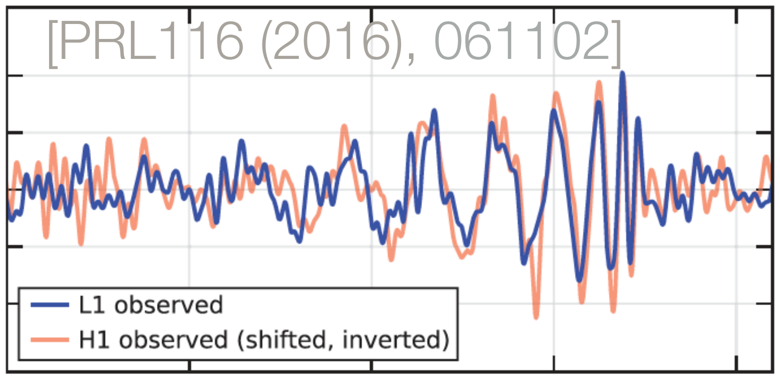
Gravitational waveforms for data analysis of spinning binary black holes

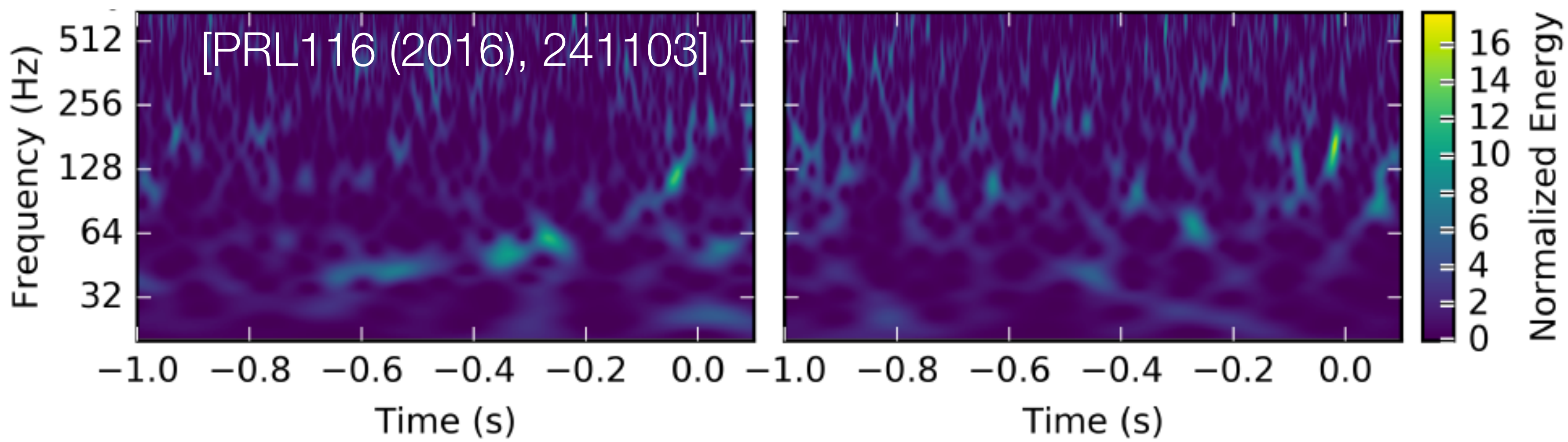
Andrea Taracchini

(Max Planck Institute for Gravitational Physics, Albert Einstein Institute — Potsdam, Germany)

[https://dcc.ligo.org/G1700243]

Numerical/analytical relativity in LIGO data analysis

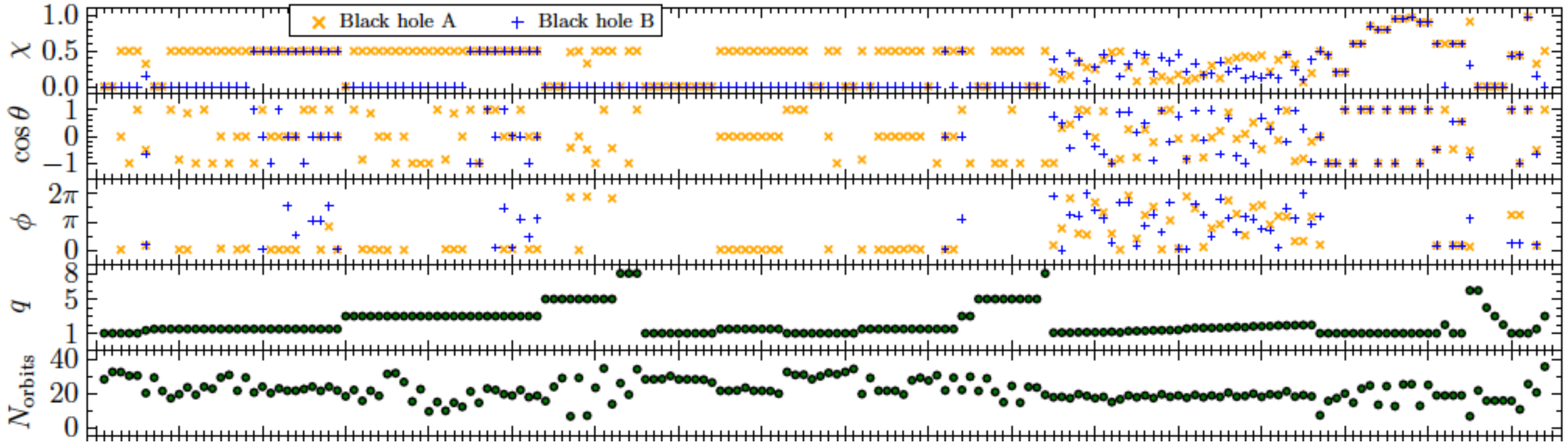




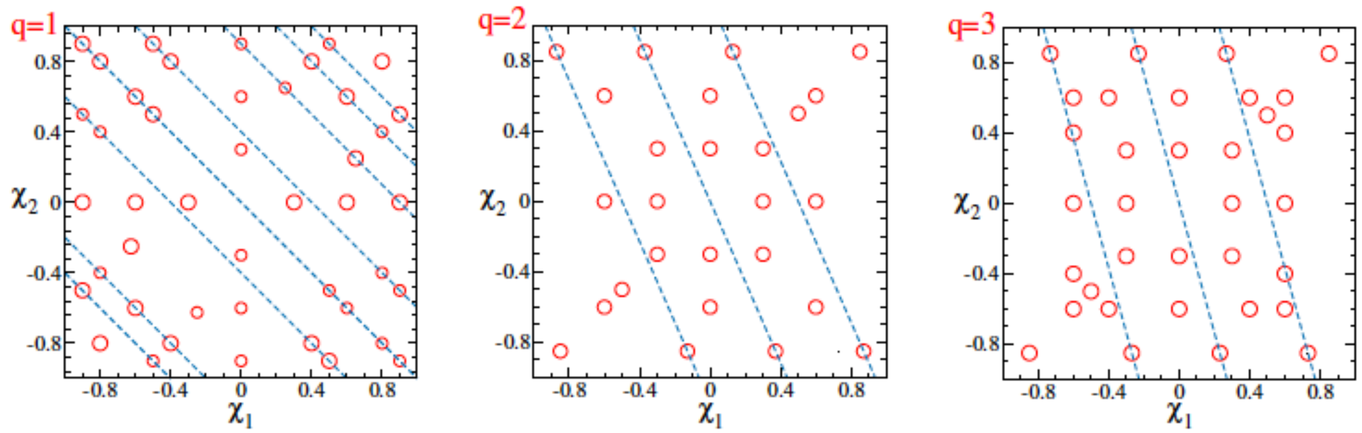
- **Synergy of numerical relativity and analytical relativity = waveform models** crucial for
  0. detecting GW151226 [LVC1606.04855]
  1. establishing 5-sigma **significance** of detections [LVC1602.03839, LVC1606.04856]
  2. measuring **properties of the source** [LVC1602.03840, LVC1606.01210, LVC1606.01262, LVC1606.04856]
  3. performing **tests of general relativity** (GR) [LVC1602.03841, LVC1606.04856]

Numerical relativity

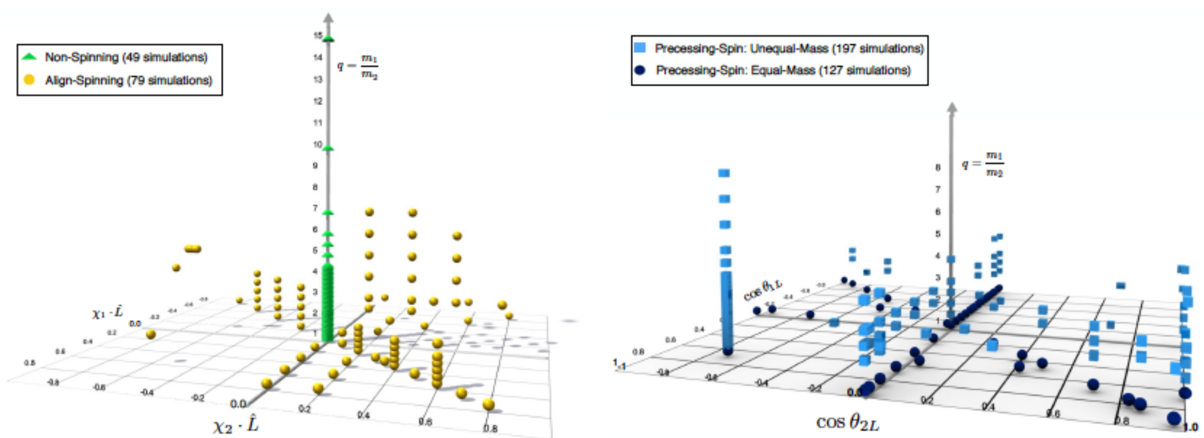
Numerical-relativity catalogs of BBHs



[Mroue+13]



[Chu+15]



[Jani+16]

… and many more NR waveforms from many groups [SXS, GATech, RIT, Cardiff-UIB, NCSA] are being computed also in response to observations

Numerical-relativity catalogs of BBHs

- Sources of **errors**: (i) resolution, (ii) extrapolation, (iii) finite length, (iv) junk radiation

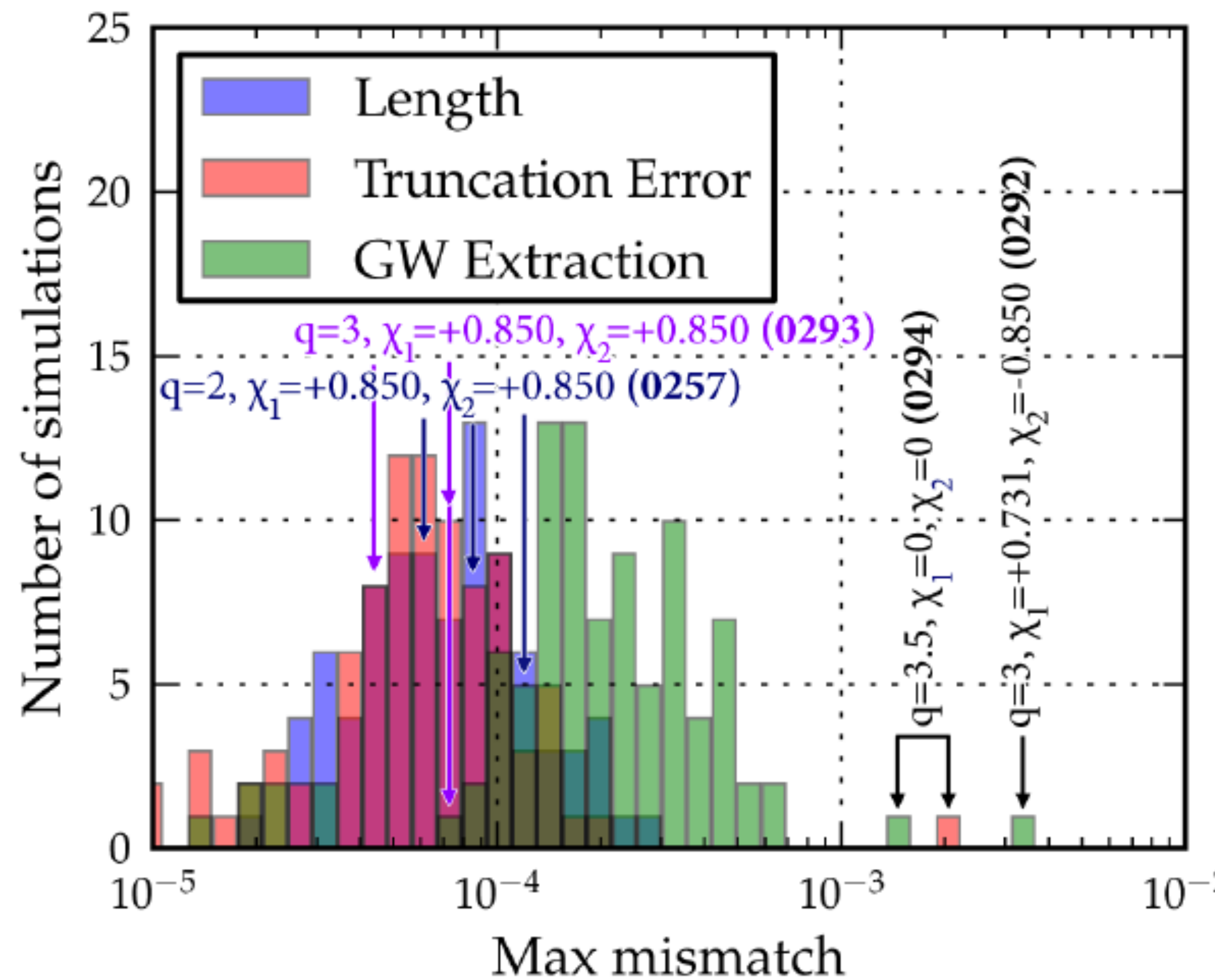
- Aligned-spin template banks accept fitting factors 3% mismatch ~ 10% loss in event rate

- Parameter estimation (sufficient) accuracy requirement [Lindblom+08]

$$1 - \mathcal{O}(h_1, h_2) < \frac{1}{2\,\mathrm{SNR}^2}$$

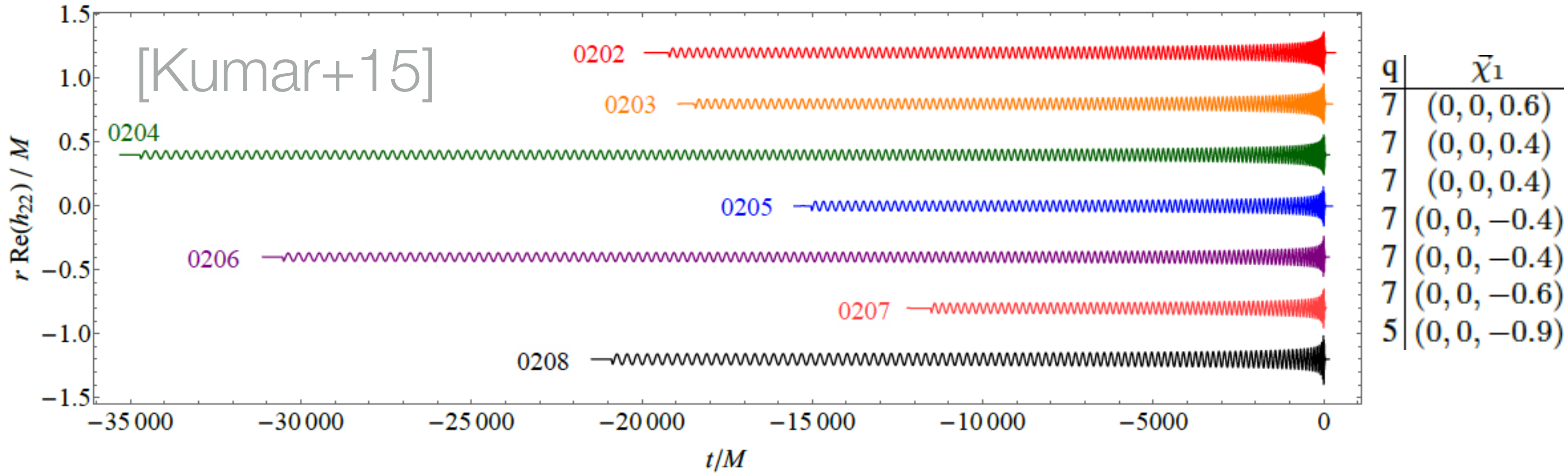
$$\langle h_1, h_2 \rangle = 4\mathrm{Re} \int_{f_{\mathrm{low}}}^{f_{\mathrm{high}}} \frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)} \mathrm{d}f$$

$$\mathcal{O}(h_1, h_2) = \max_{\phi_0, t_0} \frac{\langle h_1(\phi_0, t_0), h_2 \rangle}{\sqrt{\langle h_1, h_1 \rangle \langle h_2, h_2 \rangle}}$$



[Chu+15]

Challenging BBHs



- **Longterm** BBH simulations at mass ratio 7 [Szilagyi+14, Kumar+15]
- **Almost extremal** BBH simulations: equal-mass, aligned-spins 0.99, 0.994 [Scheel+14]
- New initial data for challenging configurations [Ossokine+15]

q	χ_1	χ_2	D_0/M
1	(0, 0, 0.9999)	(0, 0, 0.9999)	14.17
3	(0, 0.49, -0.755)	(0, 0, 0)	15.48
10	(0.815, -0.203, 0.525)	(-0.087, 0.619, 0.647)	15.09
50	(-0.045, 0.646, -0.695)	(0, 0, 0)	16

