Center for Computational Relativity and Gravitation
R·I·T

Measuring the imprint of spin in the strong field

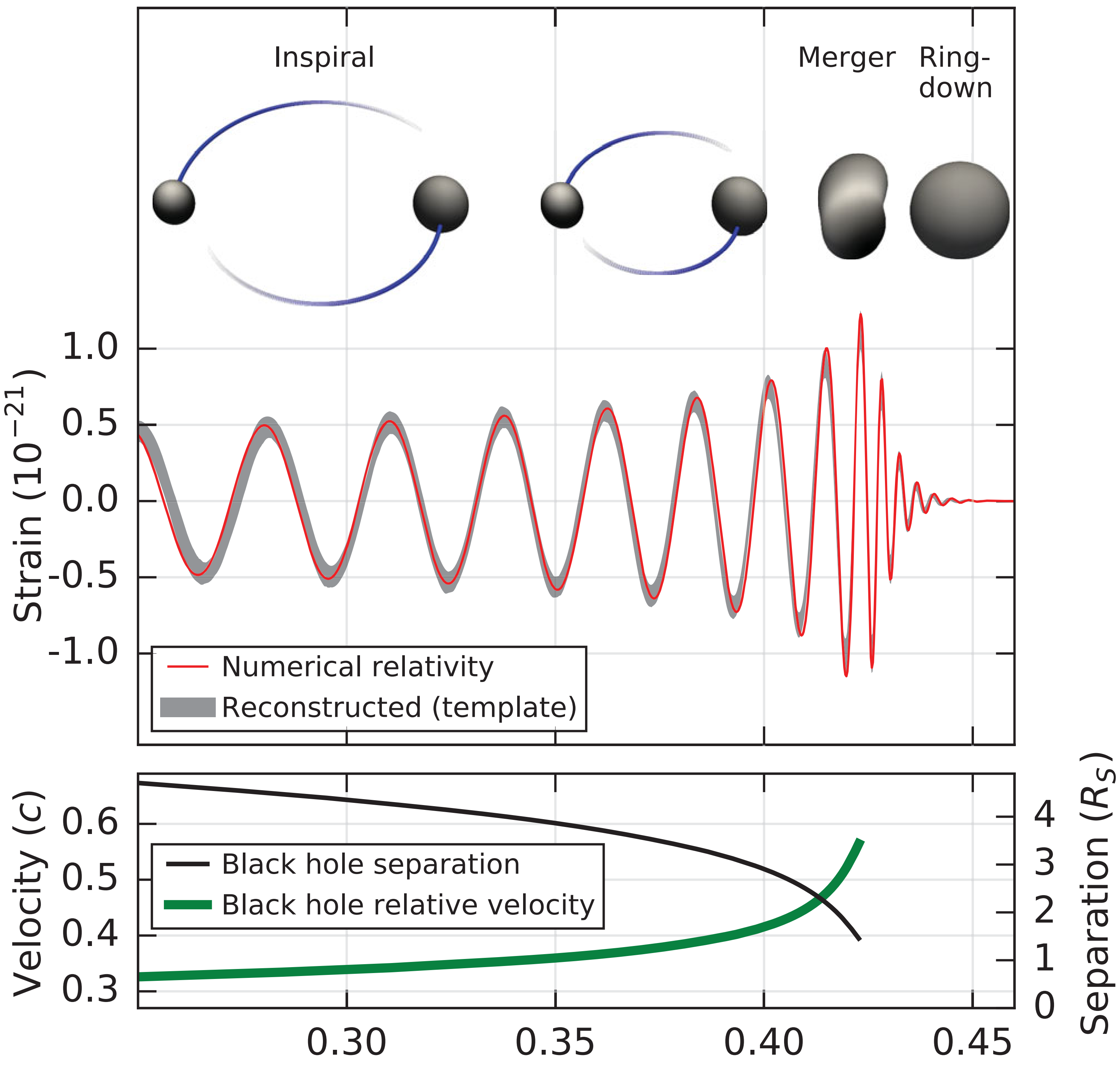
Richard O'Shaughnessy

Strong Gravity and Binary Dynamics
2017-02-28 Oxford, MS

Outline

- What is the imprint of spin on gravitational waves?
  - …and do we know what it is well enough for our purposes?
- Parameter estimation
  - Review
  - Measuring imprint of spin at low mass, with long signals
  - Measuring imprint of spin at high mass, with short signals

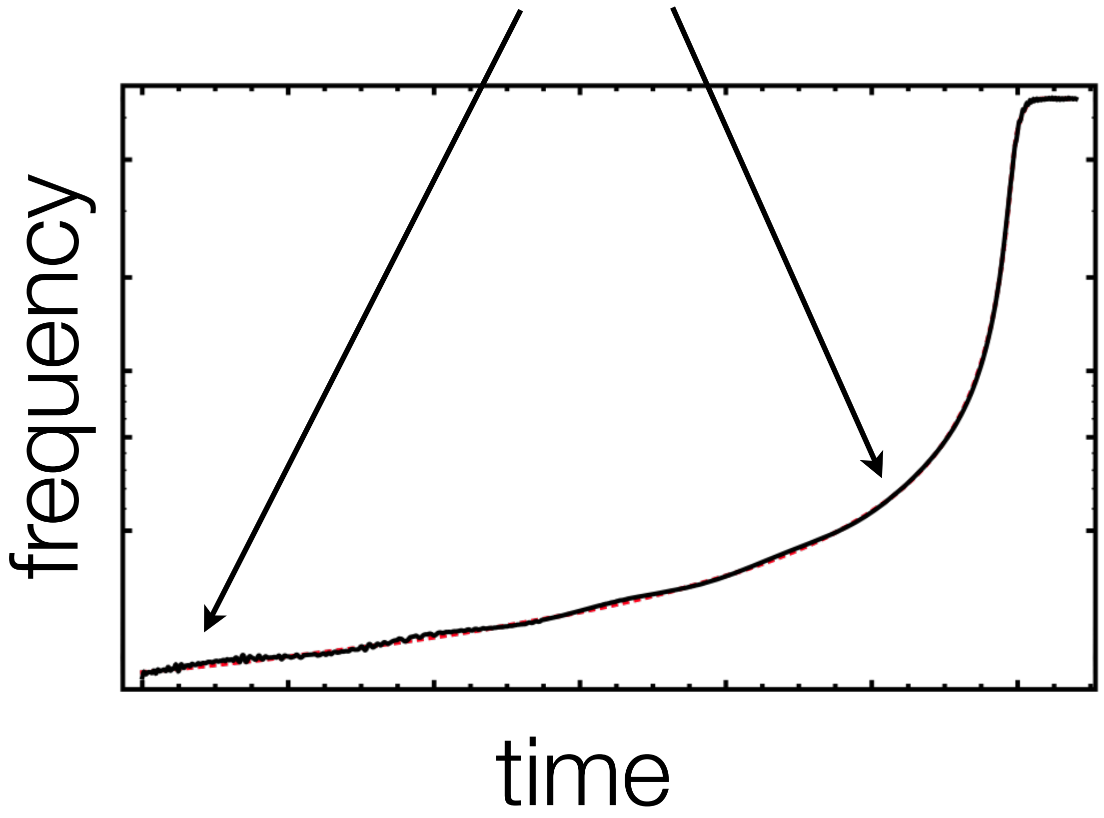
Basics of inspiral, merger, and ring down

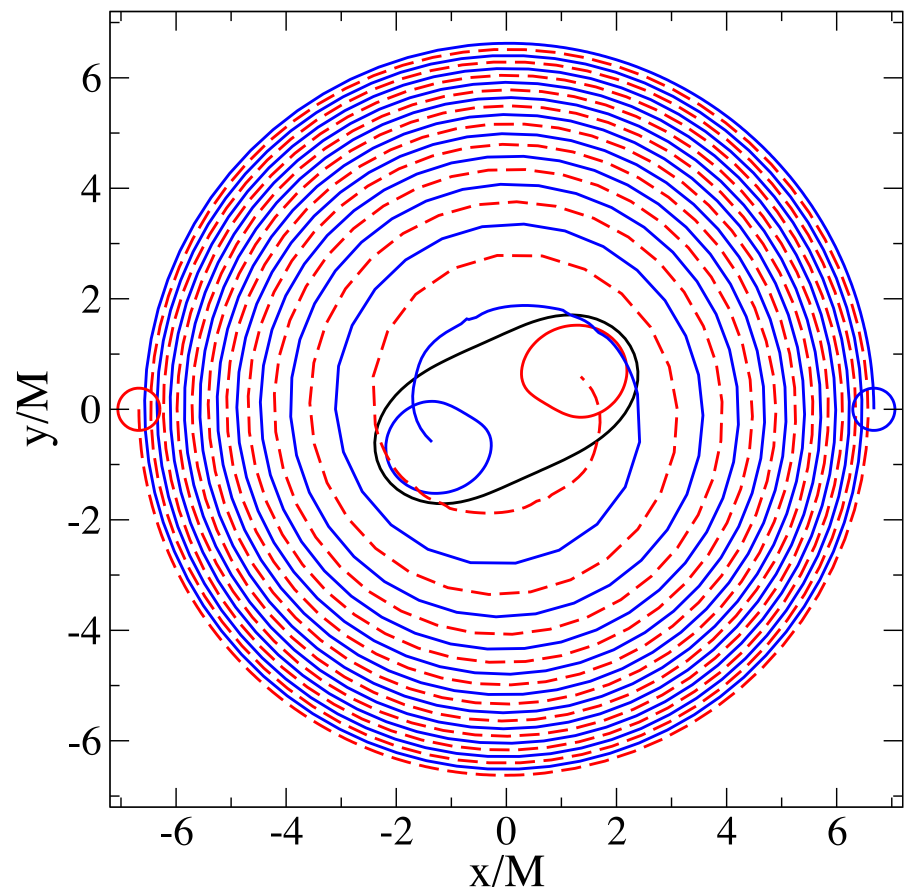


Abbott et al, PRL 116, 061102 (2016)

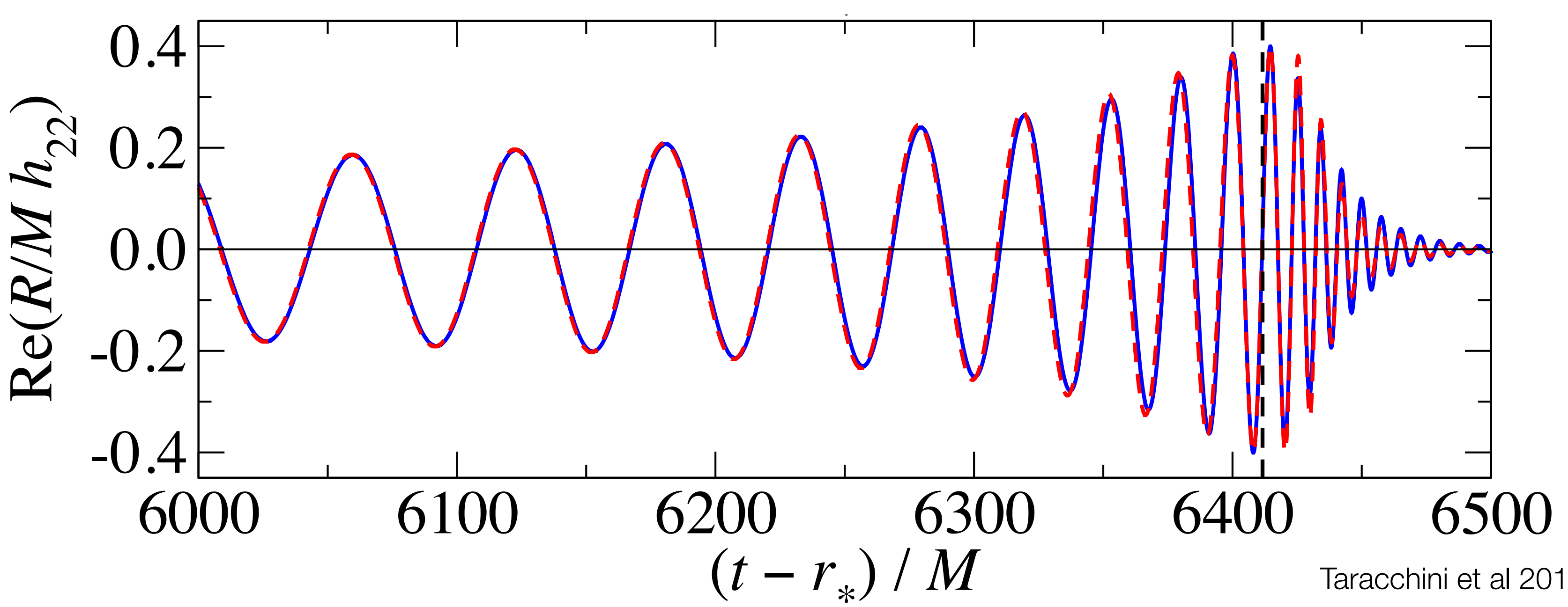
Binary inspiral and merger

- Shrinking binary spirals in, “chirping”





Chu et al 2009 (0909.1313)

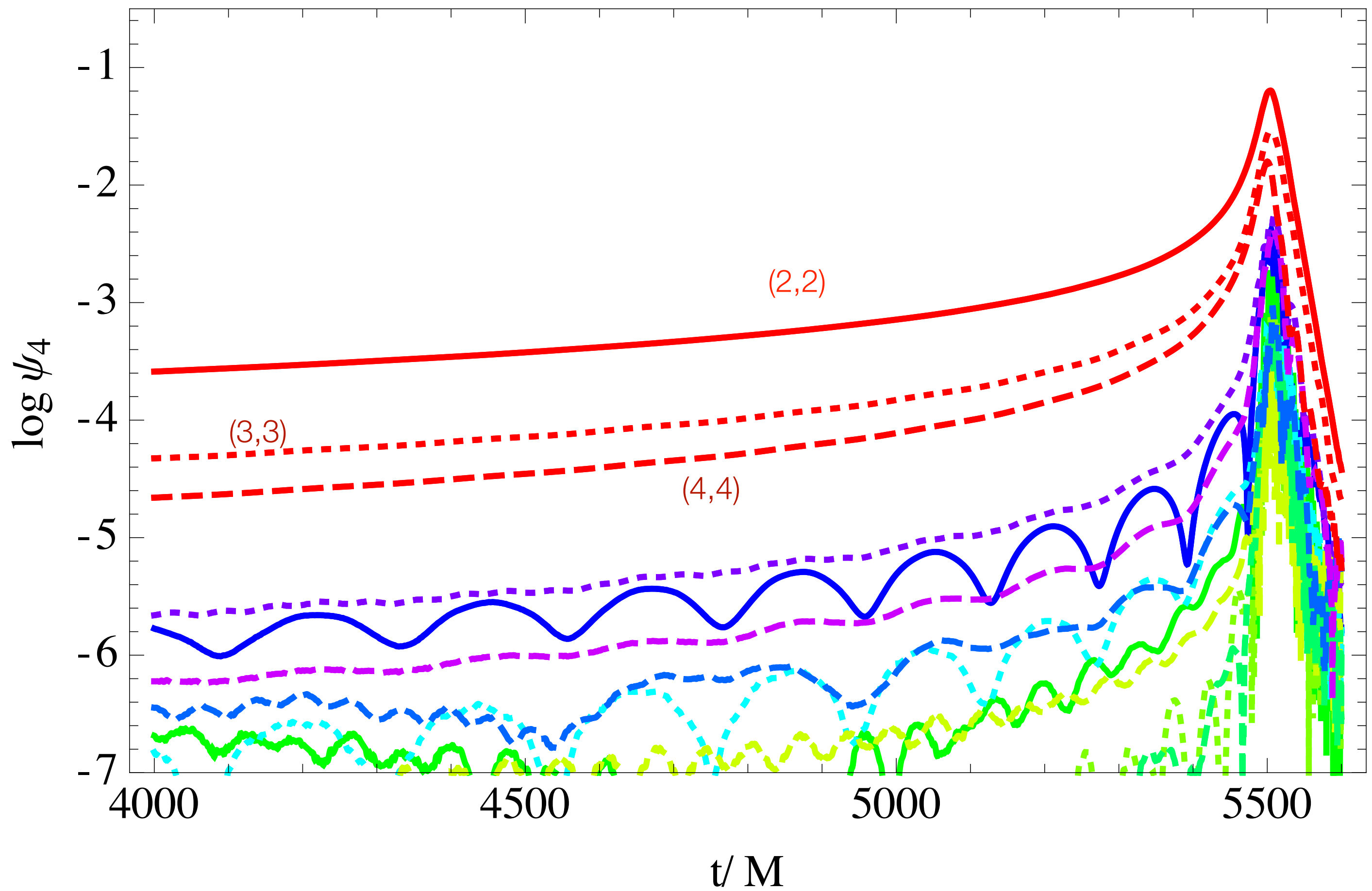


Taracchini et al 2013 (1311.2544)

