

Probing Cosmology and measuring the peculiar acceleration of binary black holes with LISA

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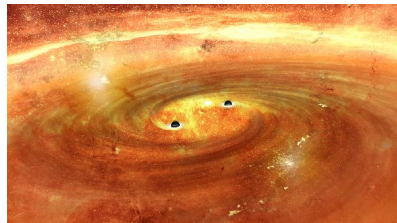
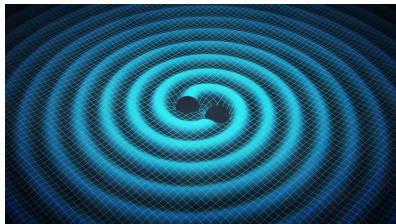


Probing cosmology with LISA

Based on:

- Tamanini, Caprini, Barausse, Sesana, Klein, Petiteau, arXiv:1601.07112
- Caprini & Tamanini, arXiv:1607.08755

- ▶ How many **standard sirens** will be detected by LISA?



- ▶ What type of sources can be used?
- ▶ For how many it will be possible to observe a counterpart?

Possible standard sirens sources for LISA:

- ▶ MBHBs ($10^4 - 10^7 M_{\odot}$)
- ▶ LIGO-like BHBs ($10 - 100 M_{\odot}$)
- ▶ EMRIs

Standard sirens for LISA

Possible standard sirens sources for LISA:

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- ▶ LIGO-like BHBs ($10 - 100 M_{\odot}$)
- ▶ EMRIs

Advantages of MBHB mergers:

- ▶ High SNR
- ▶ High redshifts (up to $\sim 10-15$)
- ▶ Merger within LISA band \rightarrow
- ▶ Gas rich environment \rightarrow *EM counterparts!*

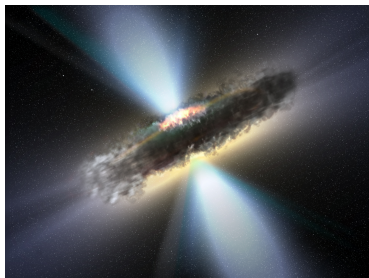
To obtain cosmological forecasts, we have adopted the following **realistic strategy**:

[NT, Caprini, Barausse, Sesana, Klein, Petiteau, arXiv:1601.07112]

- ▶ Start from simulating MBHBs merger events using **3 different astrophysical models** [arXiv:1511.05581]
 - ▶ Light seeds formation (popIII)
 - ▶ Heavy seeds formation (with delay)
 - ▶ Heavy seeds formation (without delay)
- ▶ Compute for how many of these a GW signal will be **detected by LISA** (SNR>8)
- ▶ Among these select the ones with a **good sky location accuracy** ($\Delta\Omega < 10 \text{ deg}^2$)
- ▶ Focus on **5 years** LISA mission (the longer the better for cosmology)

LISA cosmological forecasts: data simulation approach

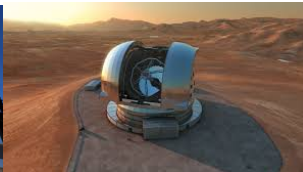
- ▶ **To model the counterpart** we generally consider two mechanisms of EM emission at merger:
(based on [\[arXiv:1005.1067\]](#))
 - ▶ A quasar-like luminosity **flare** (optical)
 - ▶ Magnetic field induced **flare** and **jet** (radio)
- ▶ Magnitude of EM emission computed using data from simulations of MBHBs and galactic evolution