Direct use of numerical relativity

Besides guiding construction of models (waveforms, remnant properties), there are other avenues to use NR:

- **Direct comparison of existing NR catalogs to observations** [LVC1602.03843, LVC1606.01262] [Richard's talk]

- **NR follow-ups to observations** [LVC detection papers, Lovelace+16]:
  1. comparisons to unmodeled reconstructions
  2. validate models

- **Surrogate waveform models** [Blackman+15,17]
  1. restricted parameter space (high mass, q<=2, spins<=0.8, one spin aligned)
  2. many NR simulations to construct basis
  3. interpolation across NR runs
  4. they do not extrapolate to low mass: need models or long NR

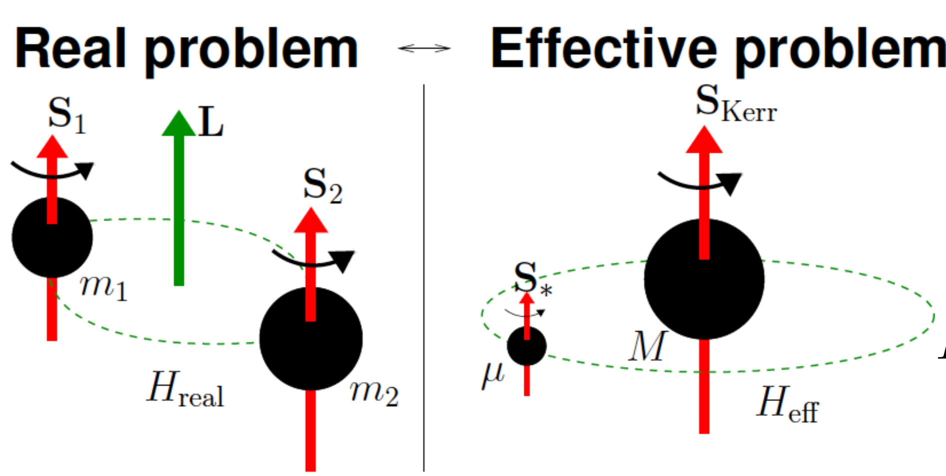
Nonprecessing models for LIGO

Effective-one-body models of nonprecessing BBHs

- Nonspinning case: particle in deformation of Schwarzschild [Buonanno & Damour99]. Spinning case: **spinning particle in deformation of Kerr** [Barausse & Buonanno10,11;Nagar+14]

- Inspiral waveforms/radiation reaction from **resummation post-Newtonian formulas** [Damour+07,09; Pan+11;Nagar+16]

- Ringdown from **superposition of quasinormal modes** of remnant BH



$$H_{\rm real} = Mc^2\sqrt{1+2\nu\left(\frac{H_{\rm eff}}{\mu c^2}-1\right)} - Mc^2$$

$$H_{\rm eff} = \mu c^2\sqrt{A(R)\left[1+\frac{\mathbf{P}^2}{\mu^2c^2}+\frac{1}{\mu^2c^2}\left(\frac{A(R)}{D(R)}-1\right)\left(\frac{\mathbf{R}\cdot\mathbf{P}}{R}\right)^2\right]}$$

$$A = \overbrace{1-2u}^{\rm Schwarzschild} + 2\nu u^3 + \left(\frac{94}{3}-\frac{42}{32}\pi^2\right)\nu u^4 + a_5 u^5 + \cdots \qquad (u = GM/Rc^2)$$

$$\nu = \frac{m_1 m_2}{(m_1+m_2)^2}$$

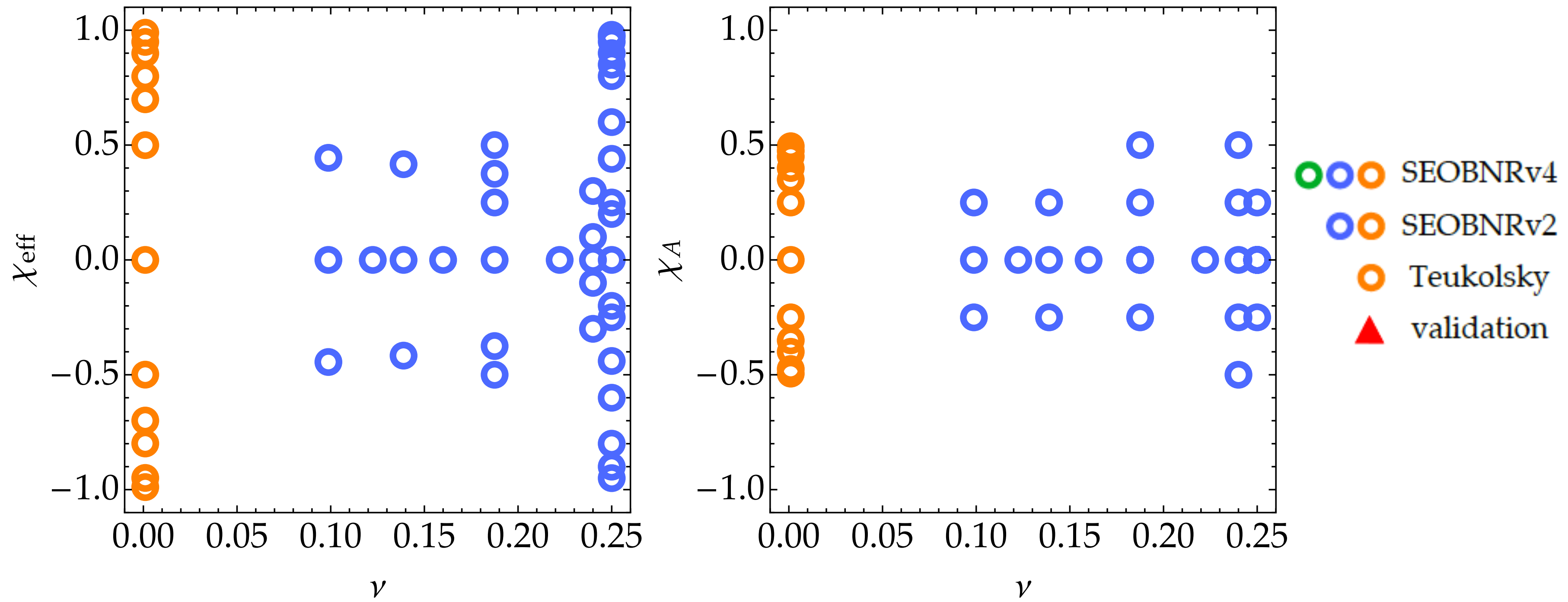
example of tuning parameter

Effective-one-body model of nonprecessing BBHs for O1

$$\nu = \frac{m_1 m_2}{(m_1+m_2)^2}$$

$$\chi_{\rm eff} = \left(\frac{\boldsymbol{S}_1}{m_1} + \frac{\boldsymbol{S}_2}{m_2}\right)\cdot \hat{\boldsymbol{L}}$$

$$\chi_{\rm A} = \left(\frac{\boldsymbol{S}_1}{m_1^2} + \frac{\boldsymbol{S}_2}{m_2^2}\right)\cdot \hat{\boldsymbol{L}}$$



- **SEOBNRv2** calibrated to better than 99% overlap with NR for design aLIGO [AT+14]
- Used in its **reduced-order-model** version [Pürrer14,15] in O1 for filtering and parameter estimation
- Similar set of calibration waveforms used in IHES models [Nagar+15,16]

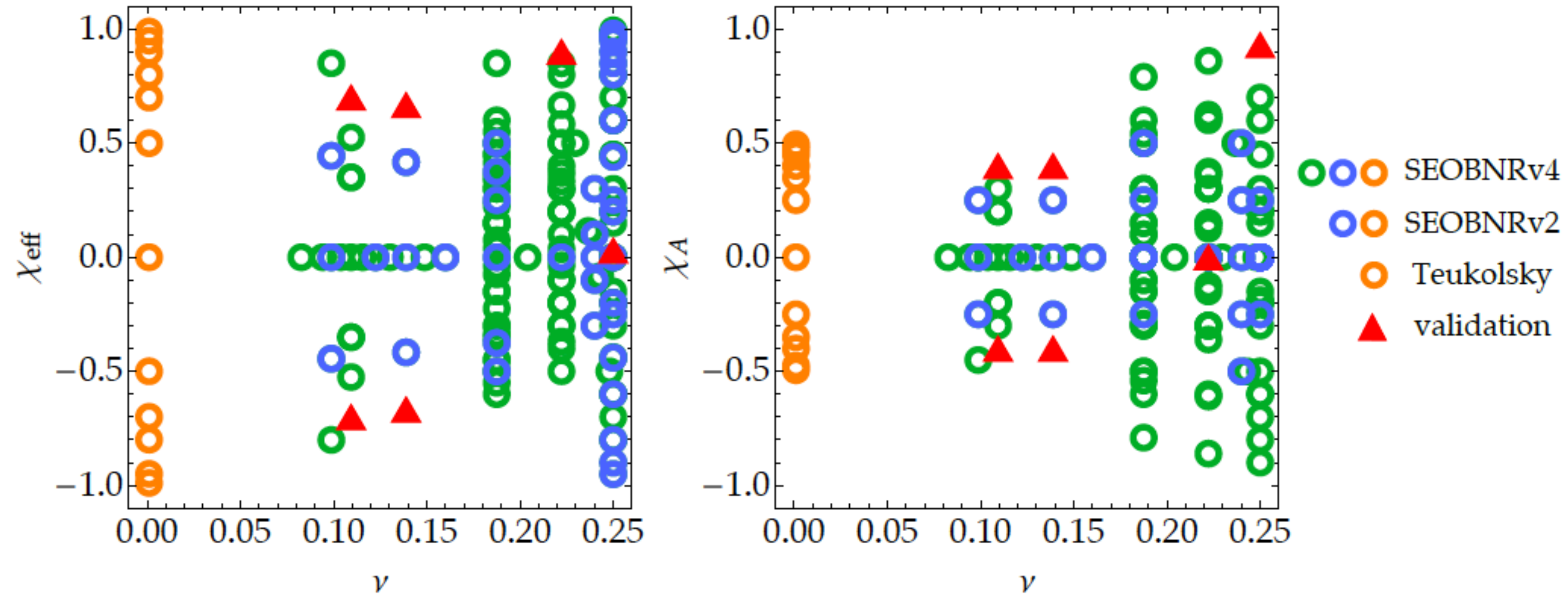
Effective-one-body model of nonprecessing BBHs for O2

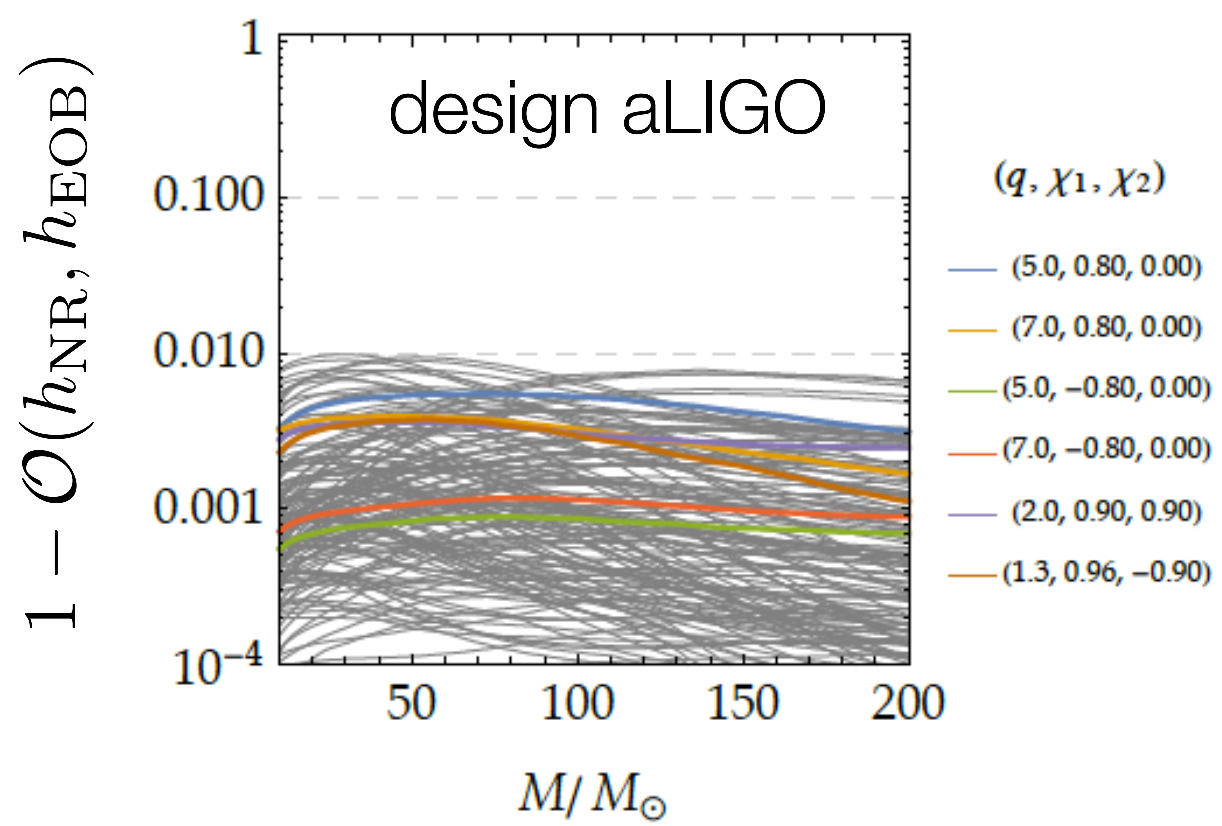
- **SEOBNRv4** [Bohe,Shao,AT+16]

$$\nu = \frac{m_1 m_2}{(m_1+m_2)^2}$$

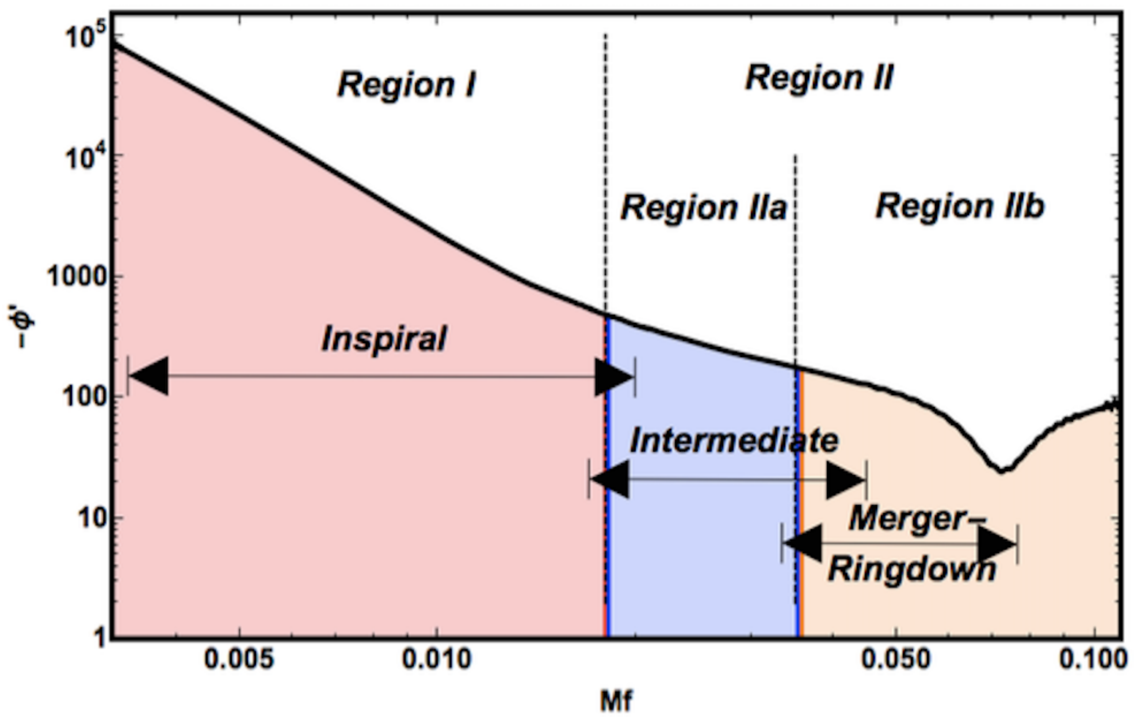
$$\chi_{\rm eff} = \left(\frac{\boldsymbol{S}_1}{m_1} + \frac{\boldsymbol{S}_2}{m_2}\right)\cdot \hat{\boldsymbol{L}}$$

$$\chi_{\rm A} = \left(\frac{\boldsymbol{S}_1}{m_1^2} + \frac{\boldsymbol{S}_2}{m_2^2}\right)\cdot \hat{\boldsymbol{L}}$$





