



FROM RINGDOWN
BACK TO INSPIRAL
NO-HAIR TESTS FROM
CONSISTENCY BETWEEN QUASI-
NORMAL MODES AND INSPIRAL

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Strong Gravity and Binary Dynamics, Ole Miss, Oxford, MS

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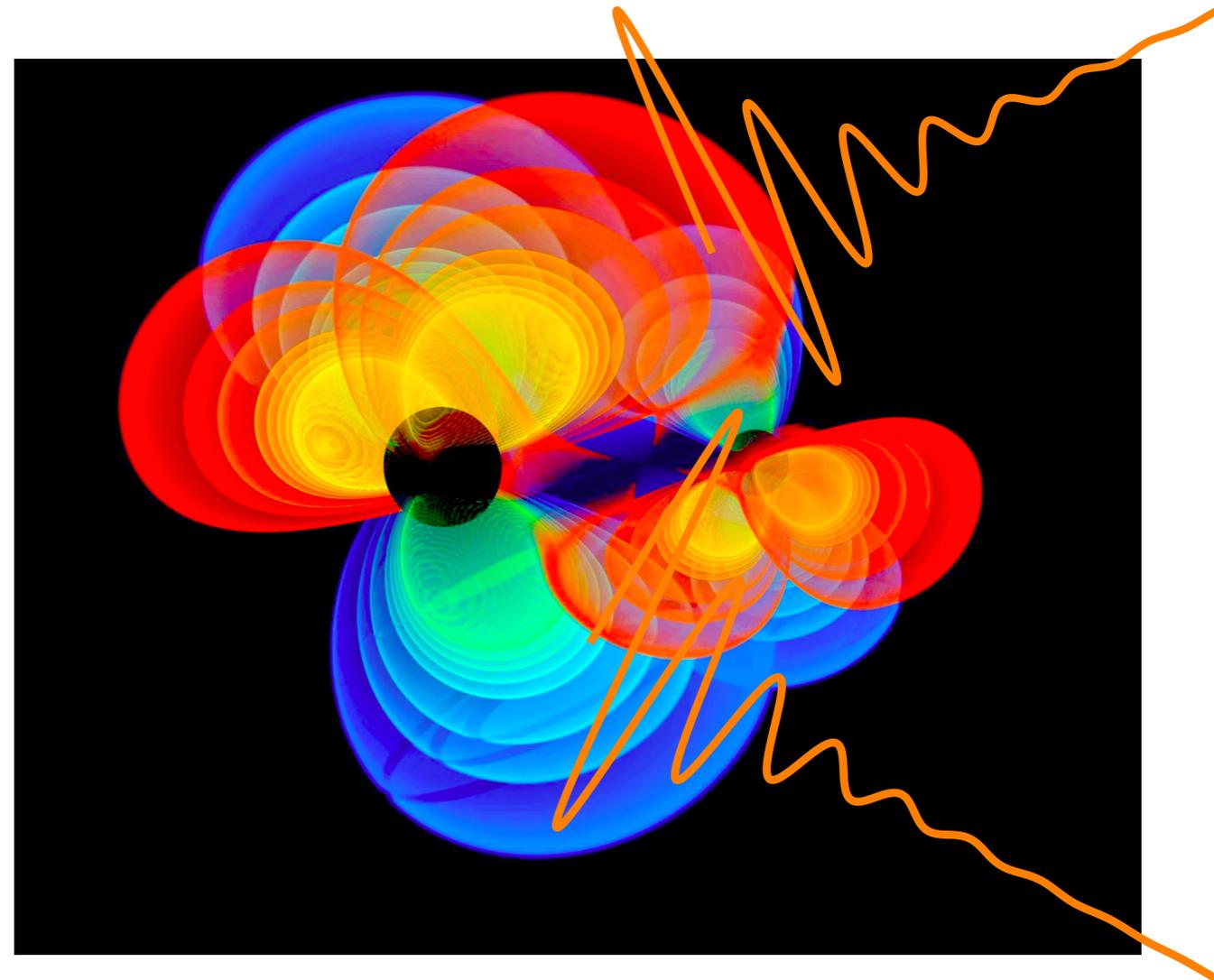
GW150914 IN FUTURE DETECTORS

Detector	GW150914 SNR	QNM SNR
O1	25	7
Advanced LIGO	80	20
LIGO-India ALIGO+ (2024)	250	80
ET (2030)	800	200
Cosmic Explorer (2034)	2400	800

LISA (2034)	10,000	2,400
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CURRENT PROPOSAL FOR TESTING "BLACK HOLE" NO-HAIR THEOREM

- measure more than one quasi-normal complex frequency
- are mass and spin inferred from different quasi-normal modes consistent with one another?
- a more powerful way of doing the same test
- measure the Bayes factor between two alternative models: one in which all modes depend on just two parameters vs another that has extra hair
- caveat: agreement b/w QNM from perturbation theory and NR sims



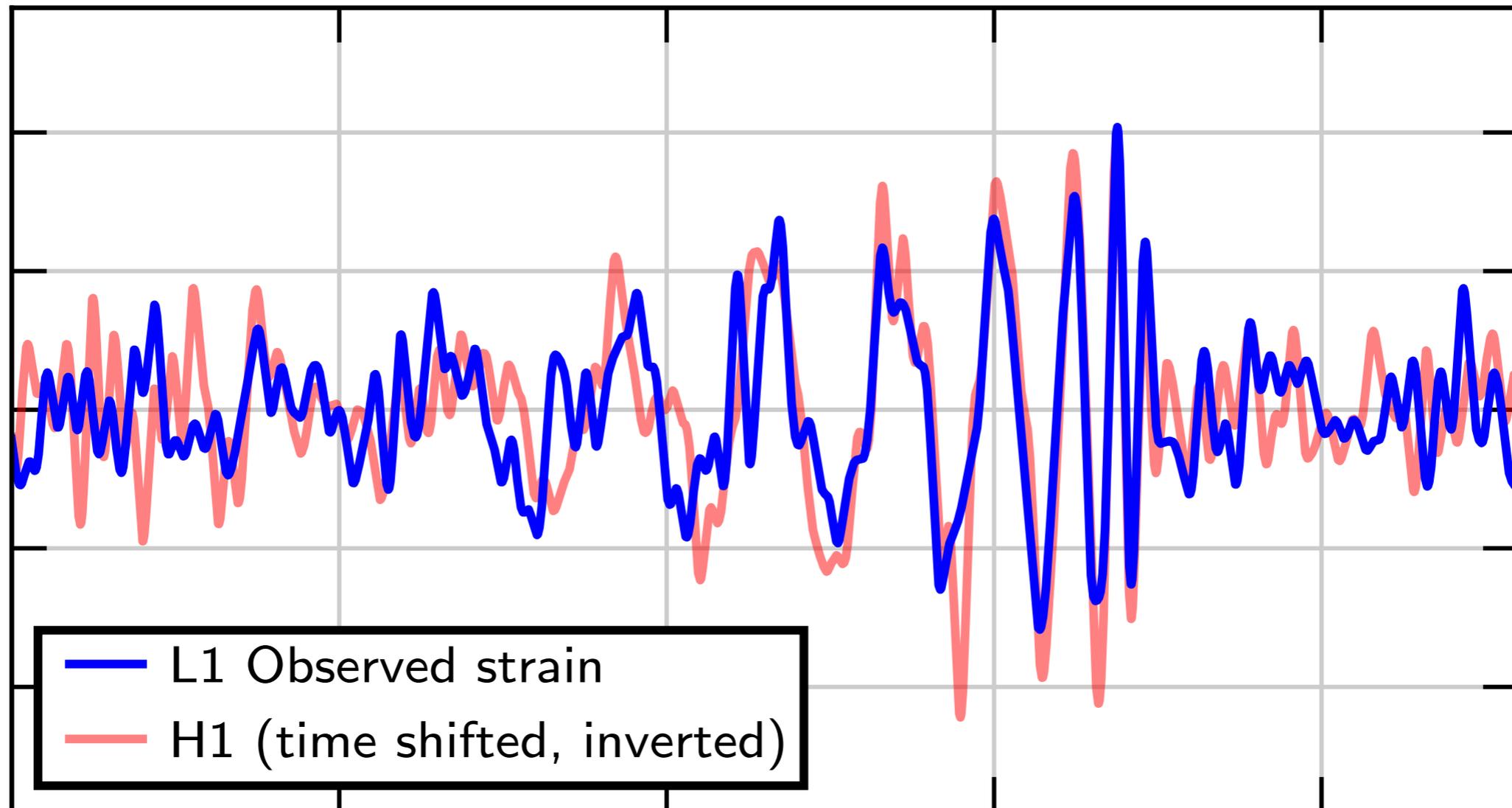
$$K = \frac{\Pr(D|M_1)}{\Pr(D|M_2)} = \frac{\int \Pr(\theta_1|M_1) \Pr(D|\theta_1, M_1) d\theta_1}{\int \Pr(\theta_2|M_2) \Pr(D|\theta_2, M_2) d\theta_2}$$

on going: George+ 2017

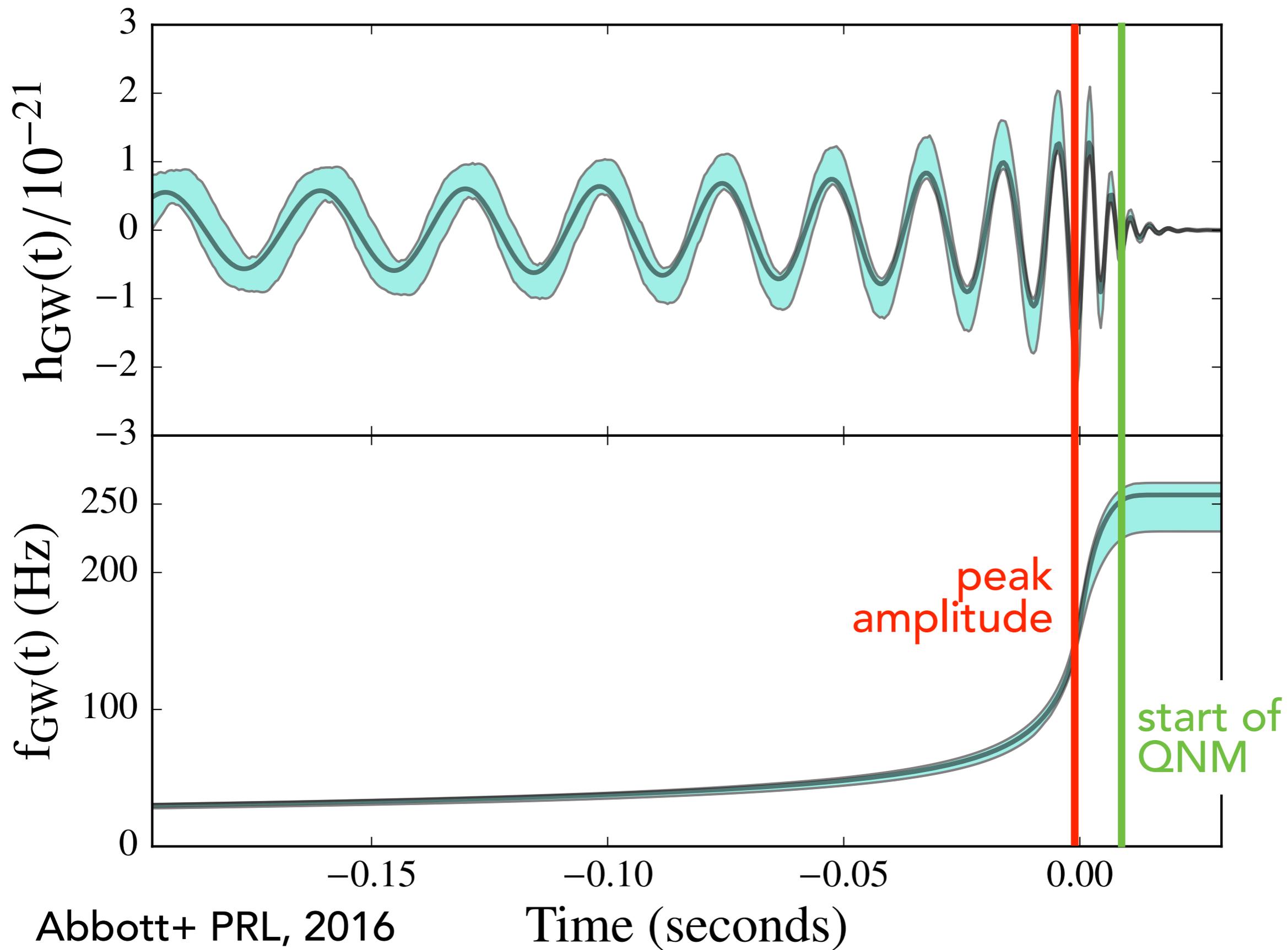
SHORTCOMINGS OF THIS PROPOSAL

- Quasi-normal mode waveforms are derived in the perturbative approximation
- need to wait for the signal to reach linear QNM regime
- this could be 10 M to 20 M after the peak strain amplitude is reached
- most visible part of the signal will be trashed in this test

