Gravitational waveforms for data analysis of spinning binary black holes

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[https://dcc.ligo.org/G1700243]
Synergy of numerical relativity and analytical relativity = waveform models crucial for
0. detecting GW151226 [LVC1606.04855]
1. establishing 5-sigma significance of detections [LVC1602.03839, LVC1606.04856]
2. measuring properties of the source [LVC1602.03840, LVC1606.01210, LVC1606.01262, LVC1606.04856]
3. performing tests of general relativity (GR) [LVC1602.03841, LVC1606.04856]
Numerical relativity
Numerical-relativity catalogs of BBHs

... and many more NR waveforms from many groups [SXS, GATech, RIT, Cardiff-UIB, NCSA] are being computed also in response to observations.
Sources of errors: (i) resolution, (ii) extrapolation, (iii) finite length, (iv) junk radiation

Aligned-spin template banks accept fitting factors 3% mismatch ~ 10% loss in event rate

Parameter estimation (sufficient) accuracy requirement \([\text{Lindblom}+08]\)

\[
1 - \mathcal{O}(h_1, h_2) < \frac{1}{2 \text{ SNR}^2}
\]
Challenging BBHs

- **Longterm** BBH simulations at mass ratio 7 [Szilagyi+14, Kumar+15]

- **Almost extremal** BBH simulations: equal-mass, aligned-spins 0.99, 0.994 [Scheel+14]

- New initial data for challenging configurations [Ossokine+15]
Direct use of numerical relativity

Besides guiding construction of models (waveforms, remnant properties), there are other avenues to use NR:

- **Direct comparison of existing NR catalogs to observations** [LVC1602.03843, LVC1606.01262] [Richard’s talk]

- **NR follow-ups to observations** [LVC detection papers, Lovelace+16]:
  1. comparisons to unmodeled reconstructions
  2. validate models

- **Surrogate waveform models** [Blackman+15,17]
  1. restricted parameter space (high mass, q<=2, spins<=0.8, one spin aligned)
  2. many NR simulations to construct basis
  3. interpolation across NR runs
  4. they do not extrapolate to low mass: need models or long NR
Nonprecessing models for LIGO
Effective-one-body models of nonprecessing BBHs

- Nonspinning case: particle in deformation of Schwarzschild [Buonanno & Damour99]. Spinning case: **spinning particle in deformation of Kerr** [Barausse & Buonanno10,11; Nagar+14]

- Inspiral waveforms/radiation reaction from **resummation post-Newtonian formulas** [Damour+07,09; Pan+11; Nagar+16]

- Ringdown from **superposition of quasinormal modes** of remnant BH

\[
H_{\text{real}} = Mc^2 \sqrt{1 + 2\nu \left( \frac{H_{\text{eff}}}{\mu c^2} - 1 \right)} - Mc^2
\]

\[
A(R) \left[ 1 + \frac{P^2}{\mu^2 c^2} + \frac{1}{\mu^2 c^2} \left( \frac{A(R)}{D(R)} - 1 \right) \left( \frac{R \cdot P}{R} \right)^2 \right]
\]

\[
A = \left( 1 - 2u \right) + 2\nu u^3 + \left( \frac{94}{3} - \frac{42}{32} \pi^2 \right) \nu u^4 + a_5 u^5 + \cdots \quad (u = GM/Rc^2)
\]

\[
\nu = \frac{m_1 m_2}{(m_1 + m_2)^2}
\]
Effective-one-body model of nonprecessing BBHs for O1

\[ \nu = \frac{m_1 m_2}{(m_1 + m_2)^2} \]
\[ \chi_{\text{eff}} = \left( \frac{S_1}{m_1} + \frac{S_2}{m_2} \right) \cdot \hat{L} \]
\[ \chi_A = \left( \frac{S_1}{m_1^2} + \frac{S_2}{m_2^2} \right) \cdot \hat{L} \]

- **SEOBNRv2** calibrated to better than 99% overlap with NR for design aLIGO [AT+14]

- Used in its **reduced-order-model** version [Pürrer14,15] in O1 for filtering and parameter estimation

- Similar set of calibration waveforms used in IHES models [Nagar +15,16]
Effective-one-body model of nonprecessing BBHs for O2

