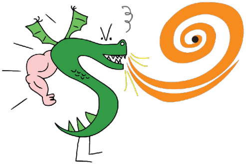
Bumpy black hole parameterizations

Leo C. Stein (TAPIR, Caltech)
Based on everybody else's work

StronG BaD — Mar. 2, 2017



The TAKE AWAY™

- Motivations
  - Clean tests in strong-field $\Longrightarrow$ black holes
  - Kerr hypothesis
  - Utility to parameterizations
- Observations of interest
  - Motion of test bodies and photons
  - Accretion disks
  - EMRI and ringdown tests
  - Caveats
- A pile of parameterizations
  - Review
  - Shortcomings
- Systematic study?

Motivations

Why test GR?

$$G_{ab} = 8\pi \hat{T}_{ab}$$

General relativity successful but incomplete

- Can't have mix of quantum/classical
- GR not renormalizable
- GR+QM=new physics (e.g. BH information paradox)

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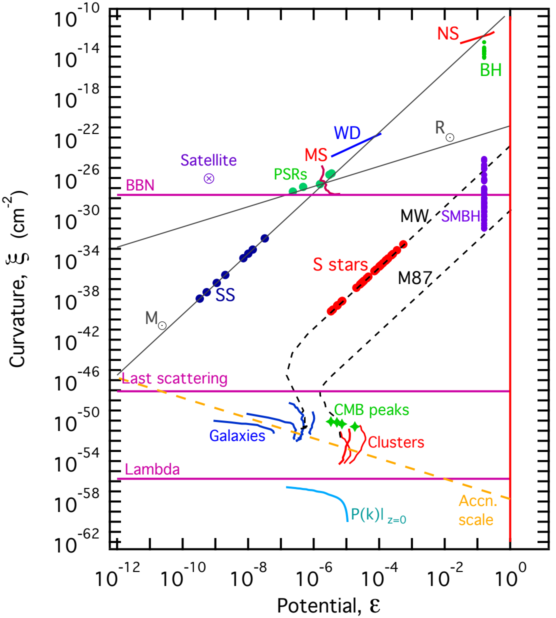
Empiricism

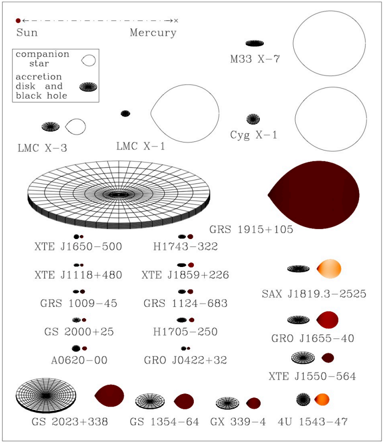
Ultimate test of theory: ask nature

[Baker, Psaltis, Skordis (2015)]



What is a black hole?

Observationally:



Very compact object

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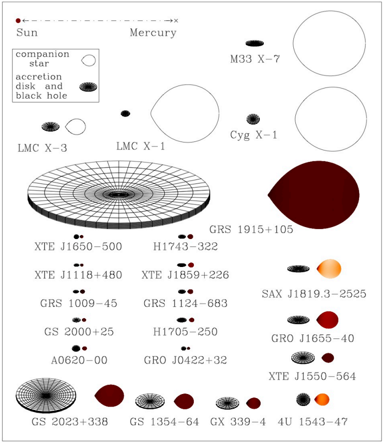

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Mathematically:

Crack open Wald,

$$B \equiv [M - J^{-}(\mathscr{I}^{+})]$$

Region that's causally disconnected from "exterior"

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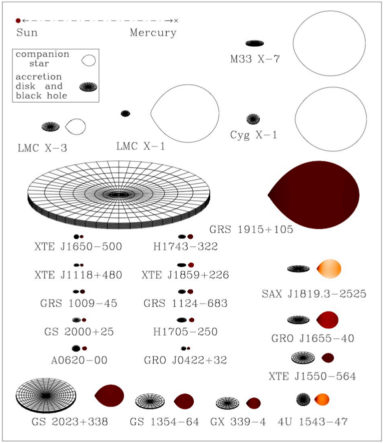

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Region that's causally disconnected from "exterior"

GR theorems say:

Subject to stationary, axisymmetric, asymptotically flat, Ricci-flatness:

Kerr is unique endpoint

$$M_\ell + iS_\ell = M(ia)^\ell$$

The Kerr hypothesis

Question: Are astronomical candidates actually Kerr black holes?