LISA cosmological forecasts: data simulation approach

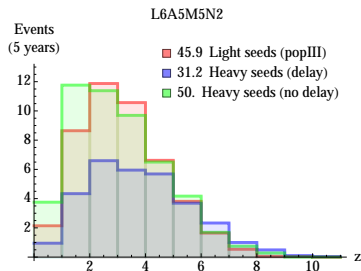
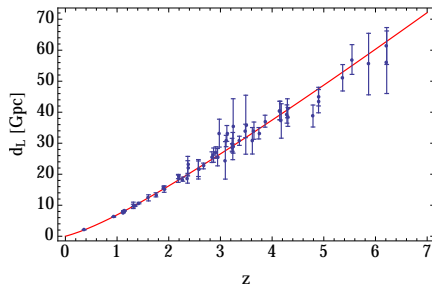
Finally **to detect the EM counterpart** of an LISA event sufficiently localized in the sky we use the following two methods:

- ▶ **LSST**: direct detection of optical counterpart
- ▶ **SKA + E-ELT**: first use SKA to detect a radio emission from the BHs and pinpoint the hosting galaxy in the sky, then aim E-ELT in that direction to measure the redshift from a possible optical counterpart either
 - ▶ Spectroscopically or Photometrically



LISA cosmological forecasts: MBHB standard sirens rate

Example of simulated catalogue of MBHB standard sirens:



Note 1: LISA will be able to map the expansion at very high redshifts (data up to $z \sim 8$), while SNIa can only reach $z \sim 1.5$
Note 2: Few MBHBs at low redshift \Rightarrow bad for DE (but one can use SNIa and other GW sources)

RESULTS: [NT et al, arXiv:1601.07112]

1 σ constraints with 5 million km armlength:

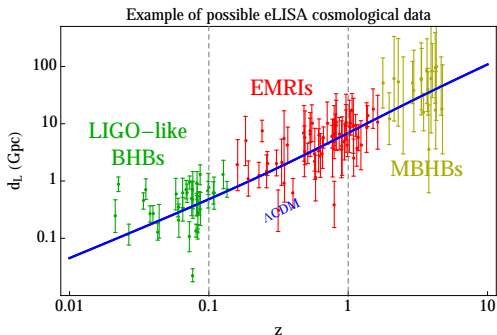
$$\begin{aligned} \Lambda\text{CDM:} & \begin{cases} \Delta\Omega_M & \simeq 0.025 & (8\%) \\ \Delta h & \simeq 0.013 & (2\%) \end{cases} \\ \Lambda\text{CDM} + \text{curvature:} & \begin{cases} \Delta\Omega_M & \simeq 0.054 & (18\%) \\ \Delta\Omega_\Lambda & \simeq 0.15 & (21\%) \\ \Delta h & \simeq 0.033 & (5\%) \end{cases} \\ \text{Dynamical DE:} & \begin{cases} \Delta w_0 & \simeq 0.16 \\ \Delta w_a & \simeq 0.83 \end{cases} \end{aligned}$$

- ▶ Similar results with 1 or 2 million km armlength

LISA cosmological forecasts: future prospects

Future work:

- ▶ Exploit other LISA GW sources for cosmology (lower z)
(this will improve the results from MBHBs only)
 - ▶ Stellar mass BH binaries ($z < 0.1$) \rightarrow
 - ▶ EMRIs ($0.1 < z < 1$) \rightarrow *no counterparts expected!*



High redshift data useful
to test alternative
cosmological models

[Caprini & NT, arXiv:1607.08755]

- ▶ Cosmology at all redshift ranges with LISA!

Measuring the peculiar acceleration of BBHs with LISA

Based on:

- [Bonvin, Caprini, Sturani, Tamanini, arXiv:1609.08093](#)
- [Inayoshi, Tamanini, Caprini, Haiman, arXiv:1702.06529](#)

- ▶ The expansion of the universe and the cosmic matter inhomogeneities affect the propagation of GWs
- ▶ We identify 3 redshift-dependent effects on the chirp signal:
 - ▶ *time variation of the background expansion of the universe*
 - ▶ *time variation of the gravitational potential at the GW source*
 - ▶ *time variation of the peculiar velocity of the GW source*
- ▶ These effects cause a phase drift during the in-spiral:
 - ▶ Not relevant for Earth-based detectors
 - ▶ Relevant for non-monochromatic LISA sources with many in-spiral cycles in band: *low chirp mass and $\tau_c \sim \Delta t_{\text{obs}}$*
- ▶ The phase drift due to the peculiar acceleration dominates:
 - ▶ Can be used to discriminate between different BBH formation channels