Phenomenological model of nonprecessing BBHs

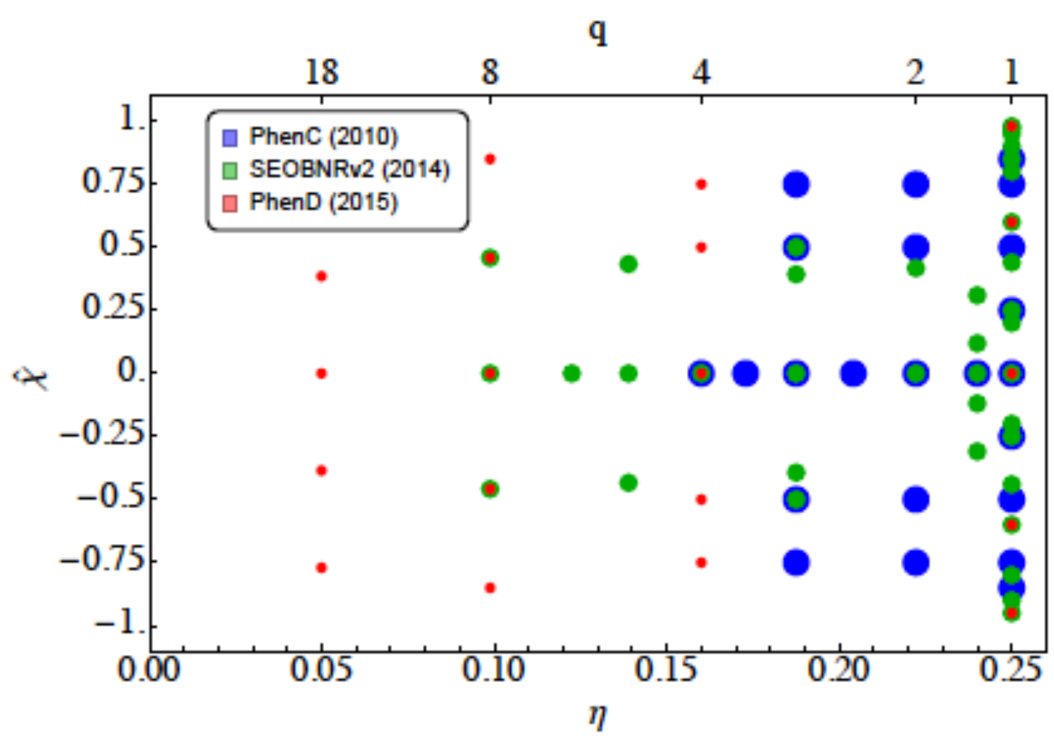


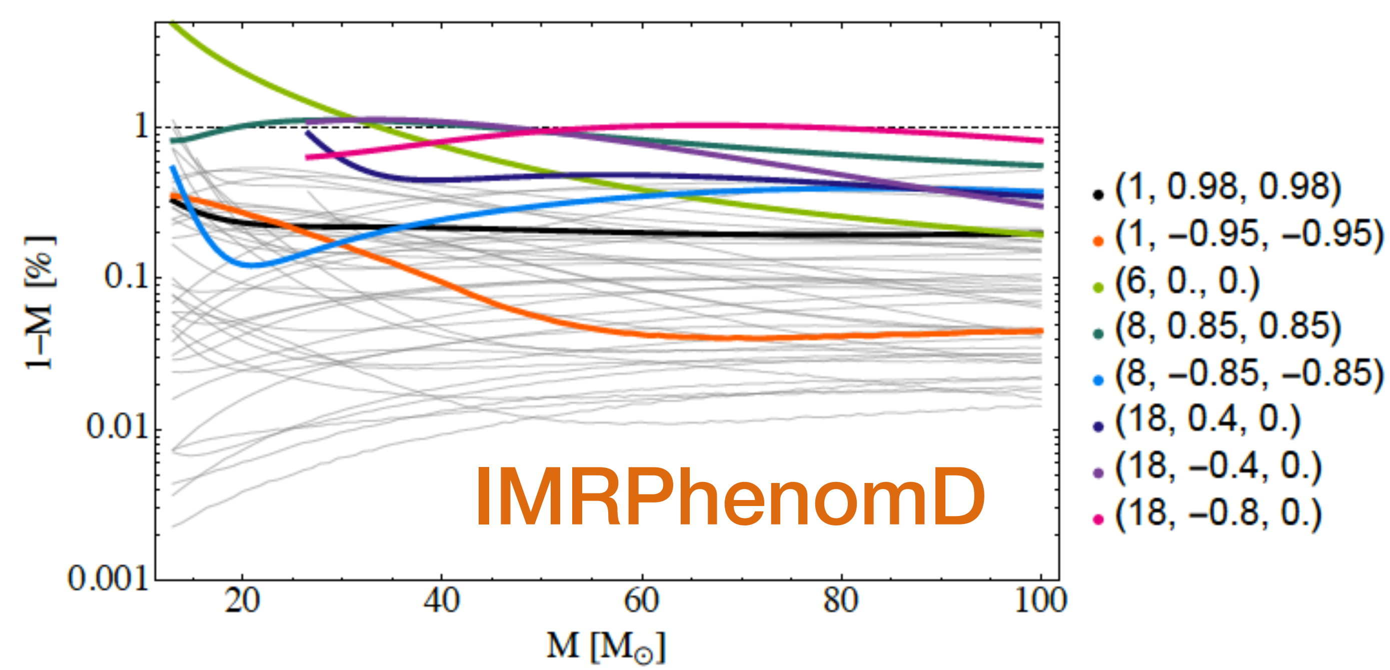
$$\phi_{\rm Ins} = \phi_{\rm TF2}(Mf;\Xi) + \frac{1}{\eta}\left(\sigma_0 + \sigma_1 f + \frac{3}{4}\sigma_2 f^{4/3} + \frac{3}{5}\sigma_3 f^{5/3} + \frac{1}{2}\sigma_4 f^2\right)$$

$$\phi_{\rm Int} = \frac{1}{\eta}\left(\beta_0 + \beta_1 f + \beta_2 \,{\rm Log}(f) - \frac{\beta_3}{3} f^{-3}\right)$$

$$\phi_{\rm MR} = \frac{1}{\eta}\left\{\alpha_0 + \alpha_1 f - \alpha_2 f^{-1} + \frac{4}{3}\alpha_3 f^{3/4} + \alpha_4 \tan^{-1}\left(\frac{f - \alpha_5 f_{\rm RD}}{f_{\rm damp}}\right)\right\}.$$

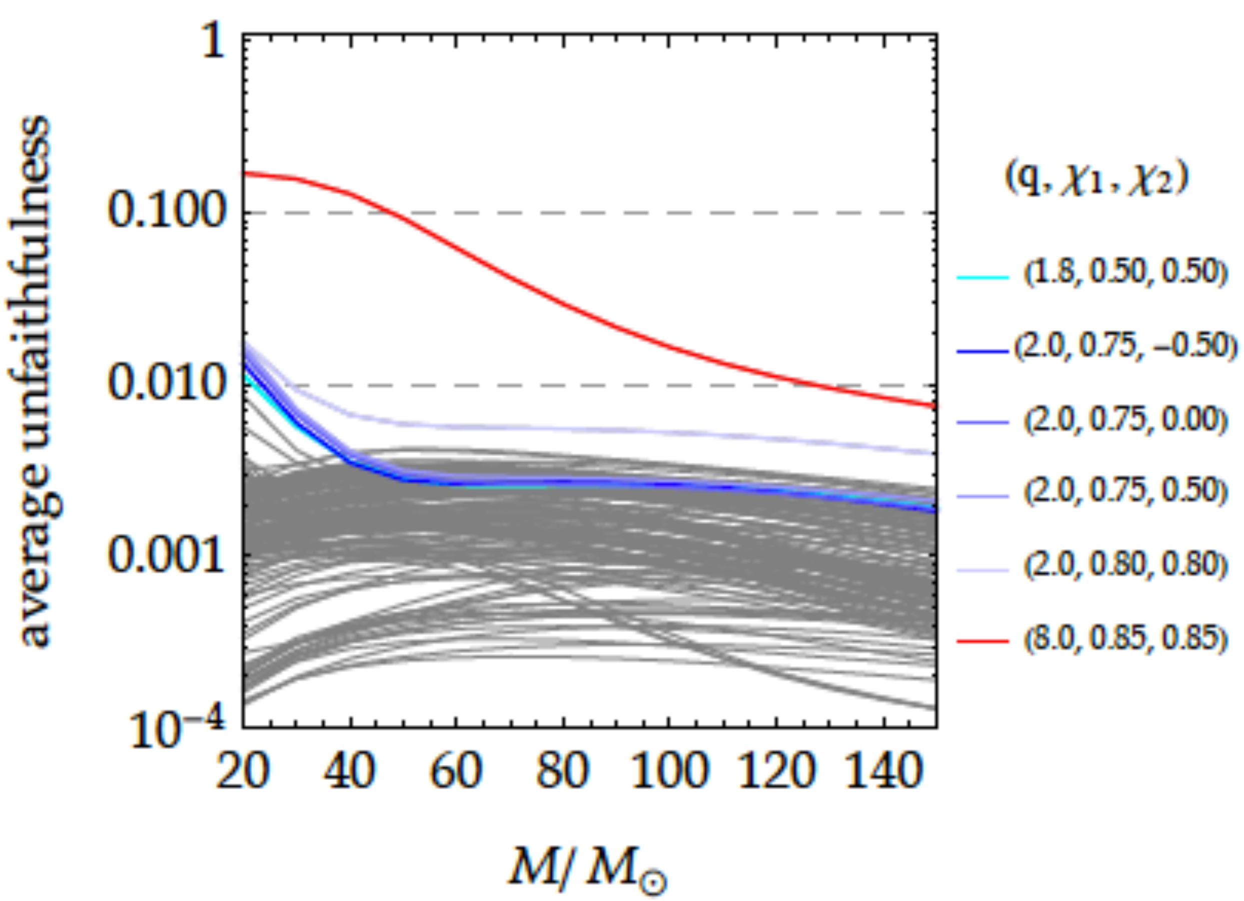
- Fit to **hybrids** of uncalibrated EOB and NR [Husa+15, Khan+15]



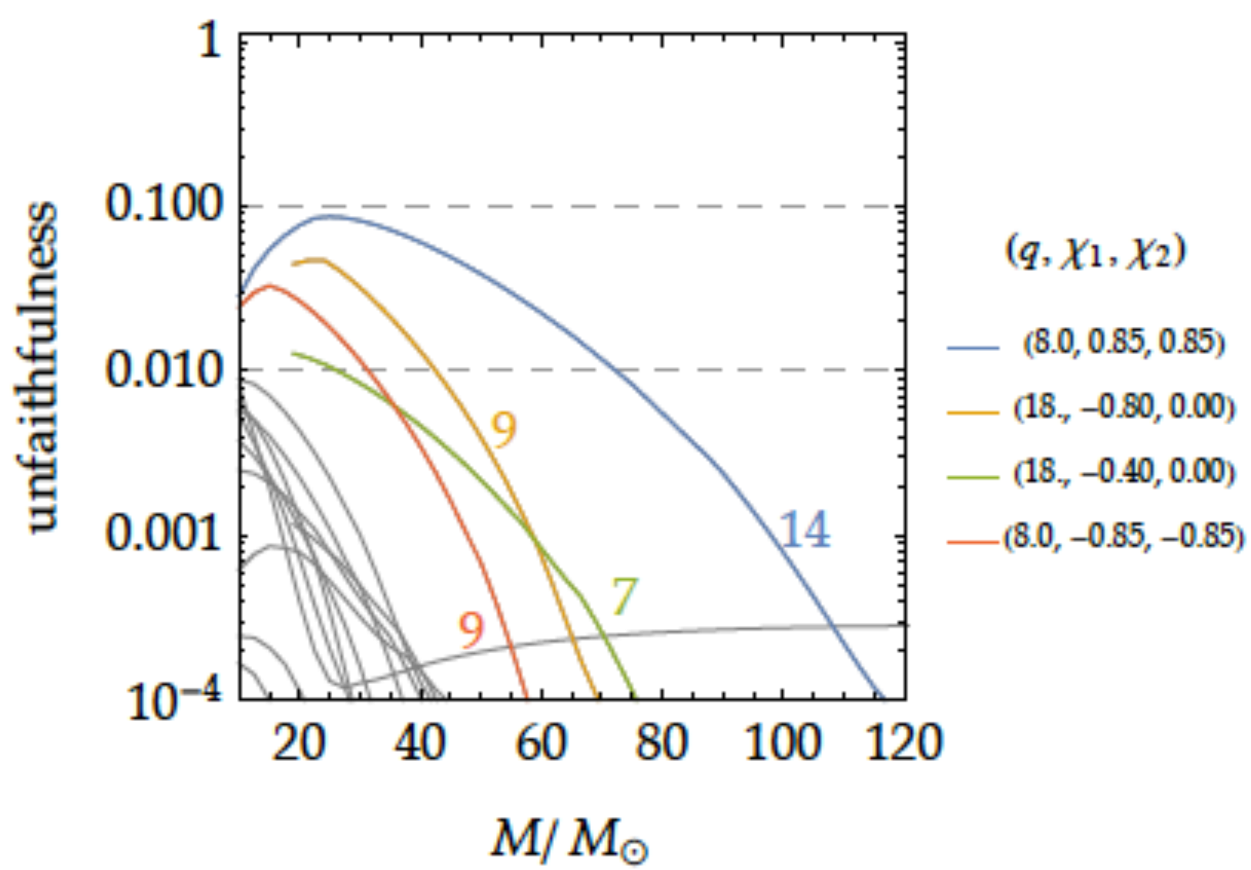


Extrapolation to low frequencies

are NR waveforms long enough to constrain down to 25Hz for moderate M?

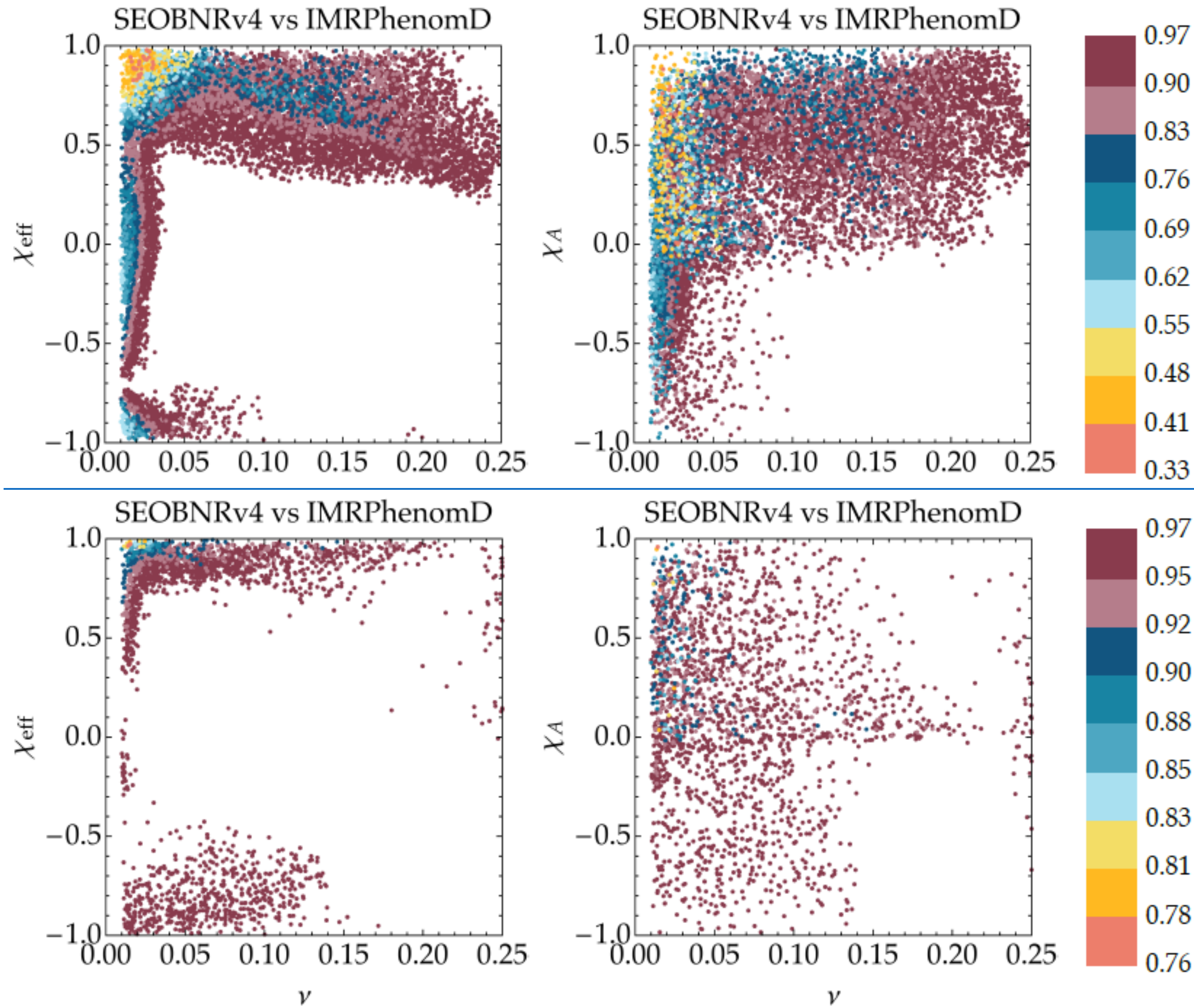


are hybrids reliable?