The Kerr hypothesis

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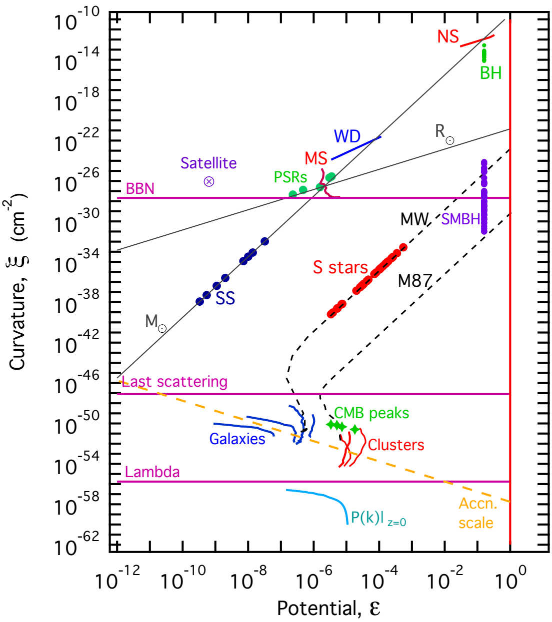
Answer: No.

Our universe is not stationary, axisymmetric, Ricci-flat, asymptotically flat . . .

Question: Are astronomical candidates *approximately* Kerr black holes? How do we test the Kerr hypothesis?

Classification of tests

[Baker, Psaltis, Skordis (2015)]



Kinematics vs. Dynamics

Kinematics: study solutions (geometry), ignore equations

Dynamics: which equations are being satisfied?

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Dynamics: which equations are being satisfied?

Caveat: Kerr a solution to many theories

Theory-specific vs. theory-independent

Theory-specific

- Pro: Easy to interpret. Bayesian model comparison (Monday)
  For models $\{GR, BGR_i\}$, can compute

$$\frac{p(\vec{d}|GR)}{p(\vec{d}|BGR_i)}$$

- Con: Lots of work for each theory

Theory-independent [e.g. PPN, PPK, PPF, PPE]

- Pro: Mapping $\implies$ reuse calculations
- Con: Interpretation unclear. Is parameterization complete?

Observations of interest

Observations of interest

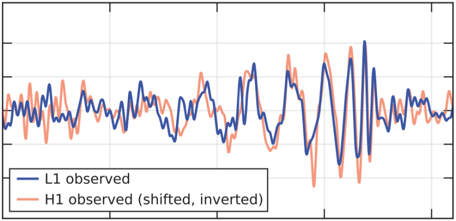
Ideal system to observe:

- quasi-stationary
- axisymmetric
- isolated
- vacuum

$\Longrightarrow$ no observational signature.

Need *weakly-coupled* probe of geometry

Binary mergers in LIGO?



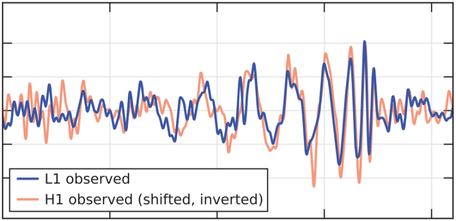
Highly dynamical, not test of Kerr

(ask me later about BBH mergers beyond GR)

Observations of interest

Stellar-mass black holes

Binary mergers in LIGO?



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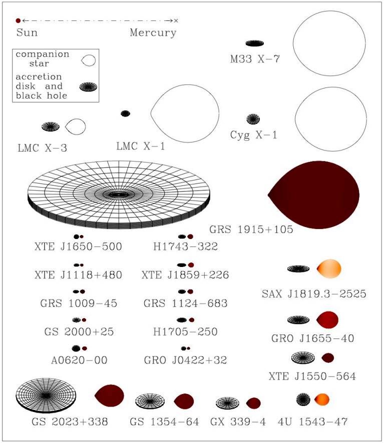
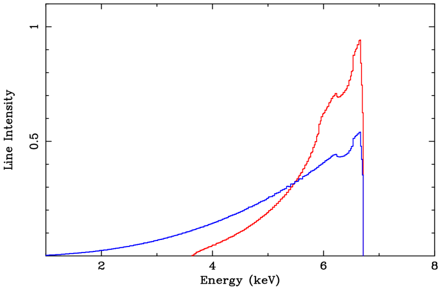
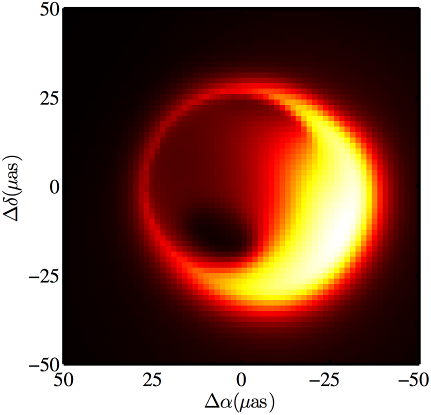
EM signatures from accretion disks



Observables: Continuum, line spectrum, radio interferometry, QPOs. . .
[Broderick, Johannsen, Narayan, Reynolds, McClintock, Steiner . . .]