Waveform for an unperturbed universe with constant z

Assuming a FRW metric, the waveform at the observer is

$$h_+(t_O) = \frac{4(G\mathcal{M}_c(z))^{\frac{5}{3}}}{d_L} (\pi f_O)^{\frac{2}{3}} \frac{1 + \cos^2 \iota}{2} \cos(\Phi_O)$$

time at the observer luminosity distance redshifted chirp mass frequency at the observer phase at the observer

Where the redshift z is assumed to be constant during the time of observation of the signal:

$$1 + z = \frac{a_O}{a_S} \quad f_O(\tau_O) = \frac{1}{\pi} \left(\frac{5}{256\tau_O} \right)^{3/8} (G\mathcal{M}_c)^{-5/8}$$

$$f_S = (1+z)f_O$$

$$\tau_O = (1+z)\tau_S$$

$$\Phi_O(\tau_O) = -2 \left(\frac{\tau_O}{5G\mathcal{M}_c} \right)^{5/8} + \Phi_i$$

Considering a varying redshift

Relax the assumption that the redshift is constant during the observational time of the GW signal

Two main effects:

- ▶ the **background expansion** of the universe varies during the time of observation of the binary
[Seto *et al* (2001), Takahashi & Nakamura (2005), Nishizawa *et al* (2012)]
- ▶ the **redshift perturbations** due to the distribution of matter between the GW source and the observer vary in time during the time of observation of the binary
[Bonvin, Caprini, Sturani, NT, arXiv:1609.08093]

Considering a varying redshift: perturbed universe

Computing the redshift perturbations:

Consider scalar perturbations on FRW:

$$ds^2 = -(1 + 2\psi) dt^2 + a^2(1 - 2\phi)\delta_{ij} dx^i dx^j$$

definition of the redshift $1 + z = \frac{f_S}{f_O} = \frac{E_S}{E_O} = \frac{(k^\mu u_\mu)_S}{(k^\mu u_\mu)_O}$

$$\frac{dk^\mu}{d\lambda} + \Gamma_{\alpha\beta}^\mu k^\alpha k^\beta = 0$$

$$u^\mu = \frac{1}{a}(1 - \psi, \mathbf{v})$$

GW wave-vector

four velocity at source and observer

Considering a varying redshift: perturbed universe

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definition of the redshift

$$1 + z = \frac{f_S}{f_O} = \frac{E_S}{E_O} = \frac{(k^\mu u_\mu)_S}{(k^\mu u_\mu)_O}$$

$$1 + z = \frac{a_O}{a_S} \left[1 + \underbrace{(\mathbf{v}_S - \mathbf{v}_O) \cdot \mathbf{n}}_{\text{Doppler term}} + \underbrace{\psi_O - \psi_S}_{\text{gravitational redshift}} - \int_{t_S}^{t_O} dt (\underbrace{\dot{\phi} + \dot{\psi}}_{\text{integrated Sachs Wolfe}}) \right]$$

↓ ↓ ↓ ↓

background expansion Doppler term gravitational redshift integrated Sachs Wolfe

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$$1 + z = \frac{a_O}{a_S} \left[1 + (\mathbf{v}_S - \mathbf{v}_O) \cdot \mathbf{n} + \psi_O - \psi_S - \int_{t_S}^{t_O} dt (\dot{\phi} + \dot{\psi}) \right]$$
$$= (1 + \bar{z})(1 + \delta z)$$