- **SEOBNRv4 [Bohe, Shao, AT+16]**

\[ \nu = \frac{m_1 m_2}{(m_1 + m_2)^2} \]
\[ \chi_{\text{eff}} = \left( \frac{S_1}{m_1} + \frac{S_2}{m_2} \right) \cdot \hat{L} \]
\[ \chi_A = \left( \frac{S_1}{m_1^2} + \frac{S_2}{m_2^2} \right) \cdot \hat{L} \]
Phenomenological model of nonprecessing BBHs

- **Fit to hybrids** of uncalibrated EOB and NR [Husa+15, Khan+15]
Extrapolation to low frequencies

are NR waveforms long enough to constrain down to 25Hz for moderate M?

are hybrids reliable?

[Bohe, Shao, AT+16]
Comparing nonprecessing IMR BBH models

\[ \mathcal{O}(h_1, h_2) \]

maximized over masses and spins (in template bank)

\[ \mathcal{O}(h_1, h_2) \]

[Bohe, Shao, AT+16]

(O1 aLIGO)
Comparing nonprecessing IMR BBH models

new, long numerical-relativity simulations are needed here

$maximized \text{ over masses and spins (in template bank)}$

$[\text{Bohe, Shao, AT+16}]$
Precessing models for LIGO
Precessing IMR BBH models

- When BH spins are not parallel to angular momentum of the binary, the orbital plane precesses

- **Precessing frame** [Buonanno+03, Schmidt+11, O’Shaughnessy+11, Boyle+11]
  1. In precessing frame, use calibrated nonprecessing model
  2. Inertial-frame modes from rotation of precessing-frame modes according to motion of orbital angular momentum

- Both effective-one-body [Pan+13, Babak, AT+16] and phenomenological [Hannam+13] models available

- Inspiral-only PN waveforms [Katerina’s talk]
Effective-one-body model for precessing BBHs

- 70 NR waveforms from SXS public catalog used to test model

[Babak, AT+16]
Effective-one-body model for precessing BBHs

$1 - \mathcal{O}(h_{\text{NR}}, h_{\text{EOB}})$ averaged over sky location and polarization

[Babak, AT+16]
Effective-one-body model for precessing BBHs

(SXS:BBH:0058)

q=5, a1=0.5, a2=0
S1 in-plane at t=0

(SXS:BBH:0058)

[Babak, AT+16]
Effective-one-body model for precessing BBHs

Testing the rotation via maximum-radiation direction

Testing the waveforms in the precessing frame

Motion of EOB angular momentum closely tracks NR direction of max radiation

(SXS:BBH:0058)

(2,2) good, (2,1) to improve, especially RD

(SXS:BBH:0058)
Effective-one-body model for precessing BBHs

- New SXS NR waveforms [Ossokine+ (in prep)] used to
  1. improve model [AEI (in prep)]
  2. assess PE systematics [AEI (in prep)]
Phenomenological model of precessing BBHs

- Start from PN and find **single effective spin** (+ phase) that dominates precessional effects [Schmidt+14]

1. Closed-form frequency domain formulas for precession of angular momentum

2. Rotate nonprecessing PhenomD directly in frequency domain

- **IMRPhenomPv2**: comparisons to many NR runs during LIGO software review

![PN+NR q=3, a1=0.75 in-plane graph]
## Differences between precessing IMR models

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<th>precessing Phenom</th>
<th>precessing EOBNR</th>
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<tr>
<td>Dof: S1z, S2z, chip, phase</td>
<td>Dof: S1x, S1y, S1z, S2x, S2y, S2z</td>
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<td>Euler angles for modes rotation derived in analytic form under approximations</td>
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<tr>
<td>Initial in-plane spin components enter final-spin formula</td>
<td>Spin-aligned formula for remnant spin evaluated at merger</td>
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</tbody>
</table>
Parameter estimation with precessing models
Nonprecessing EOBNR, precessing EOBNR, and precessing Phenom measure consistent parameters for GW150914
1. SNR
2. comparable mass
3. face off/on
4. short signal

[Image of bar charts and histograms]

[LVC1606.01262]
IMR precessing models vs GW150914

**precessing Phenom**

- $cS_1/(Gm_1^2)$
- $cS_2/(Gm_2^2)