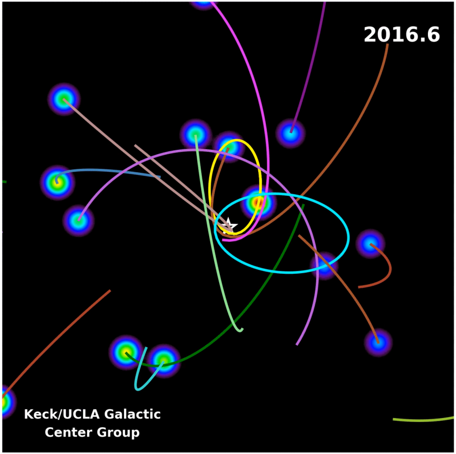


Caveats: unknown accretion physics

Observations of interest

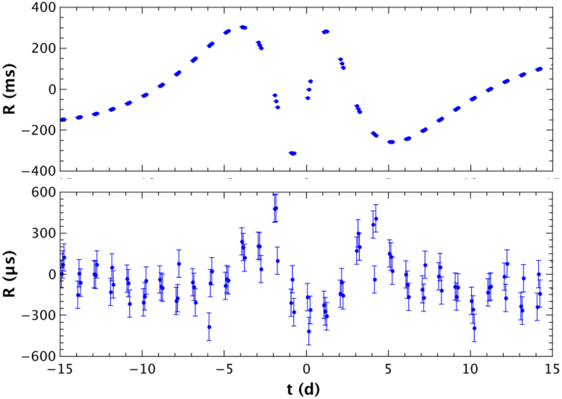
Supermassive black holes

Observable: precession of orbits of stars



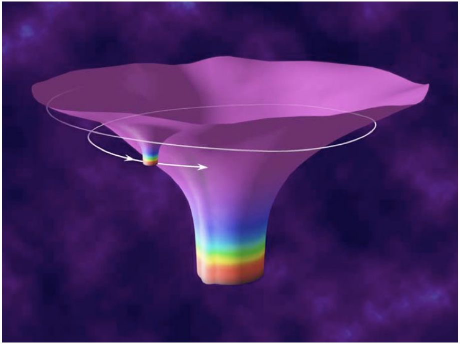
Caveats: strong equivalence principle, external perturbations?

Observable: pulsar timing [e.g. Psaltis, Wex, Kramer (2016)]



Caveats: strong equivalence principle

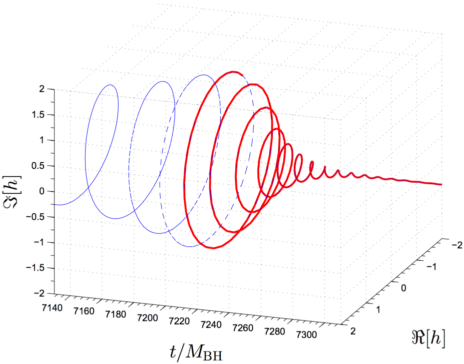
Observable: Extreme mass-ratio inspiral waveforms with LISA [Berti, Cardoso, Gair, Kesden, Pani, Yunes, . . .]



Caveats:

- strong equivalence principle
- how gravitational waves are sourced
- how gravitational waves propagate

Observable: Ringdown gravitational waveform [Berti and friends]



Caveats: what is graviton propagator? (discussion yesterday, Sathya's talk)

A pile of parameterizations

Weyl-Lewis-Papapetrou

Stationary, axisymmetric metric:

$$ds^2 = -V(dt - w d\phi)^2 + V^{-1}\rho^2 d\phi^2 + \Omega^2(d\rho^2 + \Lambda dz^2)\,. \qquad (1)$$

Specialize to Ricci-flat:

$$ds^2 = -V(dt - w d\phi)^2 + V^{-1}[\rho^2 d\phi^2 + e^{2\gamma}(d\rho^2 + dz^2)]\,, \qquad (2)$$

where $\gamma = \frac{1}{2}\ln(V\Omega^2)$.

Caveats: (1) requires $t - \phi$ reflection isometry, or integrability of $\rho - z$ planes

Con: Uncountably infinite # of degrees of freedom?