Considering a varying redshift: perturbed equations

These effects introduce additional contributions in the frequency and the phase of the chirp signal with new time dependences

$$f(\tau_O) = \frac{1}{\pi} \left(\frac{5}{256 \tau_O} \right)^{3/8} (G\mathcal{M}_c)^{-5/8} \left(1 + \frac{3}{8} Y(z) \tau_O \right)$$

$$\Phi_O(\tau_O) = -2 \left(\frac{\tau_O}{5G\mathcal{M}_c} \right)^{5/8} \left(1 - \frac{5}{8} Y(z) \tau_O \right) + \Phi_i$$

$$Y(z) = \frac{1}{2} \left(H_0 - \frac{H_S}{1 + \bar{z}} \right) + \frac{1}{2} \left[\frac{\dot{\mathbf{v}}_S \cdot \mathbf{n}}{1 + \bar{z}} - \dot{\mathbf{v}}_O \cdot \mathbf{n} + \frac{\dot{\phi}_S}{1 + \bar{z}} - \dot{\phi}_O \right]$$

variation of the
cosmological
expansion during
observation time

acceleration of the
binary and the
observer during
observation time

time variation of
the potentials
during
observation time

Considering a varying redshift: perturbed equations

$$A(f) = \sqrt{\frac{5}{24\pi^{4/3}}} \frac{(GM_c)^{5/6}}{d_L(z)} \frac{1}{f^{7/6}} \left[1 - \frac{5(GM_c)^{-5/3}}{384\pi^{8/3}} \frac{Y(z)}{f^{8/3}} \right]$$

$$\Phi(f) = 2\pi ft_c - \frac{\pi}{4} - \Phi_c + \frac{3}{128} (\pi GM_c)^{-5/3} \frac{1}{f^{5/3}} - \frac{25}{32768\pi} (\pi GM_c)^{-10/3} \frac{Y(z)}{f^{13/3}}$$

Effective $-4PN$ frequency dependence:

(but comparable to max $\sim 2PN$ once its prefactor is taken into account)

- ▶ Frequency dependent shift during the in-spiral phase
- ▶ Need observation of many cycles to be relevant
- ▶ No application to Earth-based detectors (only few cycles)
- ▶ Relevant for slowly evolving LISA sources ($\sim 10^6$ cycles)

Estimate of the amplitude of $Y(z)$

$$Y(z) = \frac{1}{2} \left(H_0 - \frac{H_S}{1 + \bar{z}} \right) + \frac{1}{2} \left[\frac{\dot{\mathbf{v}}_S \cdot \mathbf{n}}{1 + \bar{z}} - \dot{\mathbf{v}}_O \cdot \mathbf{n} + \frac{\dot{\phi}_S}{1 + \bar{z}} - \dot{\phi}_O \right]$$

variation of the
cosmological
expansion:
depends only on
the cosmology

peculiar acceleration of
the binary:
dominates over the
effect due to
cosmological expansion
for realistic situations

subdominant
 $\dot{\phi} \sim H_0 \phi \sim 10^{-5} H_0$

accounted for by eLISA motion

Estimate of the amplitude of $Y(z)$

$$Y(z) = \frac{1}{2} \left(H_0 - \frac{H_S}{1 + \bar{z}} \right) + \frac{1}{2} \left[\frac{\dot{\mathbf{v}}_S \cdot \mathbf{n}}{1 + \bar{z}} - \cancel{\dot{\mathbf{v}}_O \cdot \mathbf{n}} + \frac{\dot{\phi}_S}{1 + \bar{z}} - \cancel{\dot{\phi}_O} \right]$$