Higher-order modes

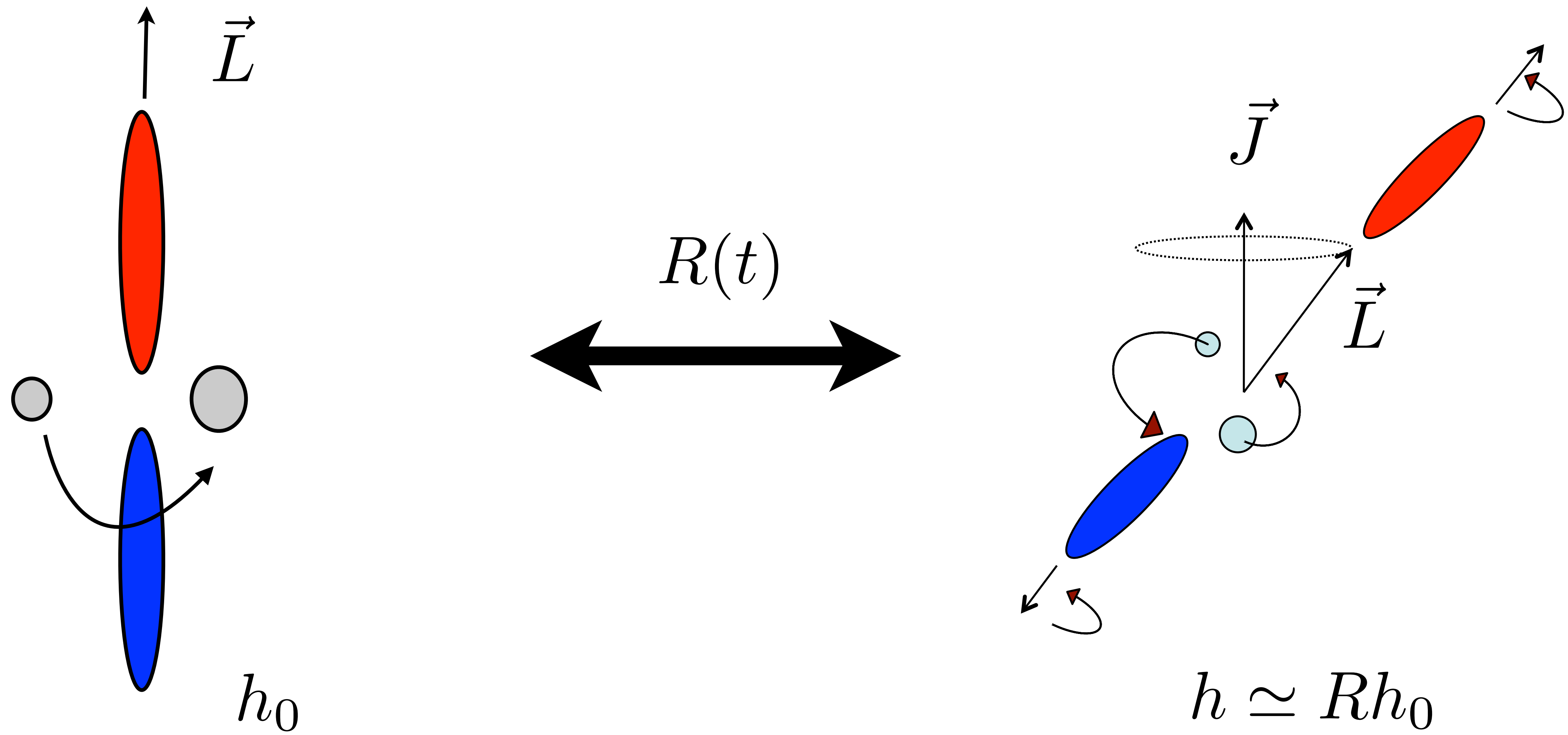
- Strong field mergers complicated: **not** simple quadrupole

$$h(t|\hat{n}) = \sum_{lm} {}_{-2}Y_{lm}(\hat{n}) h_{lm}(t)$$
$$\simeq h_{22}(t)_{-2}Y_{22} + h_{2,-2}(t)_{-2}Y_{2,-2} + 0$$



RIT GW150914-like simulation

Basics of precession-induced modulations



Radiation from precessing binary

~ rotation x (radiation from nonprecessing)

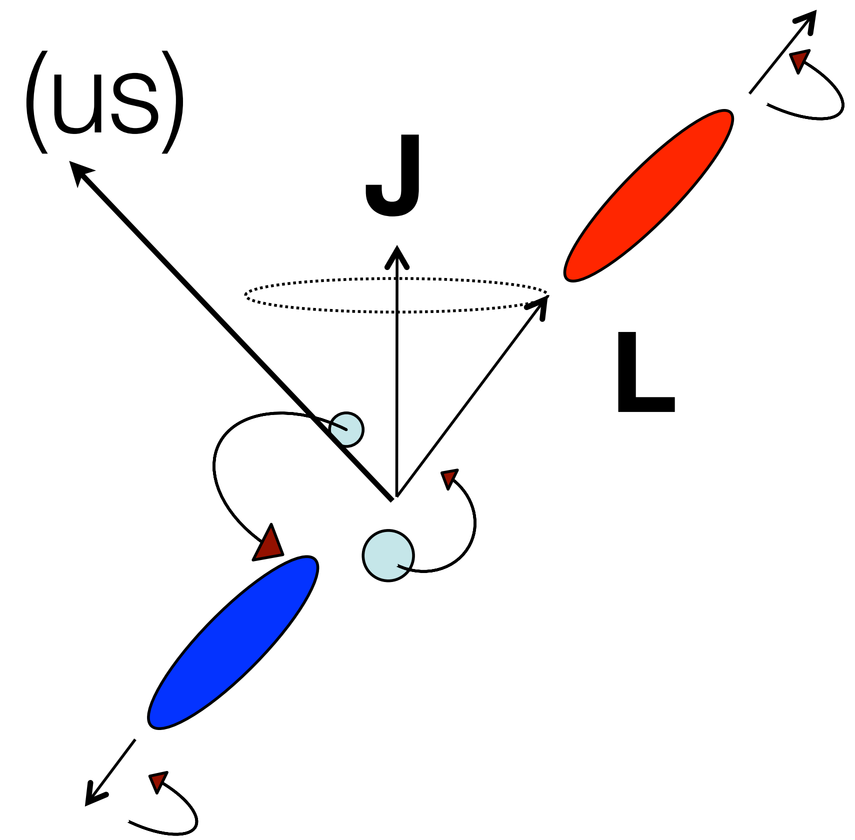
Schmidt et al 2011
ROS et al 2011 [arxiv: 1109.5224]
Boyle et al 2012
Ochsner and ROS 2012 [arxiv:1205.2287]

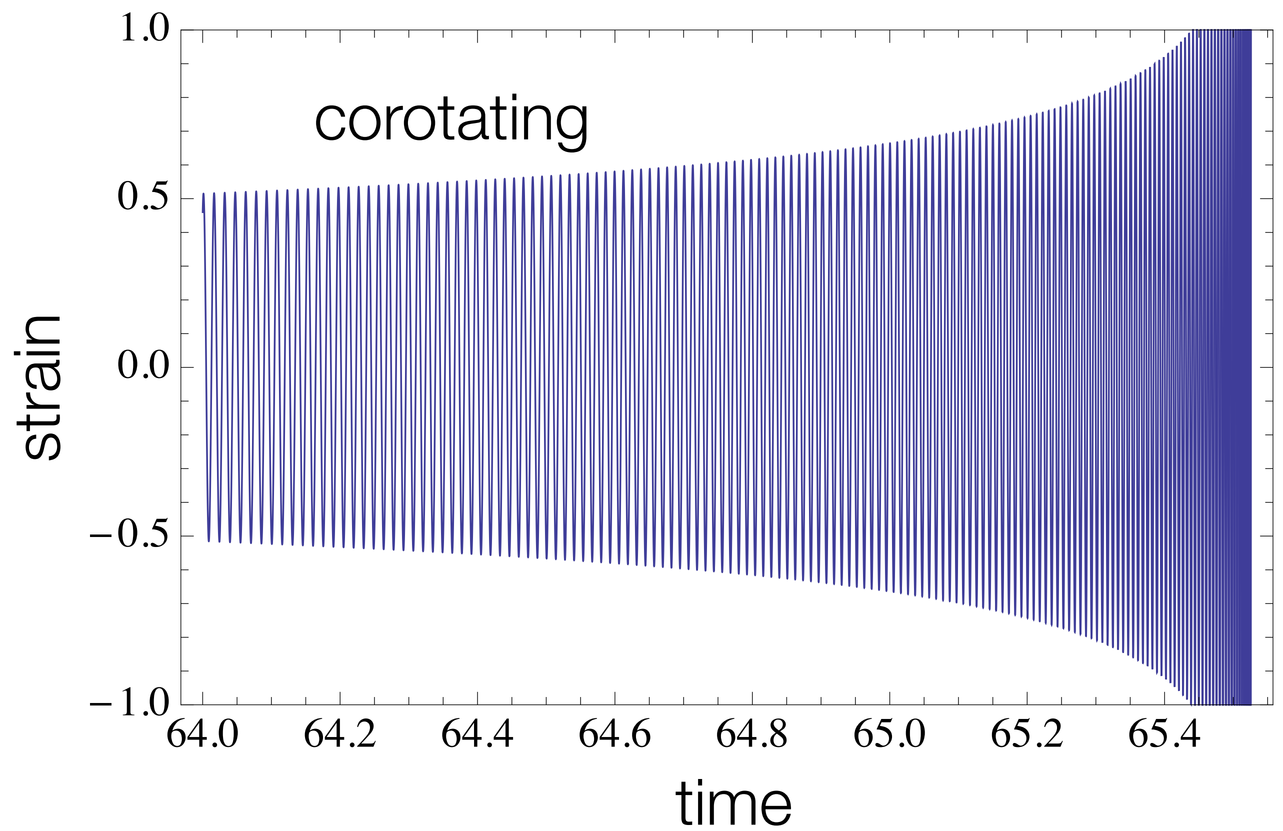
Dynamics of and GW from a BH-NS

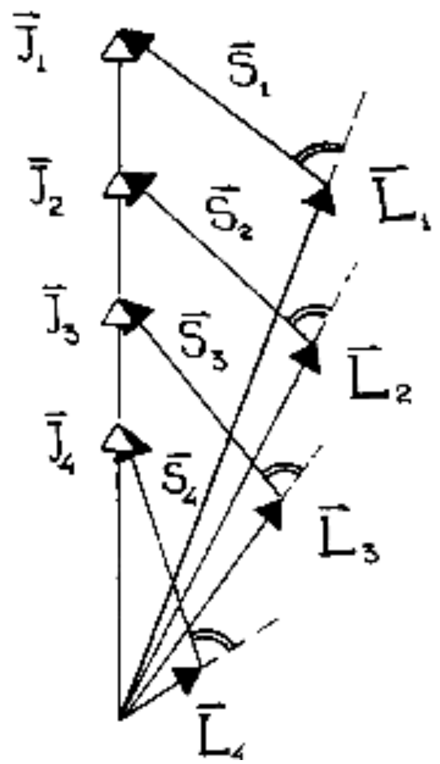
- What

$$m_1, m_2 = 10, 1.4 M_\odot \; ; \chi_1 = 1$$

- Dynamics:
  - spin, precession significant at >40 Hz
- GW
  - (corotating chirp) x (slow rotation)







Apostolatos et al 1994

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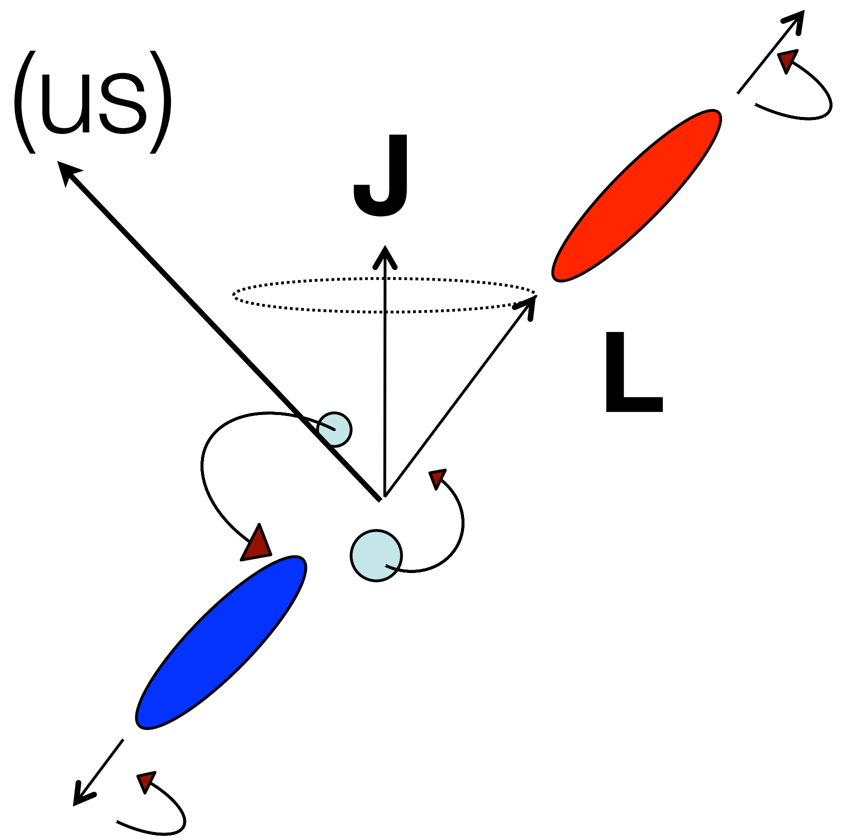
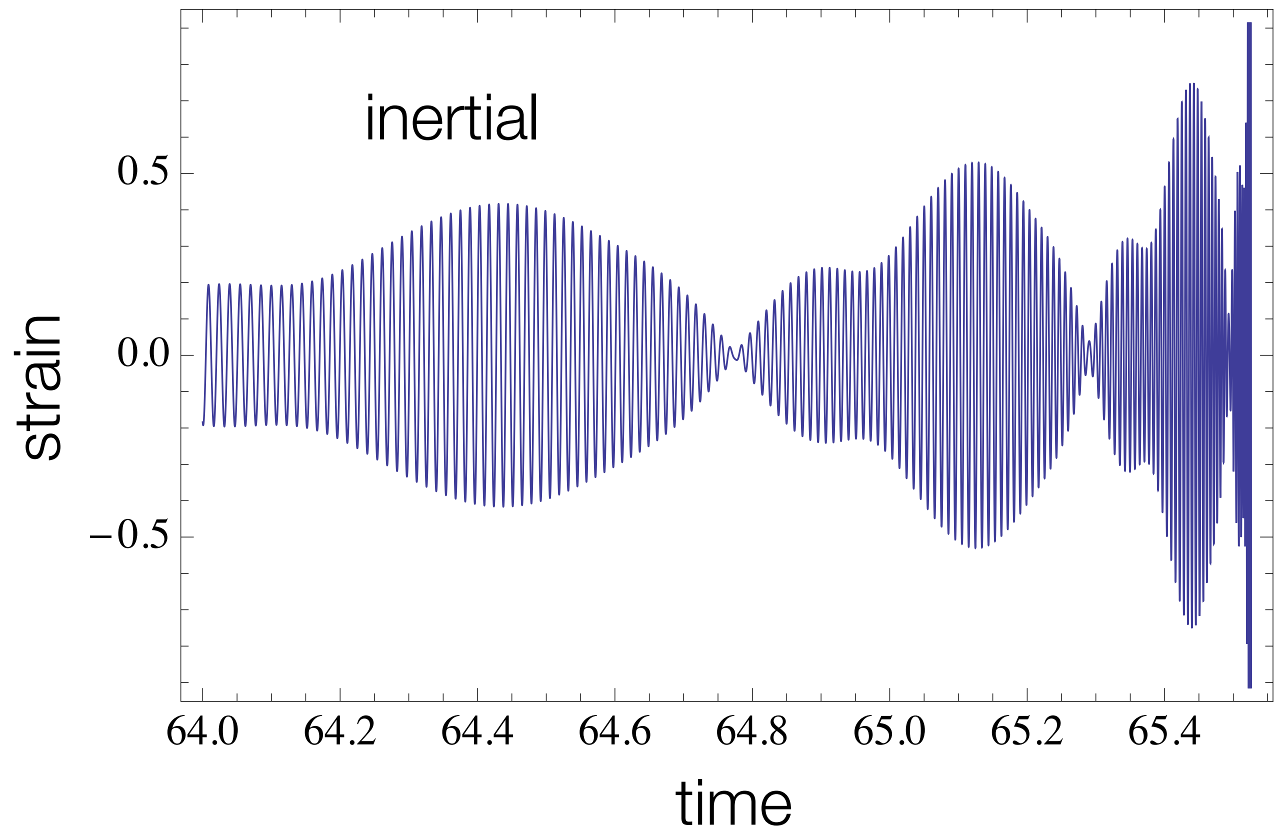
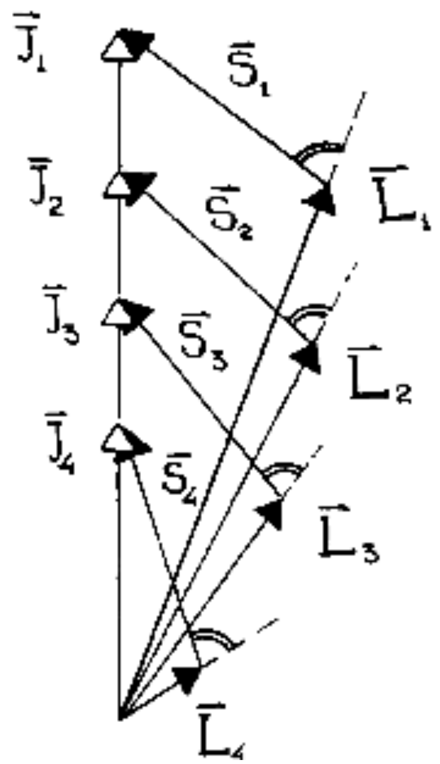