[Bohe,Shao,AT+16]

Comparing nonprecessing IMR BBH models



$\mathcal{O}(h_1, h_2)$

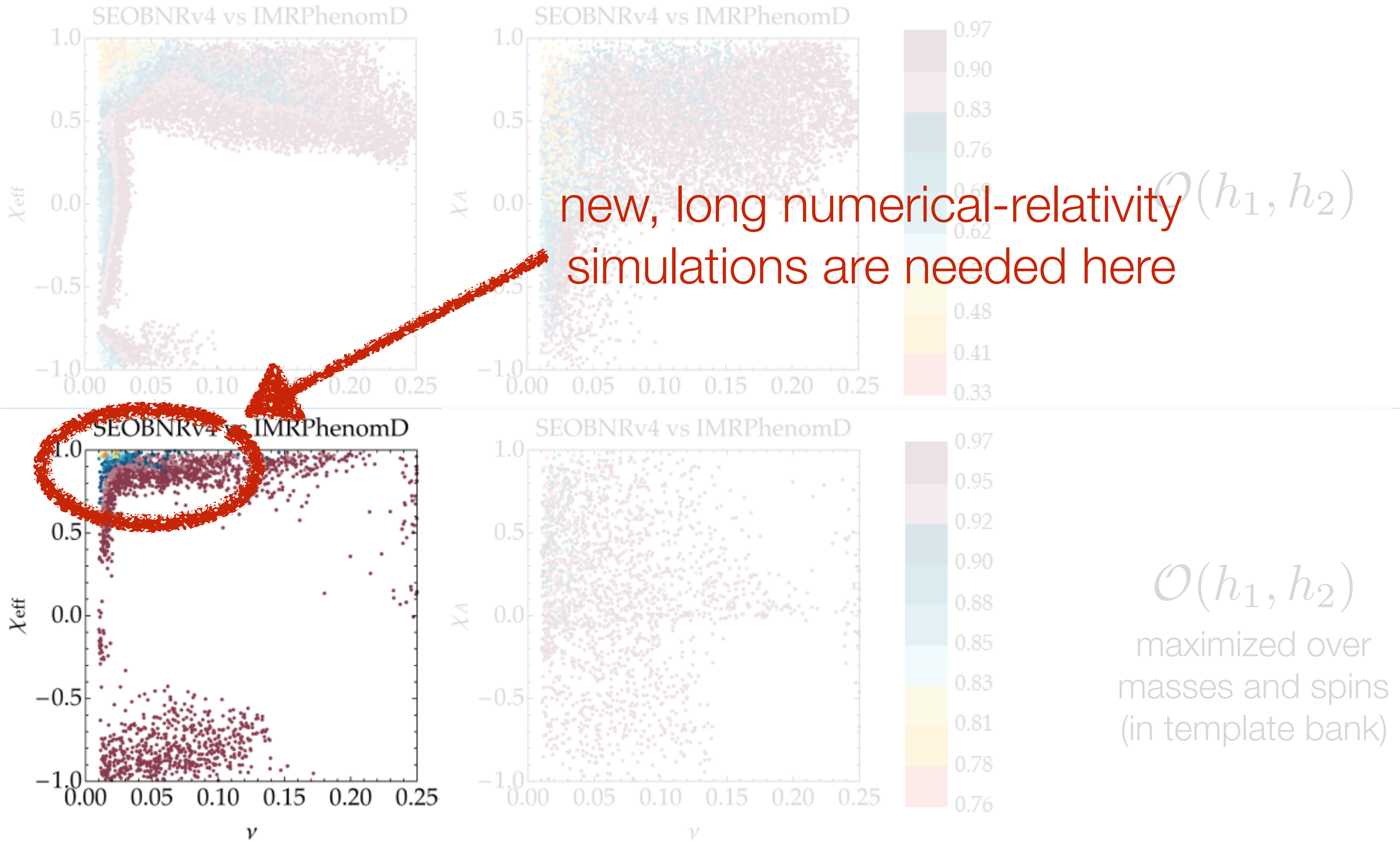
$\mathcal{O}(h_1, h_2)$

maximized over masses and spins (in template bank)

[Bohe,Shao,AT+16]

(O1 aLIGO)

Comparing nonprecessing IMR BBH models

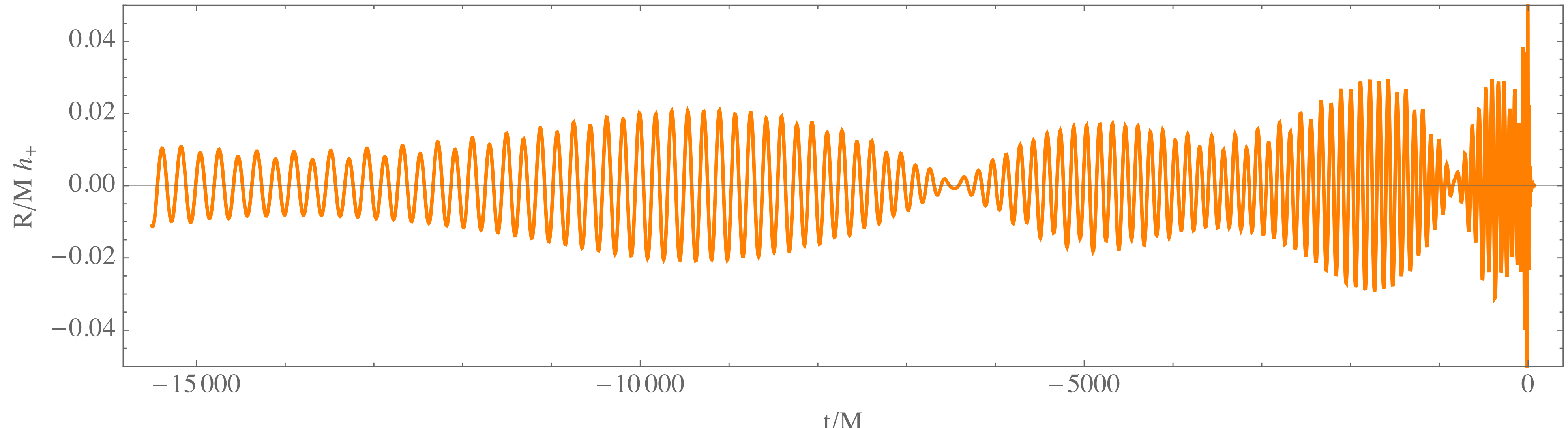


[Bohe,Shao,AT+16]

(O1 aLIGO)

Precessing models for LIGO

Precessing IMR BBH models



- When BH spins are not parallel to angular momentum of the binary, the **orbital plane precesses**

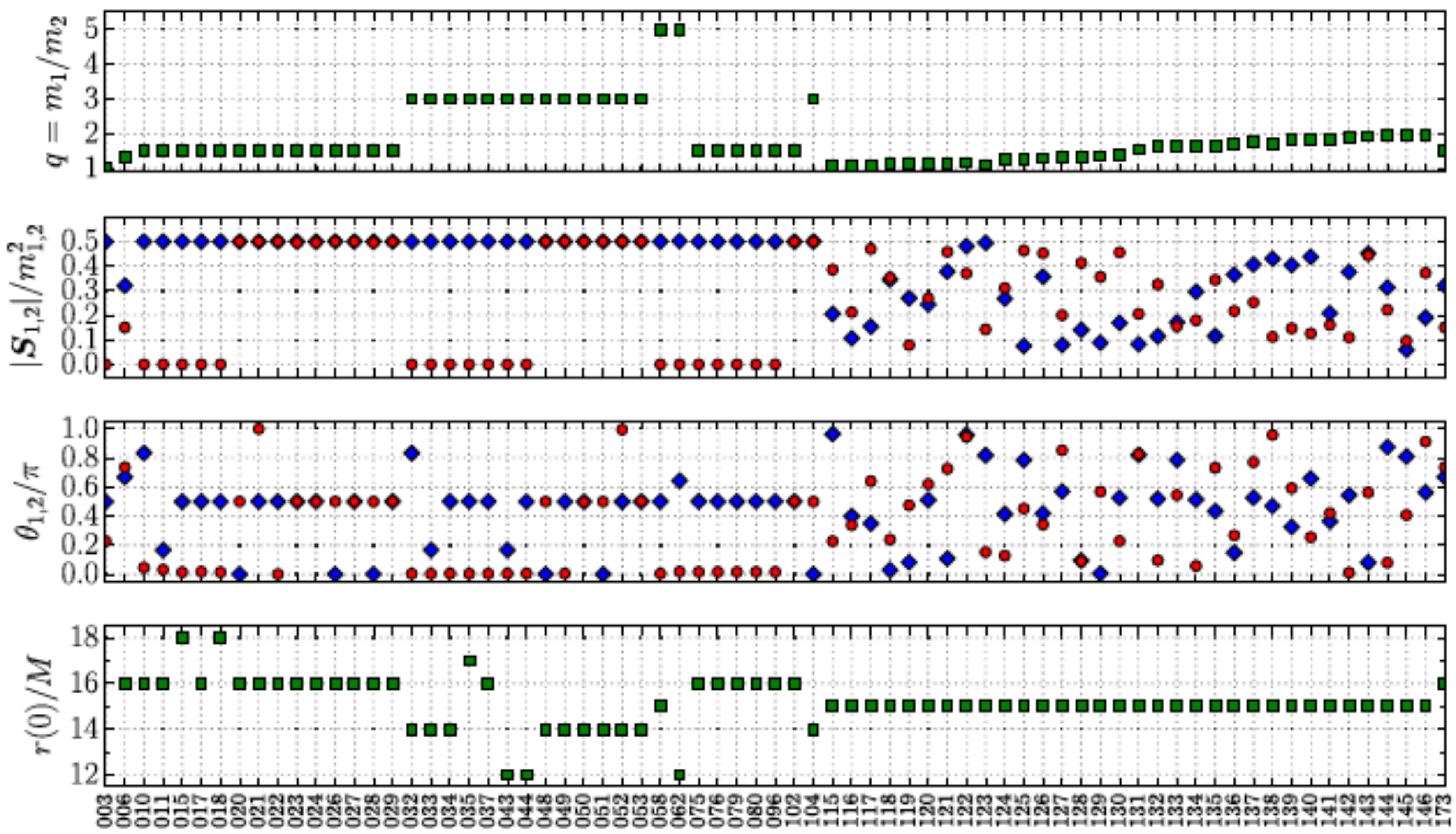
- **Precessing frame** [Buonanno+03, Schmidt+11, O'Shaughnessy+11, Boyle+11]
  1. In precessing frame, use **calibrated nonprecessing model**
  2. Inertial-frame modes from **rotation of precessing-frame modes according to motion of orbital angular momentum**

- Both effective-one-body [Pan+13, Babak, AT+16] and phenomenological [Hannam+13] models available

- Inspiral-only PN waveforms [Katerina's talk]

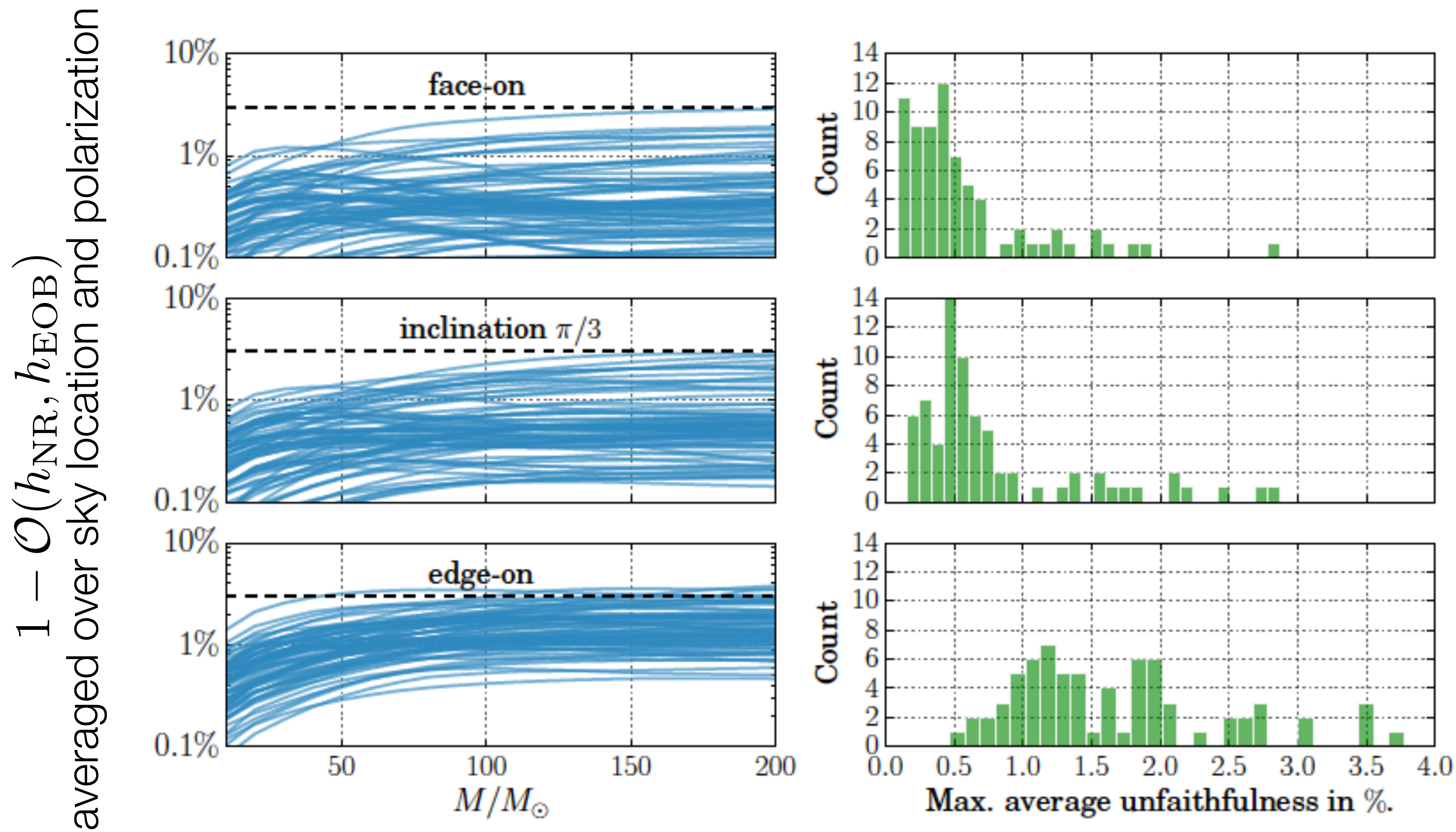
Effective-one-body model for precessing BBHs

- 70 NR waveforms from SXS public catalog used to test model



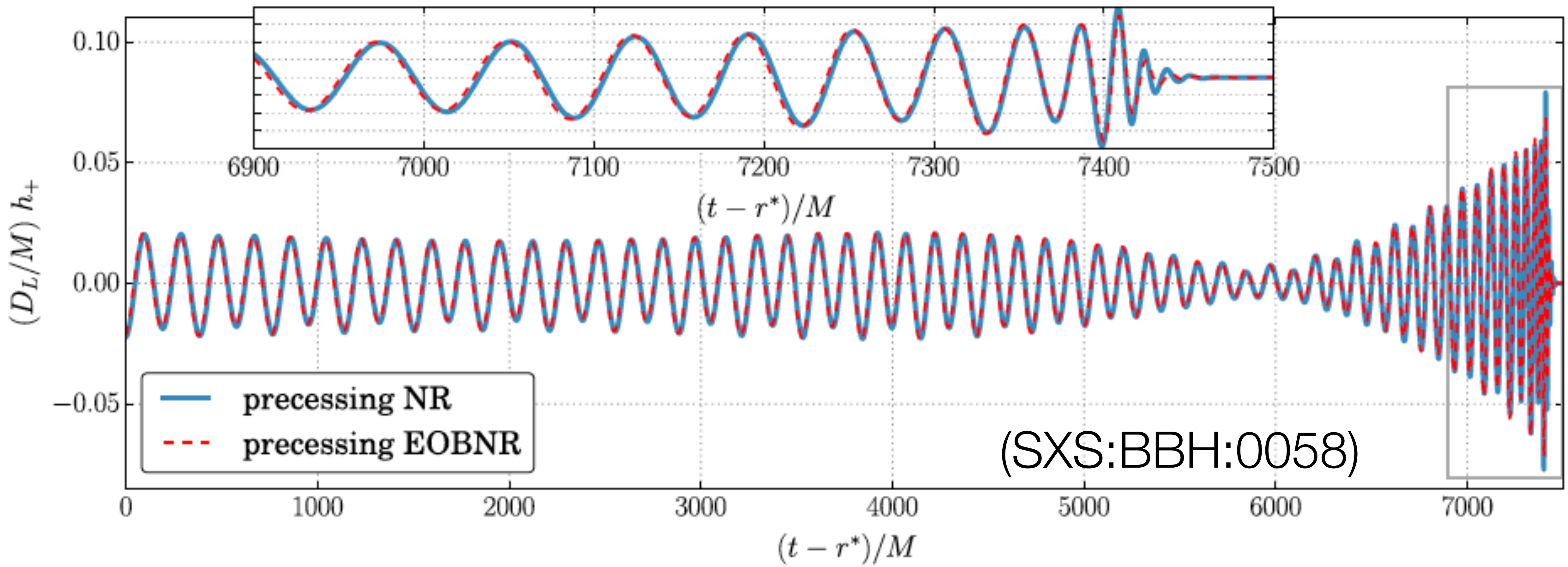
[Babak, AT+16]