**precessing EOBNR**

- $cS_1/(Gm_1^2)$
- $cS_2/(Gm_2^2)$

[**LVC1602.03840**] [**LVC1606.01262**]
\[ \chi_{\text{eff}} = \frac{c}{G} \left( \frac{S_1}{m_1} + \frac{S_2}{m_2} \right) \cdot \hat{L}_N, \]

\[ \chi_p = \frac{c}{B_1 G m_1^2} \max(B_1 S_{1\perp}, B_2 S_{2\perp}). \]
Systematics study with NR injection that had parameters close to MaP

[LVC1606.01262]
Expected uncertainties for heavy BBHs [Vitale+16]

- 200 precessing BBHs w/ m1,m2 uniform in [30,50] MSun, a1,a2 uniform in [0,0.98], isotropic sky location, uniform inclination, uniform in comoving volume, threshold network SNR=12

- Model: IMRPhenomPv2. Detectors: HLV at design sensitivity
Expected uncertainties for heavy BBHs [Vitale+16]

- $a_1 < 0.2$: can rule out $\sim$ maximal $a_1$ 90% of the times
- $a_1 > 0.8$: can rule out $\sim$ zero $a_1$ 75% of the times
- $\chi_{\text{eff}}$ better measured (90% C.I. of typical width $\sim 0.35$)
- Aligned-spins yield smaller uncertainties (90% C.I. of width $\sim 0.2$ on $a_1$)
- For unequal-mass BBHs: the more edge-on, the easier the measurement of $a_1$. For equal-mass BBHs: no dependence on inclination
- Tilts are poorly measured
- Uncertainties of GW150914 are typical of similar BBHs
Unmodeled effects
Higher-order modes

- **IMR higher-order modes** for spinning binaries are not available

- For no-spin searches, no impact for $3\text{MSun} \leq m_1, m_2 \leq 200\text{MSun}$ and $M < 360\text{MSun}$ [Capano+13]

- Higher-modes systematics $>$ statistical errors for $q>4$ and $M>100\text{Msun}$ at $\text{SNR}>8$ (orientation avg) [Calderon-Bustillo+15,16, Varma+16]
Unmodeled precessional effects

- **Precessional effects** not fully modeled
  1. mode asymmetry in precessing frame [O’Shaughnessy+13, Pekowsky +14, Boyle+14]
  2. radiation axis keeps precessing during ringdown [O’Shaughnessy+13]
  3. no calibration to precessing NR

![Graph showing precessional effects](image)
Eccentric models
Eccentric binaries

- **Dynamical formation** scenarios

- Searches for BNS using quasicircular templates ok for $e < 0.02$ (M=2.6Msun) [Huerta+13]

- Small residual eccentricity can **bias** parameter estimation [Favata14]

$e=0.4$ @ 15Hz

(1.4+1.4) Msun @ 100Mpc
Eccentric binaries

- Frequency/time-domain **PN inspiral** waveforms [Arun+09, Yunes+09, Huerta+14, Tanay+16]. Small-ecc corrections up to 3PN [Moore,Favata+16]

- IMR waveforms based on **geodesic** motion in Kerr [East+13]

- IMR waveforms based on **PN inspiral + self force + NR-informed ringdown** [Huerta+16]

- Ongoing work on eccentric IMR waveforms based on EOB/Phenom

\[ q=1.2, a_1=0.33, a_2=-0.44 \]

[QVC1611.07531](https://link-to-paper.com)

(eccentric model vs TaylorT4)

[QVC1611.07531](https://link-to-paper.com)
Conclusions
Conclusions

- Where we stand
  1. wealth of new NR simulations (calibration, surrogates, direct use)
  2. very accurate (2,2)-mode spin-aligned models for q<=6
  3. reasonably good precessing models for moderate spins (<=0.5) and q<=4
  4. spin uncertainties of GW150914 seem typical for heavy BBHs

- Open problems
  1. (large q, large spins, “low” M) domain not constrained by NR
  2. systematics against precessing NR
  3. spinning IMR models with higher harmonics still under development
  4. how many NR cycles do we need to simulate to constrain models down to 10Hz