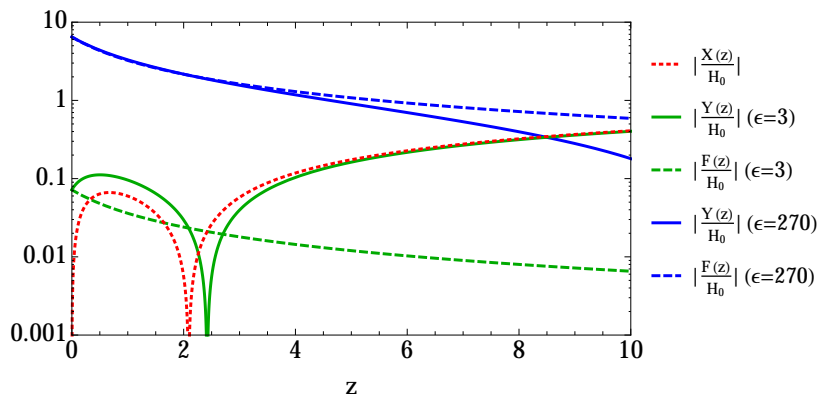
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$$2.4 \times 10^{-2} \epsilon \frac{H_0}{1 + \bar{z}}$$

$$\epsilon \equiv 10 \left(\frac{v_s}{100 \text{ kms}} \right)^2 \left(\frac{r}{1 \text{ kpc}} \right)^{-1} (\hat{\mathbf{e}} \cdot \hat{\mathbf{n}})$$

- ▶ v_s is the CoM velocity of the binary
- ▶ r is the distance from the galactic center

Estimate of the amplitude of $Y(z)$



If ϵ is not negligible, then the contribution of peculiar accelerations dominates over the ones due to expansion of the universe, especially at low redshifts

Question: What kind of peculiar accelerations of BBHs can we detect? What values of ϵ ?

To address this question we performed a Fisher matrix analysis

$$F_{ij} = \left\langle \frac{\partial h}{\partial \theta_i} \middle| \frac{\partial h}{\partial \theta_j} \right\rangle = 2 \int_{f_{\min}}^{f_{\max}} \frac{df}{S_n(f)} \left(\frac{\partial h}{\partial \theta_i} \frac{\partial h^*}{\partial \theta_j} + \text{c.c.} \right)$$

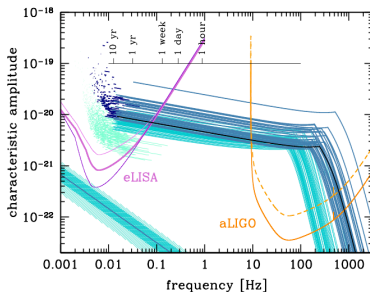
where $h(f)$ is the (sky-averaged and spin-less) 3.5PN waveform in Fourier space including the peculiar acceleration effect, which depends on the 6 parameters (for high accelerations $Y \propto \epsilon$)

$$\theta_i = (\mathcal{M}_c, \Phi_c, t_c, \eta, d_L, Y)$$

Implications for GW detection

We made two parallel error estimations:

- ▶ With LISA alone: ΔY is the 1σ error marginalized over all other waveform parameters
- ▶ With LISA + LIGO where the **time of coalescence t_c** is **fixed** by an Earth-based detection and ΔY is marginalized only over the remaining parameters



[Sesana, arXiv:1602.06951]

Implications for GW detection

Finally in order to check the validity of the Fisher matrix analysis for high values of ϵ , we also computed the SNR of the perturbed waveform:

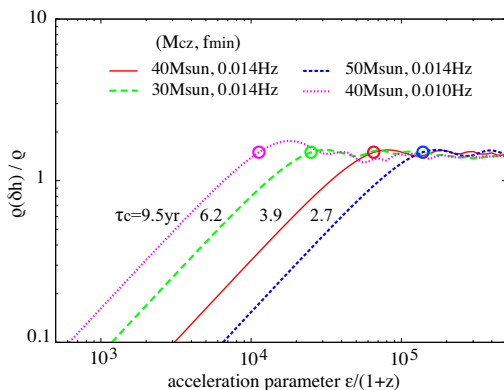
$$\rho(\delta h) = (\langle \delta h | \delta h \rangle)^{1/2} = \left[\int_{f_{\min}}^{f_{\max}} df \frac{|\delta h(f)|^2}{S_n(f)} \right]^{1/2}$$

where

$$\delta h = h(f, 0) - h(f, Y) = h(f) \left[1 - e^{i\delta\Psi_{\text{acc}}} \right]$$