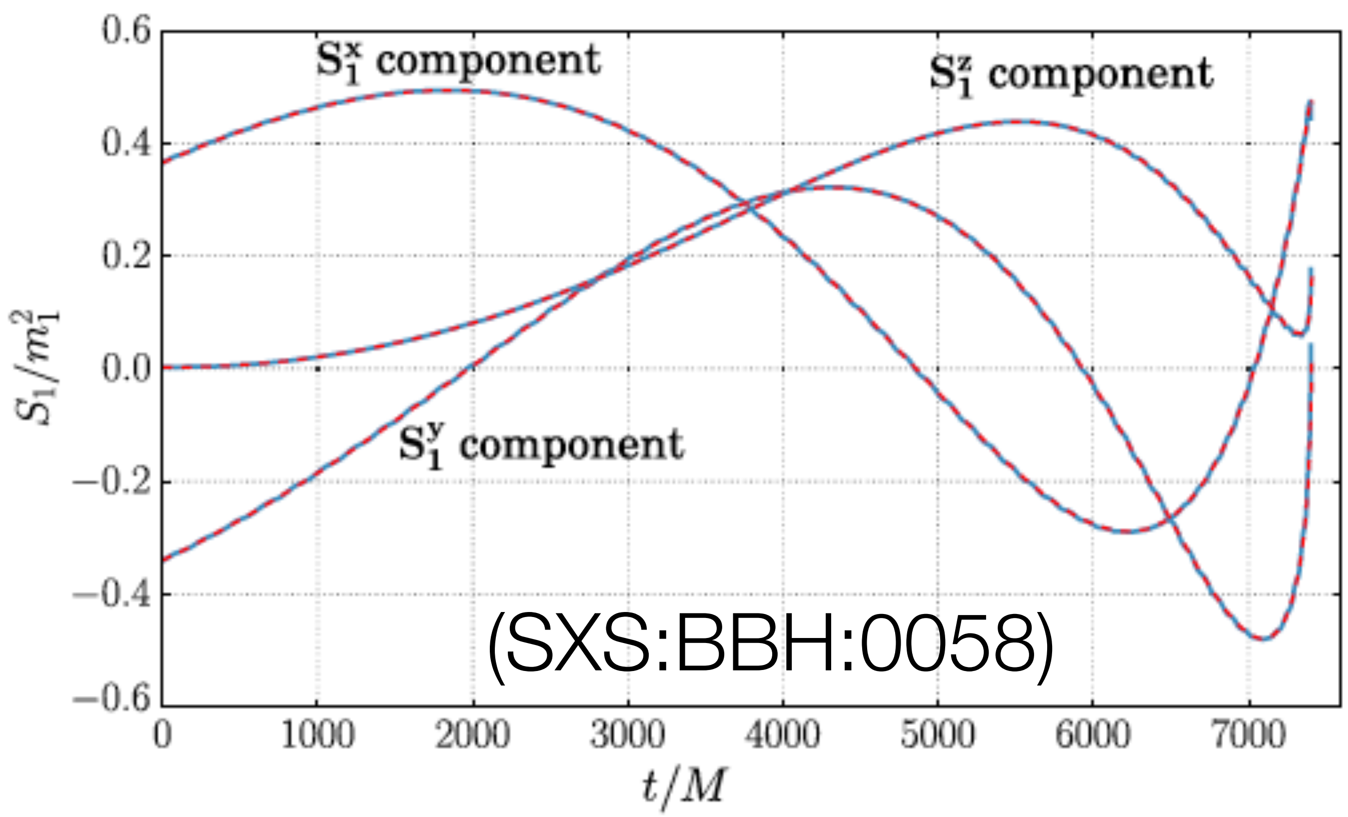
Effective-one-body model for precessing BBHs



[Babak, AT+16]

Effective-one-body model for precessing BBHs



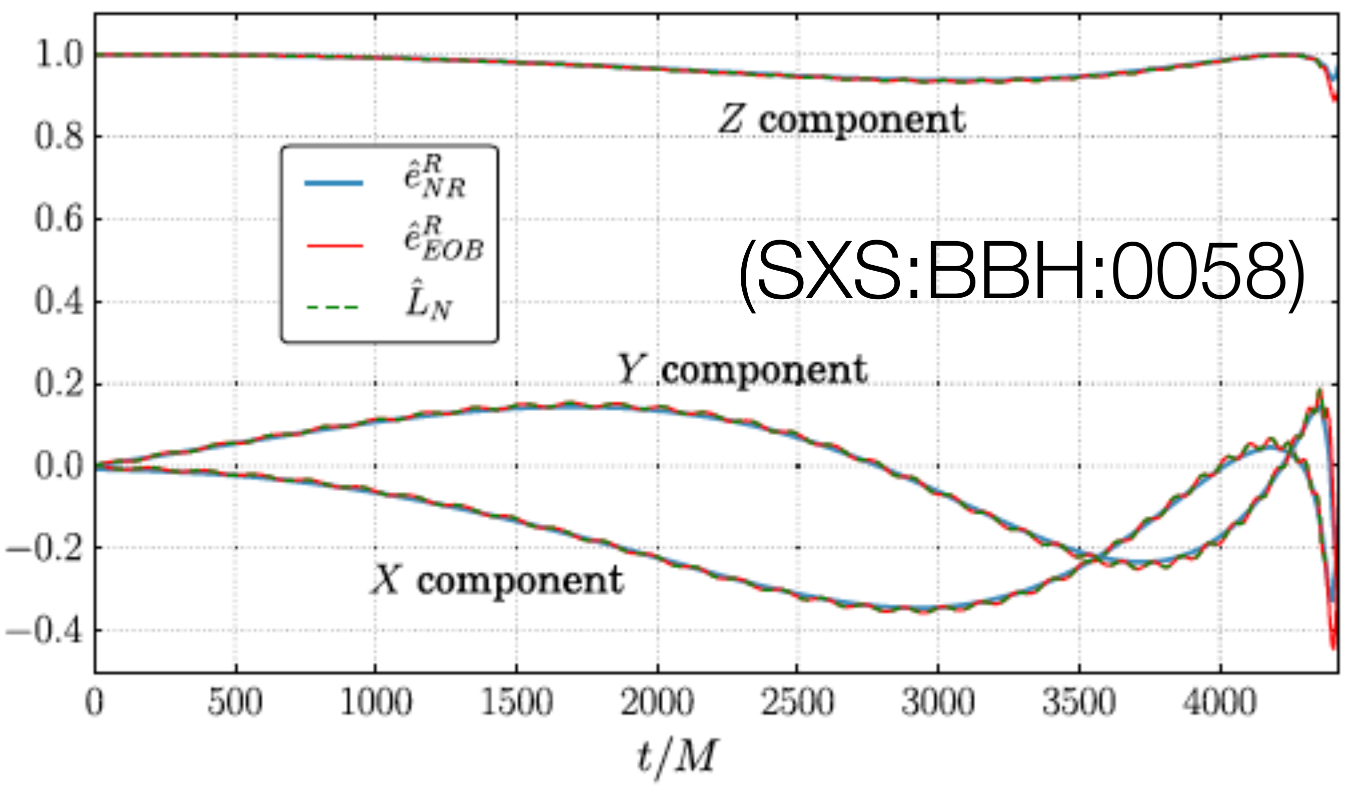


q=5, a1=0.5, a2=0
S1 in-plane at t=0

[Babak, AT+16]

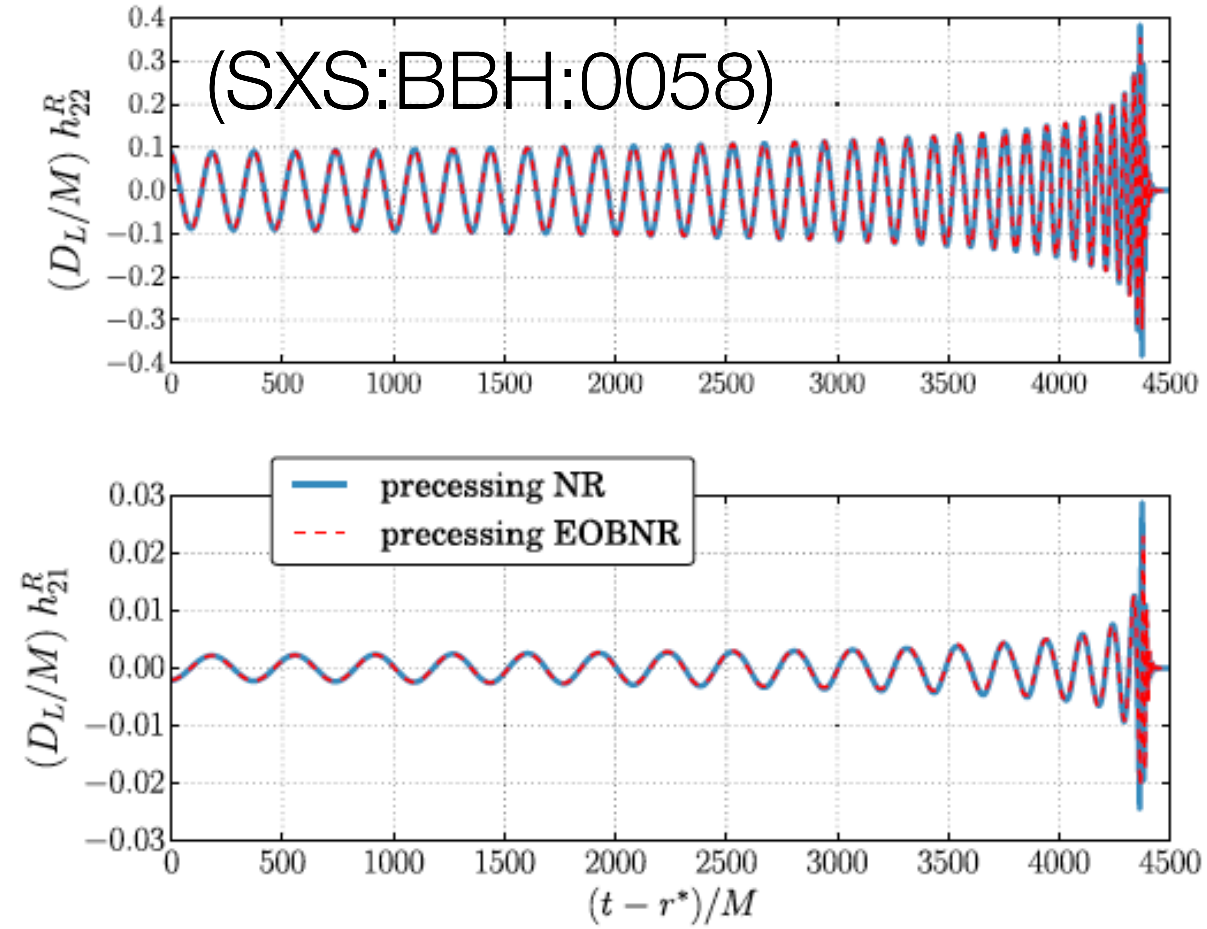
Effective-one-body model for precessing BBHs

testing the **rotation**
via maximum-radiation
direction



motion of EOB angular momentum
closely tracks
NR direction of max radiation

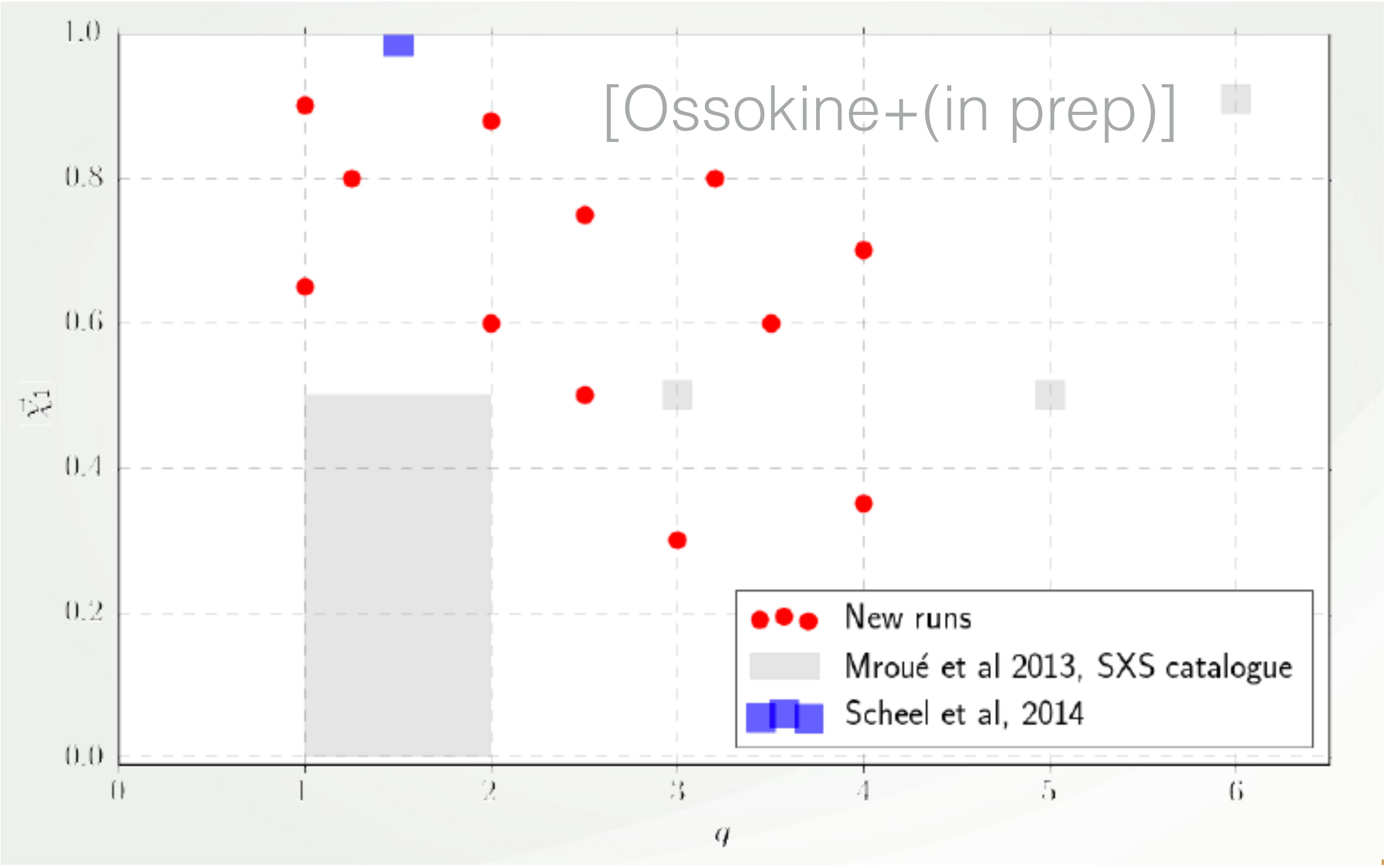
testing the waveforms
in the **precessing frame**



(2,2) good, (2,1)
to improve, especially RD

Effective-one-body model for precessing BBHs

- New SXS NR waveforms [Ossokine+(in prep)] used to
  1. improve model [AEI(in prep)]
  2. assess PE systematics [AEI(in prep)]



Phenomenological model of precessing BBHs

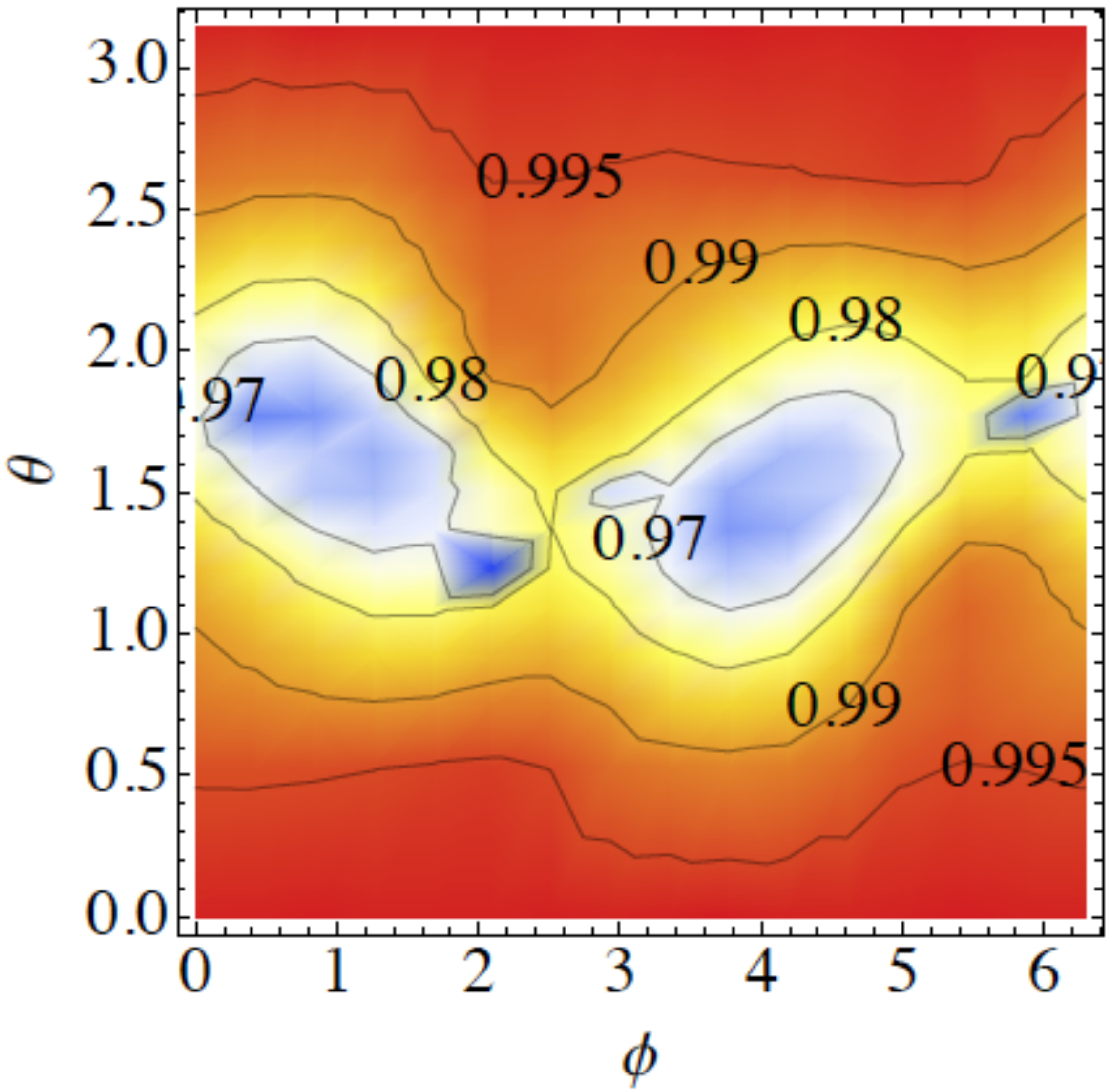
- Start from PN and find **single effective spin** (+ phase) that dominates precessional effects [Schmidt+14]