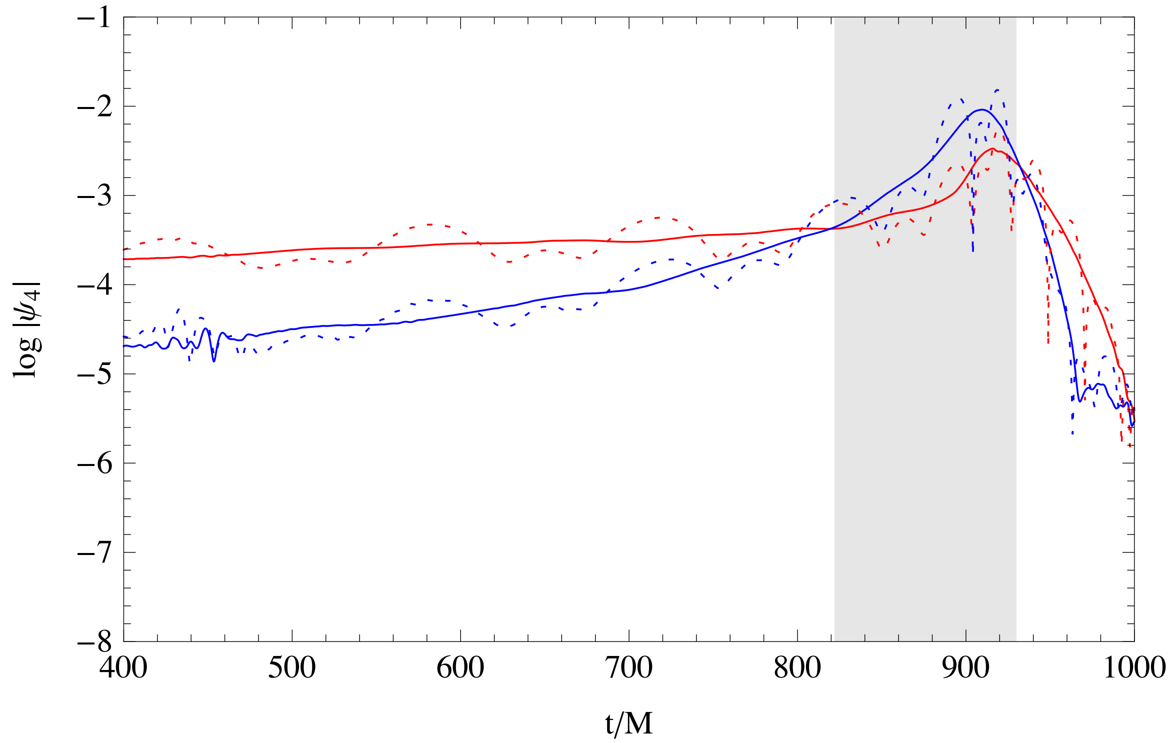


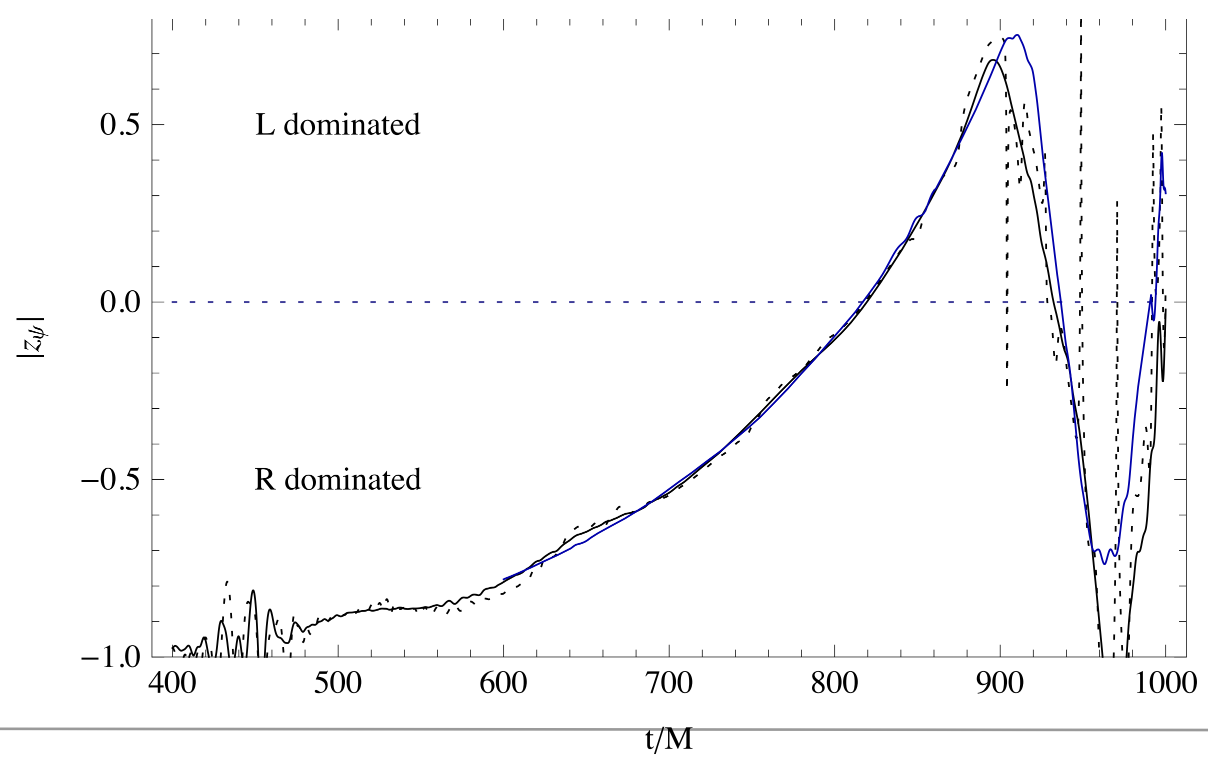


Apostolatos et al 1994

Rotation, polarization modulation robust

- **Including merger, higher modes**





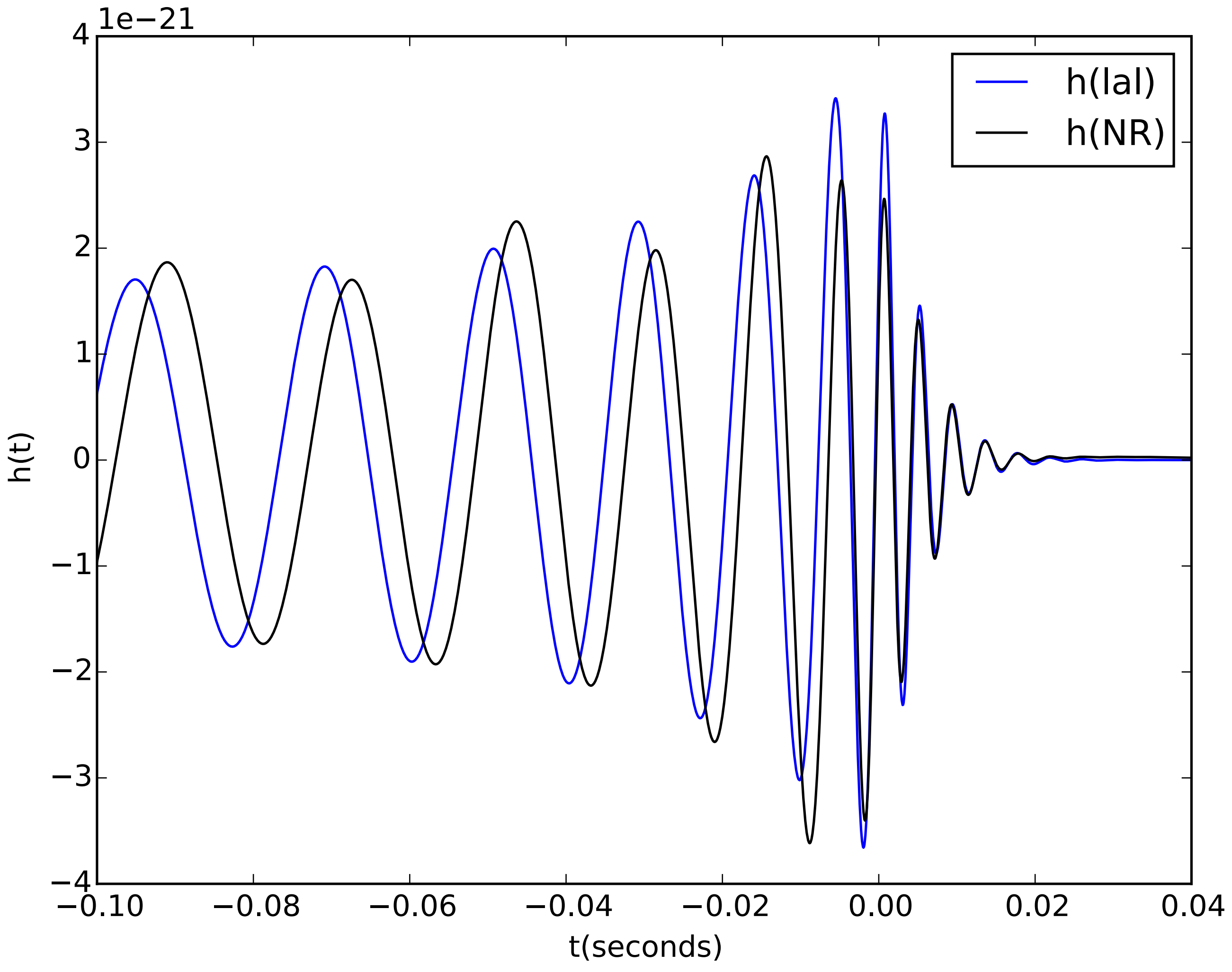
ROS et al 2013

NR solves GR more completely, accurately

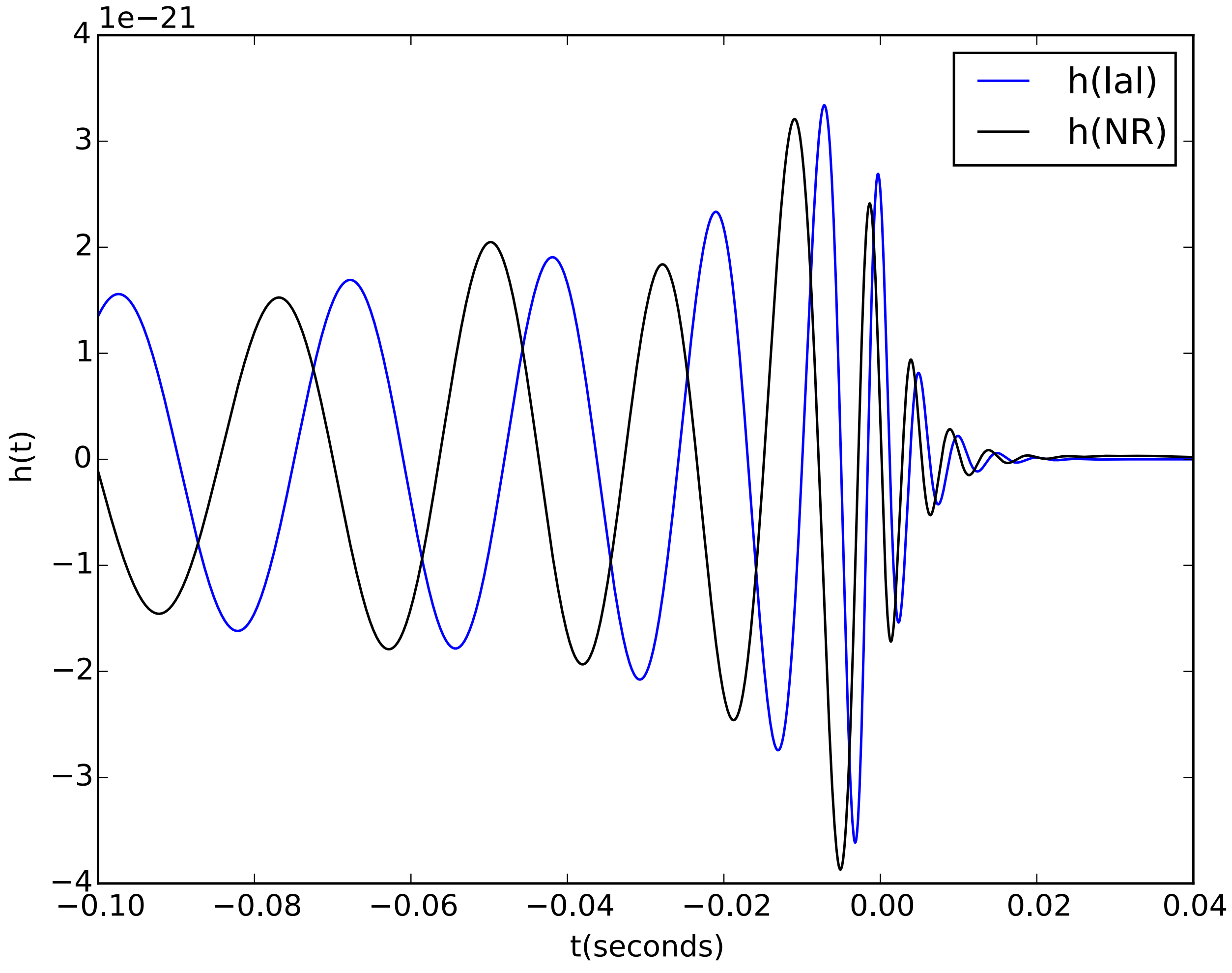
- Analytic models are good first approximations **but not perfect**

