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C I E R A

Now with 290% more interesting!

Towards Astrophysical Model Selection with Gravitational-Wave Transient Observations

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Bayesian Model Selection

- GW PE: (mostly) straightforward application of Bayes' Law — posterior distribution on binary parameters derived from (mostly uninformative, but astrophysically motivated priors) and influenced through the data + waveform model through the likelihood ratio

  - Obtain a set of samples of physical parameters of interest: chirp mass ($\boldsymbol{\mathcal{M}_c}$), mass ratio (**q**), spin orientations and magnitudes ($\mathbf{s_1}$, $\mathbf{s_2}$), and at some point probably eccentricity (not addressed here)

- Question: ***Given a set of plausible astrophysical formation channels, how do we select a model resembling nature as well as quantify any parameters of that model?***

  - Need to map {$\boldsymbol{\mathcal{M}_c}$, **q**, $\mathbf{s_1}$, $\mathbf{s_2}$} to mass/spin spectrums, progenitor metallicity, SN kick prescriptions, evolutionary pathways, etc…

Bayesian Hierarchical Modeling

- Foreman-Mackey, et al. 2014 lays out the foundation
- convert p(mod|obs) → p(mod|PE)

$$p(\{h_i\}|\beta) = \prod_i p(h_i) \int \frac{p(\theta|h_i)p(\theta|\beta)}{p(\theta)} d\theta$$

- Integral over model parameters (β) can be evaluated via **importance sampling** using parameter estimation (θ_k) samples

$$\rightarrow p(\beta|\{h_i\}) \propto \prod_i \frac{1}{N} \sum_k \frac{p(\theta_k|\beta)}{p(\theta_k)} p(\beta)$$

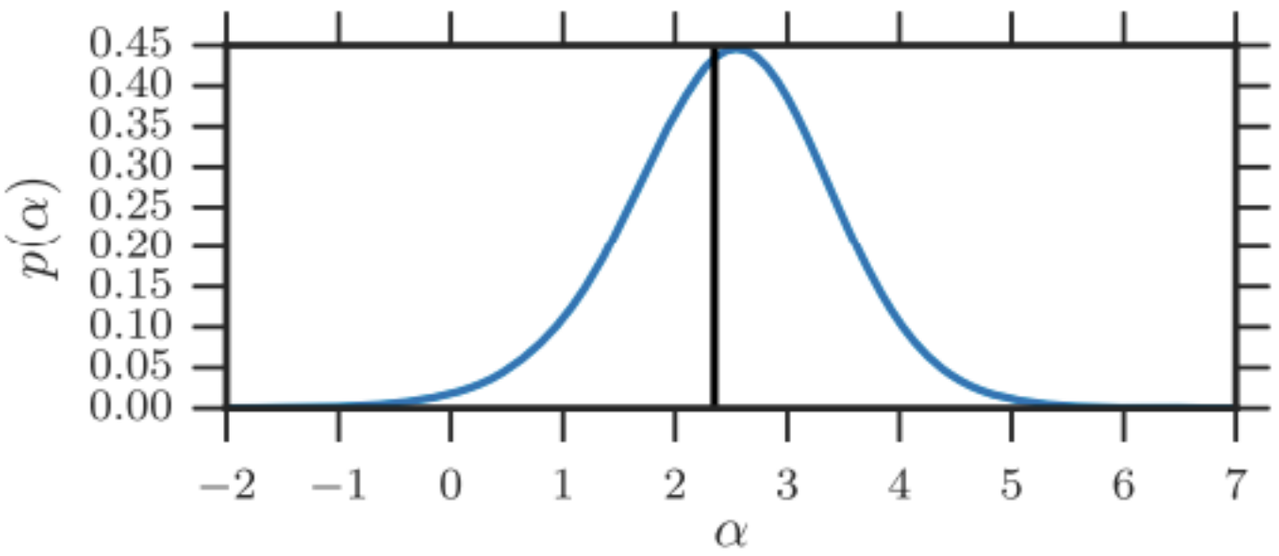
- Recasts the problem as a “higher level” parameterization with **no dependence** on original data {h_i}

Bayesian Hierarchical Modeling

- Example: O1 BBH paper, modeled primary mass distribution as a power law, and inferred the exponent hierarchically

$$p(m_1|\alpha) \propto m_1^{-\alpha}$$

- Used only **three observations** to infer the model of the distribution

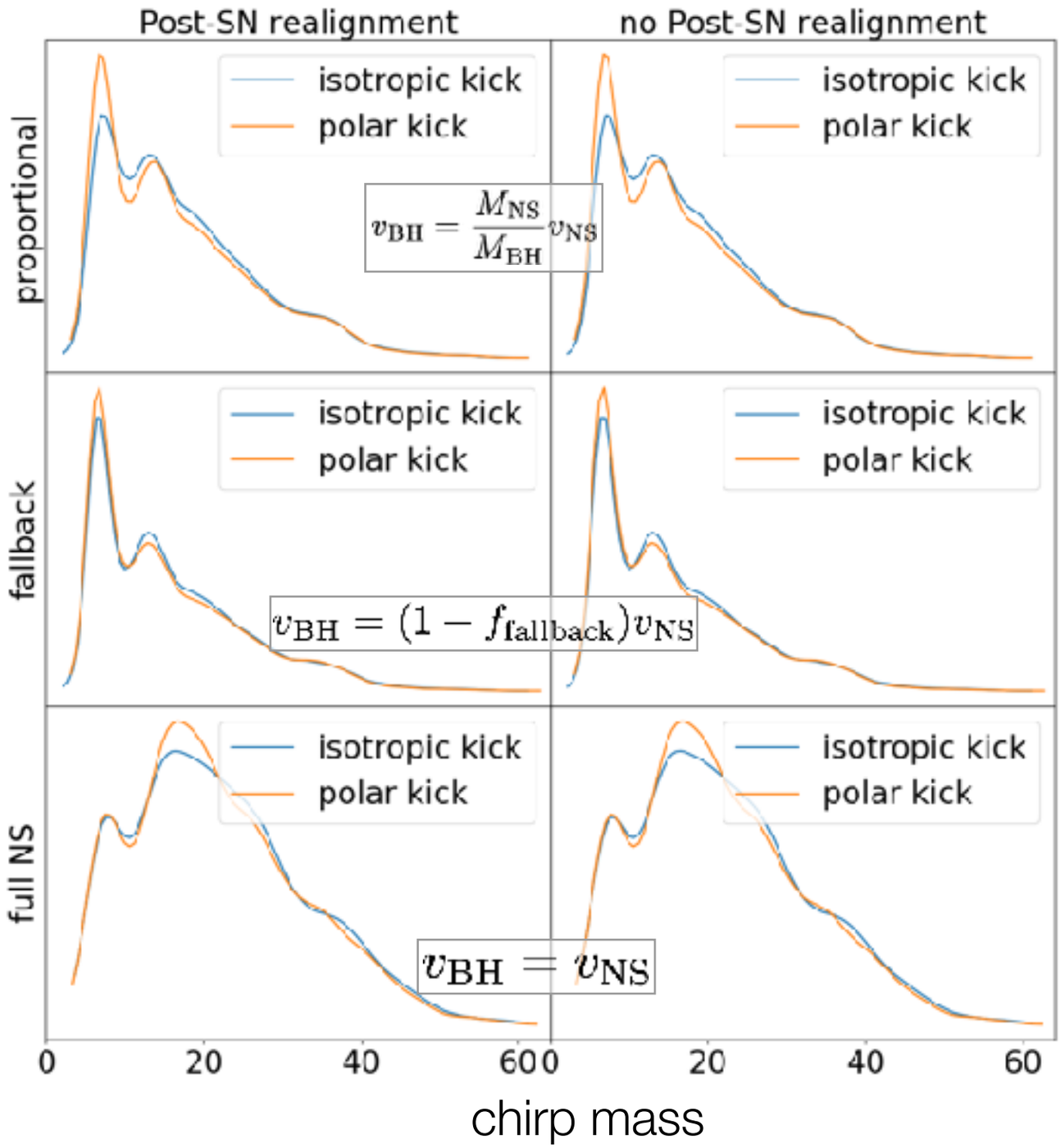


Out of Scope

- Things I won't attempt to answer:
  - How do we quantify *selection bias*: e.g. some subset of populations may be observed more often due to finite instrument sensitivity (FM, et al 2014 does address this)
  - *Catalog contamination*: Assume all events are members of an astrophysical population that we are able to parameterize (see also Farr, et al. 2013 for how to select between *astrophysical* and *terrestrial*)