EXAMPLE: GW150914 ANALYSIS

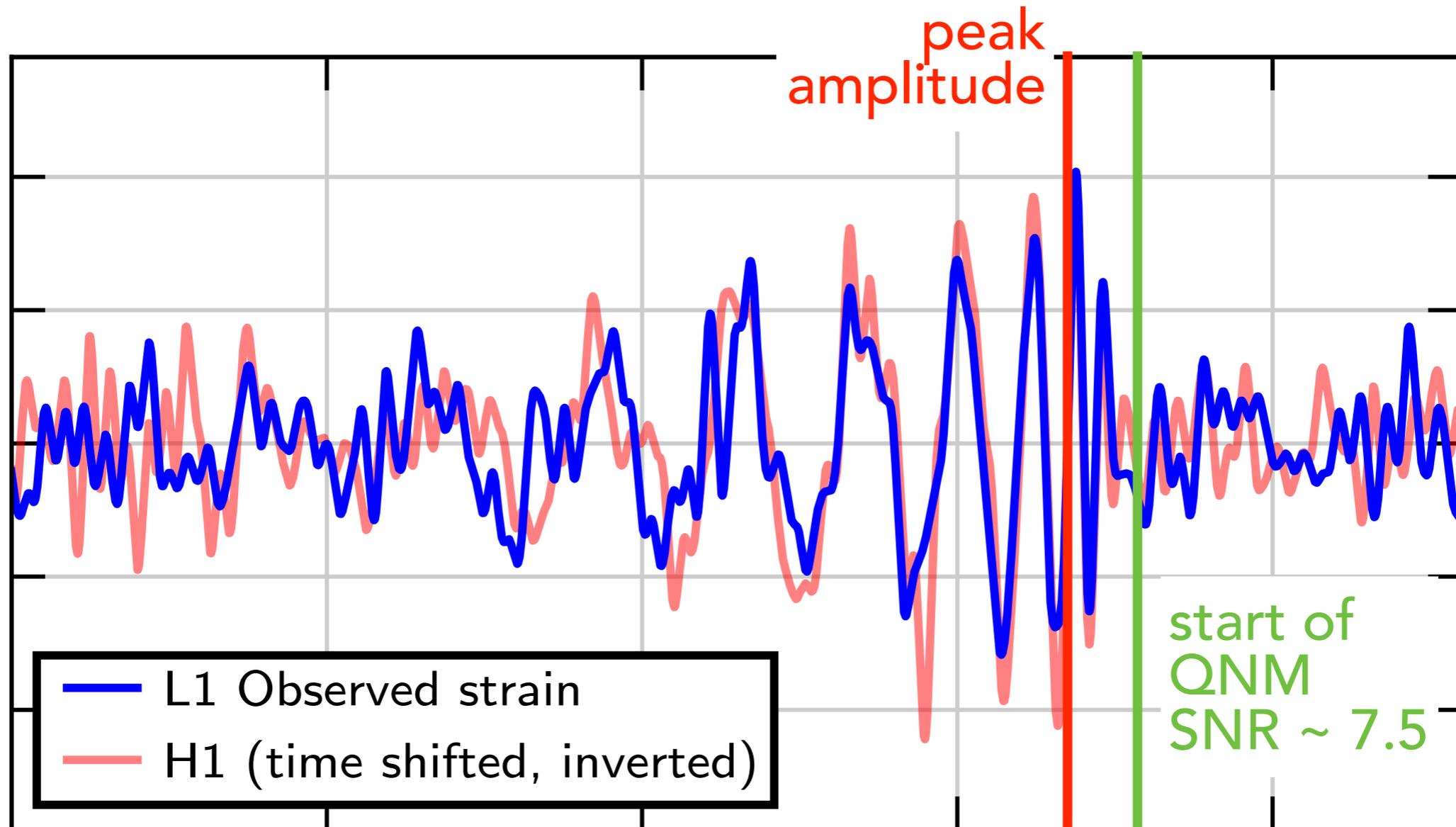


WHEN DO QNM BEGIN?



SO, DO WE SEE QNM?

YES BUT CAN'T TEST THE NO-HAIR THEOREM



SO WHAT'S THE SOLUTION?

FOUR PROPOSALS

- get a better model for post-merger
- compare the parameters of the remnant derived from inspiral waveform with those derived from QNM
 - from inspiral to ringdown
- compare the parameters of the binary derived from QNM to those derived from the inspiral
 - from ringdown back to inspiral
- search for a “binary black hole” no-hair theorem and test it

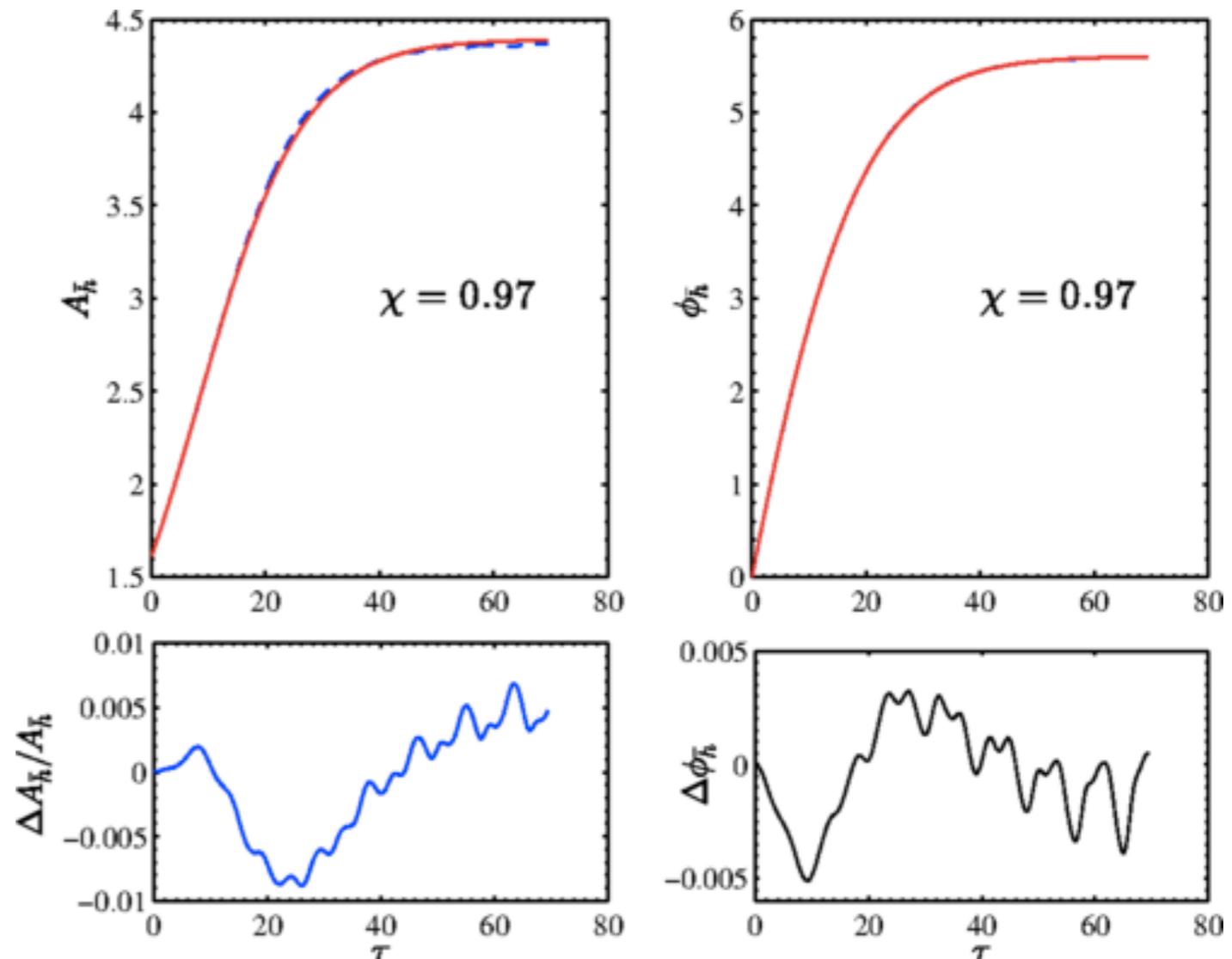
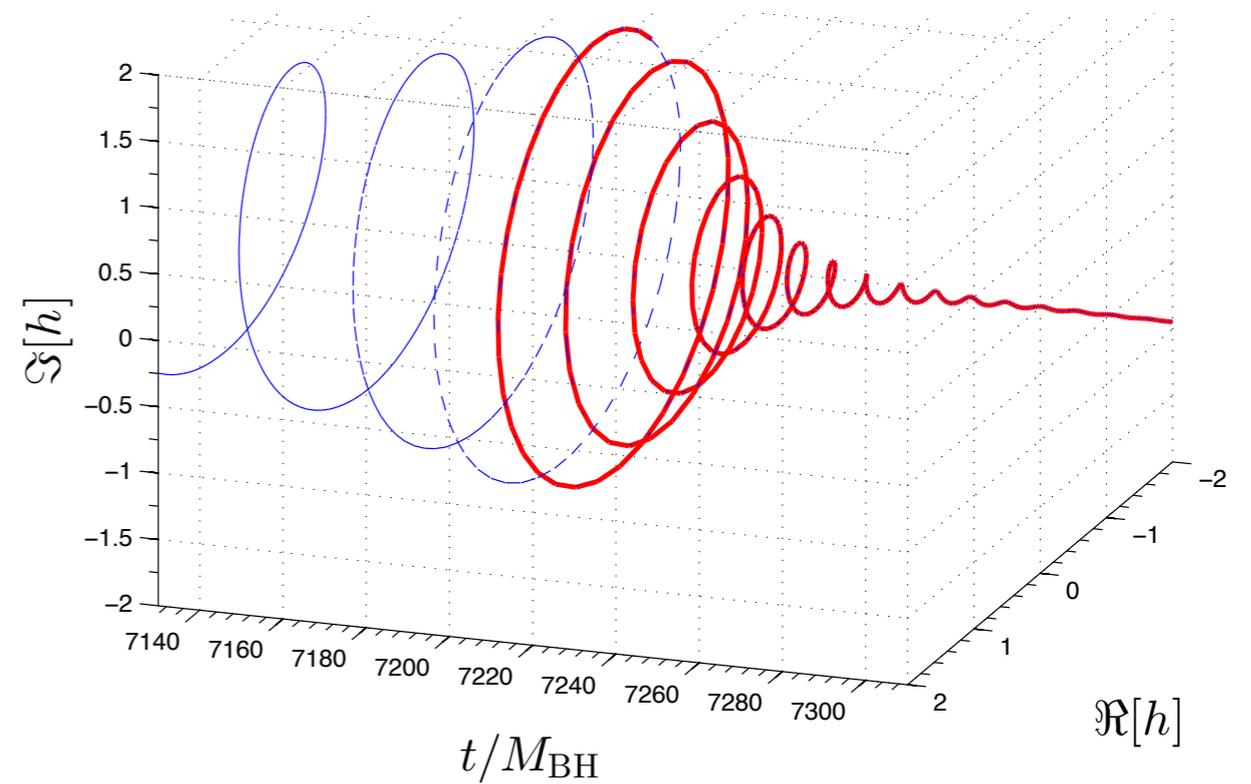
BETTER MODELS OF QNM
EOB MADE A FIRST ATTEMPT IN THIS DIRECTION