  1. Closed-form frequency domain formulas for precession of angular momentum

  2. Rotate nonprecessing PhenomD directly in frequency domain

- **IMRPhenomPv2**: comparisons to many NR runs during LIGO software review

PN+NR q=3, a1=0.75 in-plane



[Hannam+13]

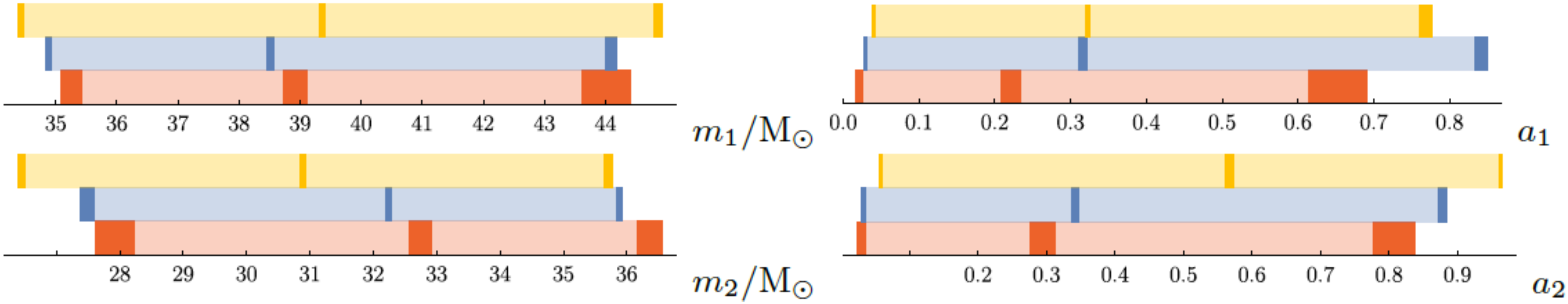
Differences between precessing IMR models

precessing Phenom	precessing EOBNR
Dof: S1z, S2z, chip, phase	Dof: S1x, S1y, S1z, S2x, S2y, S2z
Purely nonprecessing model in the precessing frame	Fully precessing conservative orbital dynamics
Ringdown built in the precessing frame	Ringdown built in final-spin frame
In the precessing frame only (2,2) mode included	In the precessing frame uncalibrated (2,1) mode included
SPA for modes rotation	Exact time-domain modes rotation
Euler angles for modes rotation derived in analytic form under approximations	Euler angles for modes rotation from motion of LN
Initial in-plane spin components enter final-spin formula	Spin-aligned formula for remnant spin evaluated at merger

Parameter estimation with precessing models

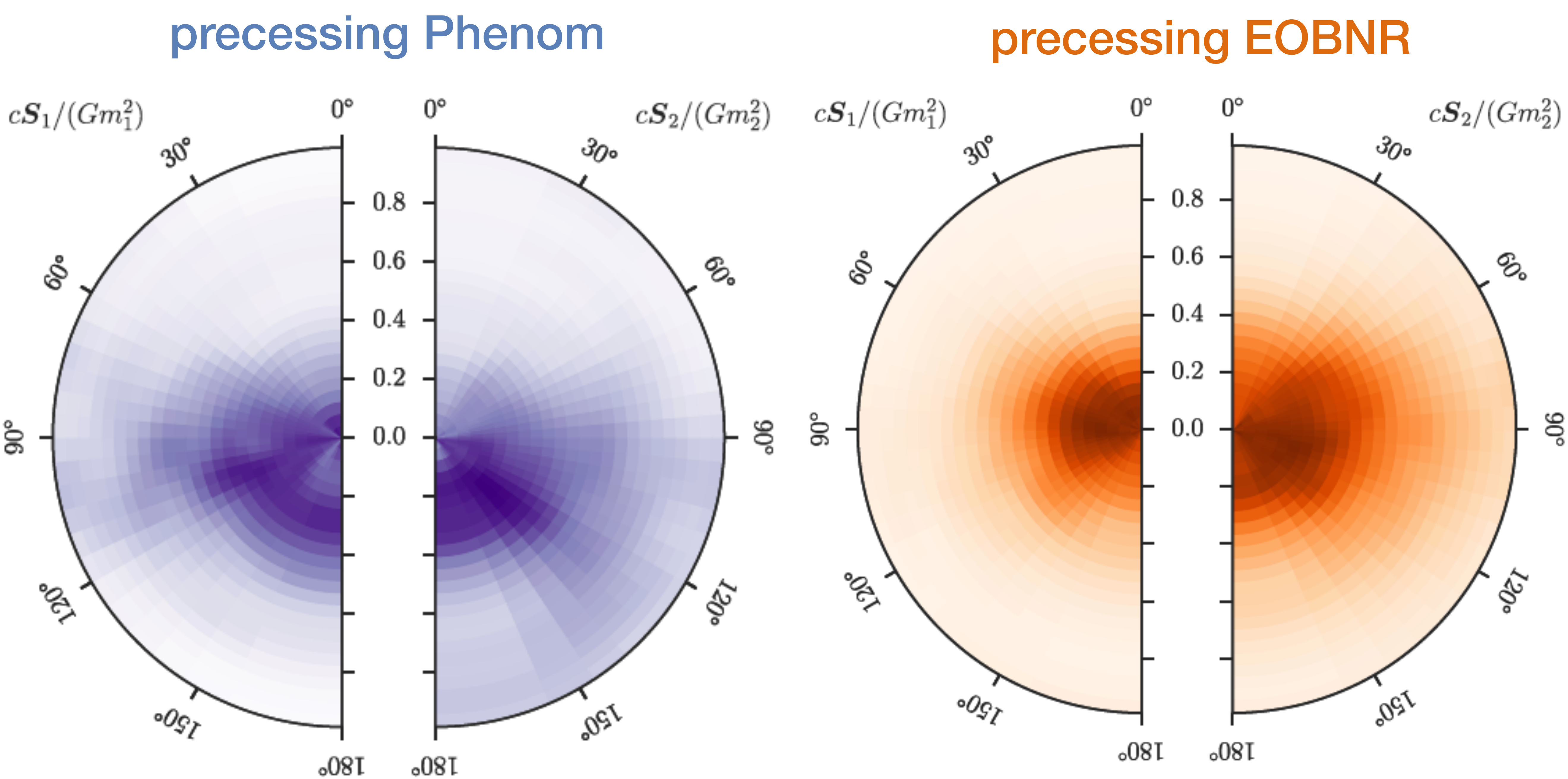
IMR precessing models vs GW150914

- Nonprecessing EOBNR, precessing EOBNR, and precessing Phenom measure consistent parameters for GW150914
  1. SNR
  2. comparable mass
  3. face off/on
  4. short signal



[LVC1606.01262]

IMR precessing models vs GW150914



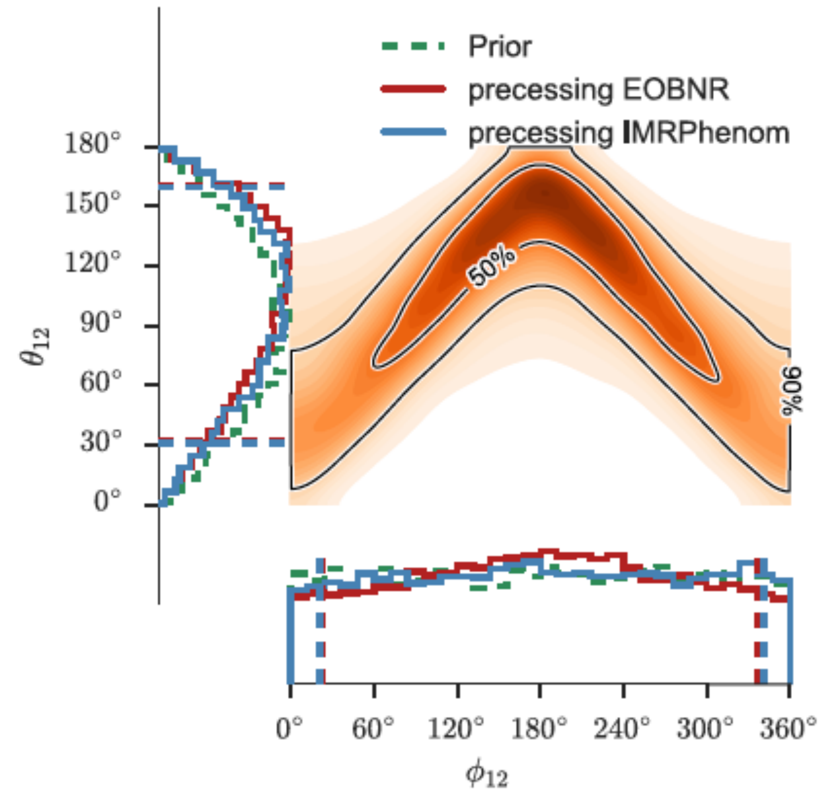
[LVC1602.03840]

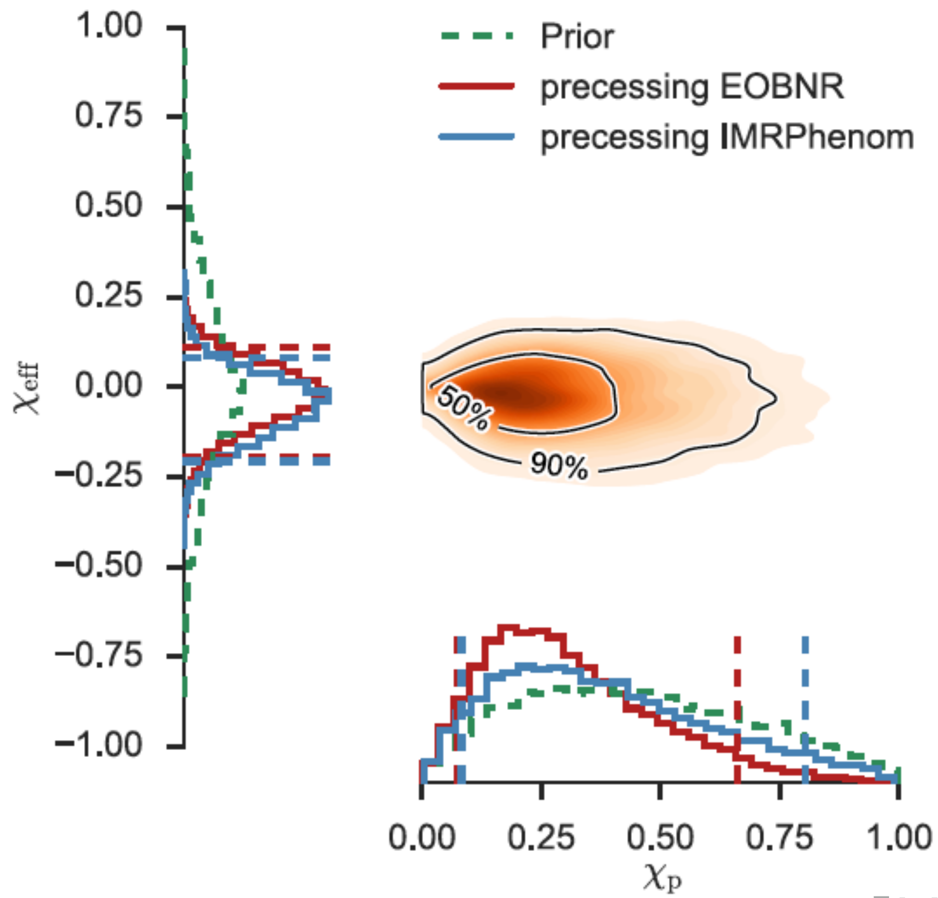
[LVC1606.01262]

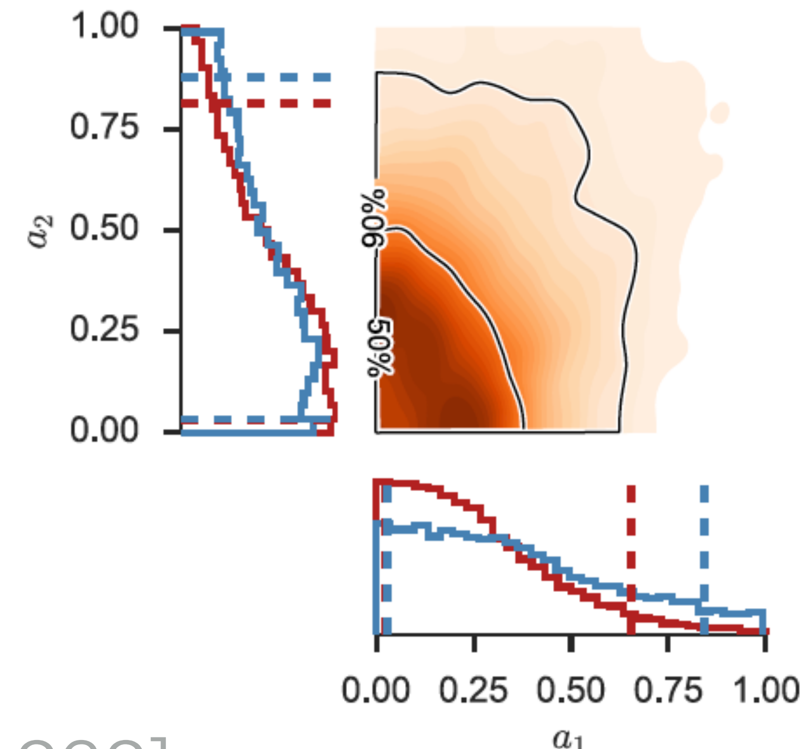
IMR precessing models vs GW150914

$$\chi_{\rm eff} = \frac{c}{G}\left(\frac{\boldsymbol{S}_1}{m_1} + \frac{\boldsymbol{S}_2}{m_2}\right)\cdot\frac{\hat{\boldsymbol{L}}_{\rm N}}{M},$$

$$\chi_{\rm p} = \frac{c}{B_1 G m_1^2}\max(B_1 S_{1\perp}, B_2 S_{2\perp}),$$



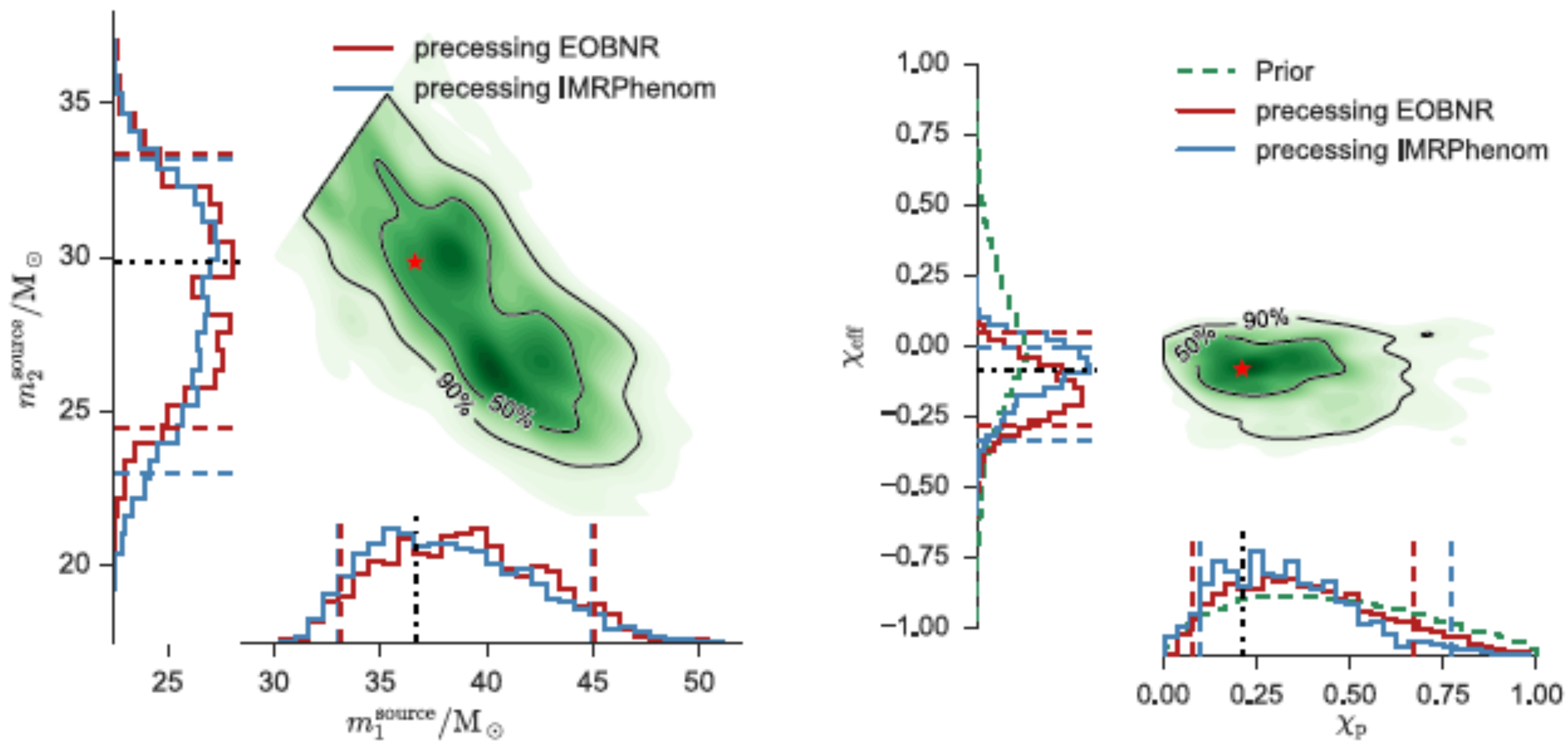




[LVC1606.01262]