Geroch-Hansen, Manko-Novikov, Ryan, Backdahl

What an awful slide

Manko-Novikov is Kerr but with different Geroch-Hansen M_ℓ

Reminder: What are M_ℓ, S_ℓ? Ricci-flat, asymptotically flat spacetime in terms of Hertz potential; analytic function given in terms of Taylor series at $r \to \infty$

Backdahl later determined conditions for convergence, how to reconstruct given arbitrary moments.

Pro: countably infinite # of DoF
Cons:

- Backdahl very complicated
- Ricci flatness
- motivated by large-r expansion – strong field?
- analyticity [counter-example: Yukawa $\exp(-mr)/r$]

Collins and Hughes, Vigeland and Hughes

Linearize metric about Schwarzschild,

$$(V, w, \gamma) = (V_{\mathsf{Schw}}, w_{\mathsf{Schw}}, \gamma_{\mathsf{Schw}}) + \epsilon(V^{(1)}, w^{(1)}, \gamma^{(1)}) + \mathcal{O}(\epsilon^2)\,.$$

Satisfy Einstein's equations through $\mathcal{O}(\epsilon)$.

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Satisfy Einstein's equations through $\mathcal{O}(\epsilon)$.

Linearized version of free Geroch-Hansen moments (Backdahl).

Vigeland and Hughes

Apply Newman-Janis trick to “rotate” near-Schwarzschild geometries into spinning geometries.

Pro: Simpler than Backdahl, spinning geometries

Con: N-J trick is ad-hoc

Glampedakis and Babak

1. Take Hartle-Thorne metric for (M, J, Q) (ignore higher order)
2. Let $Q = -J^2/M - \varepsilon M^3$
3. Cut out the $\mathcal{O}(\varepsilon)$ part of metric
4. Paste it onto the Kerr metric

Pro: Simple analytic form, only 1 extra DoF

Con: Ad-hoc, only 1 extra DoF, clearly can't capture all geometries

Johannsen and Psaltis

$$ds^2 = -f(r)[1 + h(r)]dt^2 + f(r)^{-1}[1 + h(r)]dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\,,$$

where $f(r) = 1 - 2M/r$, and

$$h(r) = \sum_{k=0}^{\infty} \epsilon_k \left(\frac{M}{r}\right)^k \,.$$

Then apply Newman-Janis trick

Pro: Simple analytic form, countably infinite # DoF

Cons:

- Ad-hoc, N-J trick
- h motivated by large-r expansion – strong field?
- Cardoso, Pani, Rico showed shortcomings

Cardoso, Pani, Rico

- Allow two power series h^t, h^r with parameters $\epsilon_i^t, \epsilon_i^r$
- Showed that all ϵ equally important in strong-field
- Argument against $1/r^n$ power series tied to weak-field
- Also showed that known beyond-GR solutions (e.g. dCS) do not fit into JP or CPR metrics

Pro: Simple analytic form, $2\times$ countably infinite # DoF

Cons:

- Ad-hoc, N-J trick
- large-r expansion unreliable

Rezzolla, Zhidenko, Konoplya

- Too long/messy to include here
- Use infinite continued fractions in metric functions to try to control convergence near strong-field and far-field

Pro: countably infinite # DoF

Con: complicated, hard to tell what is space of metrics

Vigeland, Yunes, Stein; Johannsen

- Linear perturbation $\mathcal{O}(\varepsilon)$ about Kerr
- restrict metrics to those having an $\mathcal{O}(\varepsilon)$ Killing tensor
- Later simplified by Johannsen (but still too long to show)

Pro: geodesic motion remains integrable, four functional DoF

Con: why should geodesic motion be integrable?

Systematic study?

Simplifying principles

- Weyl-Lewis-Papapetrou is nonlinear function space
- Evidence suggests BH candidates are *close* to Kerr
- Linearize to tangent function space about Kerr

$$g_{ab} = g^K_{ab} + \epsilon h_{ab} + \mathcal{O}(\epsilon^2)$$

Simplifying principles

- Evidence suggests gravity is *close* to general relativity
- Focus on theories that are perturbative deformations of GR

$$I = I_{E-H} + \epsilon I_{BGR} + \mathcal{O}(\epsilon^2)$$

- Start by studying stationary, axisymmetric form of equation

$$G^{(1)}_{ab}[h_{cd}] = S_{ab}$$

- S_{ab} can be DM cloud, axion hair, exotic matter ...
  Or beyond-GR correction to gravity

Study

Operator splits the function space into Ker[$G^{(1)}$] and Spec[$G^{(1)}$]

$$G^{(1)}[k] = 0$$

Kernel contains:

- Only Ricci-flat bits
- Pure gauge — fix
- Shifts of mass, angular momentum
- Shifts in orientation, center of mass
- Linearized Geroch-Hanson bumps (countably infinite)
- Anything else?