Note that as long as $\delta\Psi_{\text{acc}} \ll 1$ we have $\rho(\delta h)/\rho(h) \propto \epsilon$, while when the phase drift approaches a full cycle (i.e. when $\delta\Psi_{\text{acc}} \sim 1$), $\rho(\delta h)/\rho(h)$ saturates at a constant value

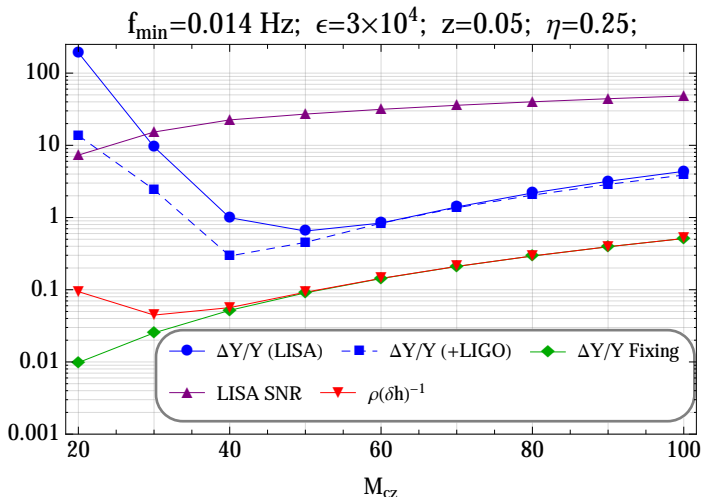
Implications for GW detection



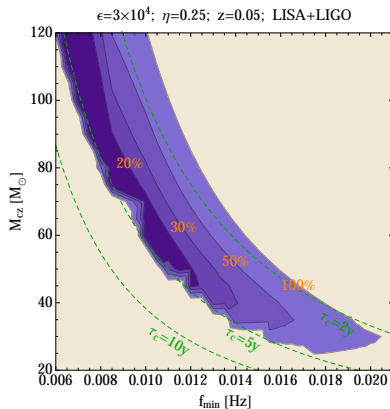
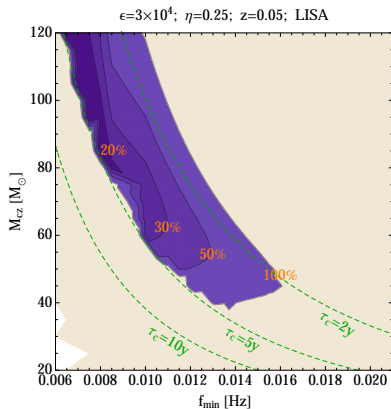
The Fisher matrix analysis is valid only in the regime $\delta\Psi_{acc} \ll 1$ since it relies on linear derivatives of $h(f)$ around the parameters' fiducial values

In this regime $\rho(\delta h)^{-1} = \Delta Y/Y$ with all other parameters fixed

Implications for GW detection

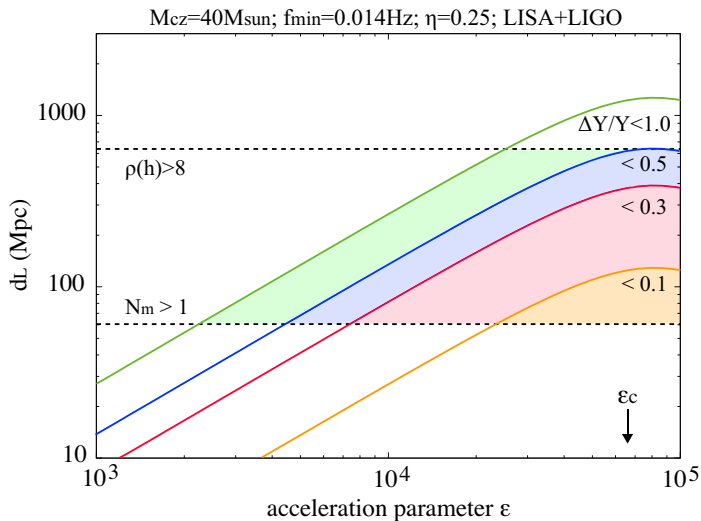


Implications for GW detection



[Inayoshi, NT, Caprini, Haiman, arXiv:1702.06529]