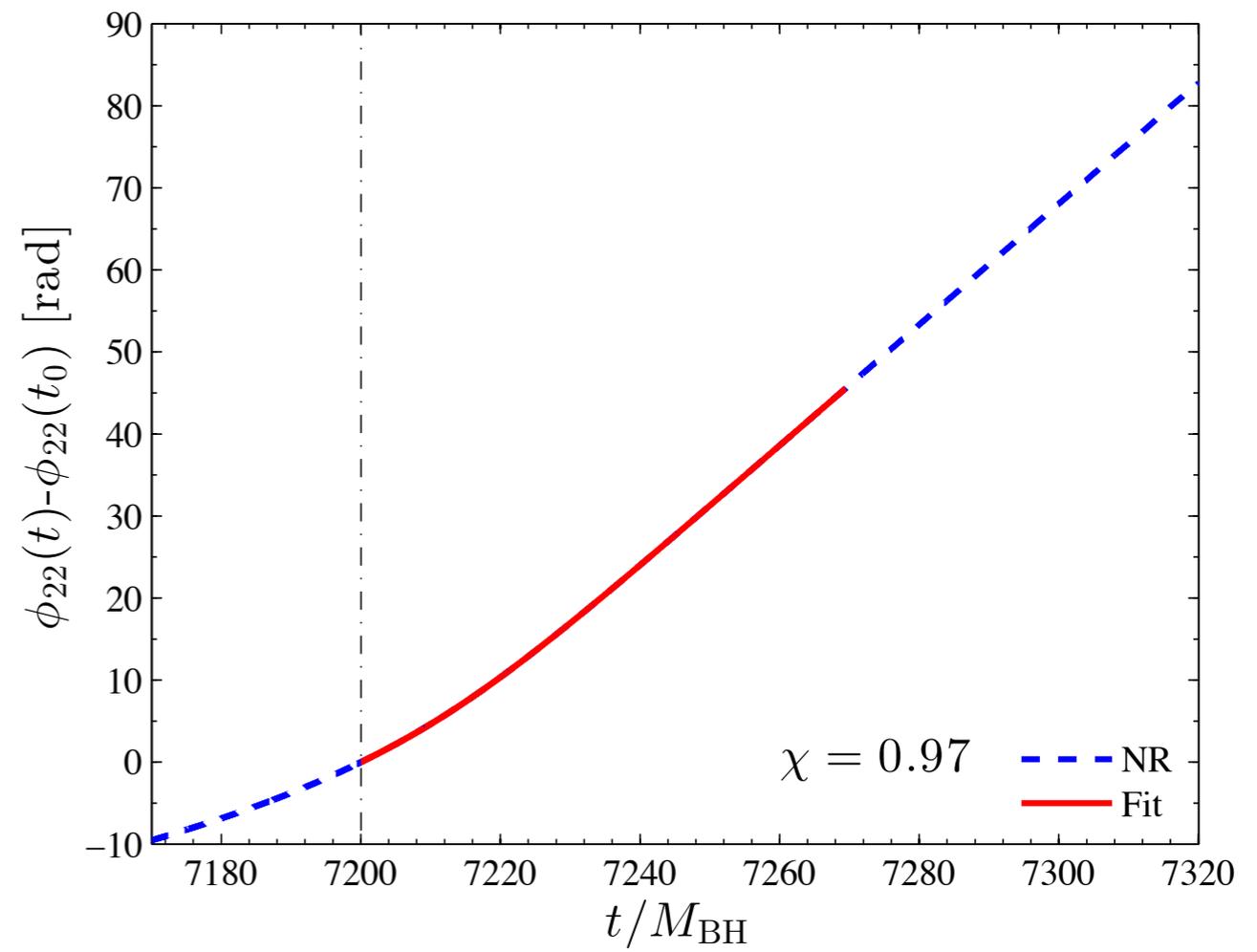
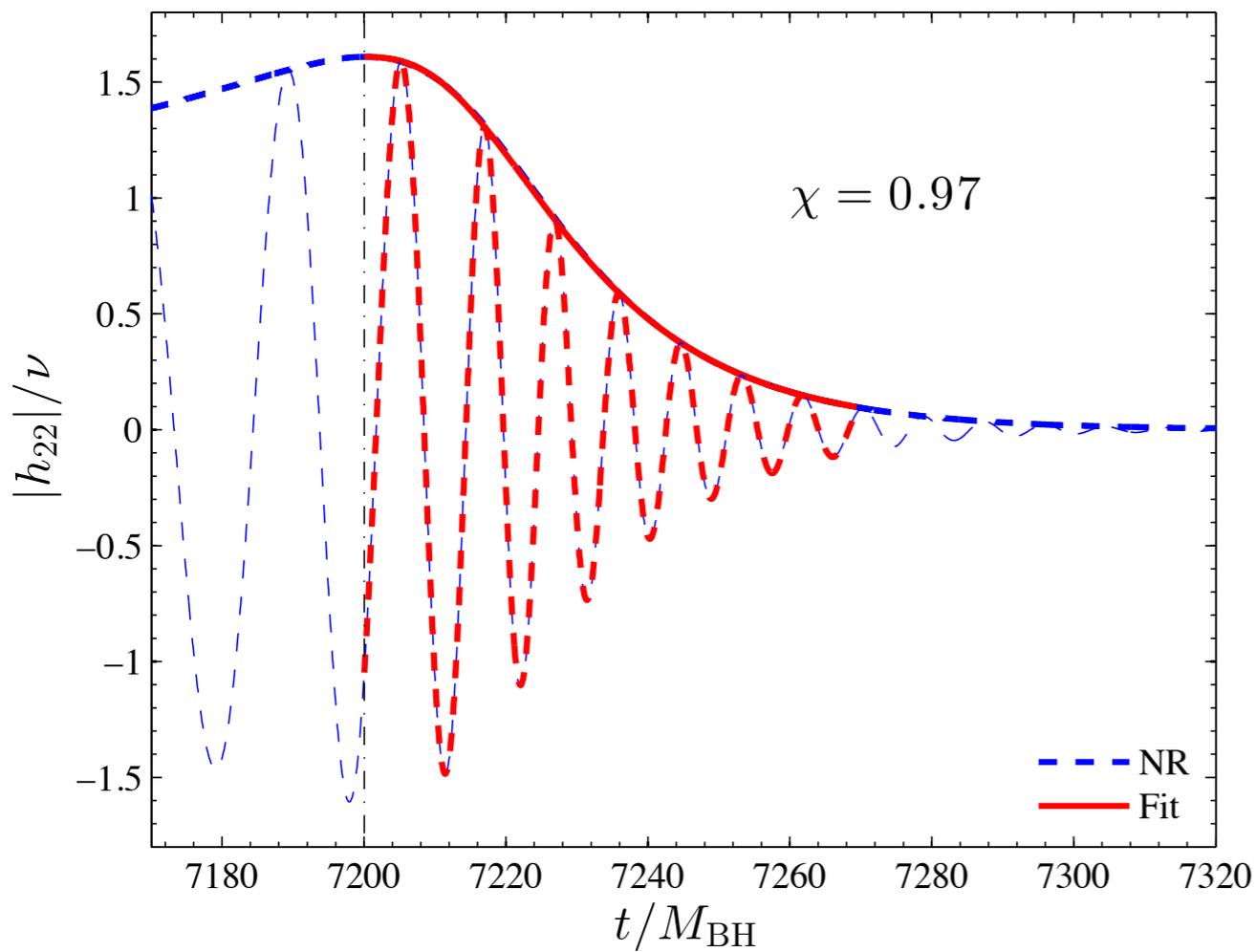
FACTORIZE QNM FROM POST-MERGER WAVEFORM

- represent QNM as “multiplicative decomposition” of $h(t)$ in contrast to linear decomposition
- residual time-dependent complex factor is fitted separately

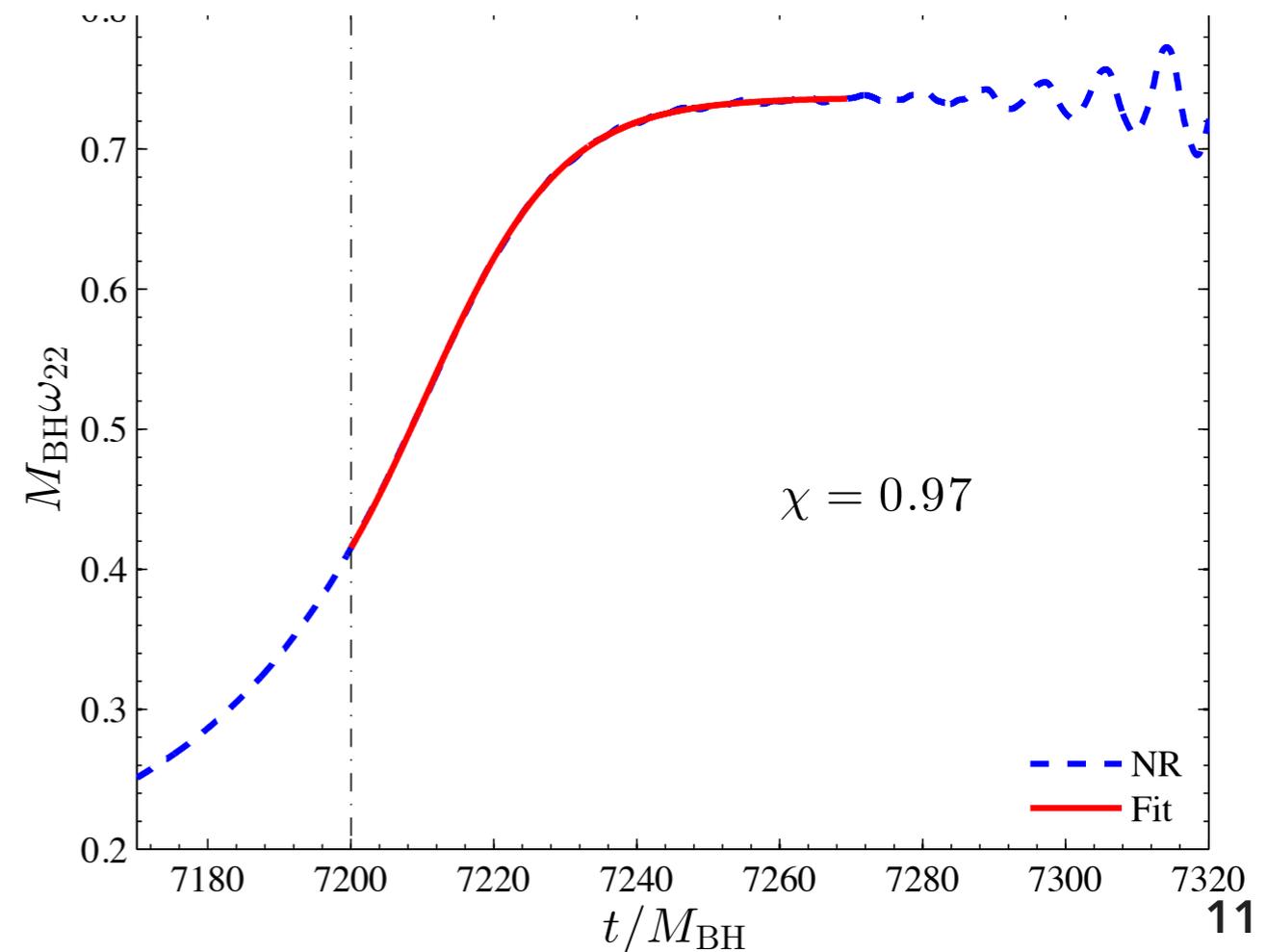
$$\bar{h}(\tau) \equiv e^{\sigma_1 \tau + i\phi_{22}^{\text{mrg}}} h(\tau),$$