Formation Scenarios (Field)

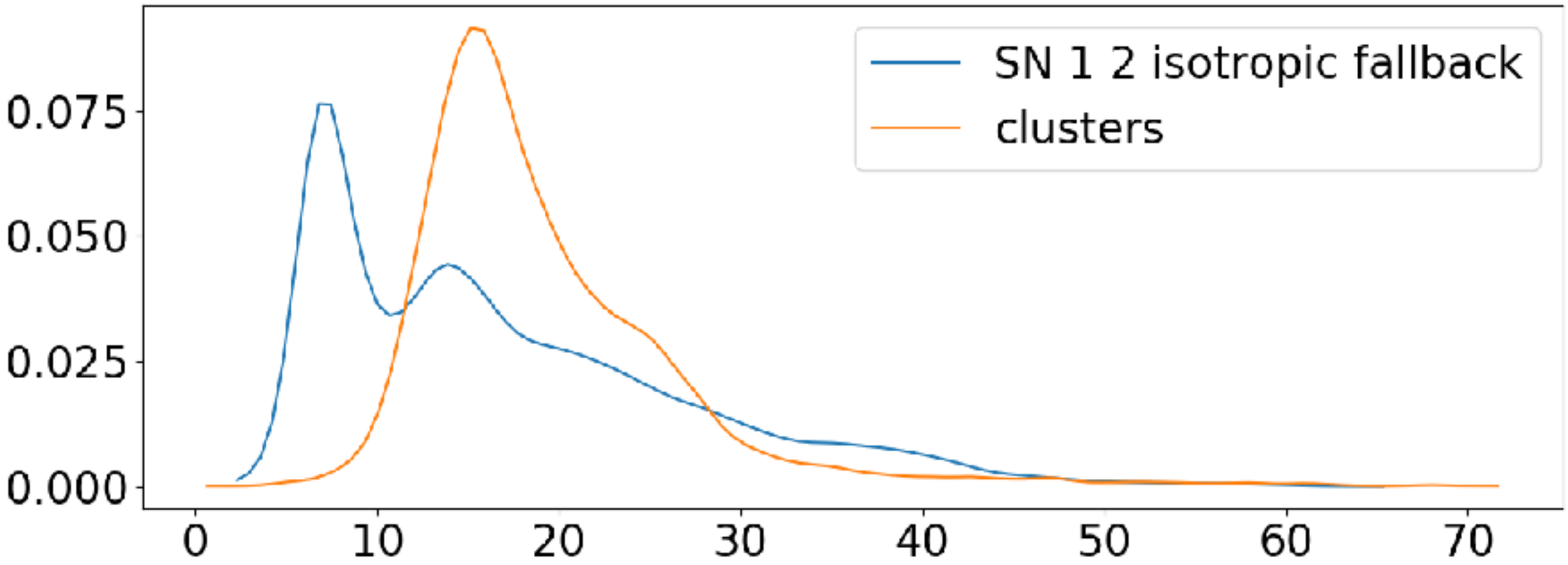
- Field Populations (assume double SN, various kick direction / magnitude prescriptions):
  - native metallicity and SN kick prescriptions lead to typically lower masses (relative to clusters)
  - We consider three kick velocity prescriptions: “full NS”, “proportional", and “fallback”



data from Rodriguez et al. 2016, see references within

Formation Scenarios (Clusters)

- Cluster Populations:
  - Higher peaked mass distribution, total mass up to ~80 $M_\odot$
  - Arbitrary spin alignment (by fiat — no compelling reason for alignment with a given direction)



data from Rodriguez et al. 2016, see references within

Method Sketch

- Parameterize models (M_i) with branching ratios (β_{ij}), quantifying relative abundance ($p(\{\beta\})$ ~ constant):

$$\sum_j \beta_{ij} p(\mathcal{M}_c|M_i) \qquad \sum_j \beta_{ij} \equiv 1$$

- For testing: generate a set of “observations”: e.g. draw binary parameters from a specific model distribution with fixed branching ratios

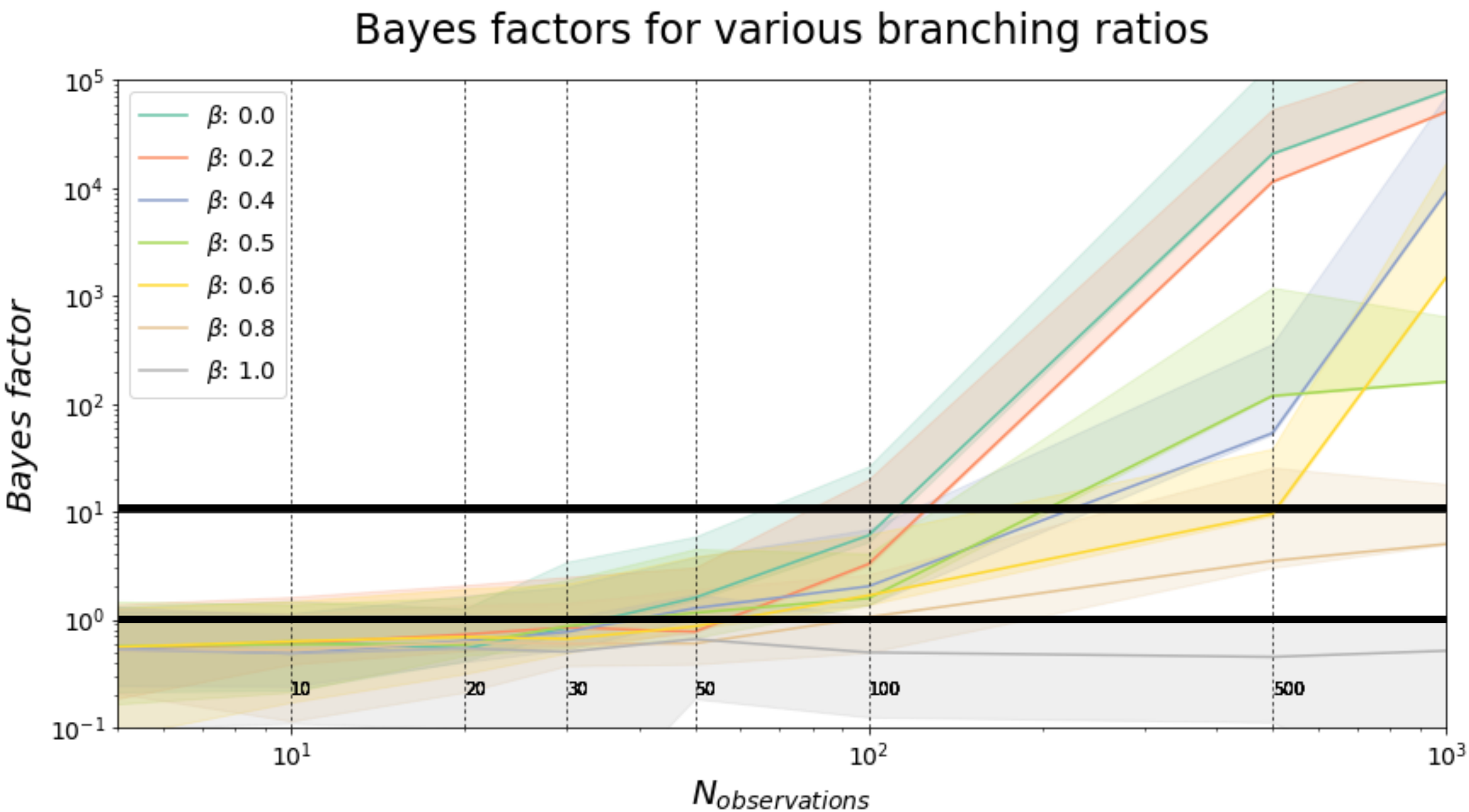
  - Generate sampling distribution, either assume δ function measurement (bad), Fisher matrix approximation (less bad), do full GW PE (computationally expensive for N > a few)