IMR precessing models vs GW150914

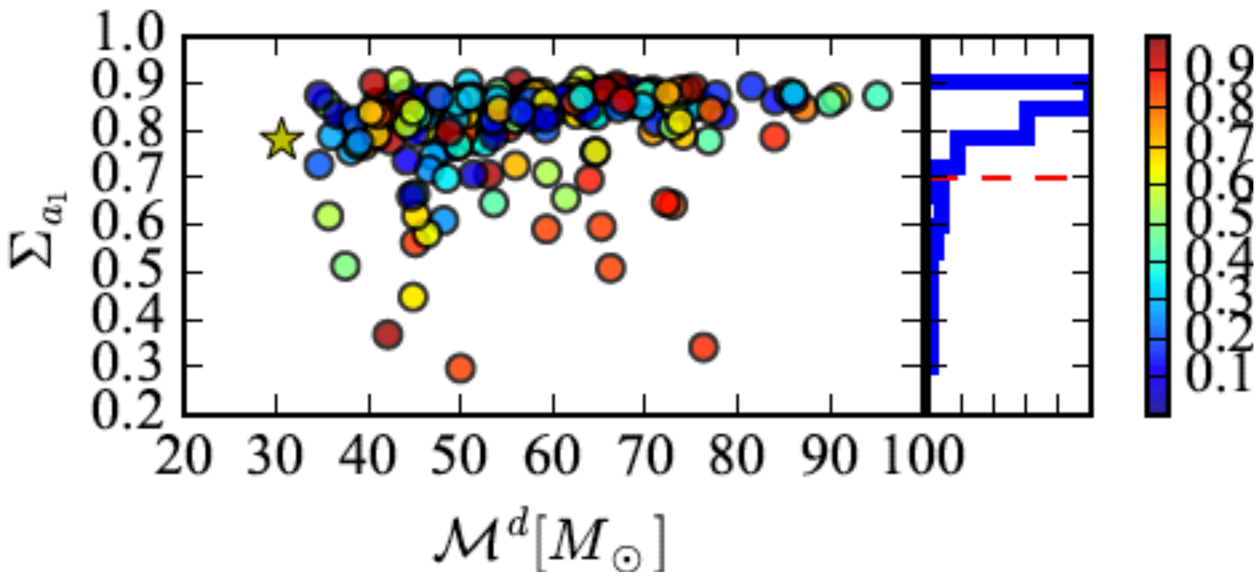
- Systematics study with NR injection that had parameters close to MaP

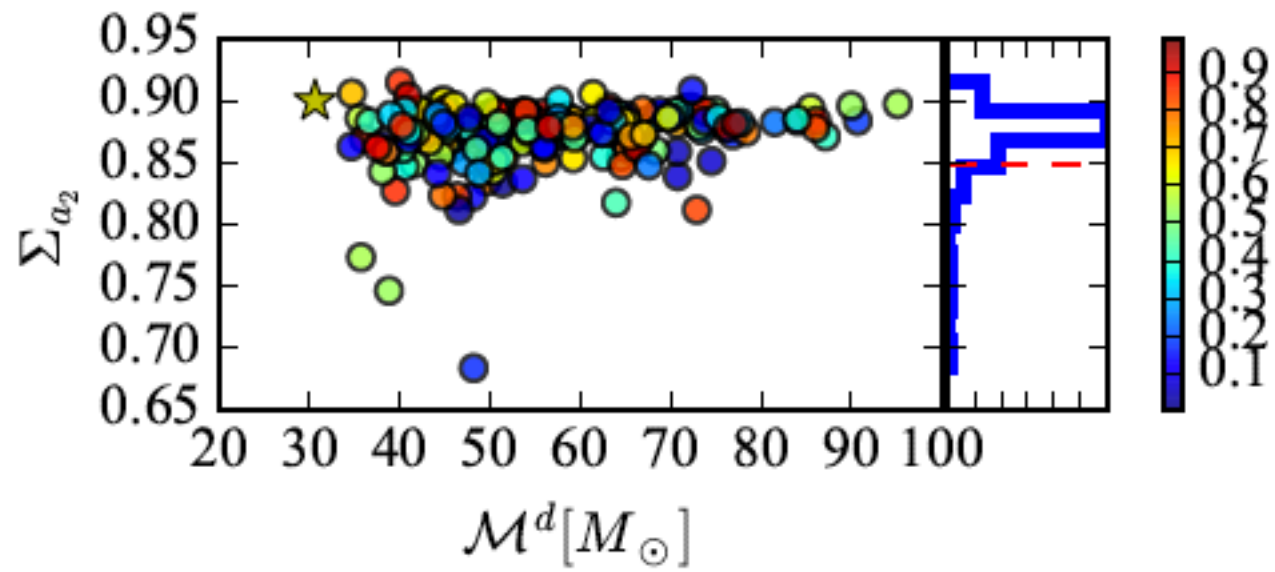


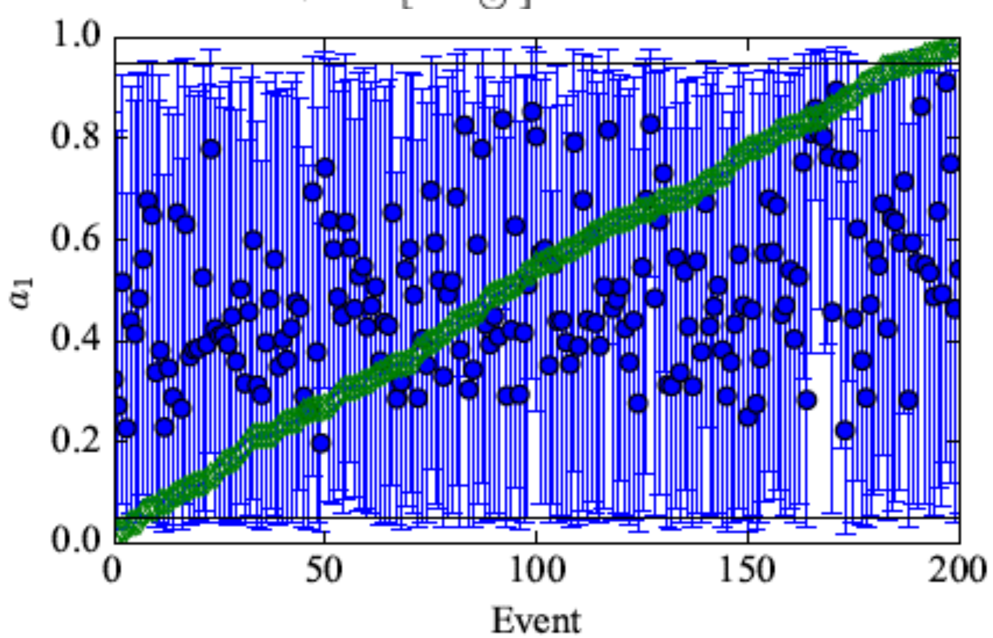
[LVC1606.01262]

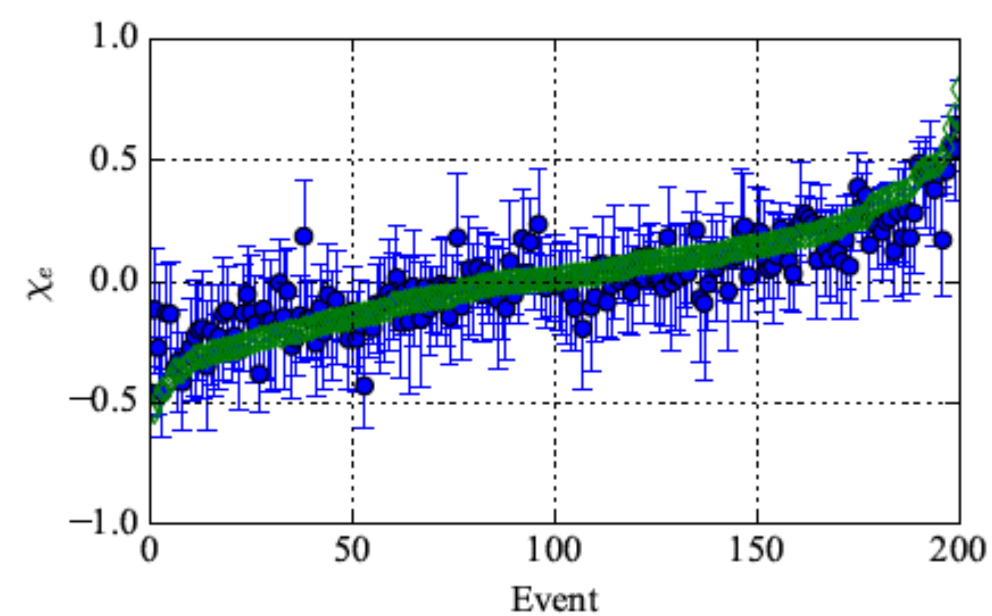
Expected uncertainties for heavy BBHs [Vitale+16]

- 200 precessing BBHs w/ m1,m2 uniform in [30,50]MSun, a1,a2 uniform in [0,0.98], isotropic sky location, uniform inclination, uniform in comoving volume, threshold network SNR=12
- Model: IMRPhenomPv2. Detectors: HLV at design sensitivity









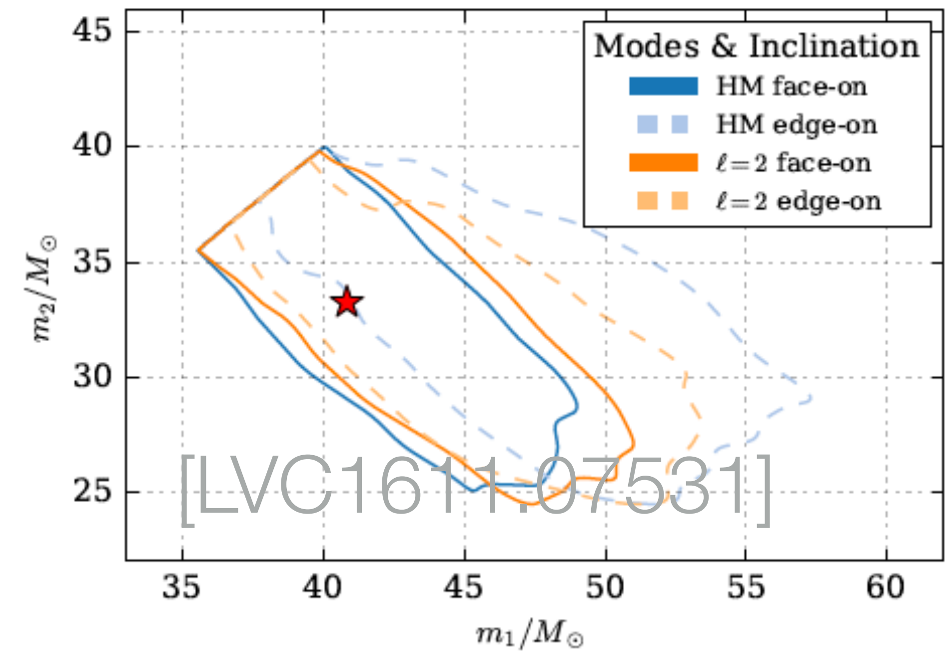
Expected uncertainties for heavy BBHs [Vitale+16]

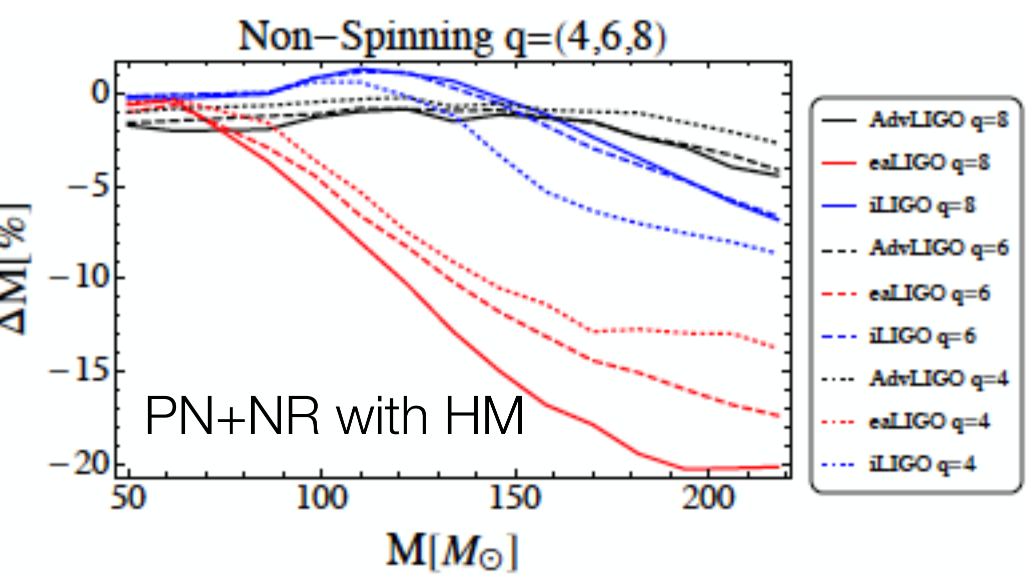
- a1<0.2: can rule out ~maximal a1 90% of the times
- a1>0.8: can rule out ~zero a1 75% of the times
- chieff better measured (90% C.I. of typical width ~0.35)
- Aligned-spins yield smaller uncertainties (90% C.I. of width ~0.2 on a1)
- For unequal-mass BBHs: the more edge-on, the easier the measurement of a1. For equal-mass BBHs: no dependence on inclination
- Tilts are poorly measured
- Uncertainties of GW150914 are typical of similar BBHs

Unmodeled effects

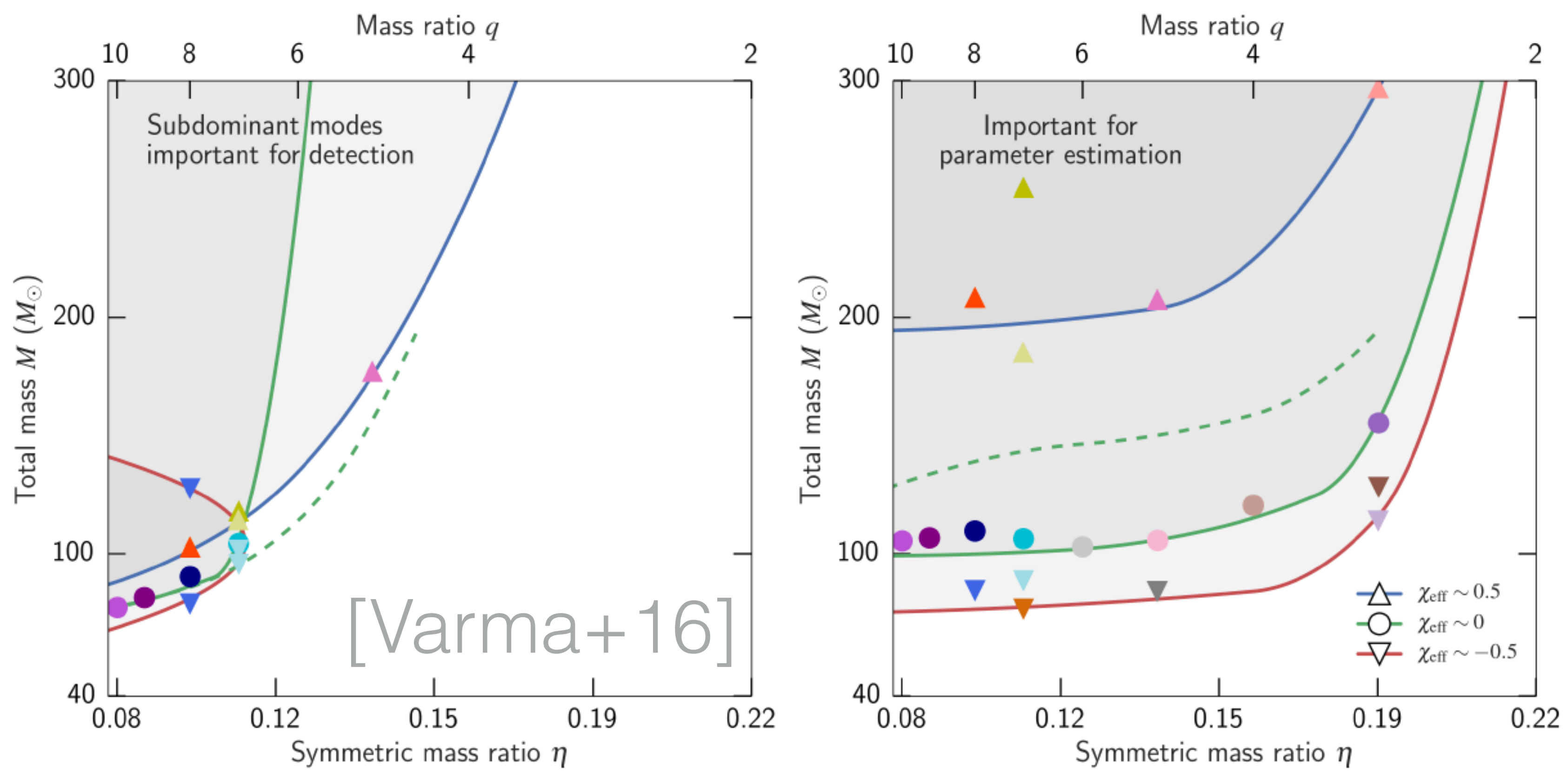
Higher-order modes

- **IMR higher-order modes** for spinning binaries are not available
- For no-spin searches, no impact for 3MSun ≤ m1, m2 ≤ 200MSun and M < 360MSun [Capano+13]
- Higher-modes systematics > statistical errors for q>4 and M>100Msun at SNR>8 (orientation avg) [Calderon-Bustillo+15,16, Varma+16]