$$\bar{h}(\tau) \equiv A_{\bar{h}} e^{i\phi_{\bar{h}}(\tau)}.$$

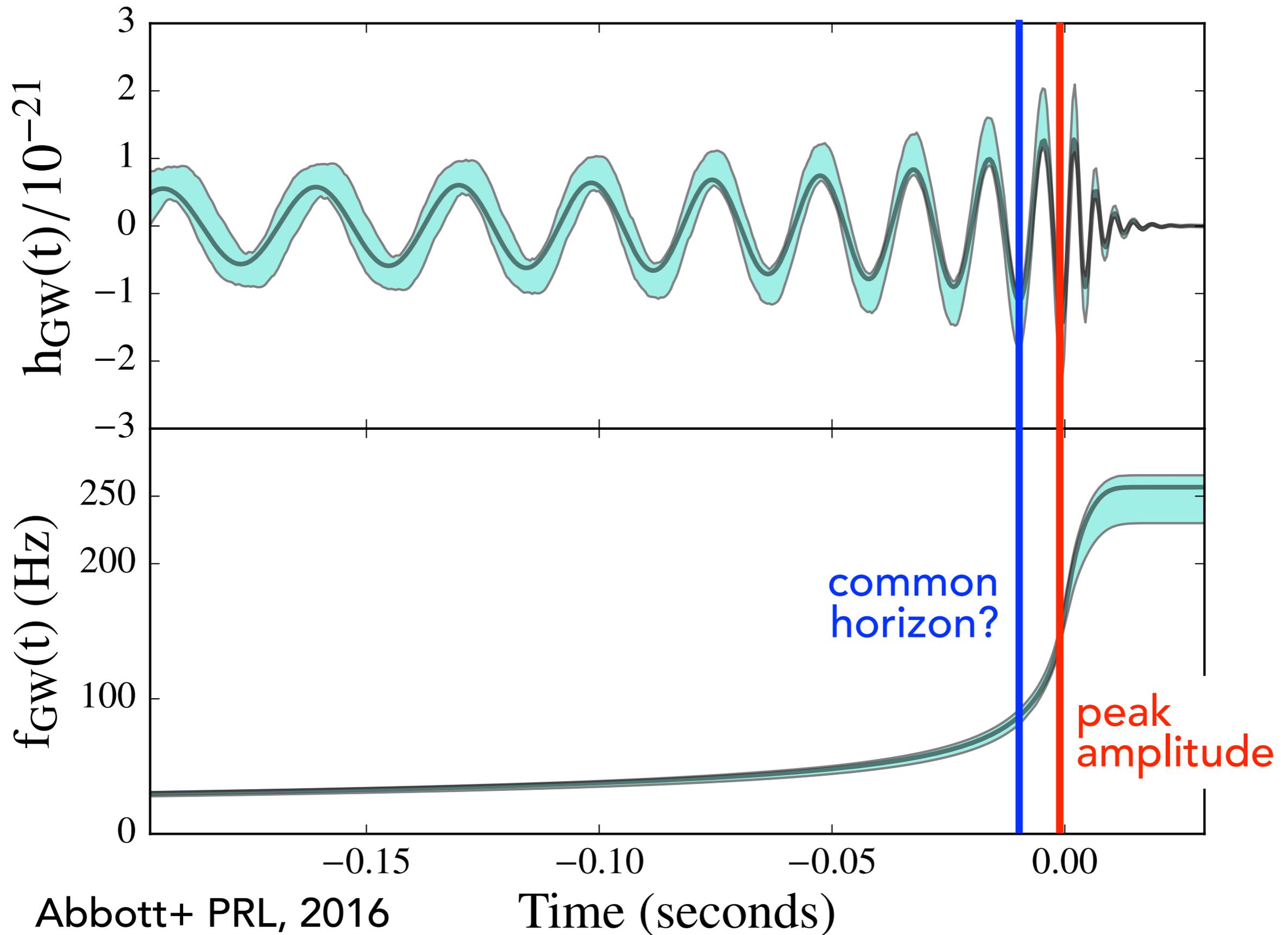




currently being
generalized to
precessing spins with
SXS data by Pratten+



WHY NOT START BEFORE THE PEAK AMPLITUDE?



Abbott+ PRL, 2016

FROM INSPIRAL TO
RINGDOWN

REMNANT MASS-SPIN CONSISTENCY

- estimate joint posterior probability for the intrinsic binary parameters (masses and spins) marginalized over all other parameters:

$$P_{\text{IMR}}(m_1, m_2, \mathbf{S}_1, \mathbf{S}_2)$$

- infer the posterior on the final mass and dimensionless spin using fitting formulas: $M_f = M_f(m_1, m_2, \mathbf{S}_1, \mathbf{S}_2)$, $\chi_f = \chi_f(m_1, m_2, \mathbf{S}_1, \mathbf{S}_2)$.

$$P_{\text{IMR}}(M_f, \chi_f)$$

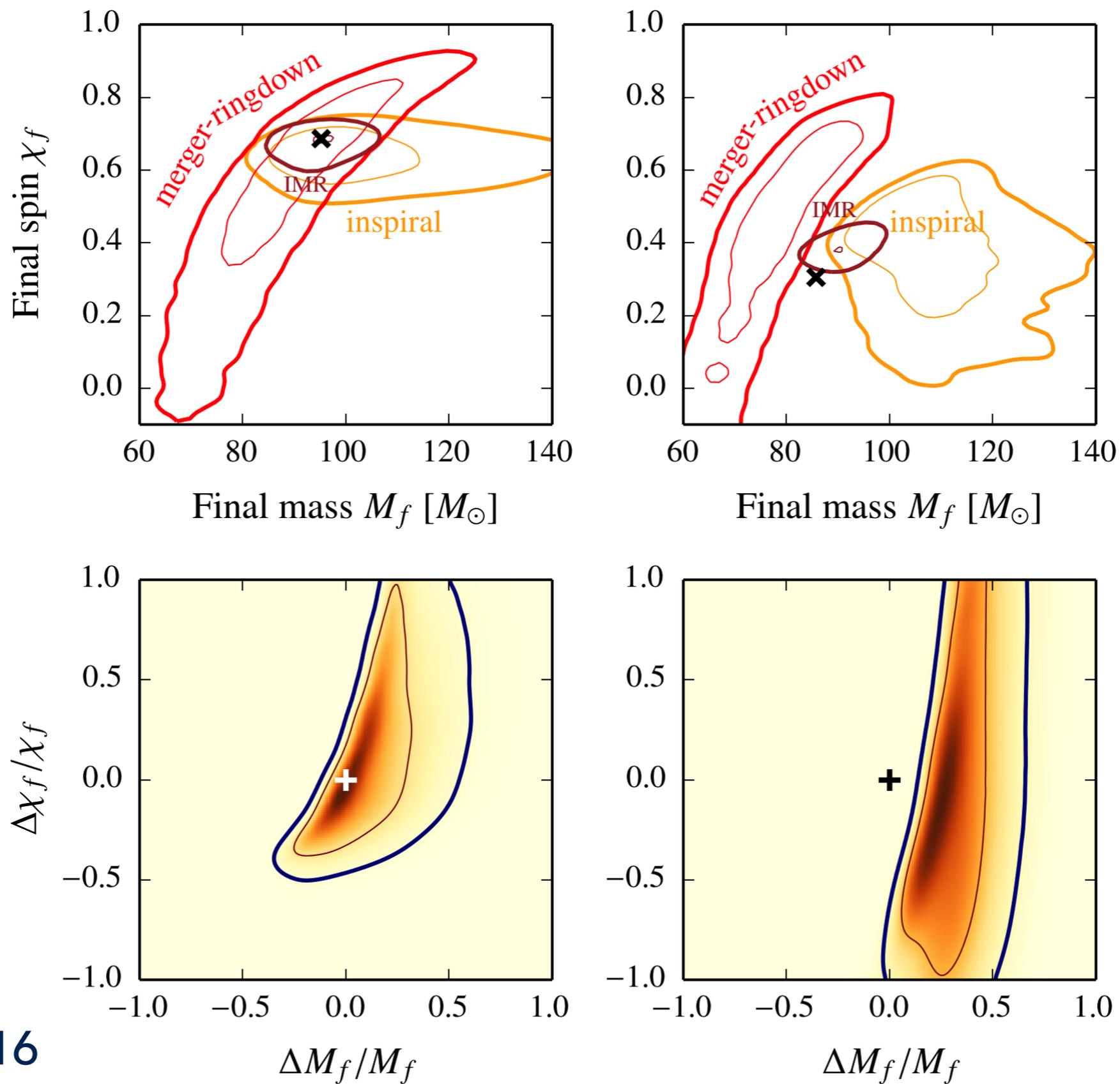
- split the waveform (in the Fourier domain) into inspiral part and merger-ringdown part and estimate the remnant mass and spin from each