Method Sketch (cont.)

- Employ a reverse jump Markov Chain Monte Carlo (RJMCMC) to simultaneously explore the models **and** model parameter space

- Quantify model "correctness" with modified Bayes factor (BF) $p(M_i)/(1-p(M_i)) \rightarrow N_i/(\Sigma_{i \neq j} N_j)$ and distributional fraction posteriors ($p(\beta_{ij}|M_i)$, assuming prior odds on model is unity)

Bayes Factor vs. N_{obs} plot

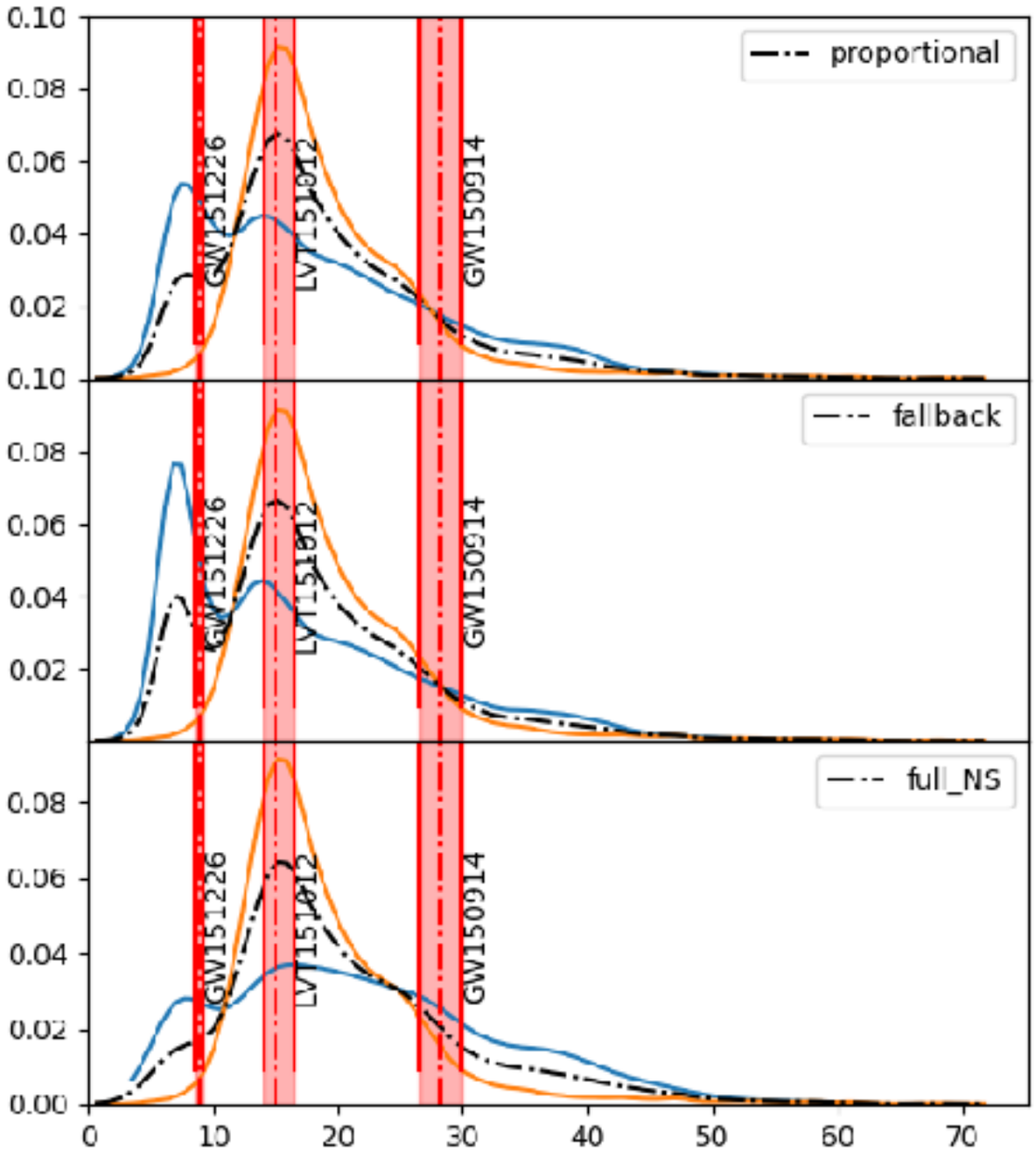


With Real Observations…

…for so few observations,
it's a bit overkill…

Current State of Affairs

- GW150914, GW151226, and LVT151012 90% confidence regions over our fiducial models if each “channel” has equal weight