- Example: Edge-on line of sight

$$q = 2.0, a = 0.0, M = 70M_\odot$$



$$q = 2.0, a = -0.8, M = 70M_\odot$$



NR solves GR more completely, accurately

- One reason: “higher modes” are missing or not calibrated

$$h(t|\hat{n}) = \sum_{lm} {}_{-2}Y_{lm}(\hat{n})h_{lm}(t)$$
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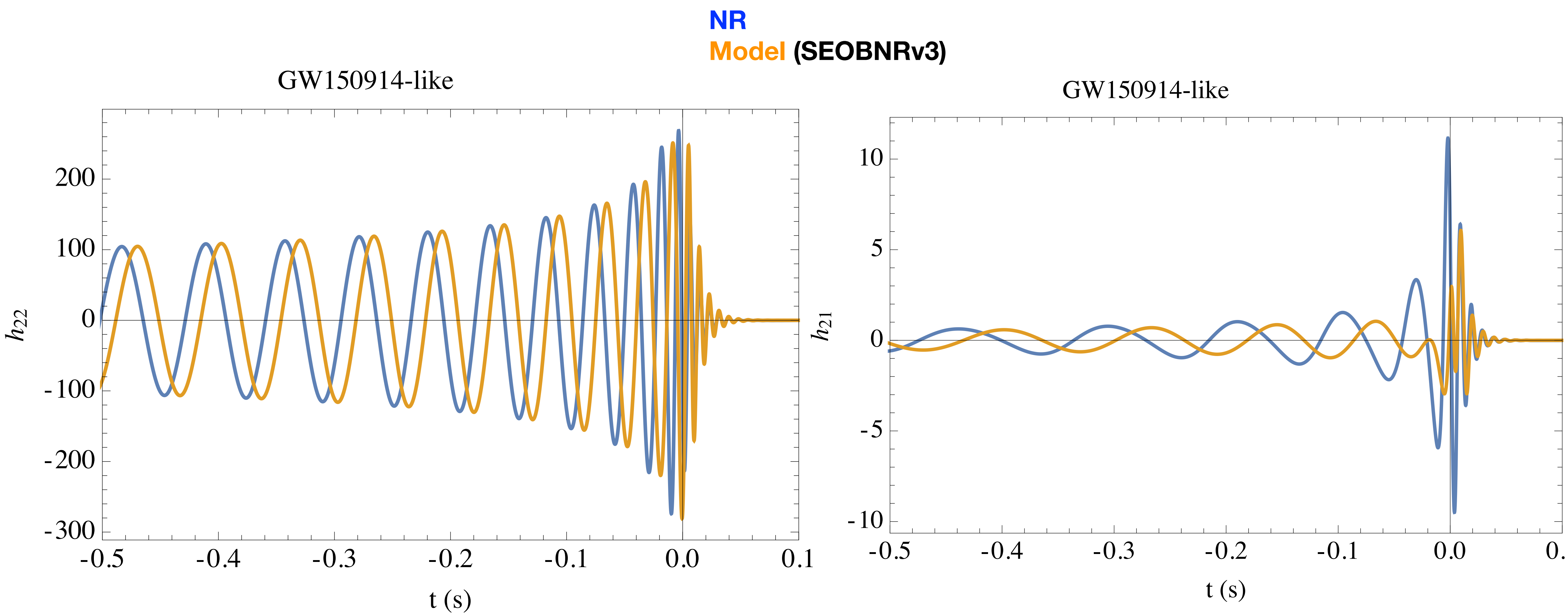
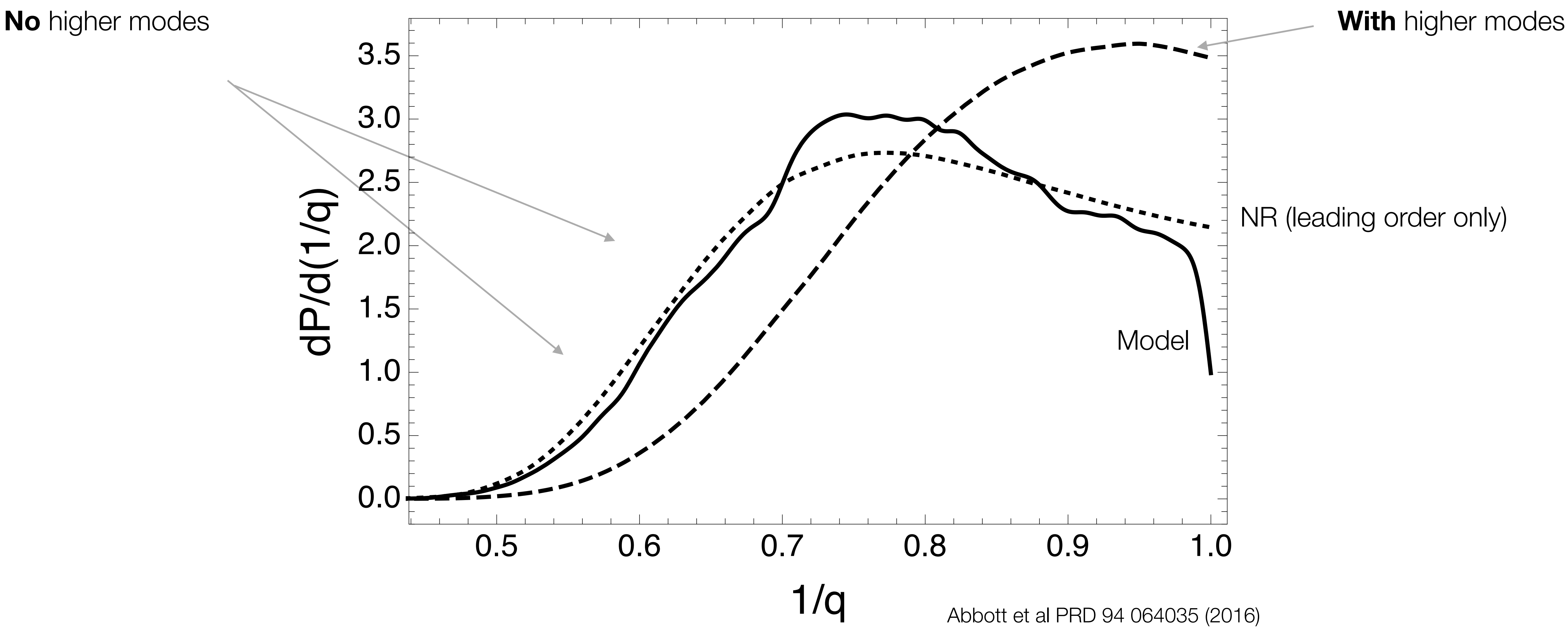


Differences matter

- Conclusions about BBH derived from NR are often slightly different
  - Even where models are "well-calibrated"

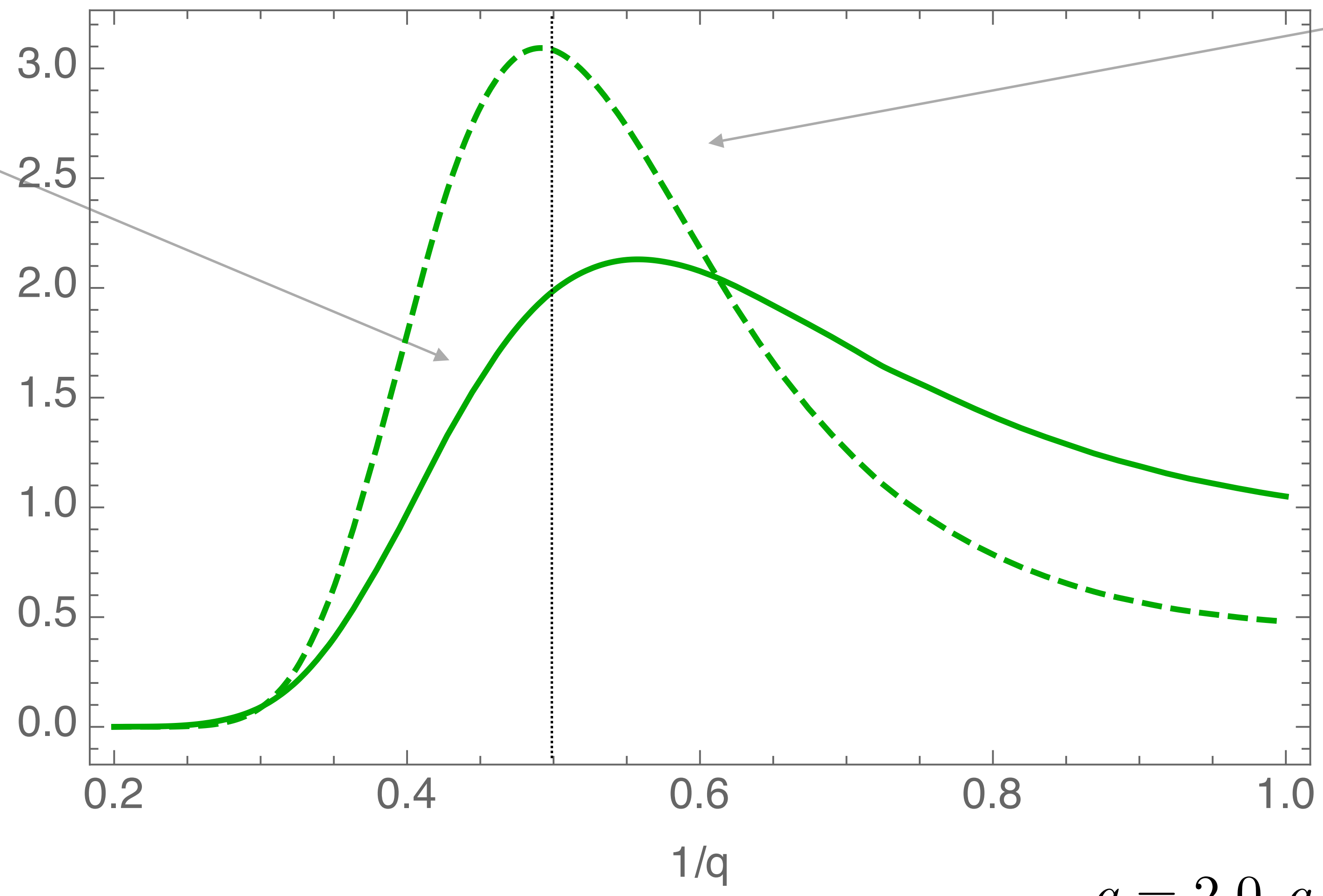


Abbott et al PRD 94 064035 (2016)
GW150914: directly comparing to NR (=with higher modes)
Nonprecessing analysis

Differences matter

No higher modes

With higher modes



Synthetic data $q = 2.0, a = 0.0, M = 70M_{\odot}$

PSD similar to GW150914-like sensitivity

Inclination ~ pi/4, SNR=20

Nonprecessing analysis

Differences matter

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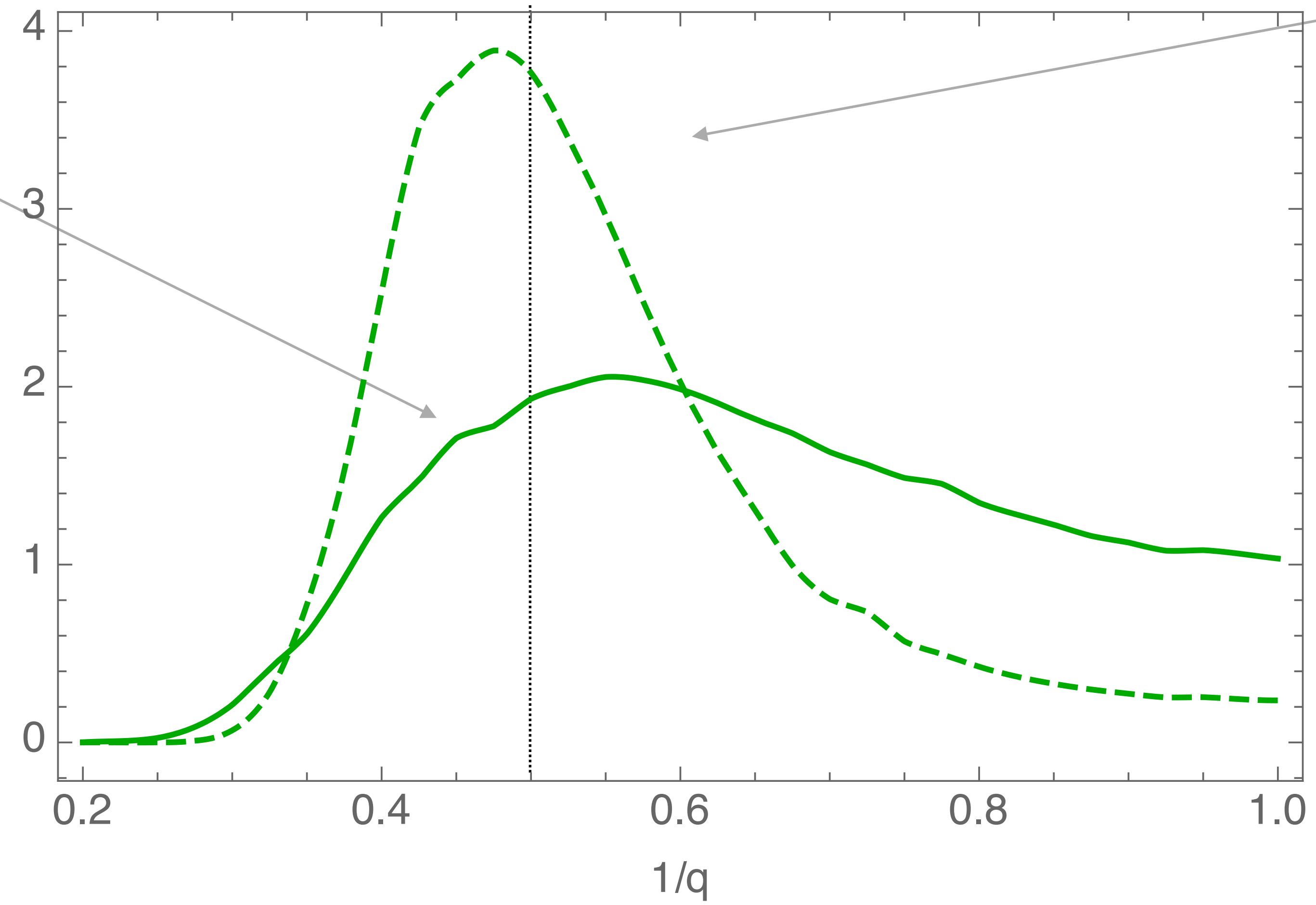

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PSD similar to GW150914-like sensitivity
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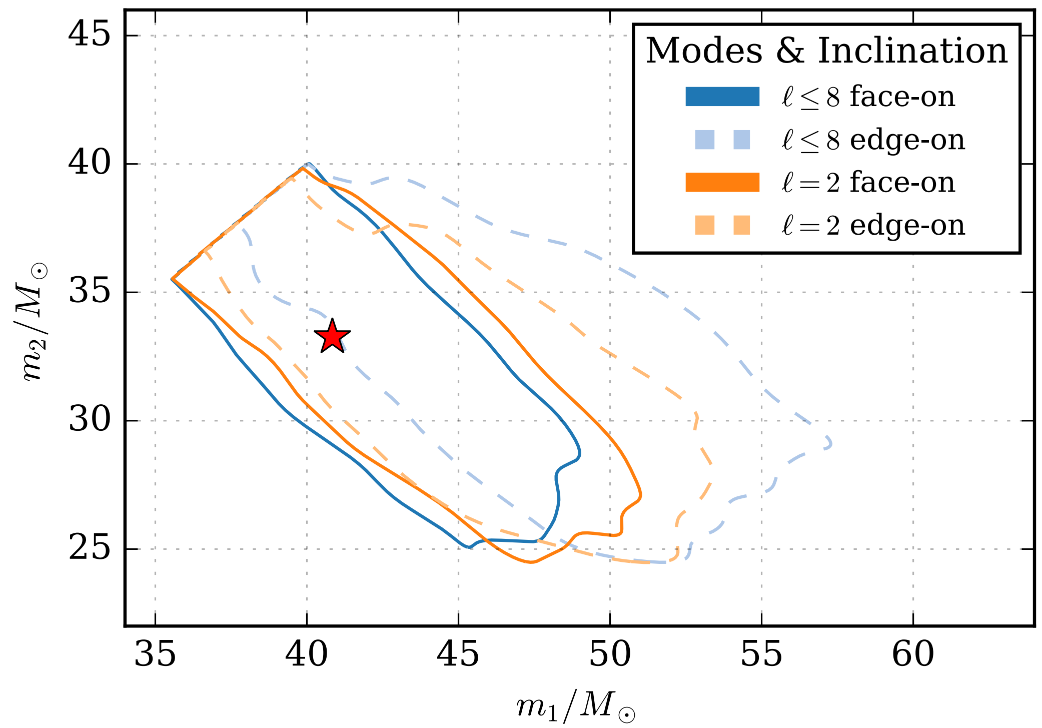
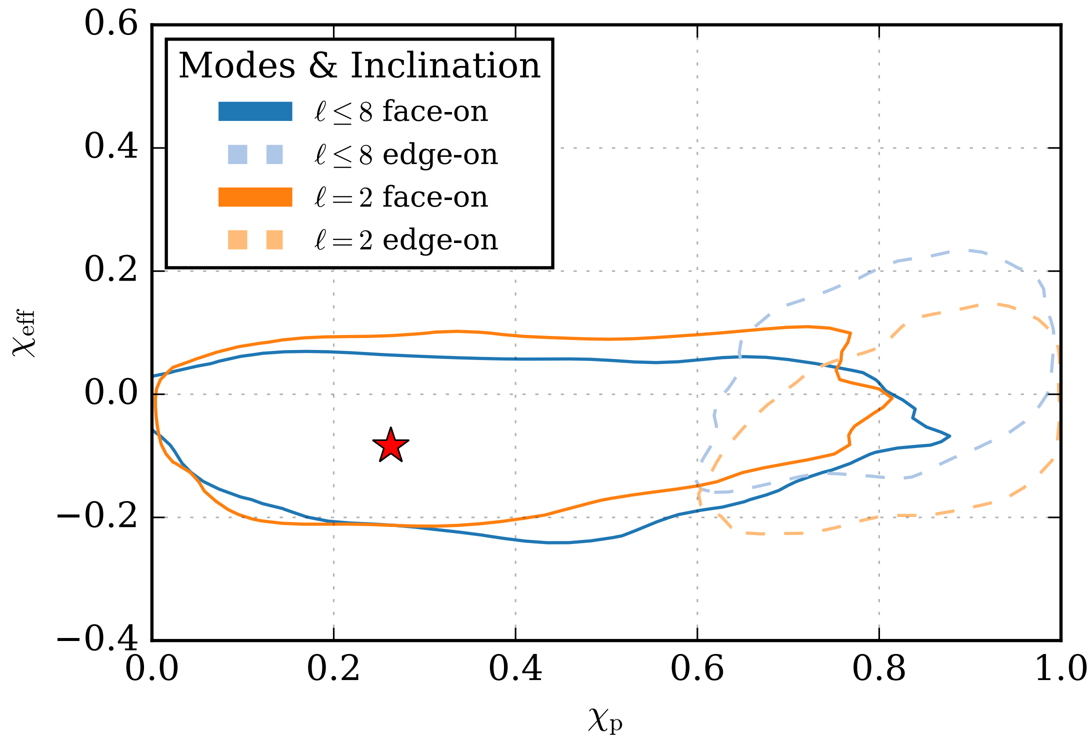
Higher-order mode effects depend on line of sight