$$P_{\text{I}}(M_f, \chi_f) \text{ and } P_{\text{MR}}(M_f, \chi_f)$$

- look for consistency of the parameters of the remnant so derived

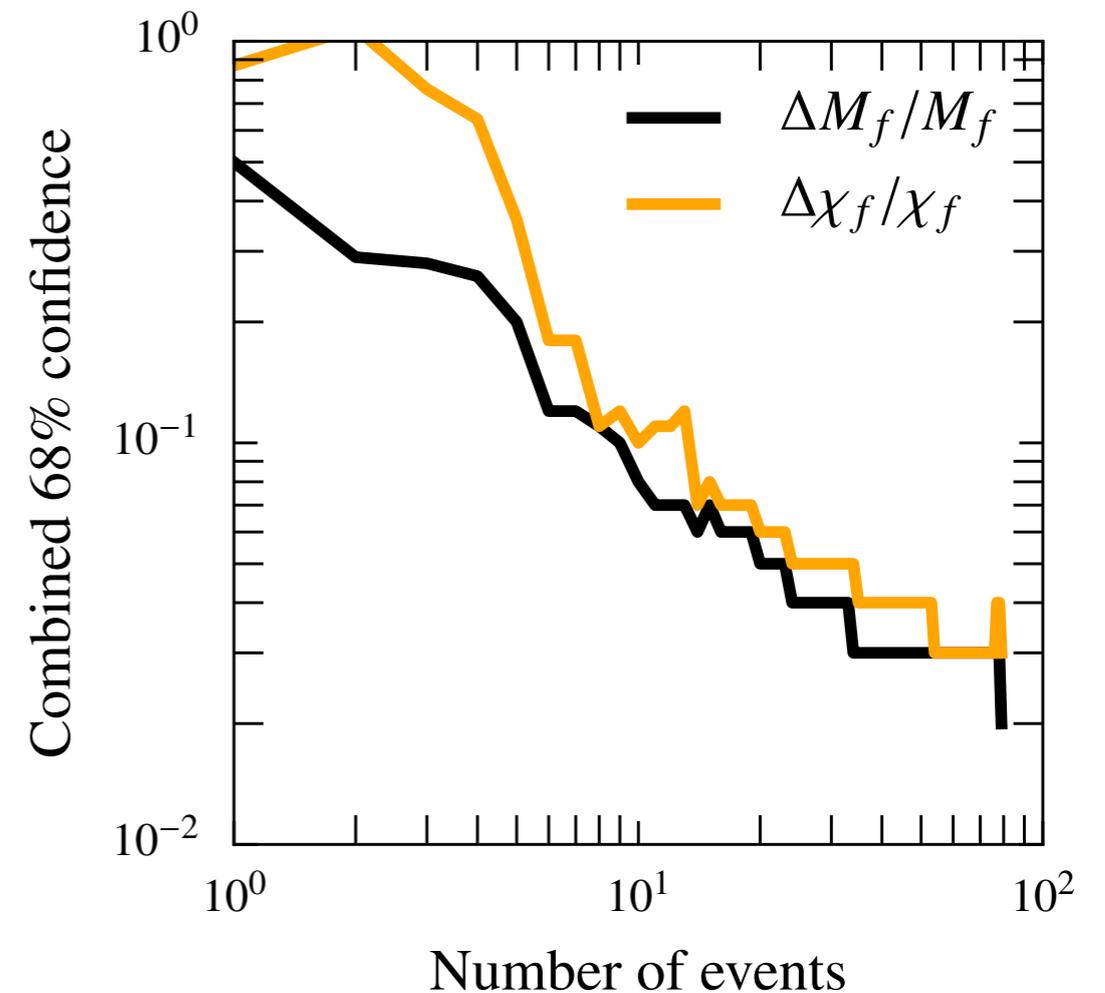
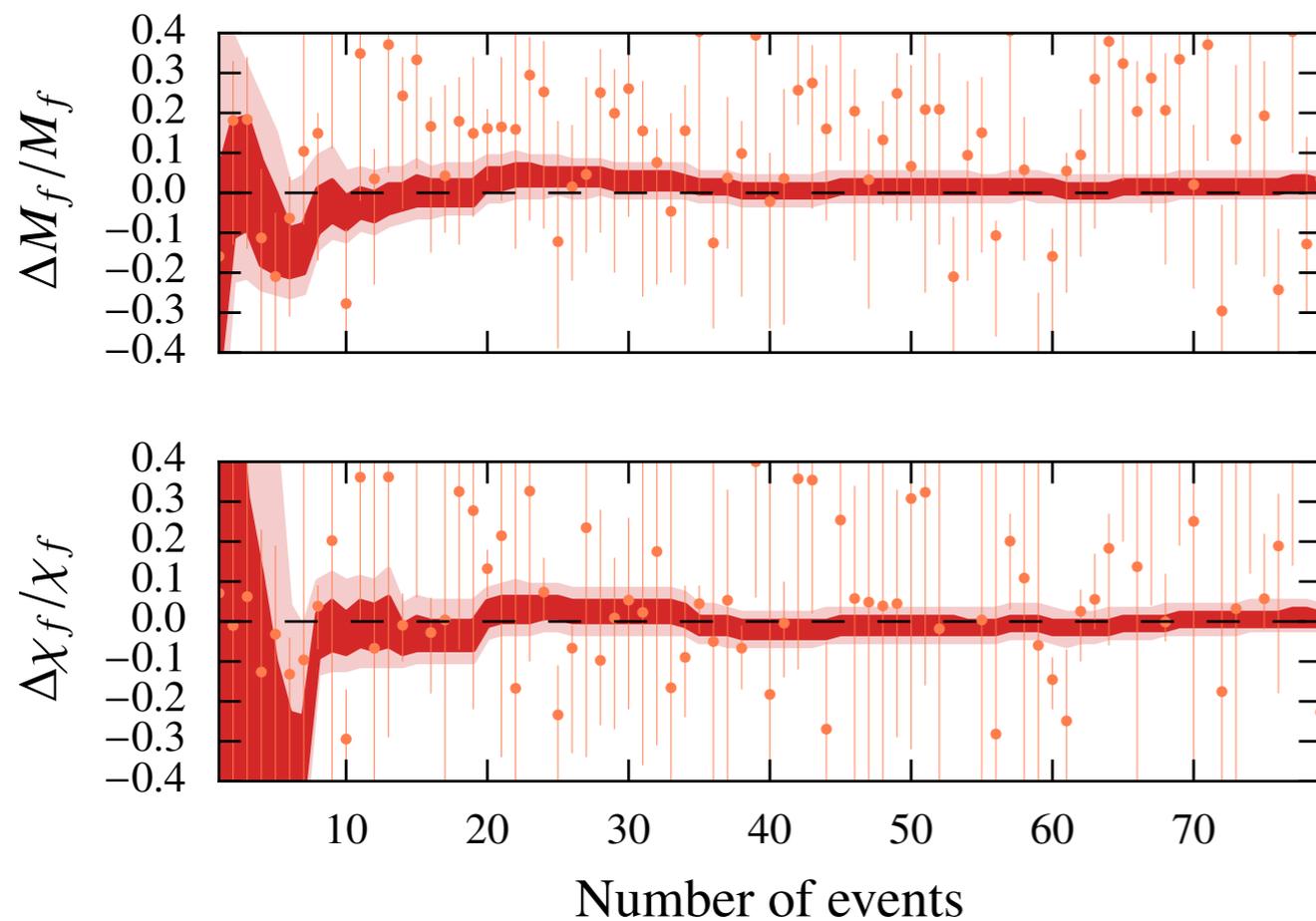
$$\Delta M_f := M_f^{\text{I}} - M_f^{\text{MR}}, \quad \Delta \chi_f := \chi_f^{\text{I}} - \chi_f^{\text{MR}},$$

GR INJECTION (LEFT) NON-GR (RIGHT)

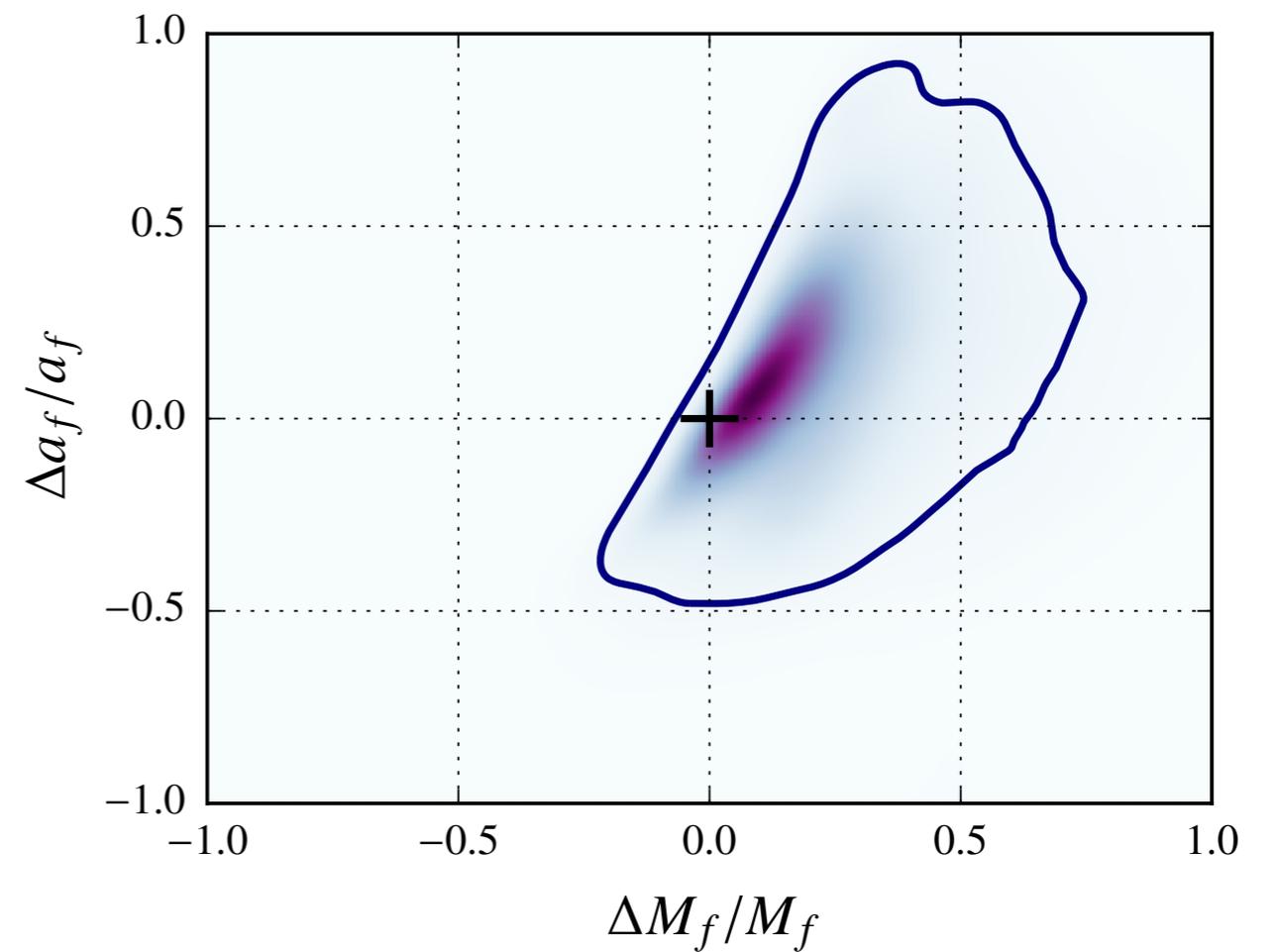
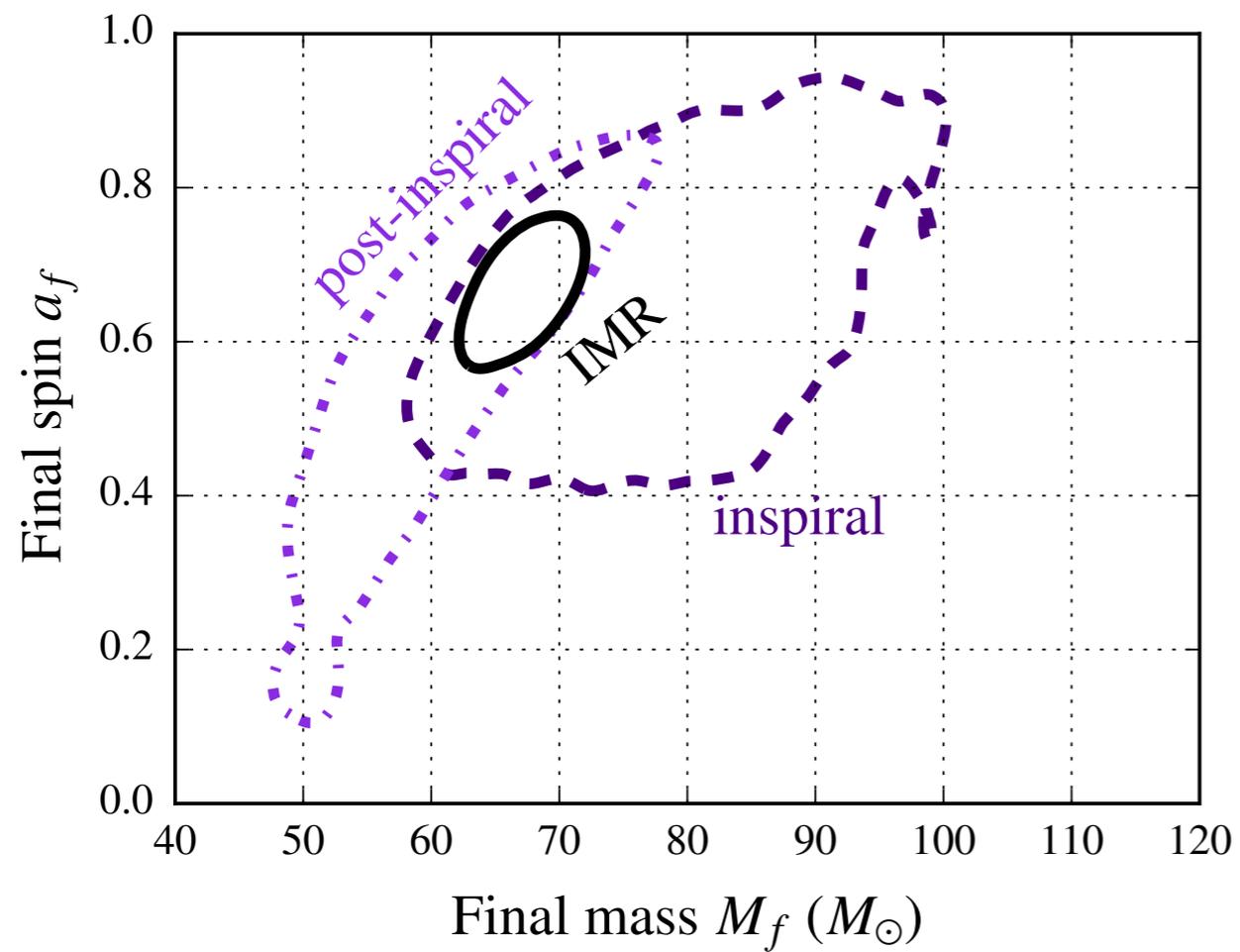