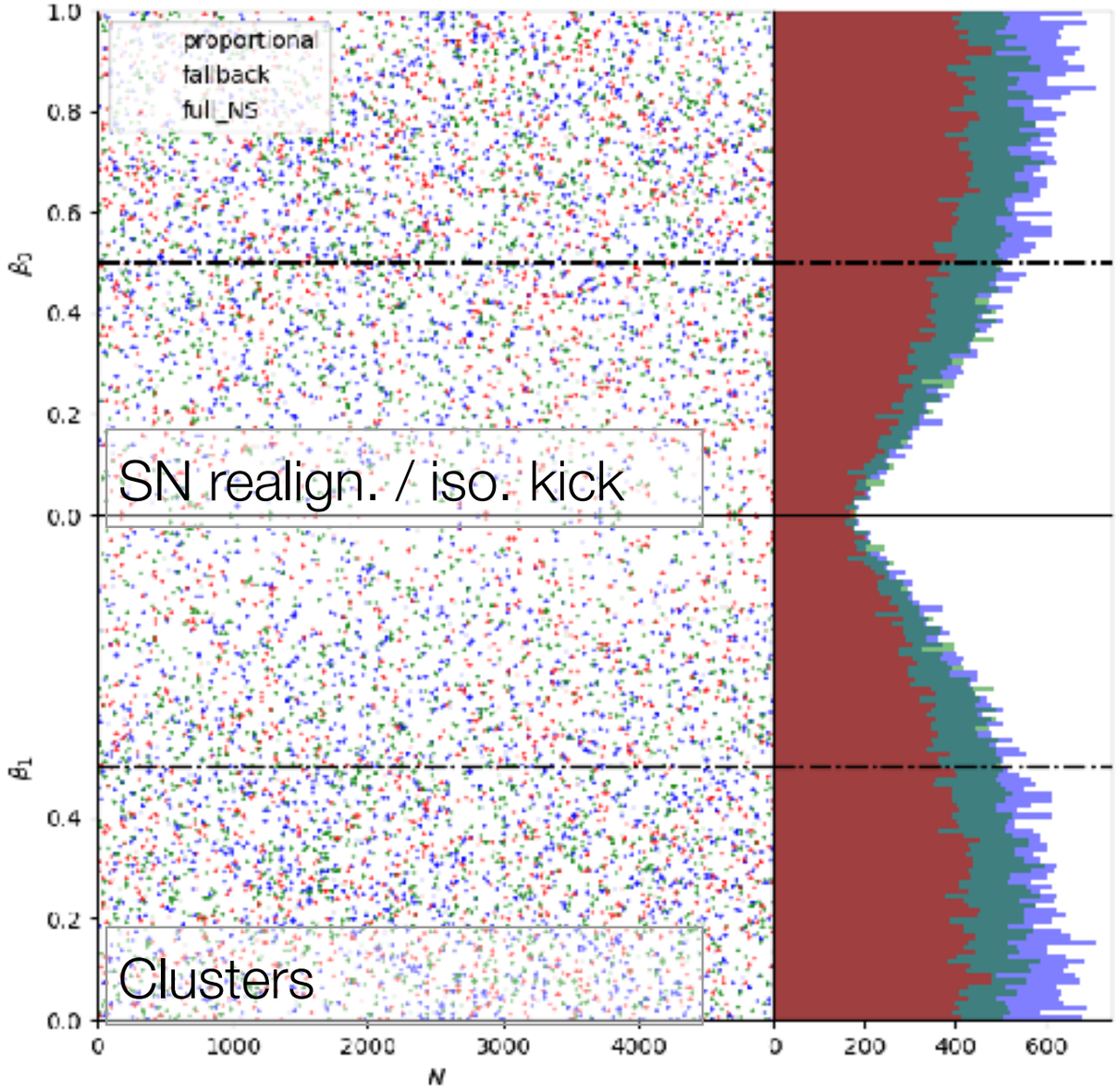


90% conf. interv.	GW150914	GW151226	LVT151012
$\mathcal{M}_c$	$28.1^{+1.8}_{-1.5}$	$8.9^{+0.3}_{-0.3}$	$15.1^{+1.4}_{-1.1}$

Data from PRX 6 *0141015* (2016)

Current State of Affairs

- 2 parameters: clusters + isotropic field model



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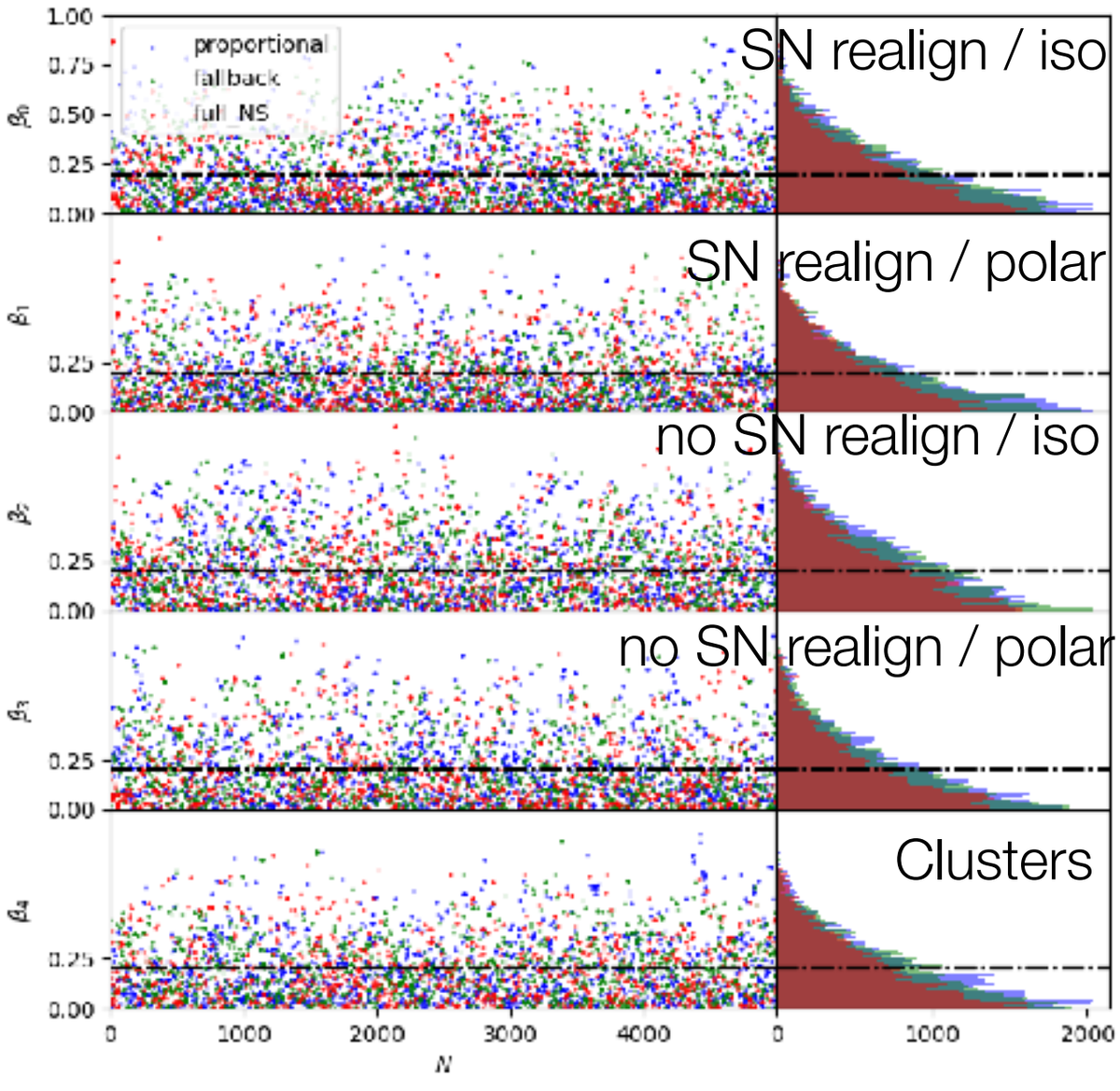
Data from PRX 6 0141015 (2016)