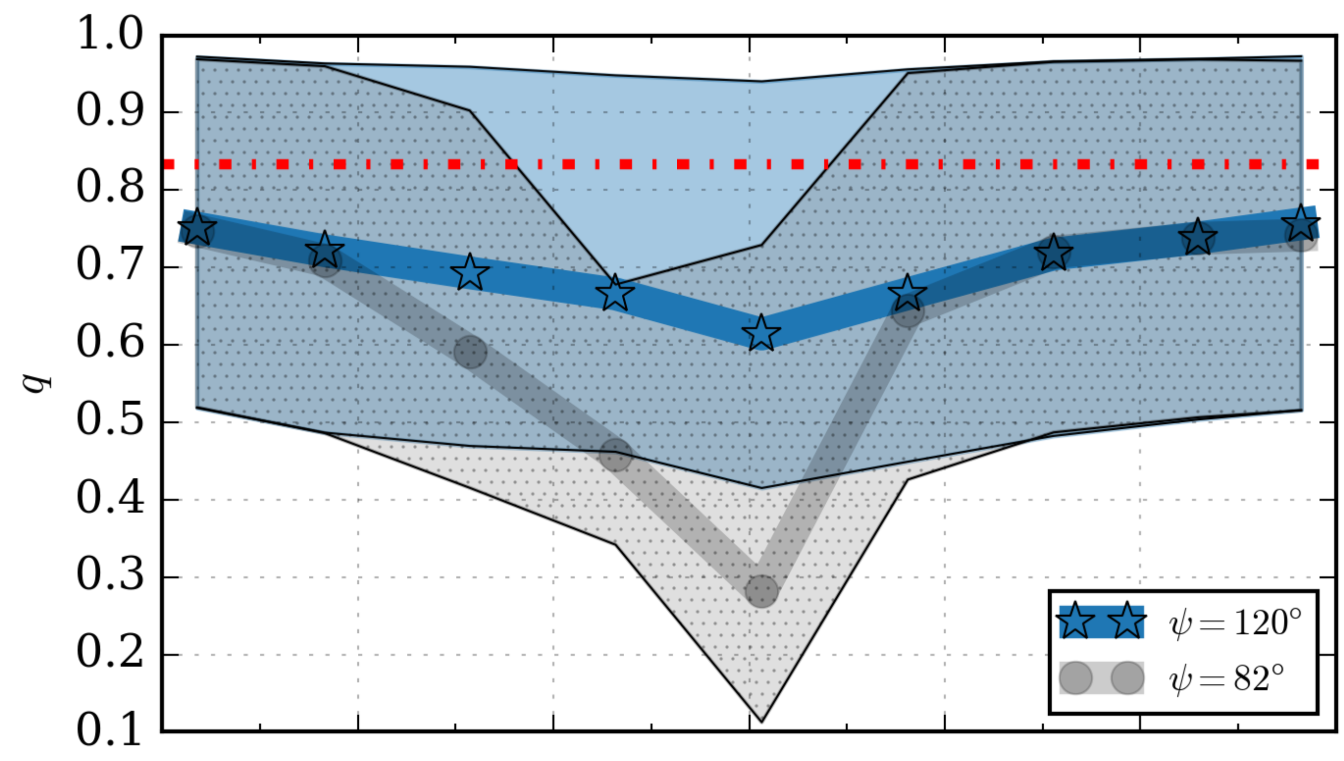
- Some lines of sight are more likely to be detected than others





Abbott et al (1611.0753)

Some lines of sight lead to biased reconstructions
(if performed without higher order modes)

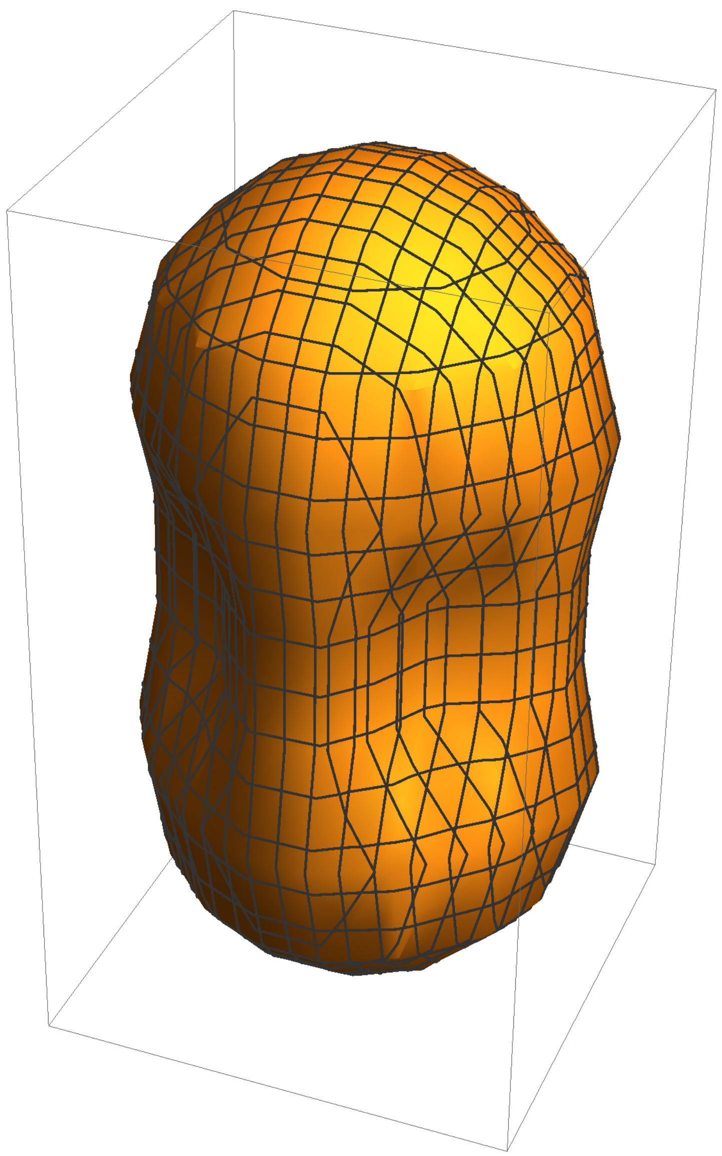


Reconstructions on this slide all done without higher modes

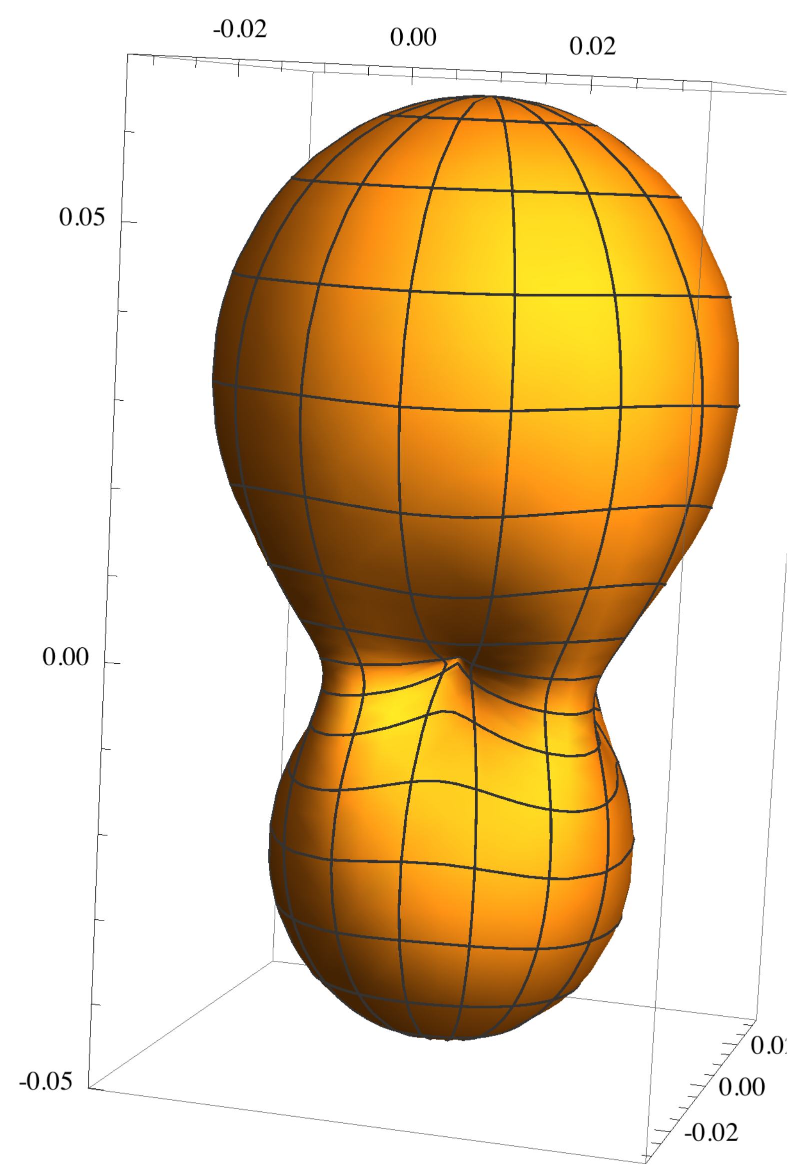
Higher modes missing & matter

- **Example**: Current quadrupole (sourced by orbiting misaligned spins)
  - Strong, well-known effect (e.g., recoil kicks)…providing unique access to spin info

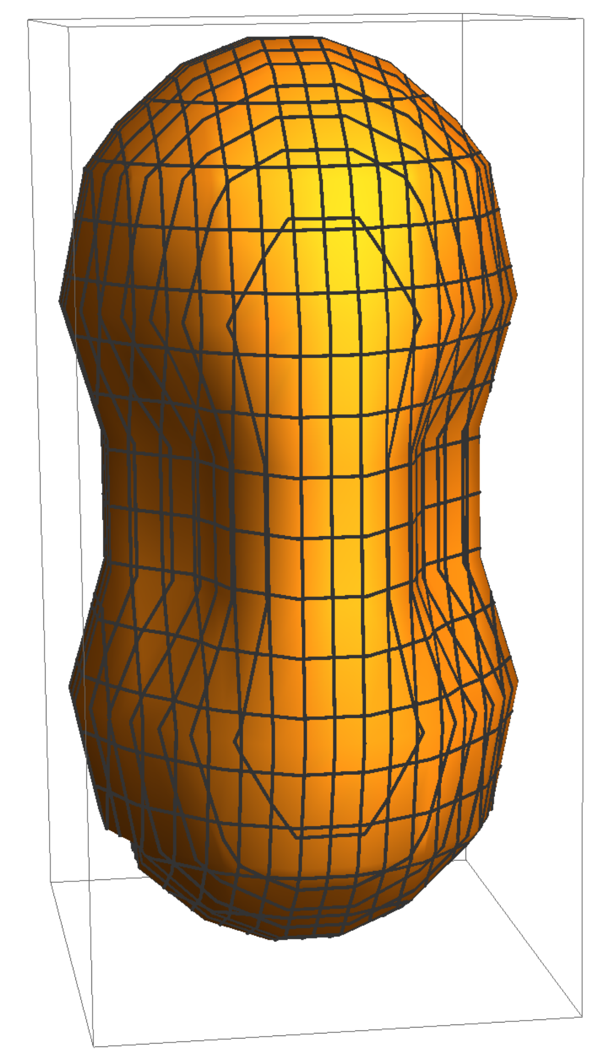
Nonprecessing



Precessing, NR



Precessing, SEOB



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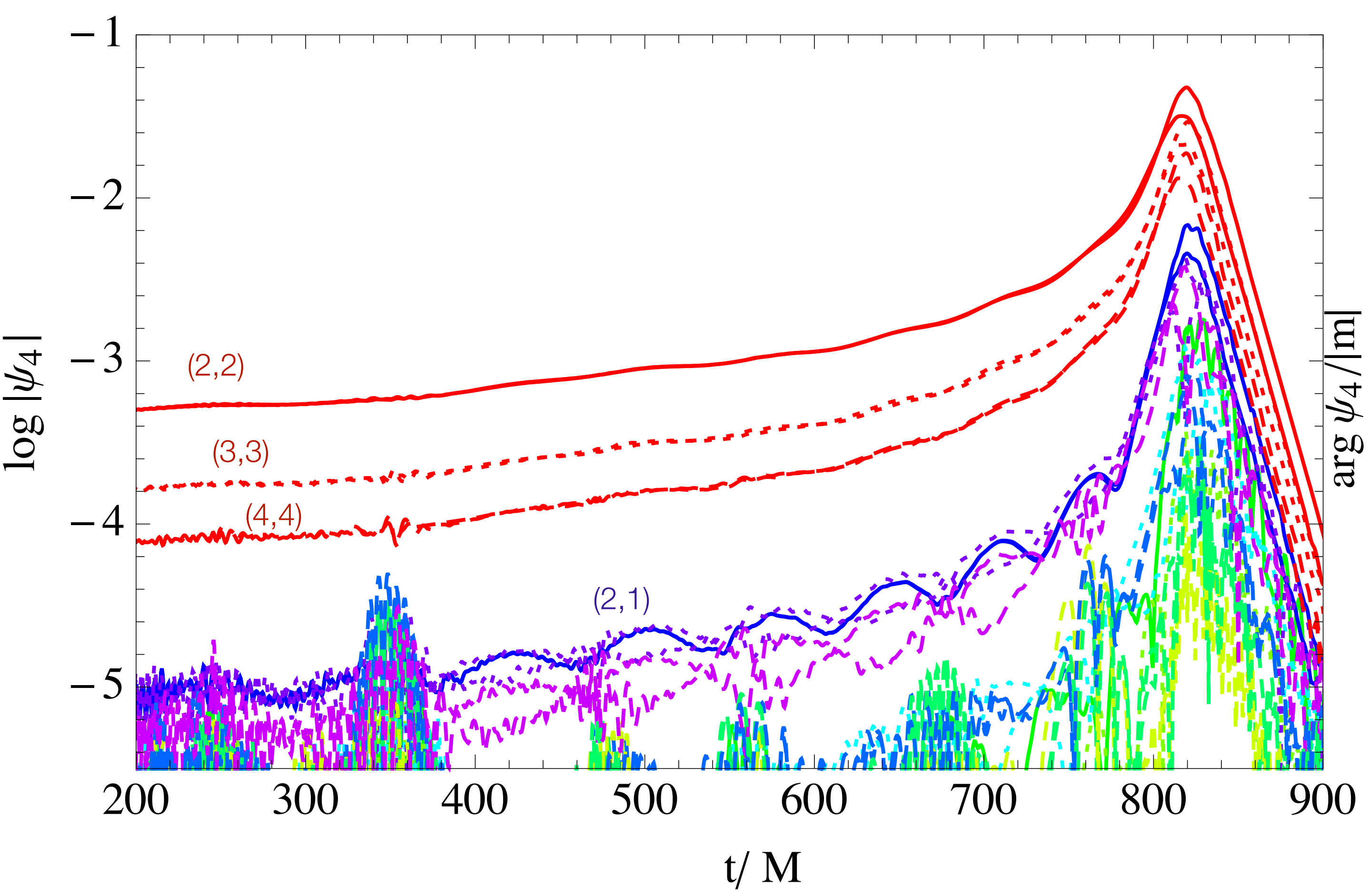
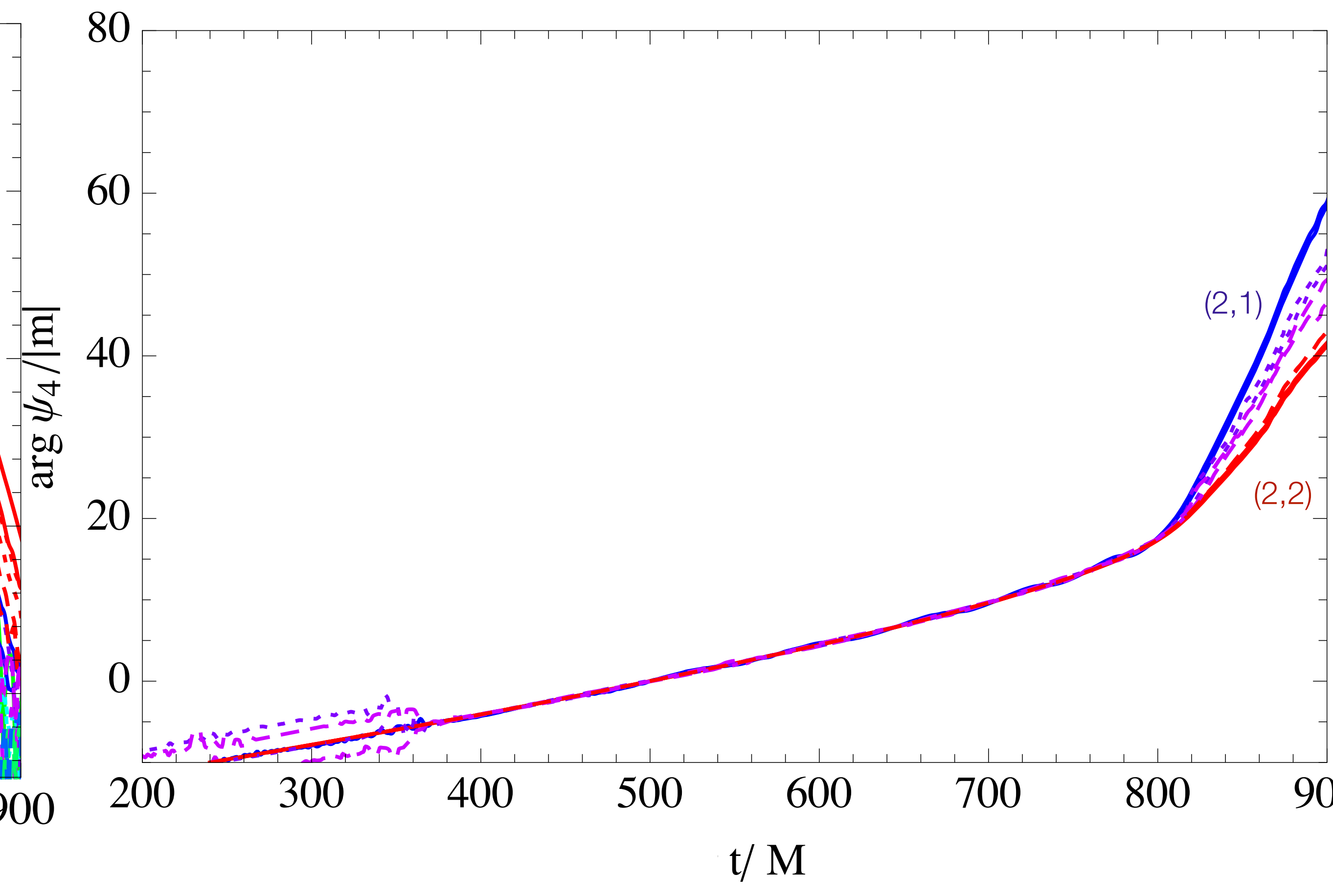
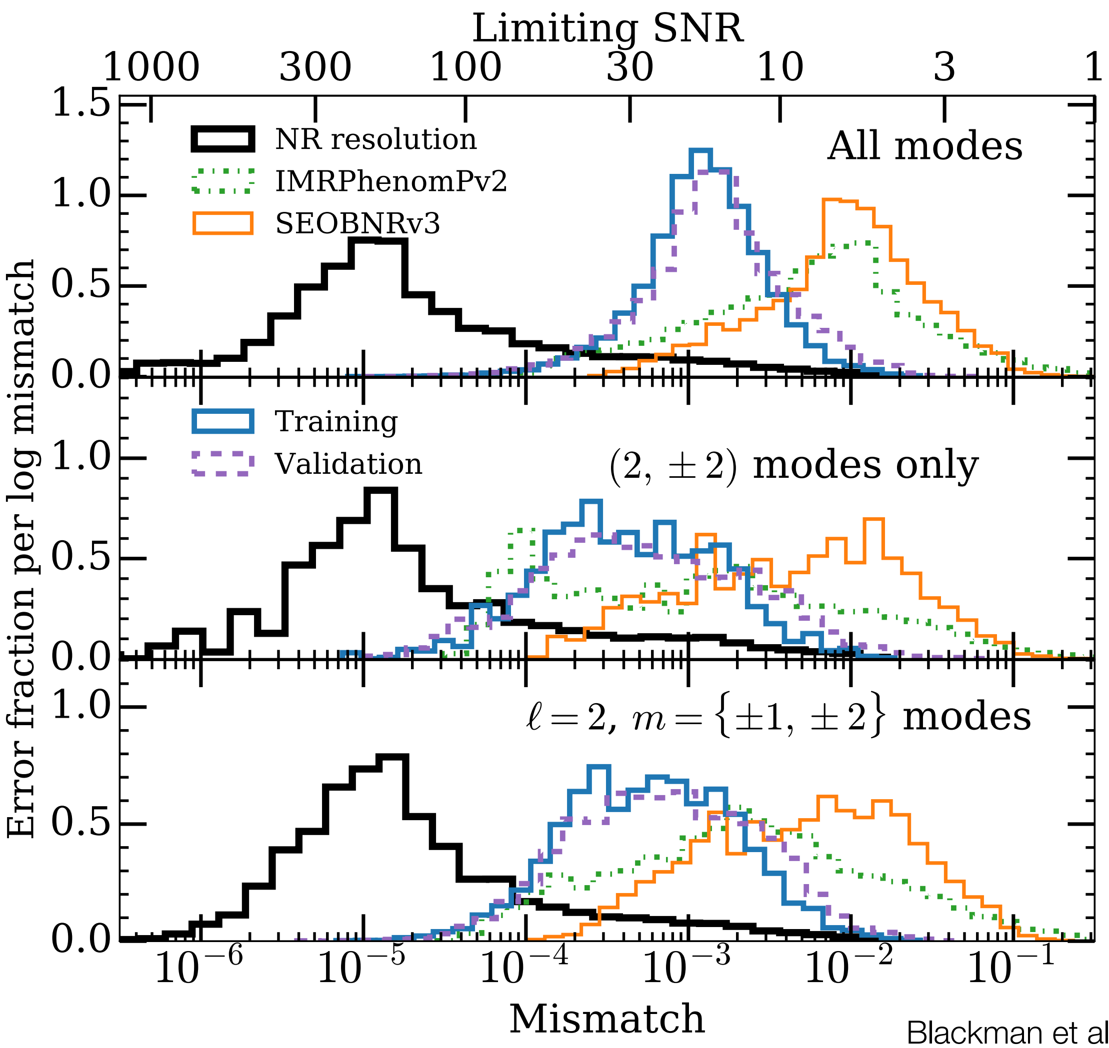