COMBINING EVENTS

AFTER ~ 100 EVENTS WIDTH OF THE MARGINALIZED POSTERIOR IS BELOW 1%



APPLICATION TO GW150914: FINAL BH MASS AND SPIN

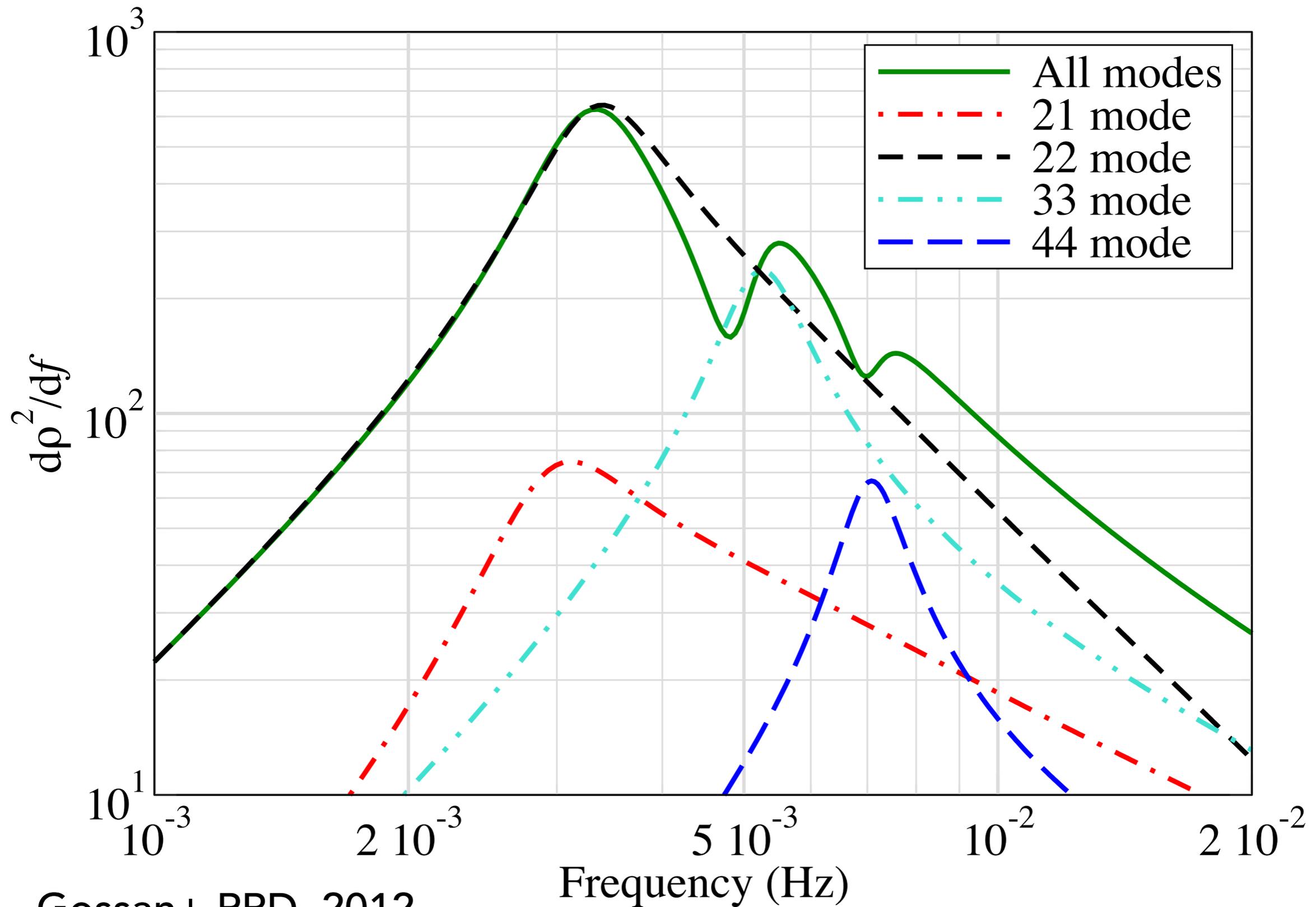


Abbott+ PRL, 2016

FROM RINGDOWN BACK TO INSPIRAL

PROGENITOR BINARY PARAMETERS FROM RINGDOWNS

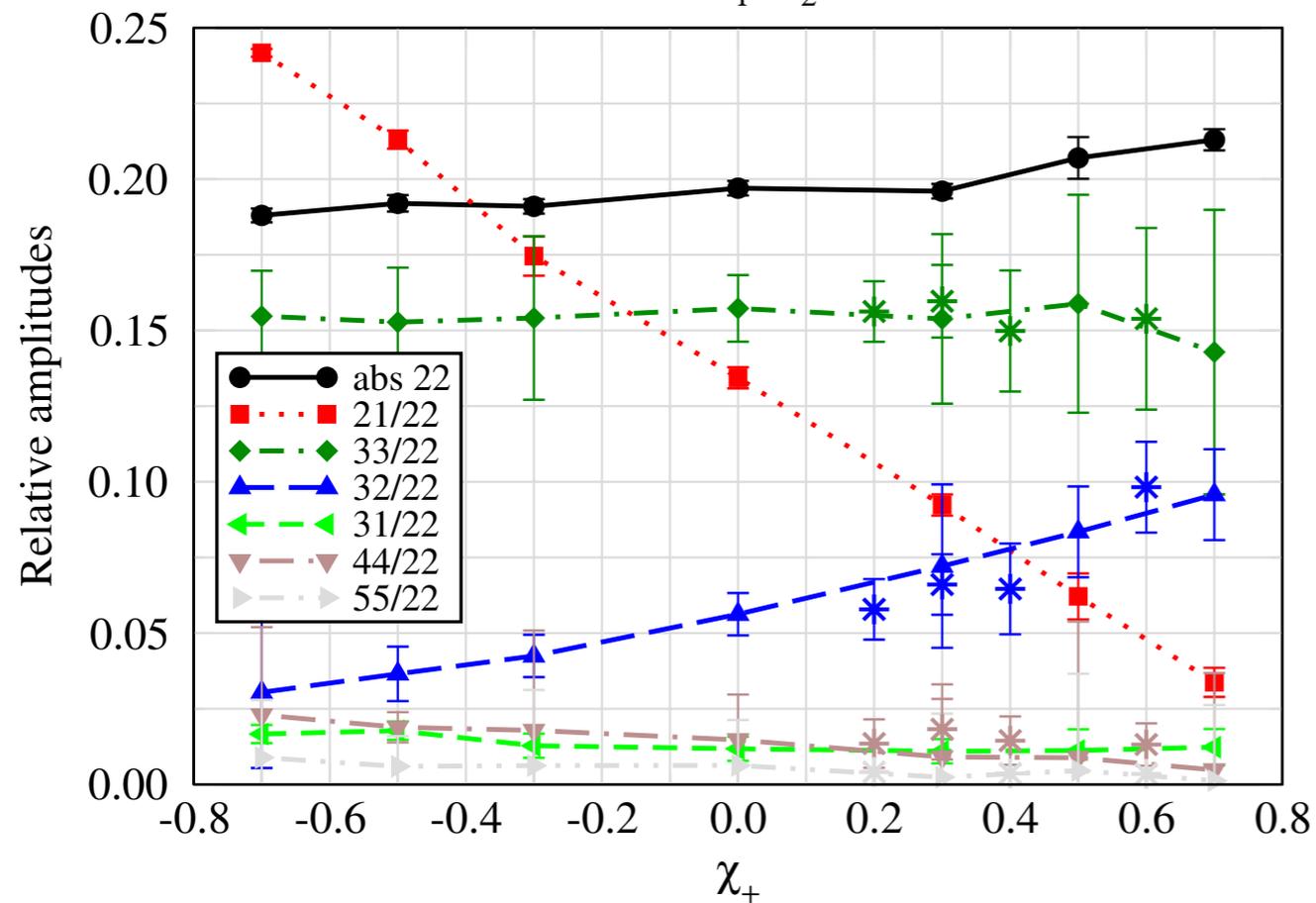
BLACK HOLE RAINBOWS



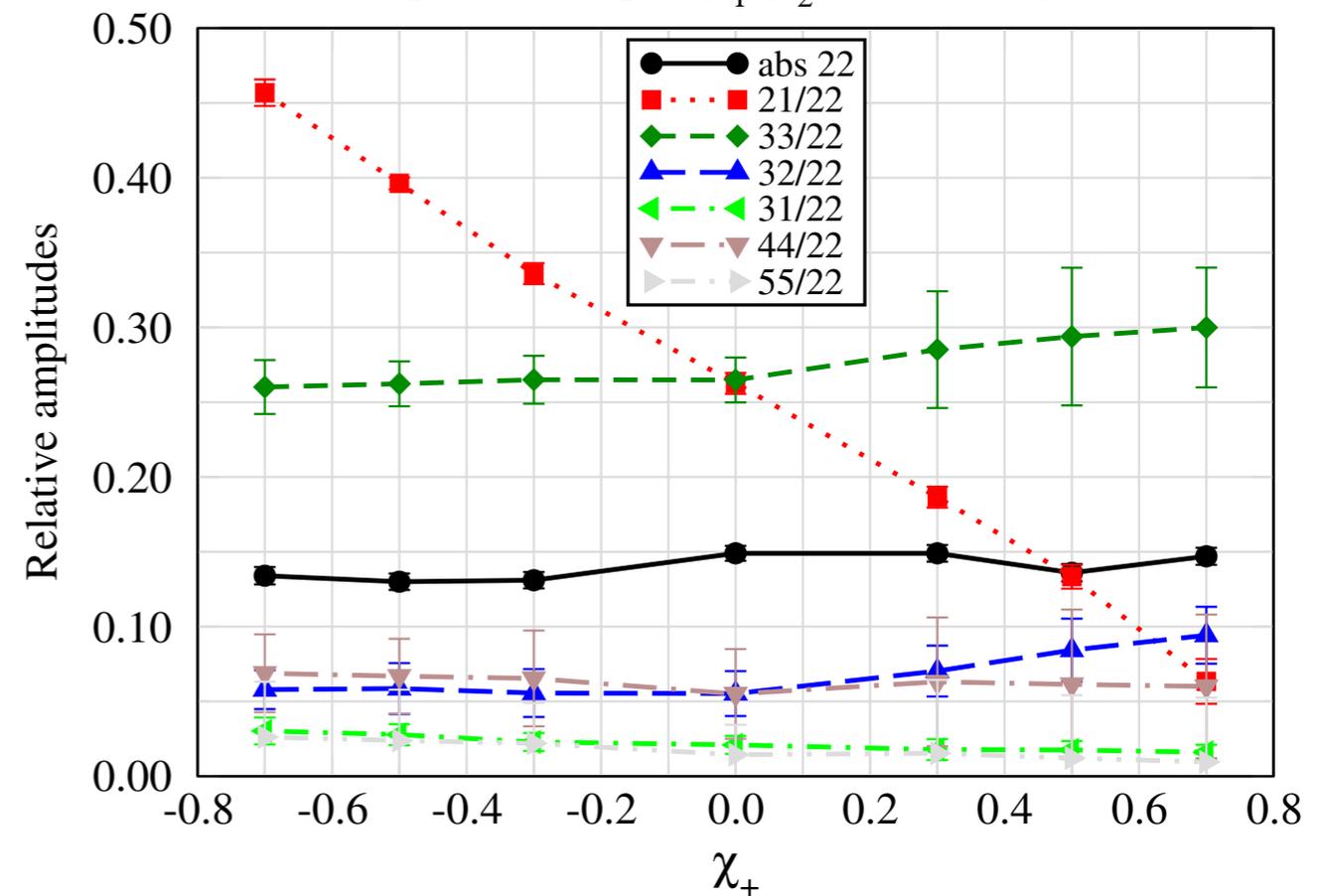
Gossan+ PRD, 2012