	proportional	fallback	full NS
samples acquired	48743	44641	34616

Current State of Affairs

- Five parameter clusters + 4 field prescriptions



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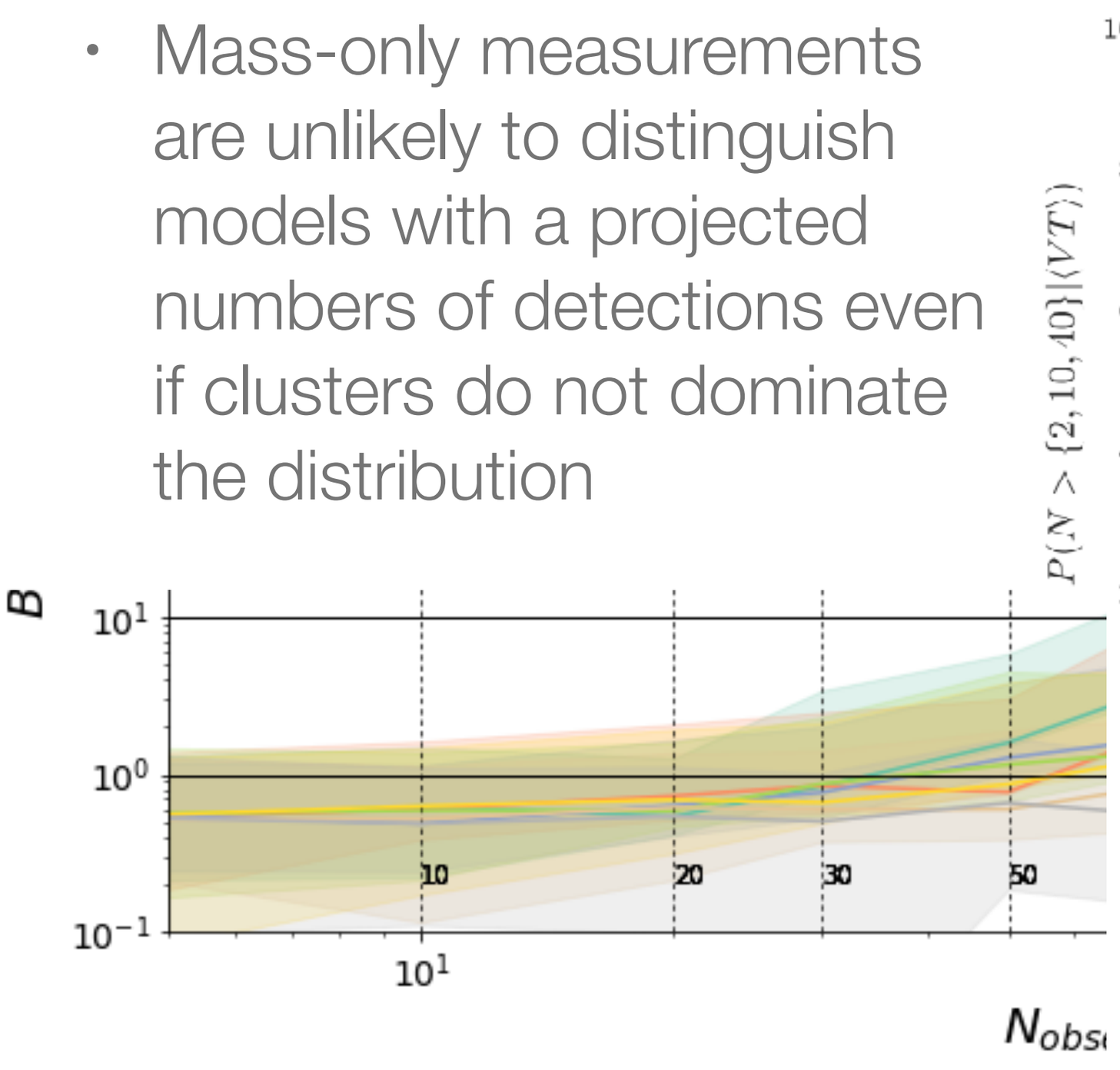
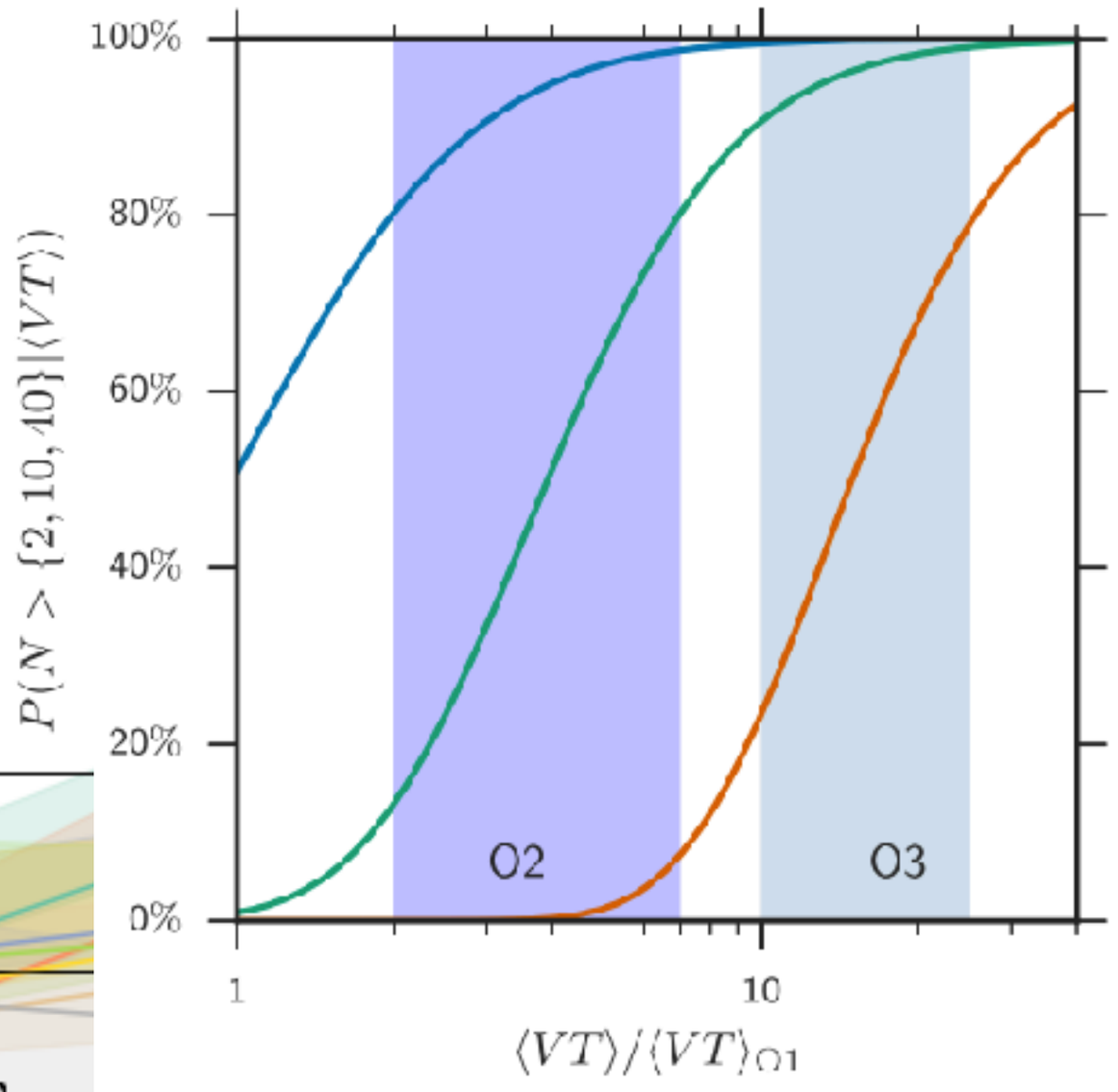
Data from PRX 6 0141015 (2016)

	proportional	fallback	full NS
samples acquired	46772	45168	36060

Ramifications

- Mass-only measurements are unlikely to distinguish models with a projected numbers of detections even if clusters do not dominate the distribution