Study

Operator splits the function space into Ker[$G^{(1)}$] and Spec[$G^{(1)}$]

$$G^{(1)}[E_I] = \lambda_I E_I$$

Spectrum contains:

- All non-Ricci-flat bits

Unknowns:

- Is the spectrum discrete, continuous, or both? [Ex: Hydrogen atom]
- Is there a complete basis of eigenfunctions?
- Is there an inner product and are eigenfuncs orthogonal?

N.B. Do not confuse spectrum with QNMs!

Eigenfunction approach?

$$G^{(1)}[h] = S$$

- Suppose we know eigenfunctions $\{E_I\}$ and they are complete
- Given a source S, resolve into sum*

$$S = \sum_I S_I E_I$$

where S_I's are just numbers

- Solution is now

$$h = \sum_I \lambda_I^{-1} S_I E_I + \text{hom.}$$

where homogeneous solutions determined by boundary conditions

Best possible mathematical setting

Most you could ask for:

- Positive-definite symmetric inner product $(\,,\,)$ on Hilbert space
- $G^{(1)}$ is self-adjoint with respect to $(\,,\,)$
- Source must be divergence-free
- Source must be orthogonal to every element of Ker$[G^{(1)}]$

Might also be possible with anti-symmetric product

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Backup slides

Only 10 numbers in parametrized post-Newtonian

PPN formalism for metric theories of gravity

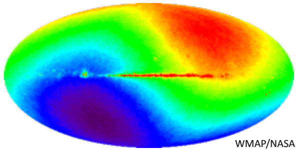
Metric:

$$g_{00} = -1 + 2U - 2\beta U^2 - 2\xi\Phi_W + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 + 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\Phi_2 + 2(1+\zeta_3)\Phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi)\Phi_4 - (\zeta_1 - 2\xi)\mathcal{A} - (\alpha_1 - \alpha_2 - \alpha_3)w^2 U - \alpha_2 w^i w^j U_{ij} + (2\alpha_3 - \alpha_1)w^i V_i + \mathcal{O}(\epsilon^3),$$

$$g_{0i} = -\frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)W_i - \frac{1}{2}(\alpha_1 - 2\alpha_2)w^i U - \alpha_2 w^j U_{ij} + \mathcal{O}(\epsilon^{5/2}),$$

$$g_{ij} = (1 + 2\gamma U)\delta_{ij} + \mathcal{O}(\epsilon^2).$$

w: motion w.r.t. preferred reference frame



Metric potentials:

$$U = \int \frac{\rho'}{|\mathbf{x}-\mathbf{x}'|}\,d^3x', \quad \text{(Newtonian potential)}$$

$$U_{ij} = \int \frac{\rho'(x-x')_i(x-x')_j}{|\mathbf{x}-\mathbf{x}'|^3}\,d^3x',$$

$$\Phi_W = \int \frac{\rho'\rho''(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3}\cdot\left(\frac{\mathbf{x}'-\mathbf{x}''}{|\mathbf{x}-\mathbf{x}''|} - \frac{\mathbf{x}-\mathbf{x}''}{|\mathbf{x}'-\mathbf{x}''|}\right)d^3x'\,d^3x'',$$

$$\mathcal{A} = \int \frac{\rho'[\mathbf{v}'\cdot(\mathbf{x}-\mathbf{x}')]^2}{|\mathbf{x}-\mathbf{x}'|^3}\,d^3x',$$

$$\Phi_1 = \int \frac{\rho' v'^2}{|\mathbf{x}-\mathbf{x}'|}\,d^3x',$$

$$\Phi_2 = \int \frac{\rho' U'}{|\mathbf{x}-\mathbf{x}'|}\,d^3x',$$

$$\Phi_3 = \int \frac{\rho'\Pi'}{|\mathbf{x}-\mathbf{x}'|}\,d^3x',$$

$$\Phi_4 = \int \frac{p'}{|\mathbf{x}-\mathbf{x}'|}\,d^3x',$$

$$V_i = \int \frac{\rho' v'_i}{|\mathbf{x}-\mathbf{x}'|}\,d^3x',$$

$$W_i = \int \frac{\rho'[\mathbf{v}'\cdot(\mathbf{x}-\mathbf{x}')](x-x')_i}{|\mathbf{x}-\mathbf{x}'|^3}\,d^3x'.$$

[Will 1993, *Will 2014, Living Reviews in Relativity*]

Keplerian orbits: parameters - observables = 2

7 - 2 = 5 tests of GR