AMPLITUDES OF HIGHER MODE CARRY THE SIGNATURE OF INSPIRAL

Equal initial spins $\chi_1=\chi_2$, mass ratio $q=2$



Equal initial spins $\chi_1=\chi_2$, mass ratio $q=4$



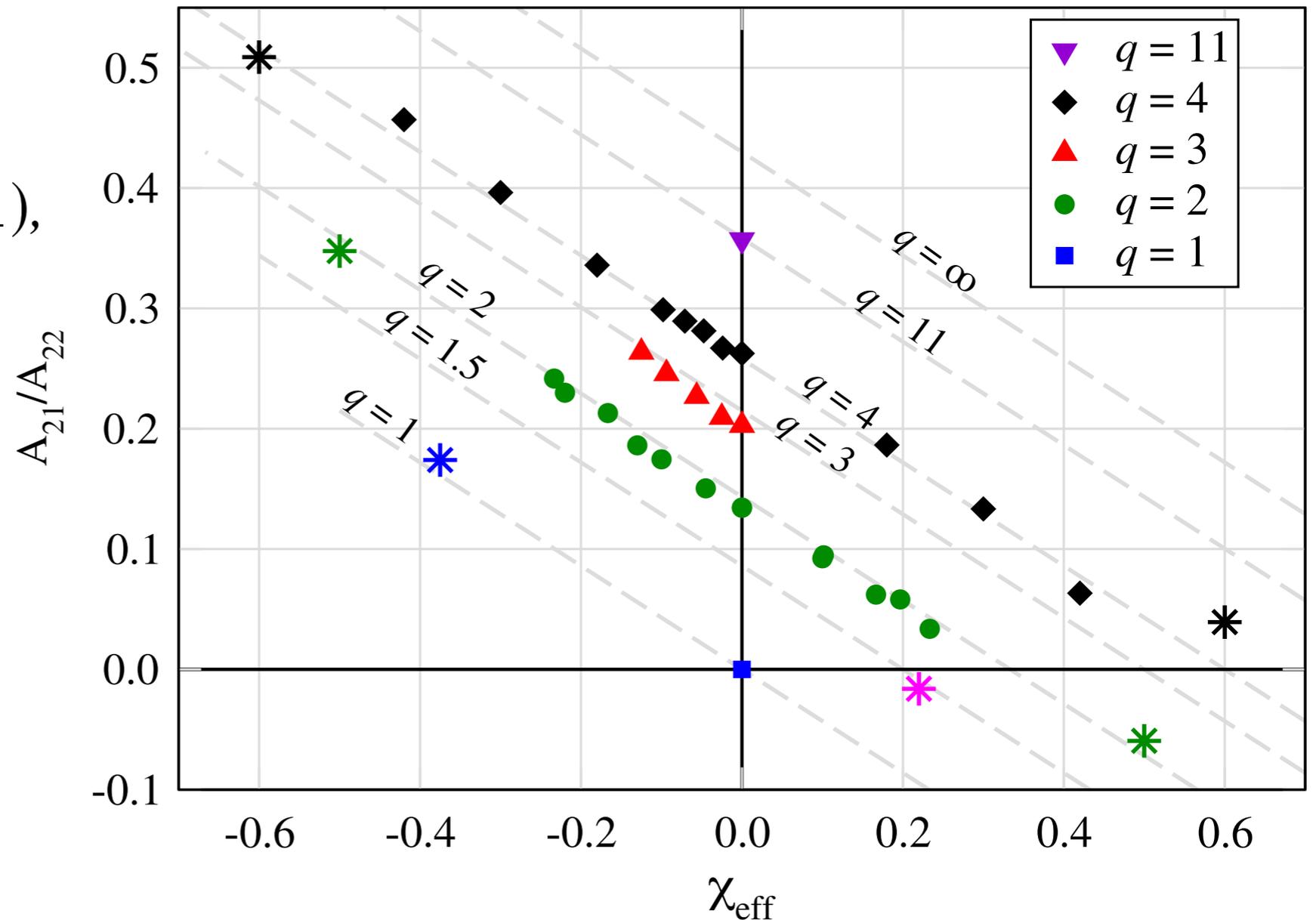
A21 AS A FUNCTION OF EFFECTIVE SPIN

CURRENTLY BEING GENERALIZED TO PRECESSING BINARIES
AND IMPLEMENTATION OF THE TEST BY BORHANIAN+

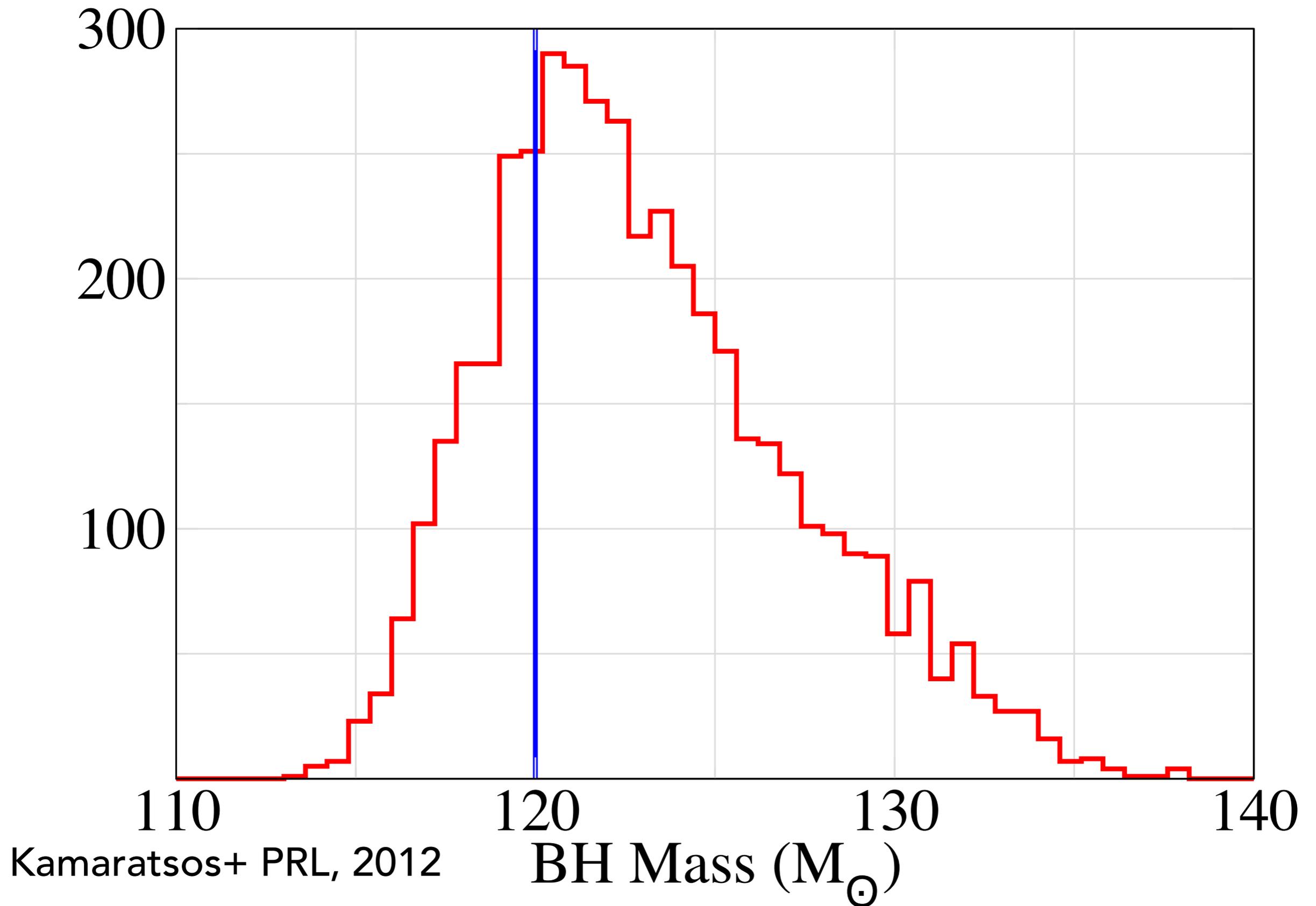
$$\chi_{\text{eff}} = \frac{1}{2}(\sqrt{1 - 4\nu}\chi_1 + \chi_-),$$