NR-calibrated surrogate models

- Surrogate models can
  - interpolate between NR simulations directly
  - include most higher modes & precession **approximately**

Blackman et al 2015,2017
ROS et al 2017



Blackman et al 2017

- Limitations so far
  - Placement (exploration in ‘q’; spins), **duration**

Parameter estimation: foundations

- Evidence for signal

$$Z(d|H_1) \equiv \frac{p(\{d\}|H_1)}{p(\{d\}|H_0)} = \int d\lambda p(\vec{\lambda}|H_1)\frac{p(\{d\}|\vec{\lambda},H_1)}{p(\{d\}|H_0)}$$

posterior distribution

H_1 : with signal
H_0 : no signal

- Inputs:
  - Prior knowledge $p(\lambda|H_1)$ about distribution of λ
  - Signal model $h(\lambda)$
  - Noise model $p(\{d\}|H_0)$ $\quad p(\{d\}|\vec{\lambda},H_1) = p(\{d-h(\vec{\lambda})\}|H_0)$
  - Algorithm for integral/exploration in many dimensions

- Noise model: Gaussian

$$\mathcal{L} \equiv p(\{d\}|\vec{\lambda},H_1)/p(\{d\}|H_0)$$

$$= \frac{e^{-\langle h(\lambda)-d|h(\lambda)-d\rangle/2}}{e^{-\langle d|d\rangle/2}}$$

How to explore the space?

- Grids?

$$\mathcal{L}_{\rm red}(\lambda) = \int \mathcal{L}(\lambda, \theta) p(\theta) d\theta$$

- Monte Carlo
  - Trivial theory & convergence. **Embarassingly parallel.**

$$\mathcal{L}_{\rm red}(\lambda) = \int \frac{\mathcal{L}(\lambda, \theta) p(\theta)}{p_s(\theta)} p_s(\theta) d\theta \simeq \frac{1}{N} \sum_{i=1}^{N} \frac{\mathcal{L}(\lambda, \theta_i) p(\theta_i)}{p_s(\theta_i)}$$

  - Many adaptive variants
- MCMC: Oracle for independent samples
  - Easy to get started: write likelihood+prior
  - “Walker” with jumps satisfying detailed balance + ergodicity. **Serial.**
  - Results follow by histograms. Coordinate transformations trivial.
  - Many adaptive variants
  - Practical efficiency and convergence tests rare, tricky

Example: Two integration-based strategies

- Parameter estimation for GW sources: Compare models and data, using gaussian statistics

$$\ln \mathcal{L}(\lambda;\theta) = -\frac{1}{2}\sum_k \langle h_k(\lambda,\theta)-d_k|h_k(\lambda,\theta)-d_k\rangle_k - \langle d_k|d_k\rangle_k$$

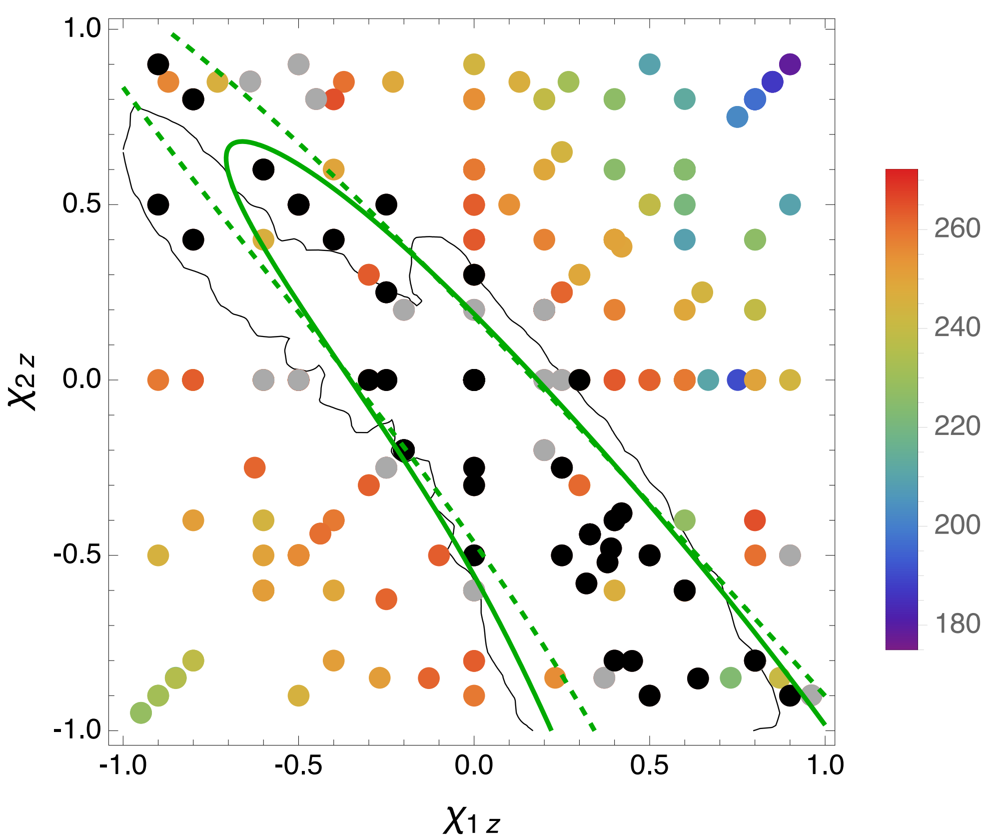
- Method 1: **grid** : [e.g., Pankow et al 2015 (1502.04370)]
  - Integrate over extrinsic parameter space [**NR can't vary intrinsic params**]

$$\mathcal{L}_{\rm marg}(\lambda) \equiv \int \mathcal{L}(\lambda,\theta)p(\theta)d\theta$$

  - Stitch likelihood from discrete evaluations $\mathcal{L}_{\rm marg}(\lambda_k)$
    - **Currently**: Aligned spin via fit (or GP)
  - Posterior via Bayes

$$p_{\rm post}(\lambda) = \frac{\mathcal{L}_{\rm marg}(\lambda)p(\lambda)}{\int d\lambda \mathcal{L}_{\rm marg}(\lambda)p(\lambda)}$$

Abbott et al 2016, PRD 94 4035



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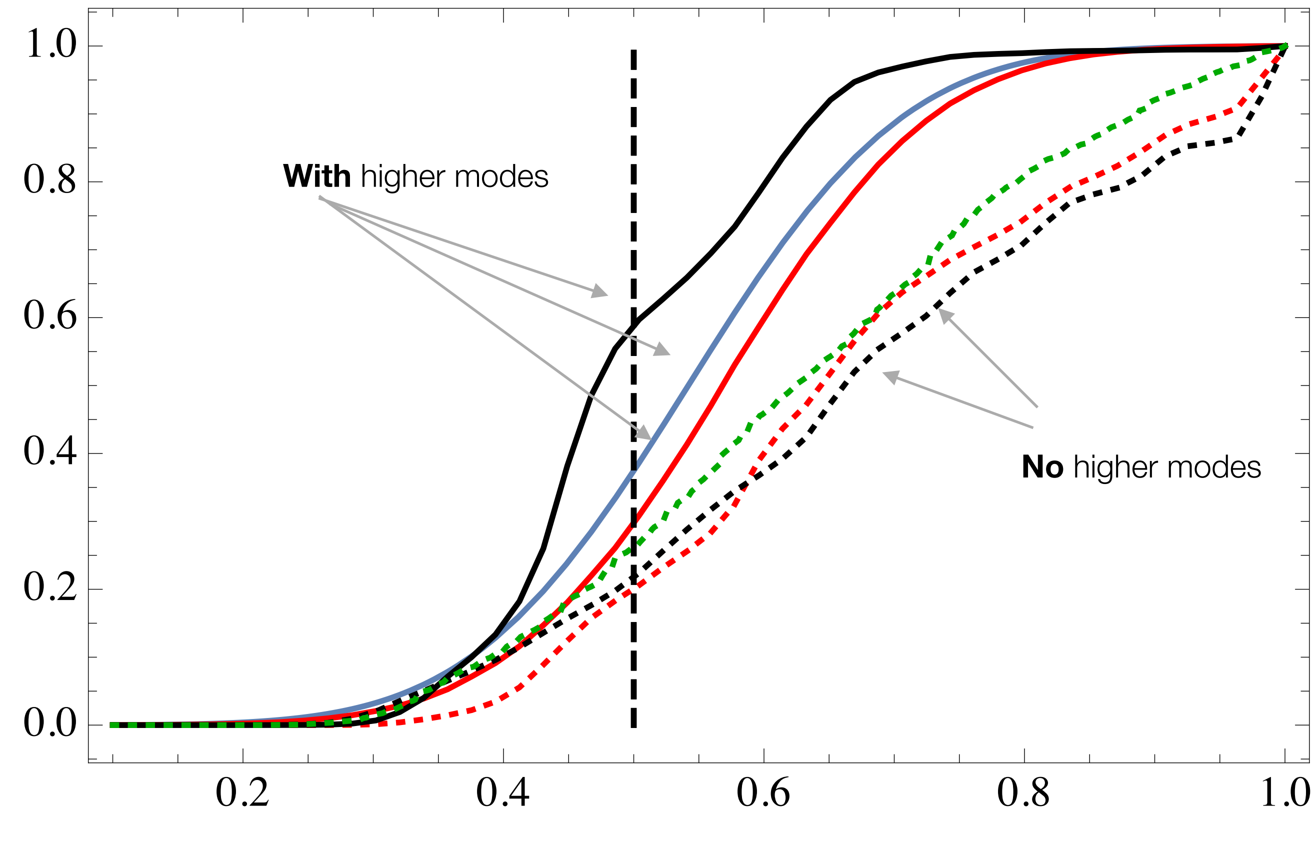
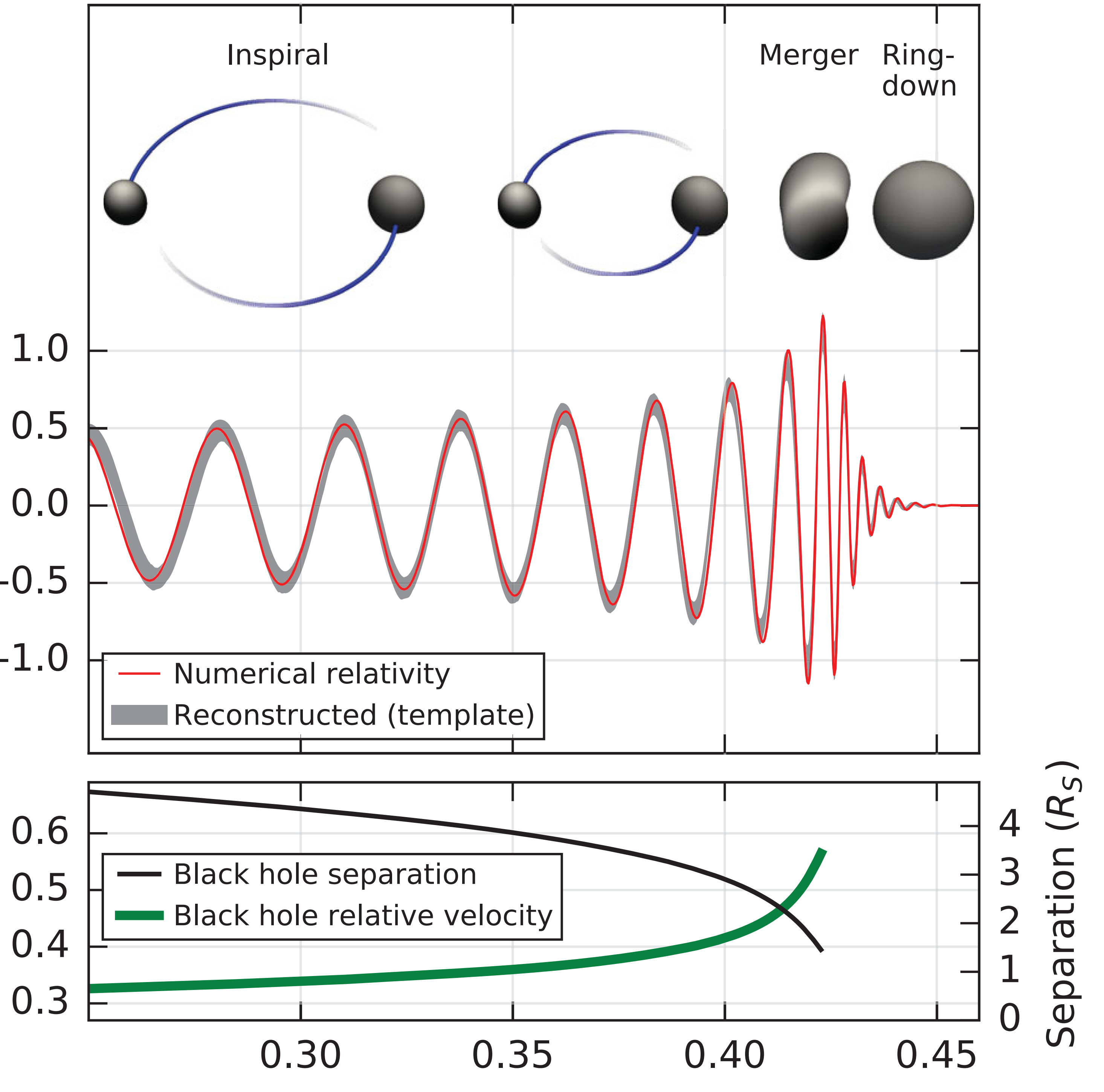
- Method 1: **grid** : [e.g., Pankow et al 2015 (1502.04370)]
  - Integrate over extrinsic parameter space [**NR can't vary intrinsic params**]
- Method 2: **pure Monte Carlo** [e.g., ROS et al 2017]
  - Use a model which can be evaluated everywhere
  - Posterior = histogram