Implications for GW detection



Implications for BBH formation models

Result: the phase drift can be measured for BBHs with high enough peculiar acceleration ($\epsilon \gtrsim 10^4$)

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Question: what values of the peculiar acceleration (i.e. of ϵ) are expected for stellar mass BBHs? Are there any BBH formation channels leading to high peculiar accelerations?

Implications for BBH formation models

Result: the phase drift can be measured for BBHs with high enough peculiar acceleration ($\epsilon \gtrsim 10^4$)

Question: what values of the peculiar acceleration (i.e. of ϵ) are expected for stellar mass BBHs? Are there any BBH formation channels leading to high peculiar accelerations?

Answer: Yes! The following BBH formation models predict that BBHs orbit very close to the galactic center \Rightarrow high acceleration!

- ▶ Dynamical formation in dense stellar systems: mass segregation through dynamical friction pushes BBHs towards nuclear star clusters or galactic nuclei
- ▶ BBH formation in AGN disks: BBH could form either from massive stellar binaries in the AGN disk itself or at migration traps located closer in; moreover pre-existing binaries in the 3D bulge can also be captured in the inner regions of the disk

Implications for BBH formation models

scenario	v (kms)	r (pc)	ϵ	d_L
<i>Field binaries (A)</i>				
formed at $z \simeq 0$	~ 200	$> 5 \times 10^3$	< 10	
formed at $z \simeq 3$	~ 300	$10^3 - 10^4$	$10 - 100$	
<i>Dense stellar systems (B)</i>				
globular clusters	~ 200	$\sim 5 \times 10^4$	~ 1	
nuclear star clusters	$30 - 100$	~ 1	$10^3 - 10^4$	
<i>AGN disks (C)</i>				
formed in disk	~ 200	~ 1	$\sim 10^4$	
captured or migrated in	~ 600	~ 0.1	$\sim 10^5$	
<i>Very high-redshift (D)</i>				
Population III	~ 200	$\lesssim 10^3$	$10 - 100$	

Table from [\[Inayoshi, NT, Caprini, Haiman, arXiv:1702.06529\]](#)

Conclusion: *the phase drift in the GW waveform produced by the peculiar acceleration of BBHs can be used as a robust discriminator between different BBH formation channels by LISA (+LIGO)*

Interestingly the hosting galaxy of BBHs associated with AGN disks can uniquely identified within the sky localization error box of LISA, exactly because AGNs are rare objects in the universe
⇒ *the redshift of BBHs for which the connection with AGNs is verified by the peculiar acceleration effect, can be determined uniquely*

Probing cosmology with LISA

- ▶ MBHBs will be excellent **standard sirens** for LISA
- ▶ Direct probe of the cosmic expansion at **very high redshift**
- ▶ Constraint on H_0 **down to 1%**
- ▶ Useful to constrain **alternative models** at high redshift

Measuring BBH peculiar accelerations with LISA

- ▶ The GW signal is affected by the evolution of the **redshift perturbations** during the observational time
- ▶ This produces a phase drift which is dominated by the **peculiar acceleration** contribution
- ▶ The effect is relevant for **low mass LISA sources** ($30M_{\odot} \lesssim \mathcal{M}_c \lesssim 100M_{\odot}$) with $\tau_c \sim \Delta t_{\text{obs}}$
- ▶ It can be used to **discriminate between different BBH formation channels**