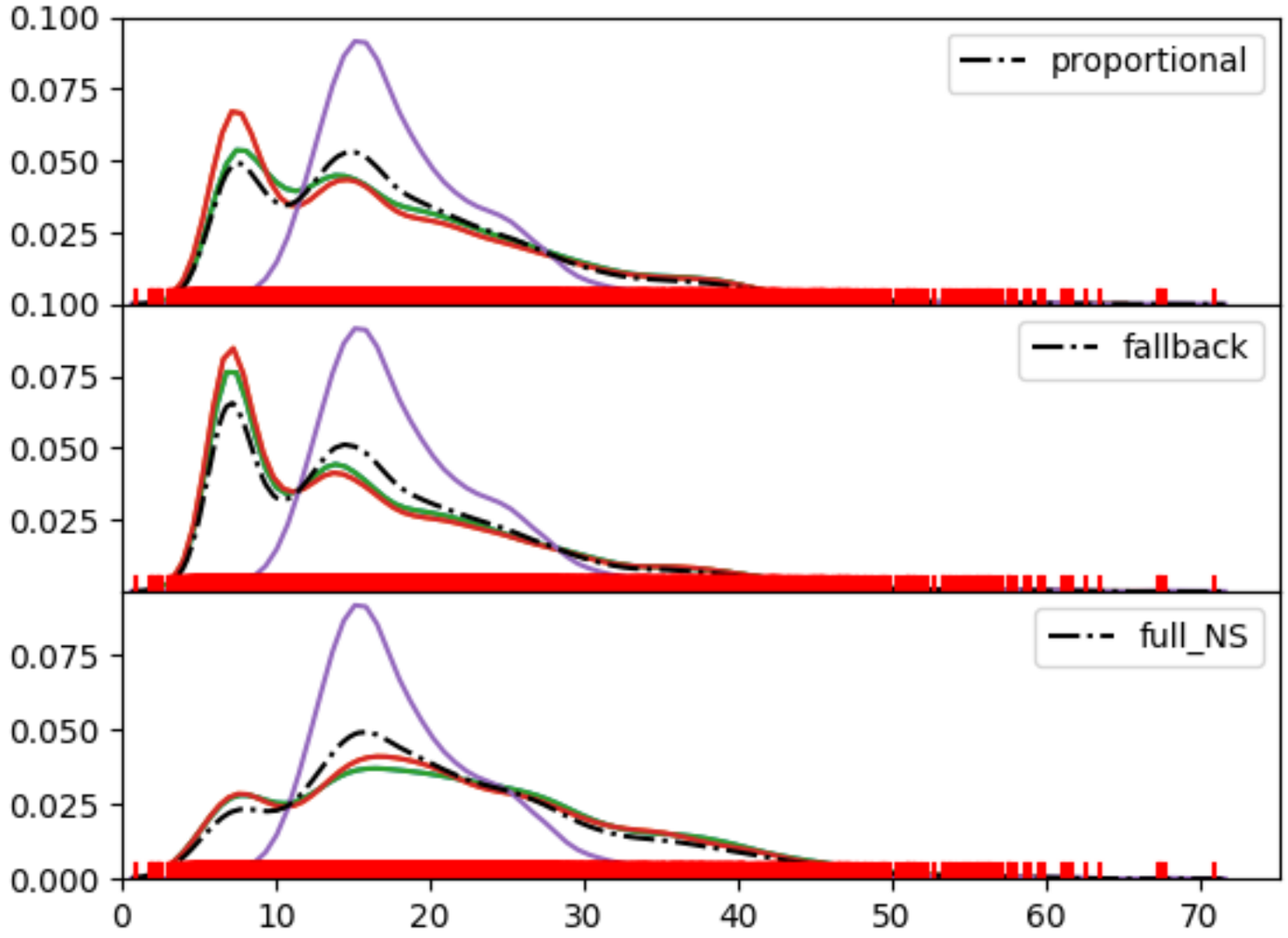




Beyond Two Parameter Models

- Are kick *direction* prescriptions (**isotropic / polar**) measurable at the level of mass spectrums?