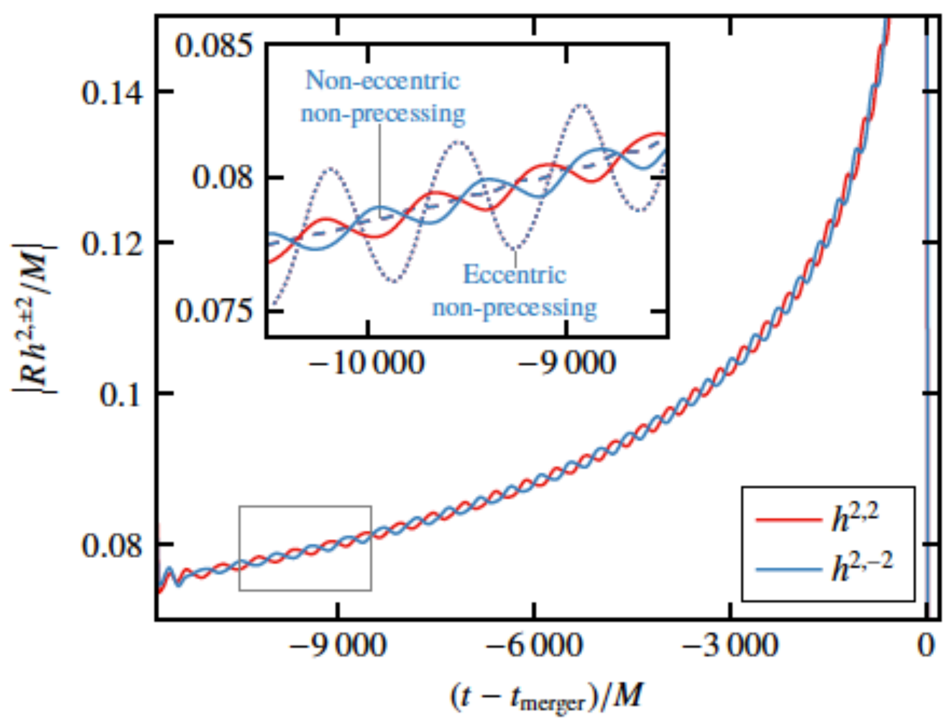


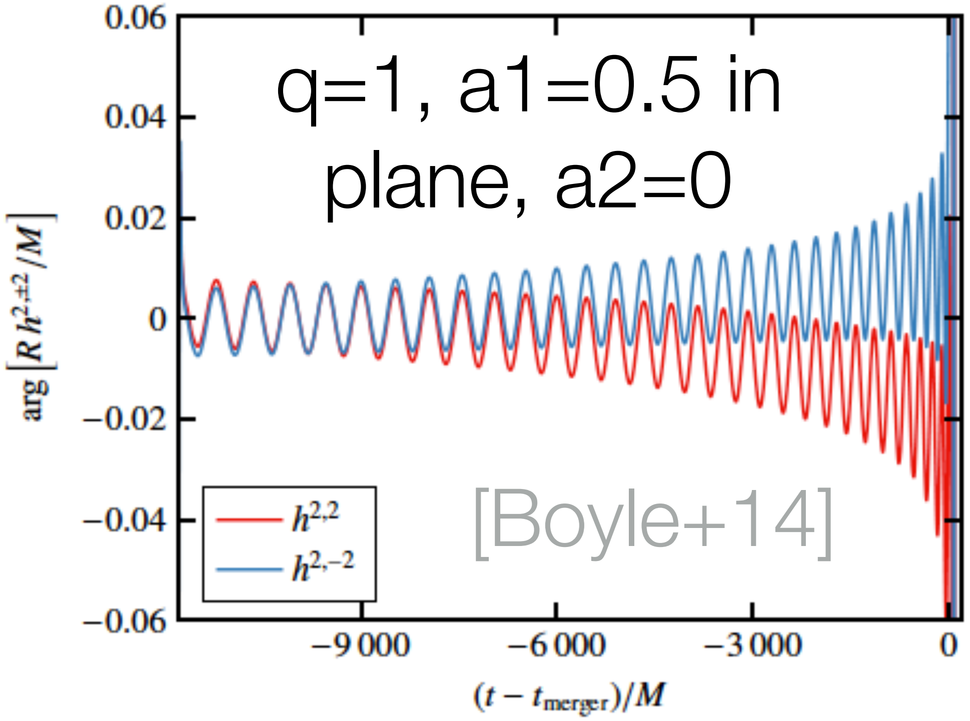
[Calderon-Bustillo+15,16]



Unmodeled precessional effects

- **Precessional effects** not fully modeled
  1. mode asymmetry in precessing frame [O'Shaughnessy+13, Pekowsky +14, Boyle+14]
  2. radiation axis keeps precessing during ringdown [O'Shaughnessy+13]
  3. no calibration to precessing NR



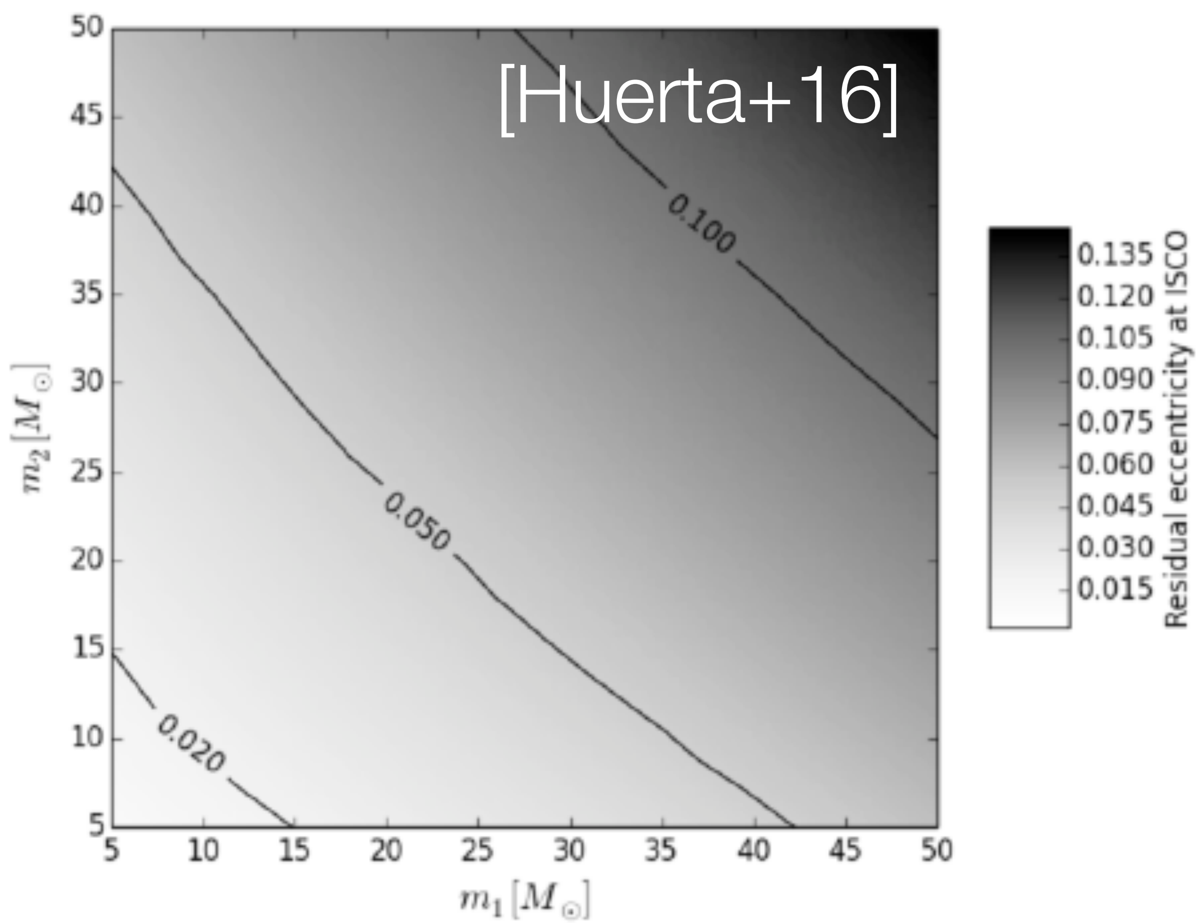


Eccentric models

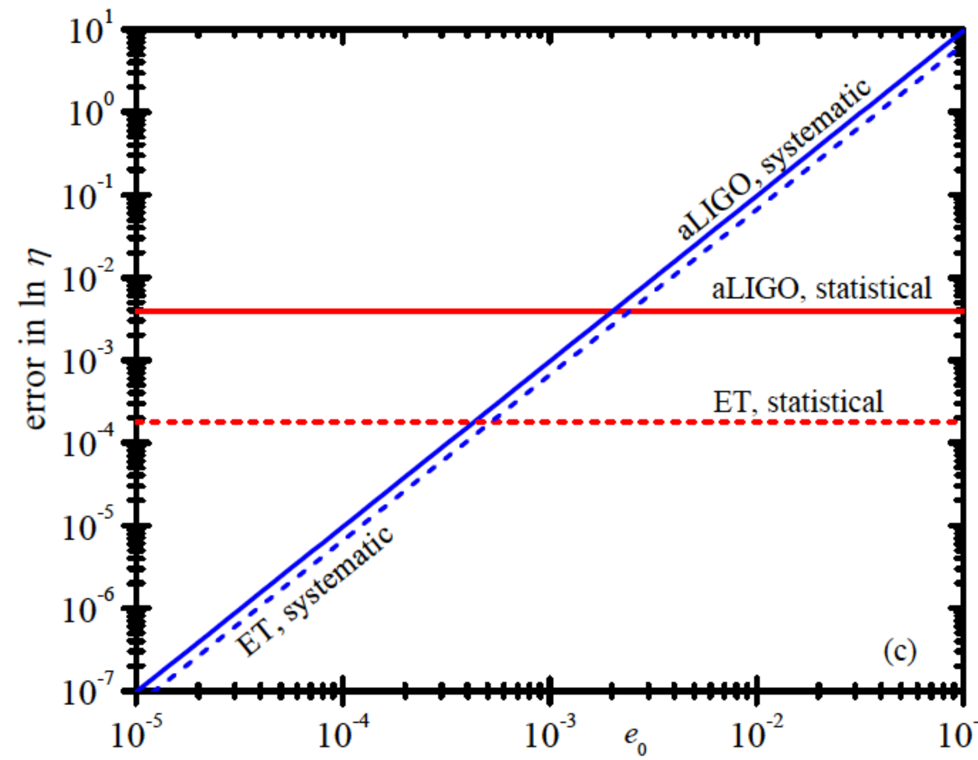
Eccentric binaries

- **Dynamical formation** scenarios
- Searches for BNS using quasicircular templates ok for e<=0.02 (M=2.6Msun) [Huerta+13]
- Small residual eccentricity can **bias** parameter estimation [Favata14]

(e=0.4 @ 15Hz)



(1.4+1.4) Msun @ 100Mpc

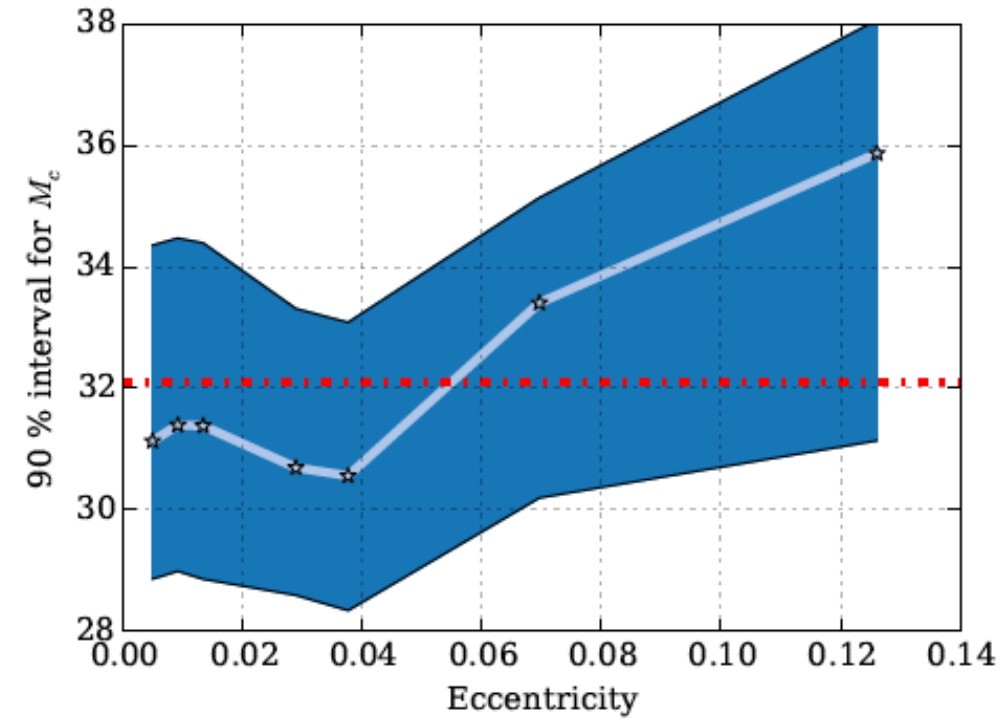


[Favata14]

Eccentric binaries

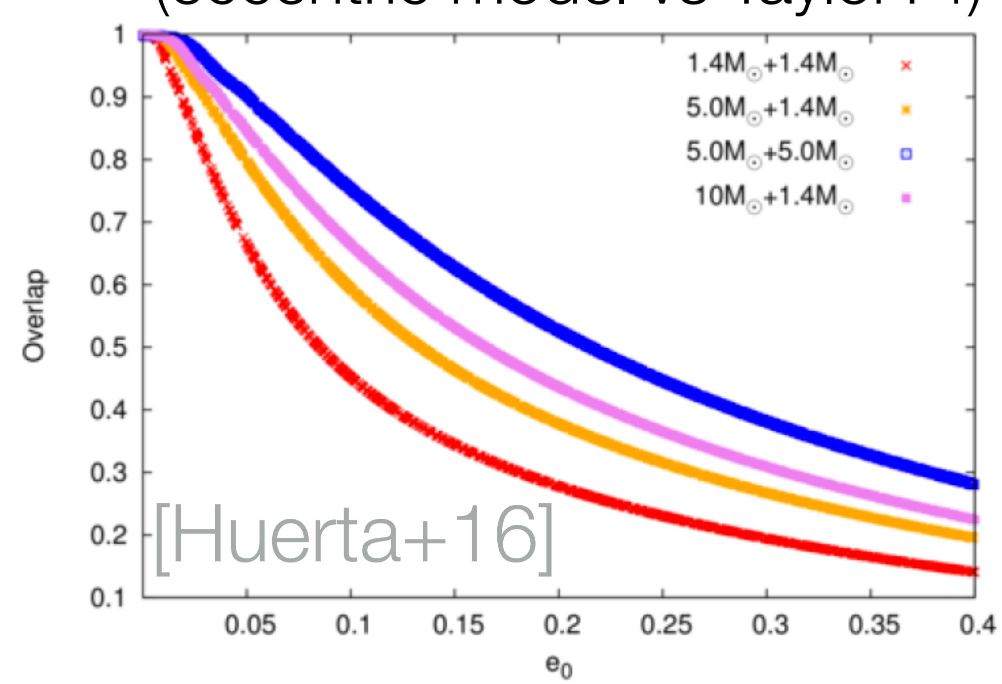
- Frequency/time-domain **PN inspiral** waveforms [Arun+09, Yunes+09, Huerta+14, Tanay+16]. Small-ecc corrections up to 3PN [Moore,Favata+16]
- **IMR** waveforms based on **geodesic** motion in Kerr [East+13]
- **IMR** waveforms based on **PN inspiral + self force + NR-informed ringdown** [Huerta+16]
- Ongoing work on eccentric IMR waveforms based on EOB/Phenom

q=1.2, a1=0.33, a2=-0.44



[LVC1611.07531]

(eccentric model vs TaylorT4)



Conclusions

Conclusions

- Where we stand
  1. wealth of new NR simulations (calibration, surrogates, direct use)
  2. very accurate (2,2)-mode spin-aligned models for q<=6
  3. reasonably good precessing models for moderate spins (<=0.5) and q<=4
  4. spin uncertainties of GW150914 seem typical for heavy BBHs

- Open problems
  1. (large q, large spins, “low” M) domain not constrained by NR
  2. systematics against precessing NR
  3. spinning IMR models with higher harmonics still under development
  4. how many NR cycles do we need to simulate to constrain models down to 10Hz