$$\chi_- = \frac{m_1\chi_1 - m_2\chi_2}{M_{\text{in}}}.$$

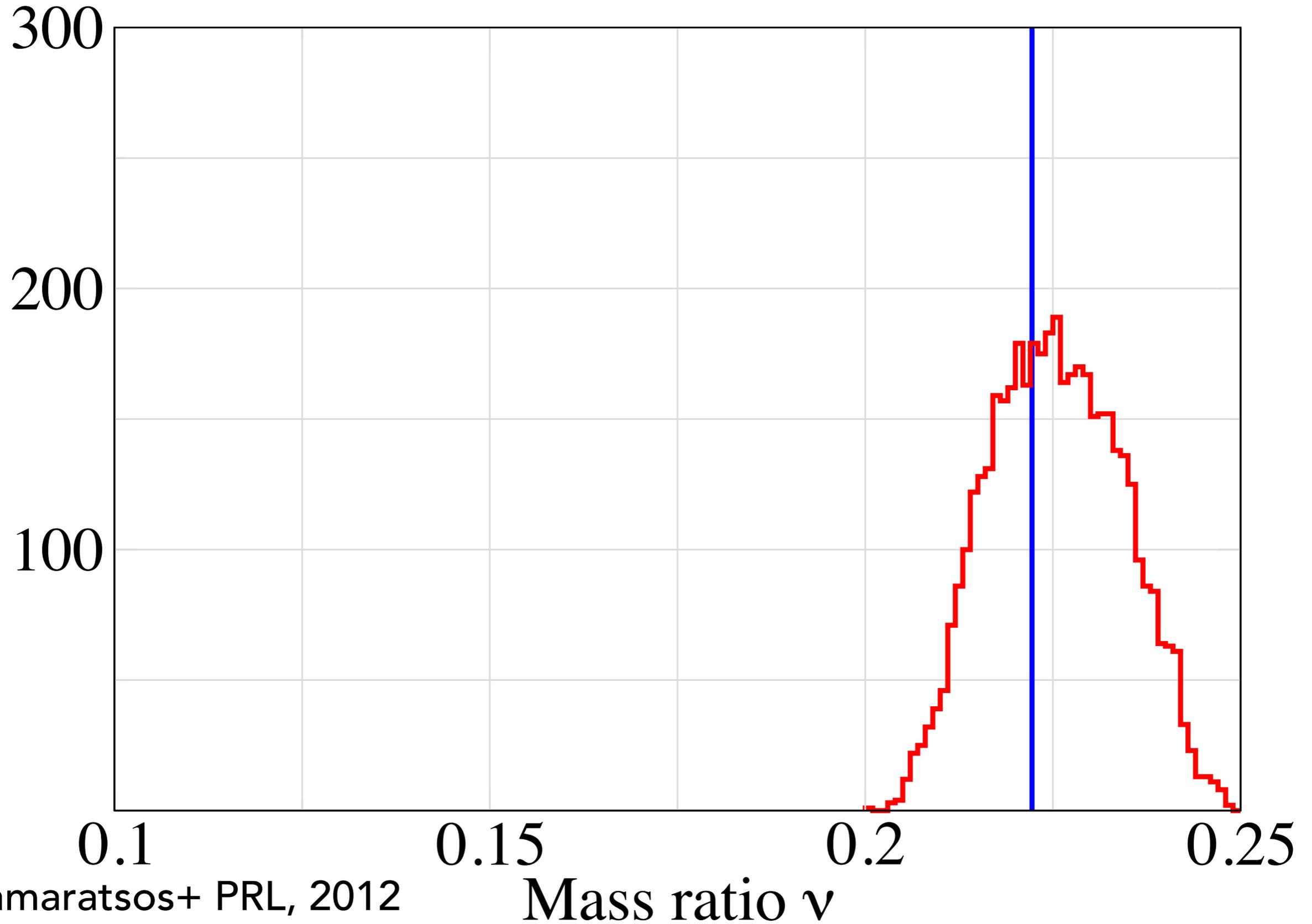
$$\hat{A}_{21} \equiv A_{21}/A_{22} = 0.43[\sqrt{1 - 4\nu} - \chi_{\text{eff}}],$$



POSTERIOR DENSITY OF BINARY BLACK HOLE MASS



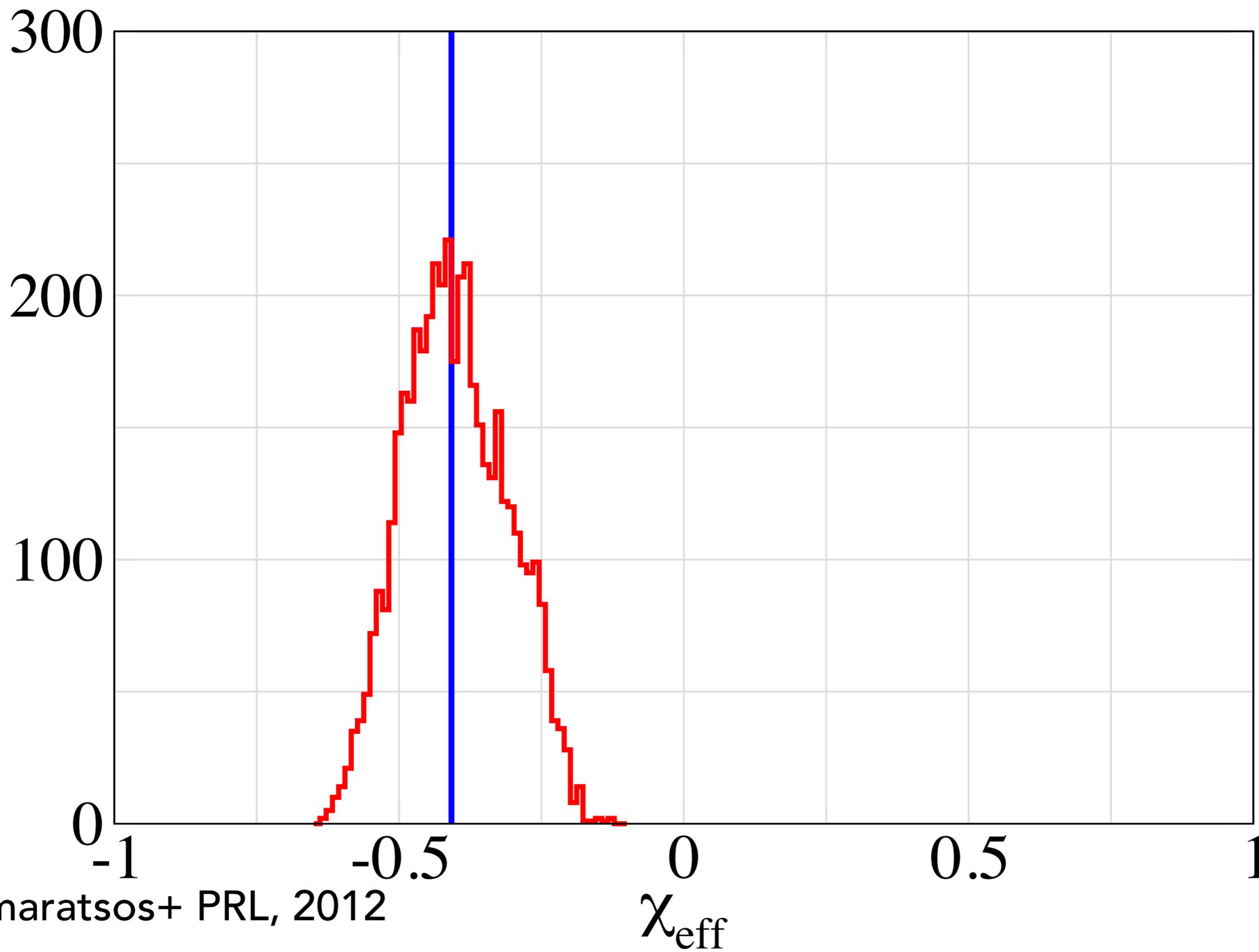
POSTERIOR PDF OF BINARY MASS RATIO



Kamaratsos+ PRL, 2012

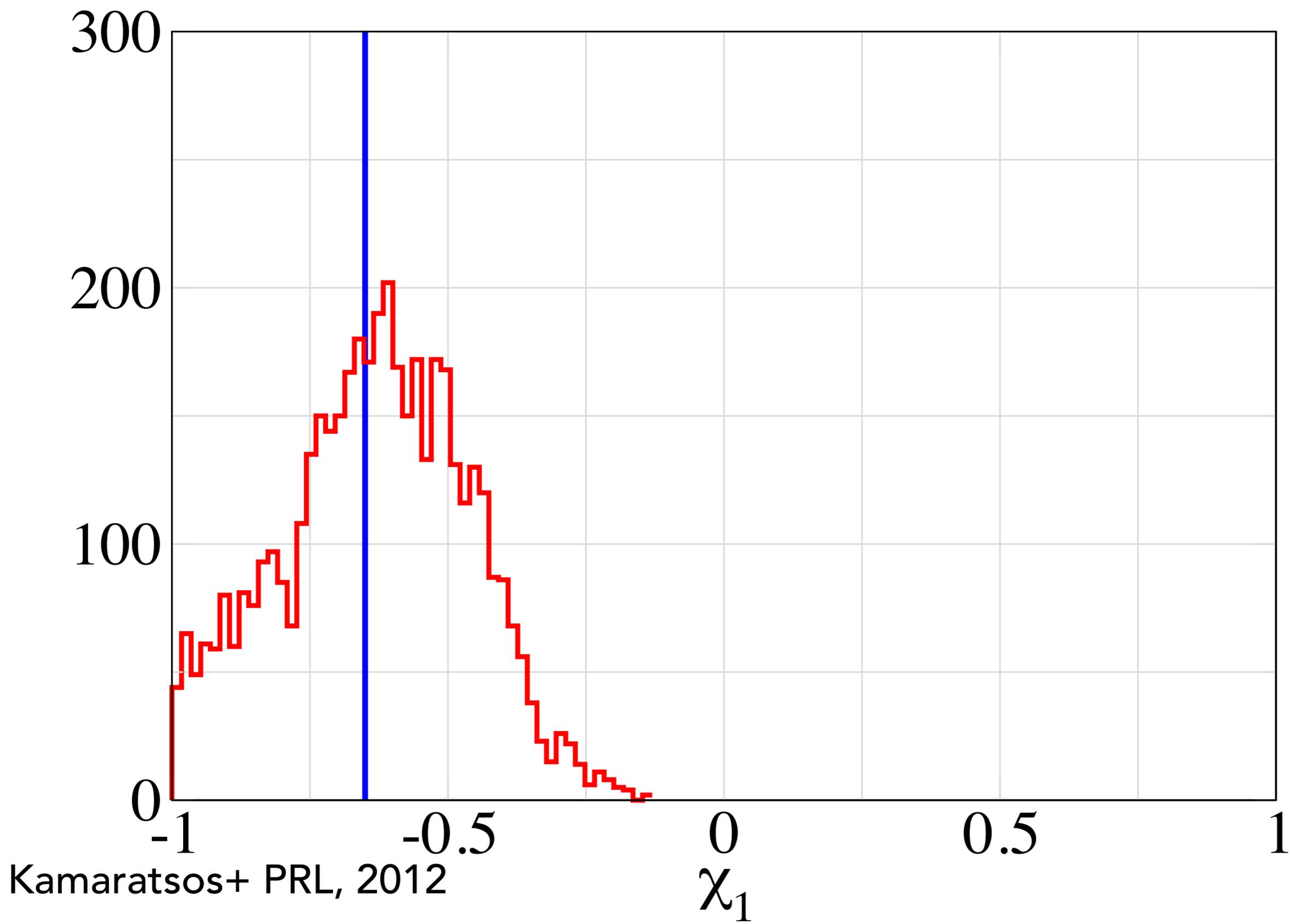
Mass ratio ν

POSTERIOR PDF OF EFFECTIVE SPIN



Kamaratsos+ PRL, 2012

POSTERIOR PDF OF COMPONENT SPIN



GENERALIZING THE NO-
HAIR THEOREM TO
BINARY BLACK HOLES

CONSISTENCY BETWEEN DIFFERENT MULTIPOLES OR MODES

- originally proposed in the context of EMRI's for a single black hole by Ryan
 - in that case one avoids the problem of having to do with the spacetime of two black holes
 - the small black hole is a test body orbiting supermassive black hole under radiation reaction; primarily interested in measuring the multipoles of the big blackhole
- the current proposal is to measure the multipole structure of a binary black hole
 - identify a set of parameters that most accurately determined by higher modes
 - look for parameter-independent description - multipole structure of the binary
- pose the test as consistency between different modes

Borhanian+ 2017