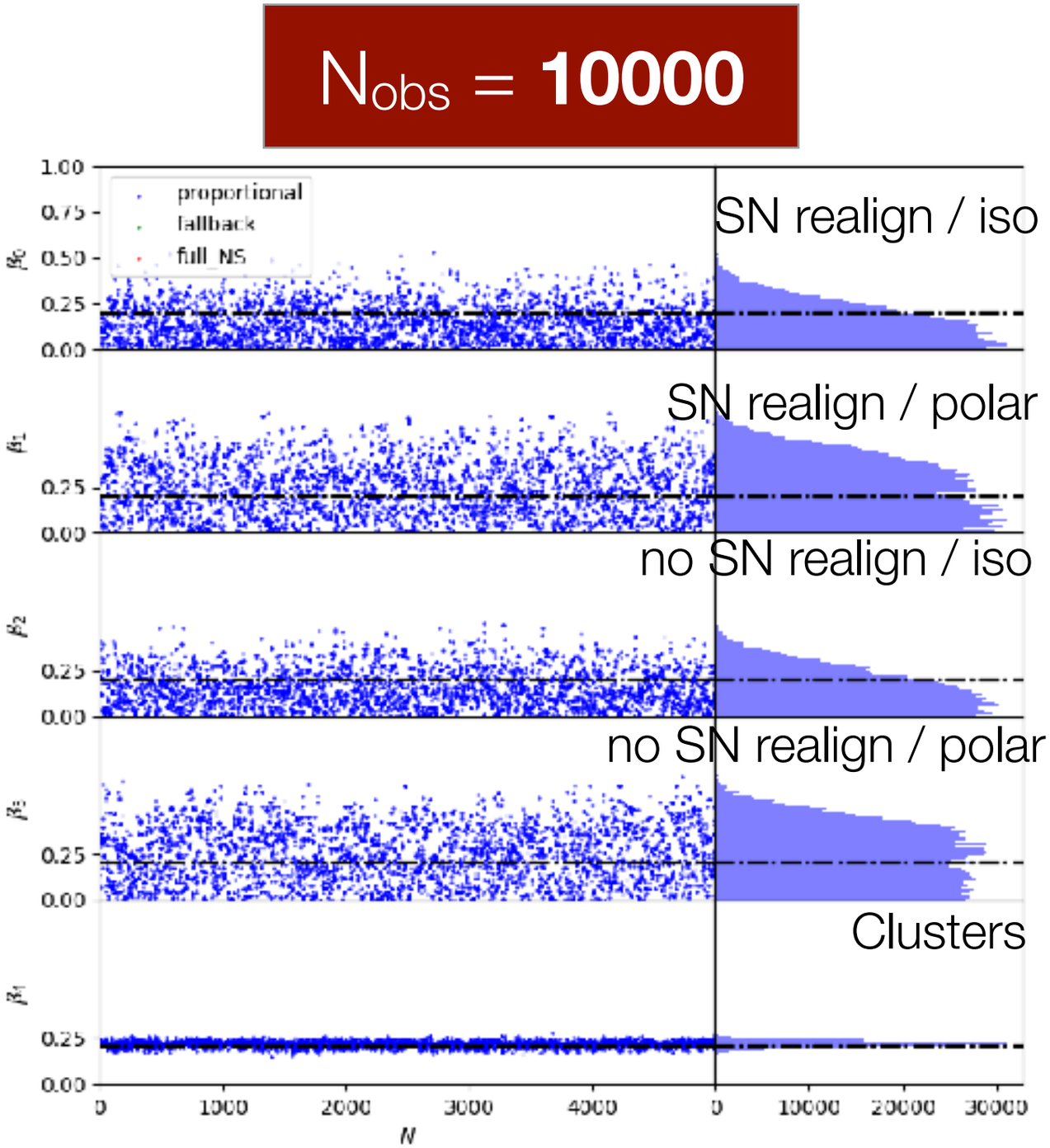
$N_{obs} = 10000$



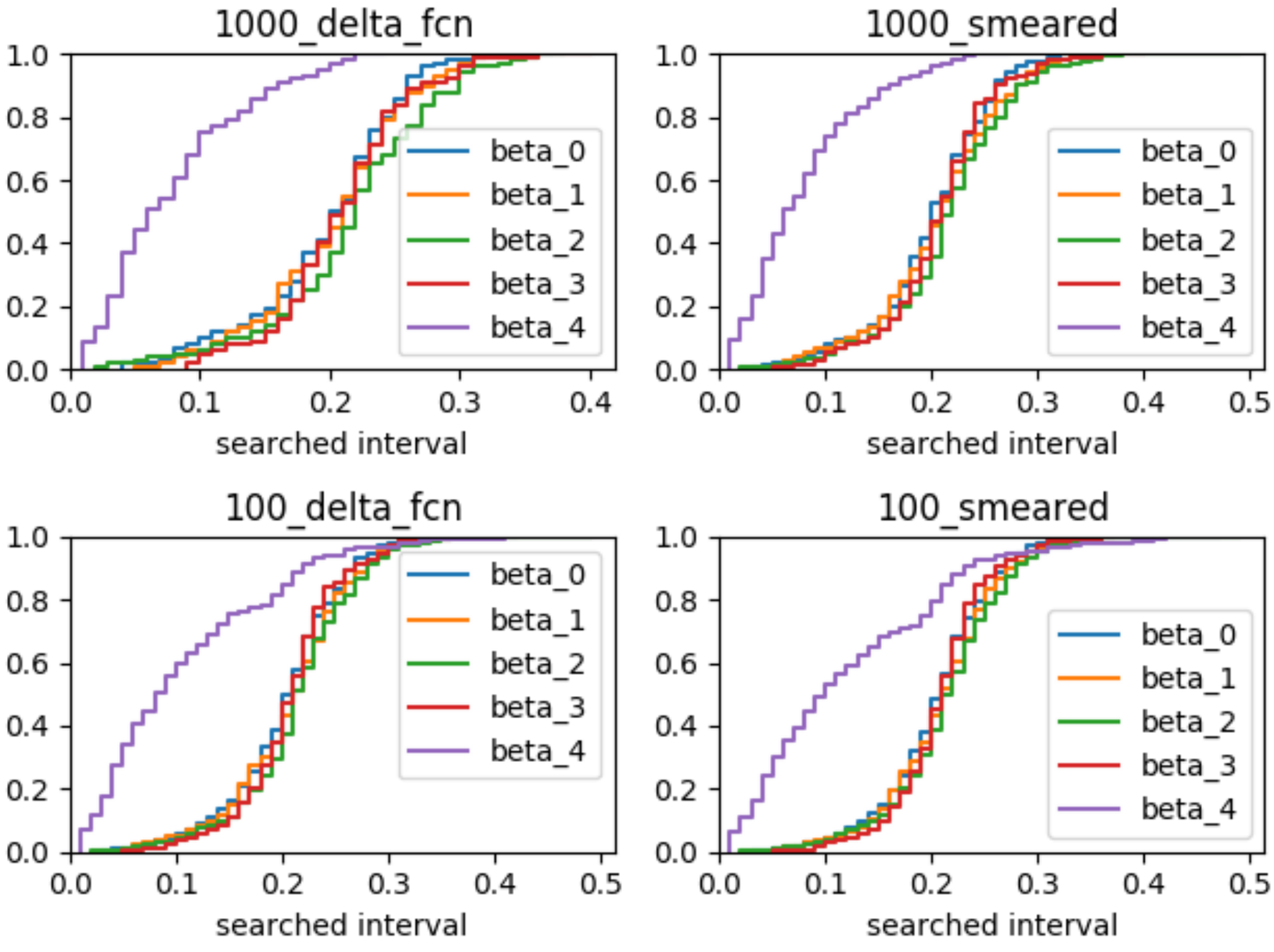
Beyond Two Parameter Models

- Are kick *direction* prescriptions (**isotropic / polar**) measurable at the level of mass spectrums?

- **Spoilers**: *No*. Most mass spectrums are degenerate, and spins (Stevenson, et al. 2017, Rodriguez, et al. 2016) are required



Estimation Accuracy



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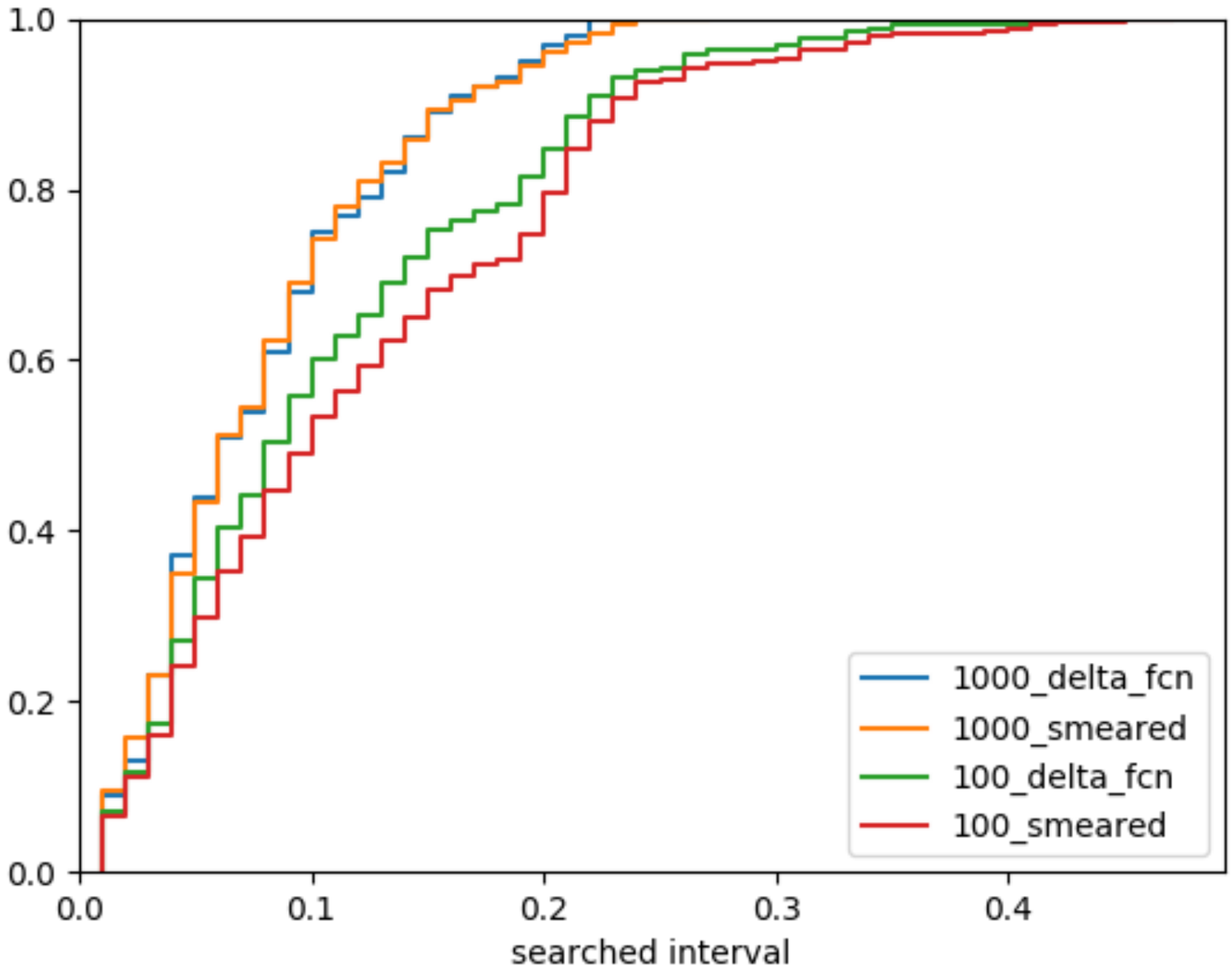
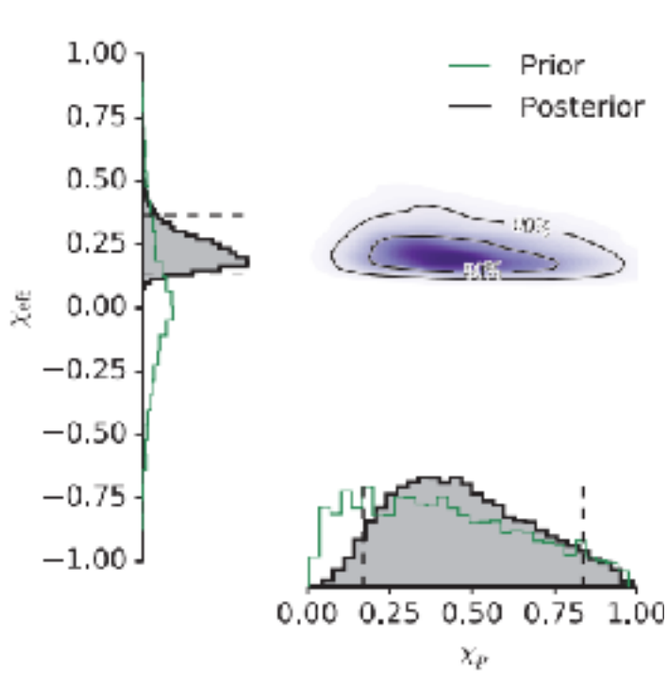
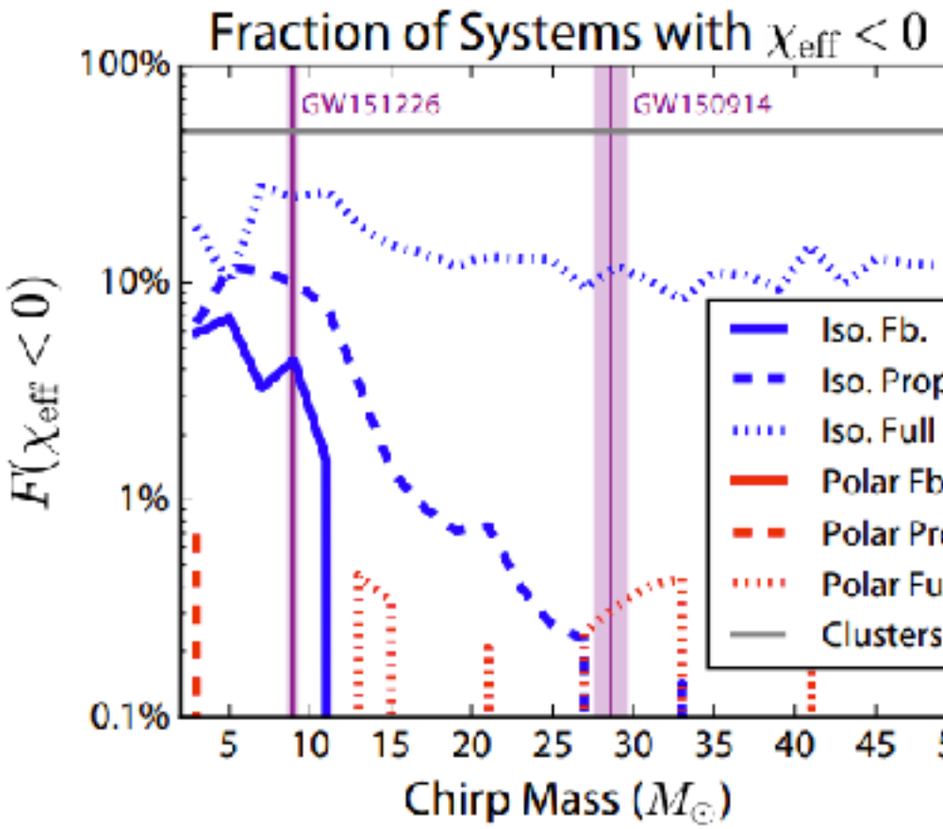
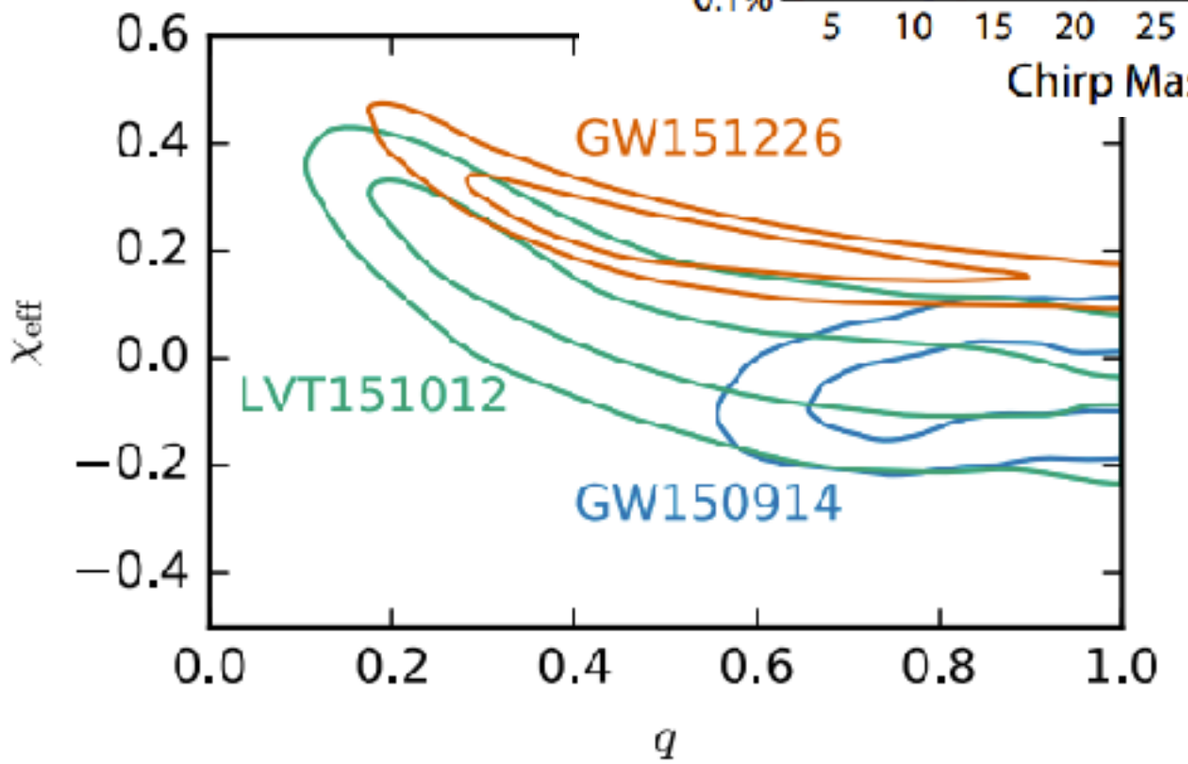


Including Other Parameters

- Rodriguez, et al. 2016 (earlier), Stevenson et al. 2017 (next talk!)
- Isotropic spin directions in clusters vs near alignment in field models
- Mass ratios probably have similar degeneracies, but could be sufficiently different
- Break many degeneracies, but χ_{eff} is not bounded away from 0 in most GW observations







Next Steps / Speculation

- Include mass ratio and spin
- Include more prescription effects in clusters (correlate prescriptions across formation channels)
- Translate fractions to actual rate estimates
- Incorporate redshift (e.g. R(z) dependence!)
- Residual eccentricity — but GW PE does not yet robustly measure these quantities (see limits discussed in GW astrophysics paper)
- Folding in p(astro)/p(terrestrial) — downweight contamination from terrestrial false alarms in GW interferometers

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