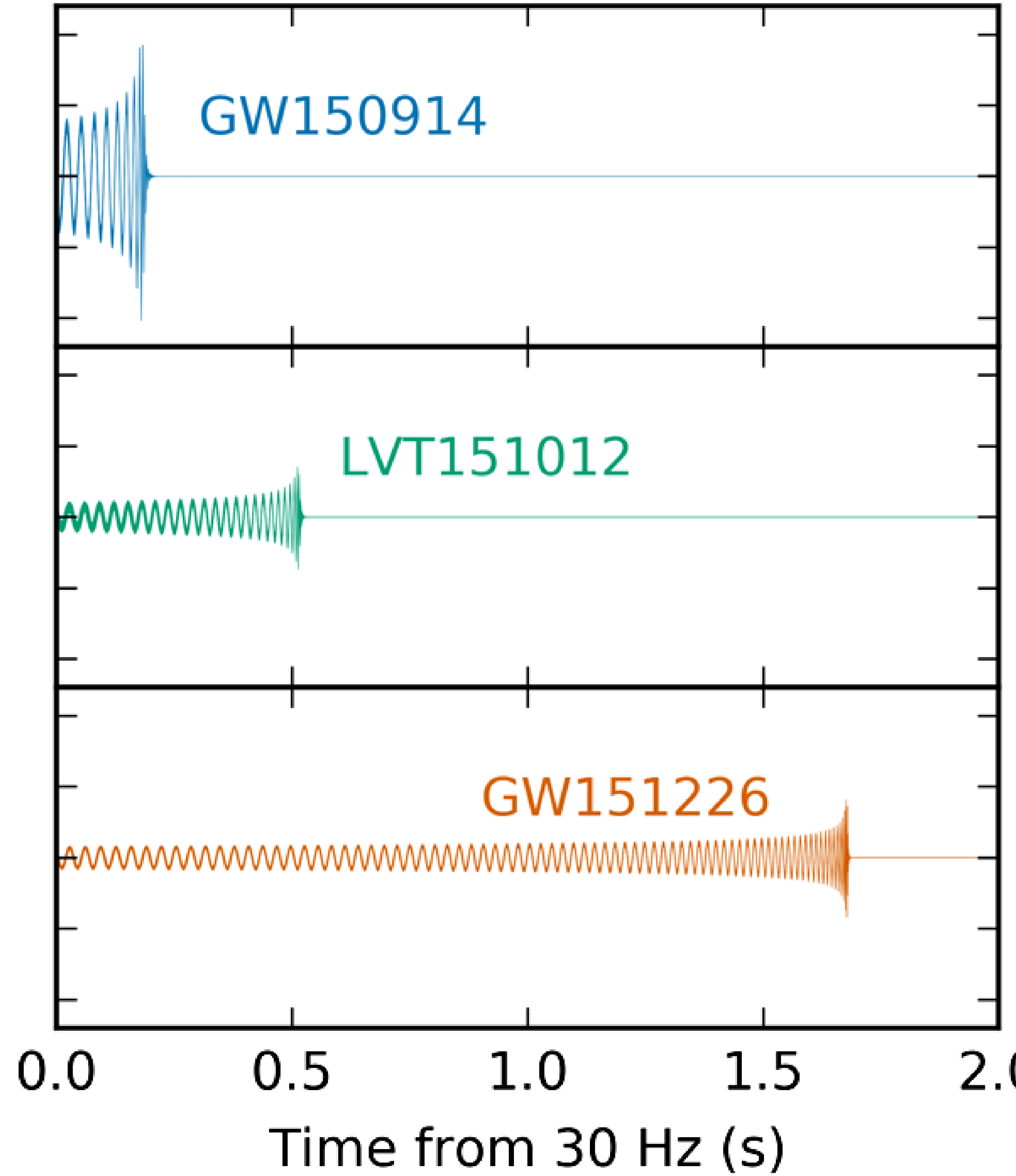
ILE+ EOBNRv2HM [Reference]
ILE + ROM **on grid**
ILE+ROM **Monte Carlo**



O'Shaughnessy, Blackman, Field 2017 (1701.01137)
M=150, q=2, aLIGO SNR=25, zero spin

Short, high-mass signals





Abbott et al, PRL 116, 061102 (2016)

Short, high-mass signals

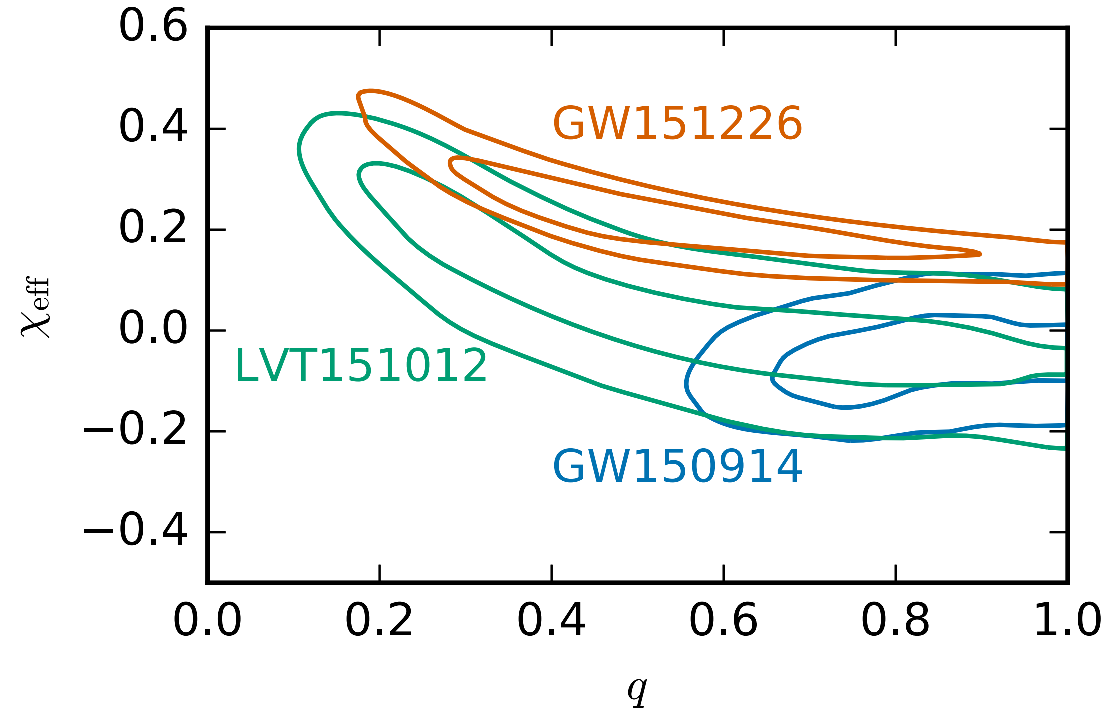
- **Few cycles for leverage**, and fewer precession cycles

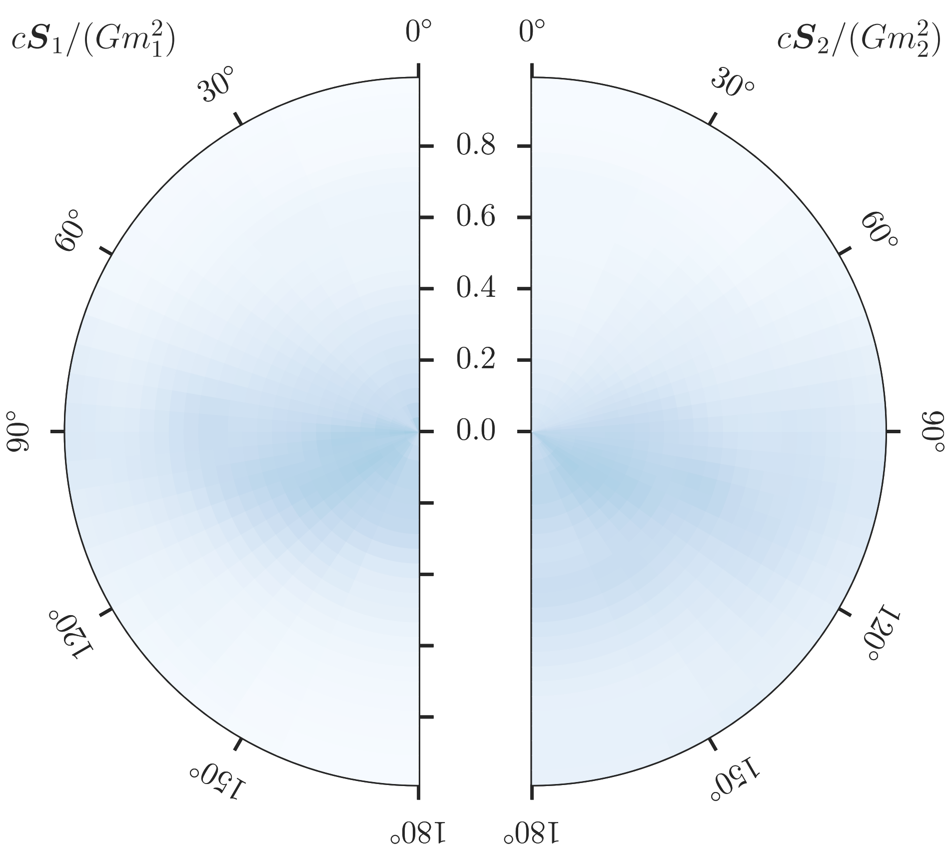
$$N_P \simeq \int_{\pi f_{min}}^{\pi f_{max}} df_{orb} \frac{dt}{df_{orb}} \Omega_p$$

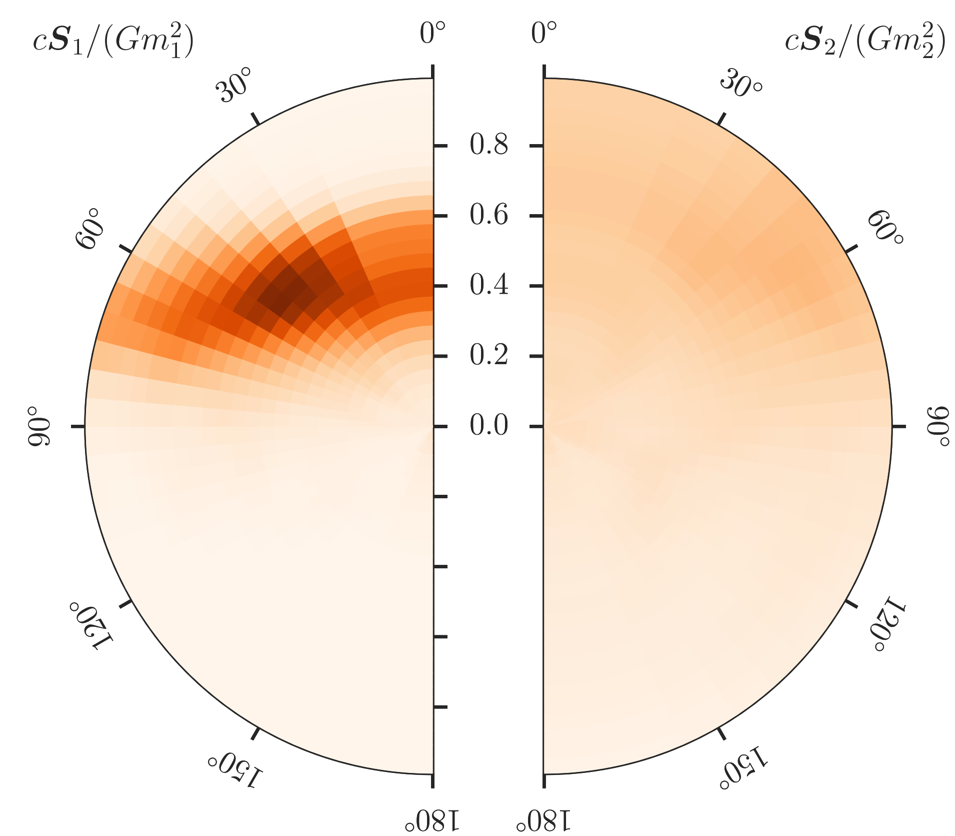
$$= \frac{5}{96}(2 + 1.5\frac{m_2}{m_1})[(M\pi f_{min})^{-1} - (M\pi f_{max})^{-1}]$$

$$\approx \frac{27(1 + 0.75 m_2/m_1)}{M/10M_\odot} \quad \text{above 20 Hz}$$

Abbott et al, PRX 2016

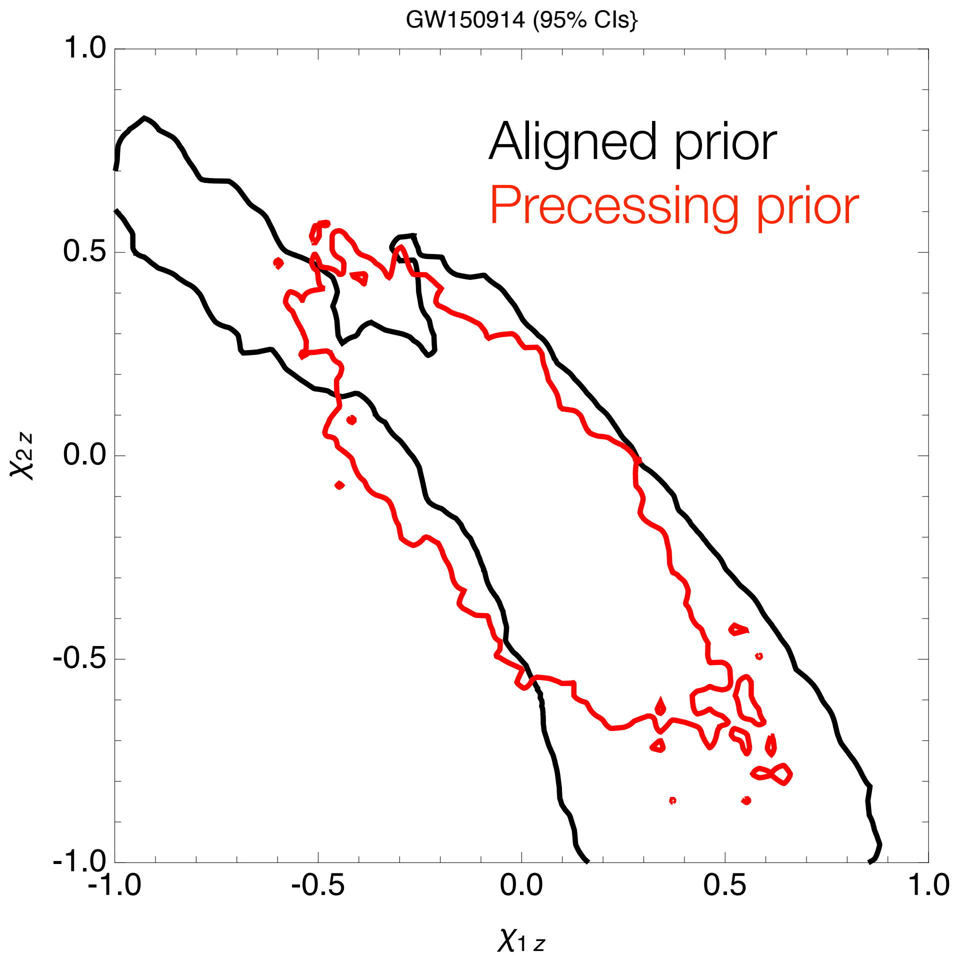


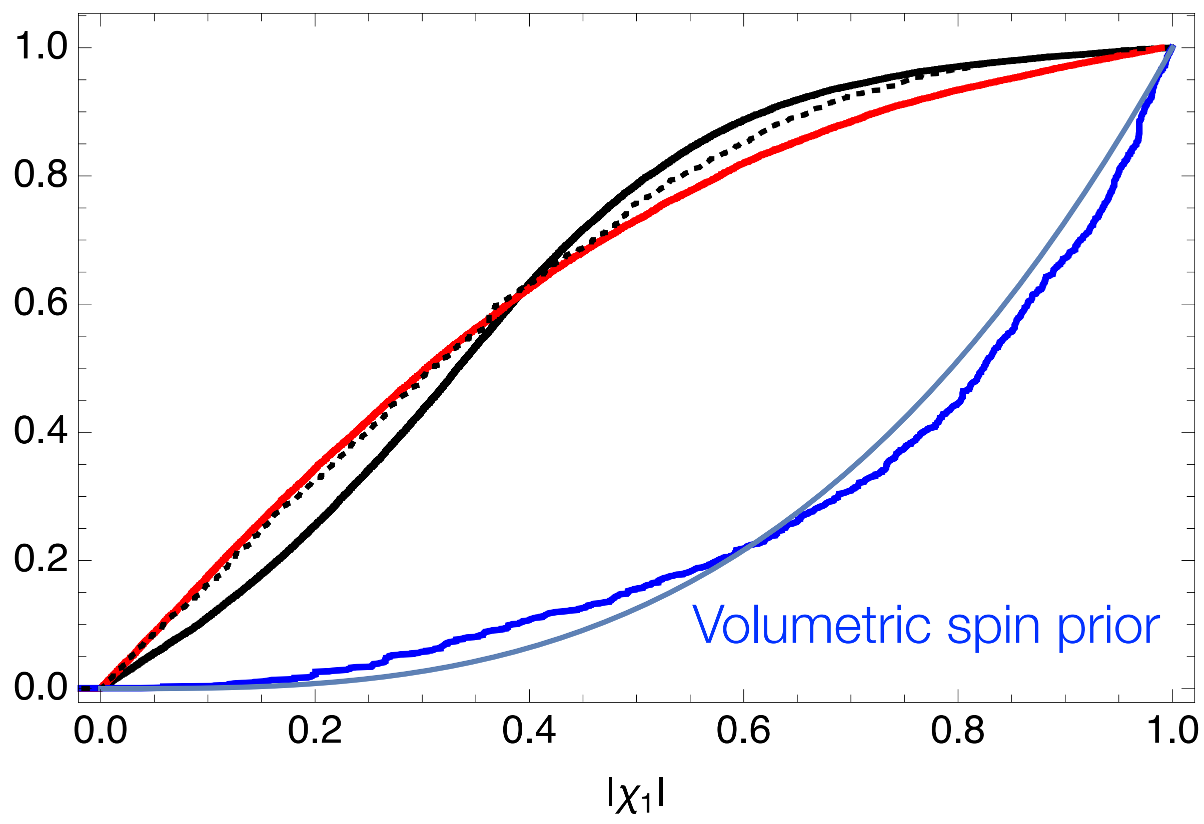




Priors: Parse statements about spin with care

- **Issue**: Likelihood alone not compelling, so prior choices matter
- **Notes**: Current prior is uniform in spin **magnitude** and both **masses**
    - Large aligned spins unlikely (alignment+magnitude: doubly special)
    - Configurations with two dynamically-significant spins **very** unlikely
- **Example**:

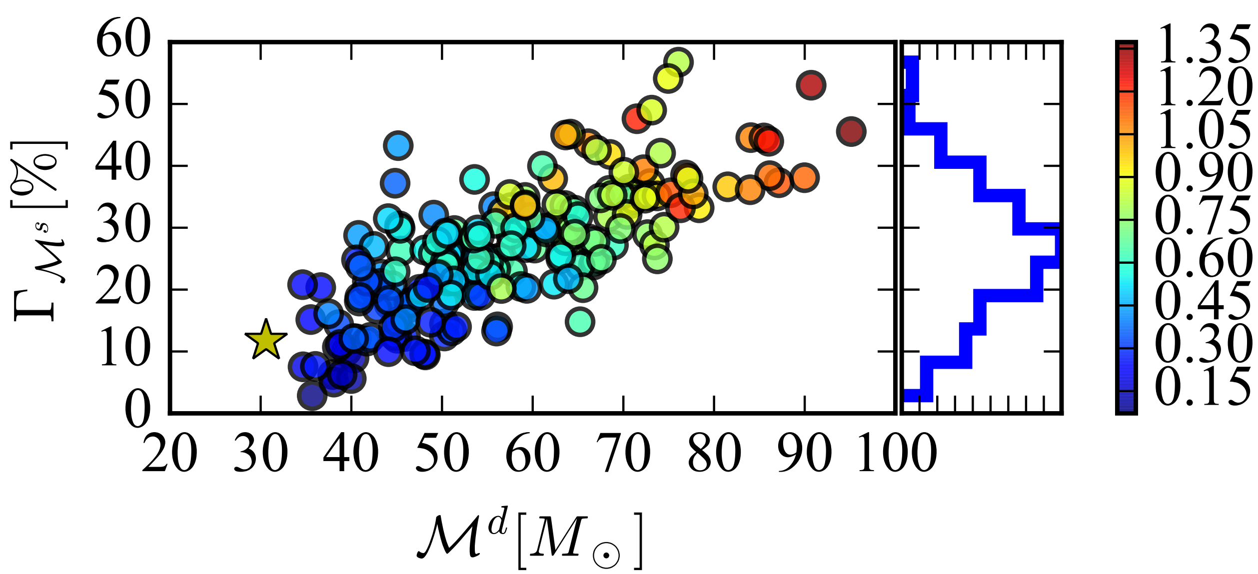


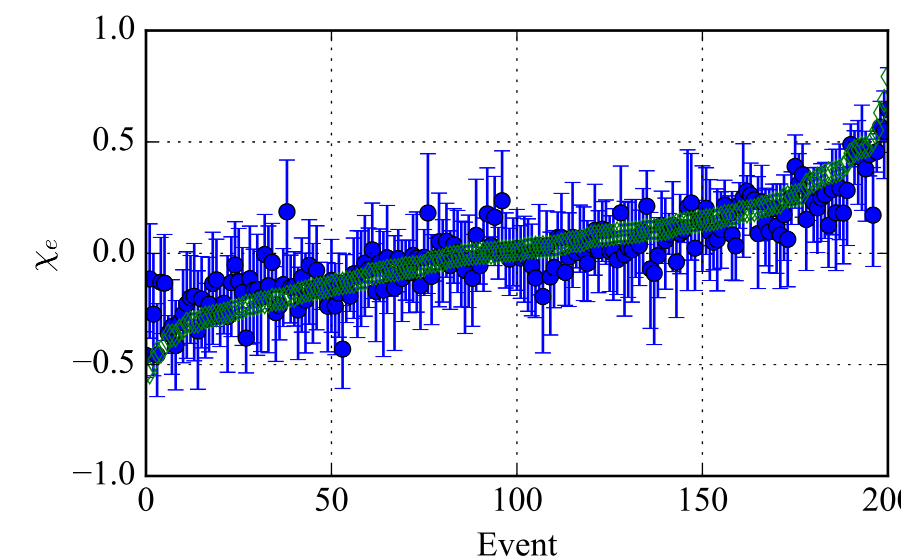


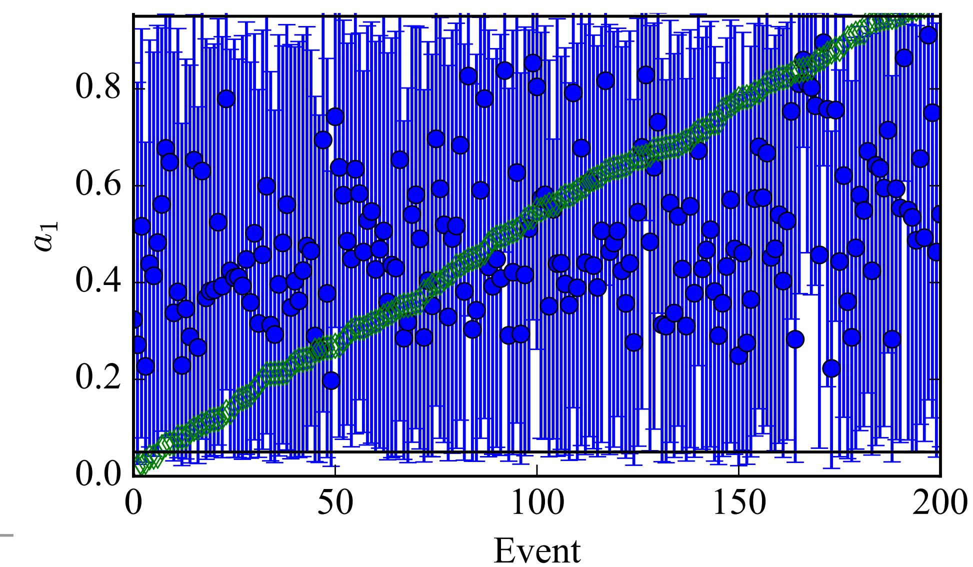
How accurately can we measure parameters?

- Model adequacy:
  - At low SNR / face on, we won't do terribly with what we have [Abbott et al 2017; Varma et al 2017]
  - If nature is kind, we could need better models soon [A. Taracchini today]

- Assessments based on one model (IMRP) with a single spin [Vitale et al 1611.01122; a few masses & spins]
  - Effectively spins: fairly reliably
  - Masses: few - several tens percent total
  - Individual spins: poor
- What about systematics ? ….

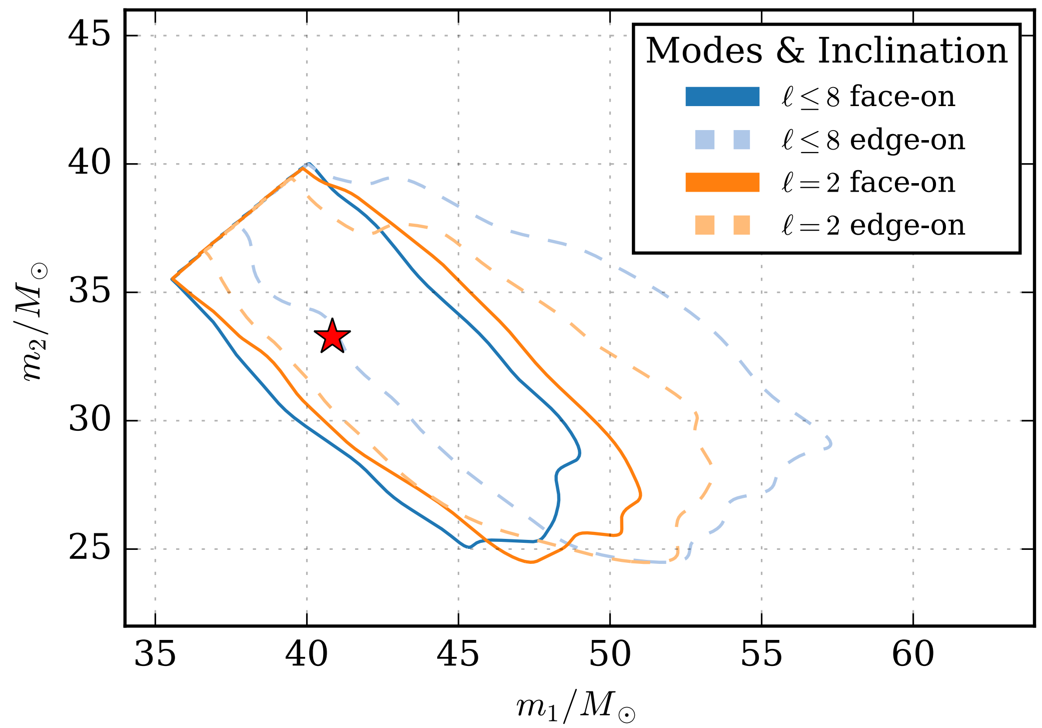


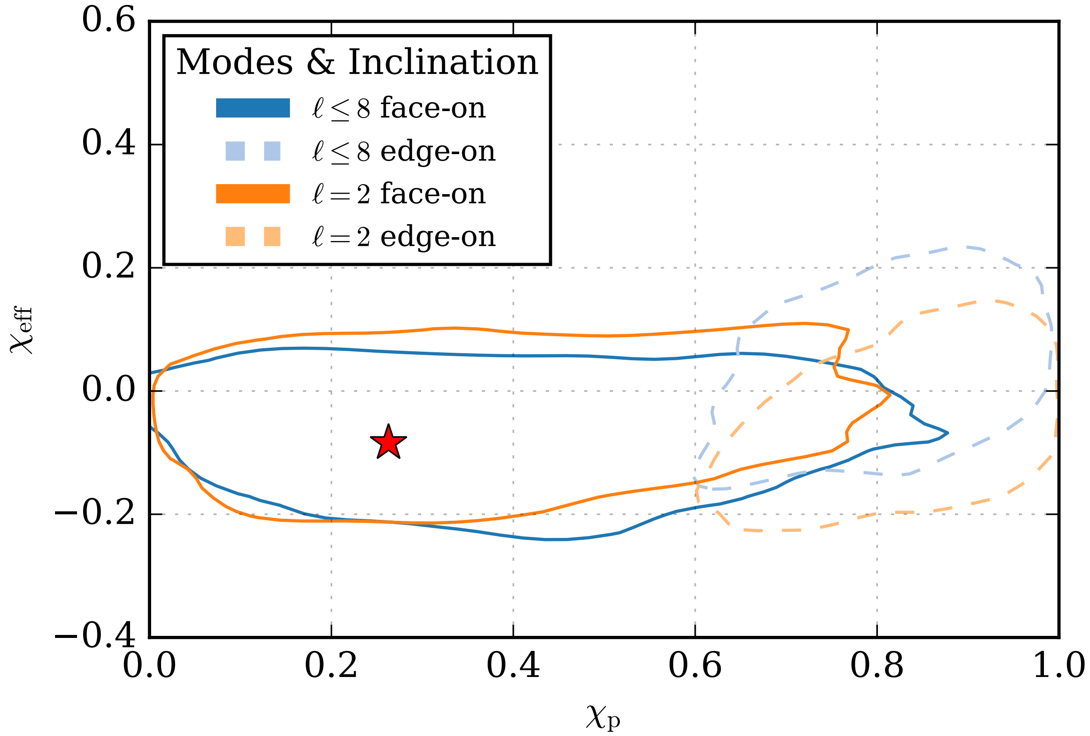




How accurately can we measure parameters?

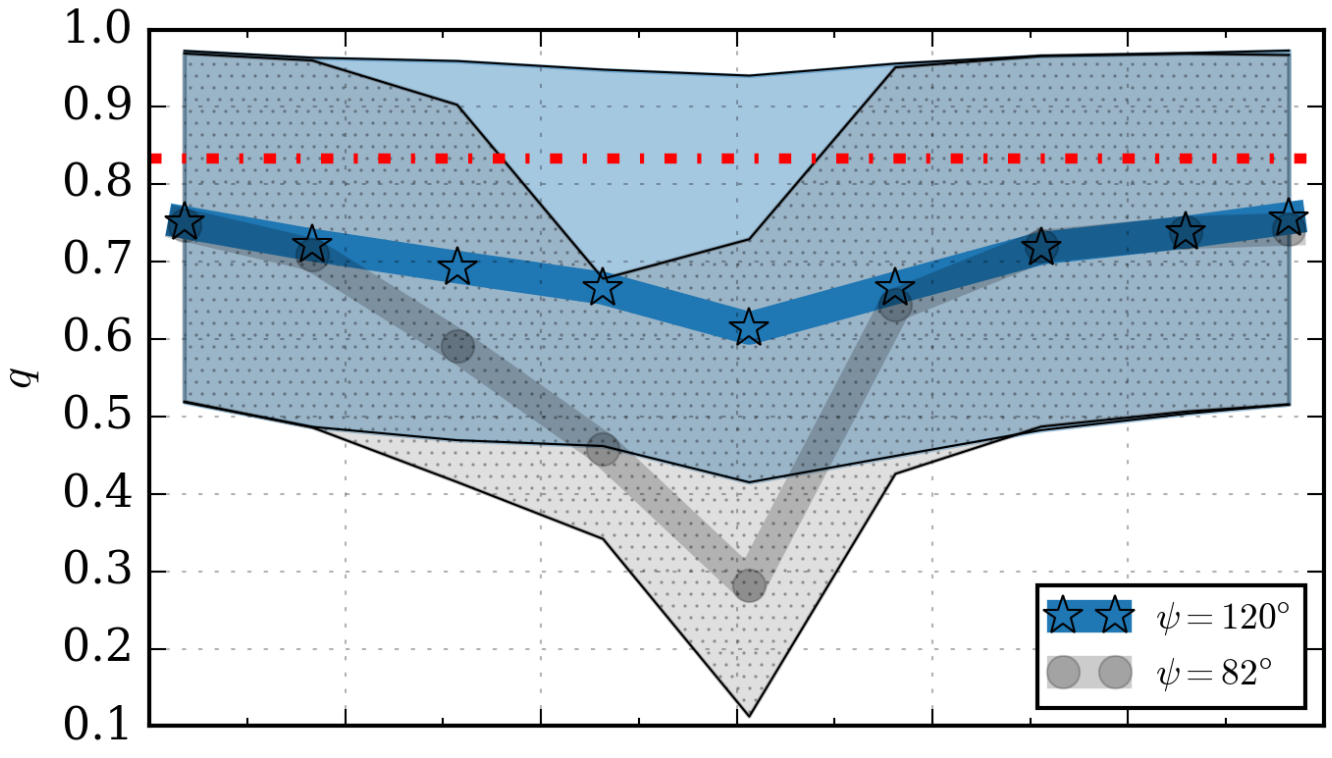
- Some lines of sight are more likely to be detected than others





Abbott et al (1611.0753)

Edge-on lines of sight lead to biased reconstructions
(if performed without higher order modes)



Reconstructions on this slide all done without higher modes

How accurately can we measure parameters?

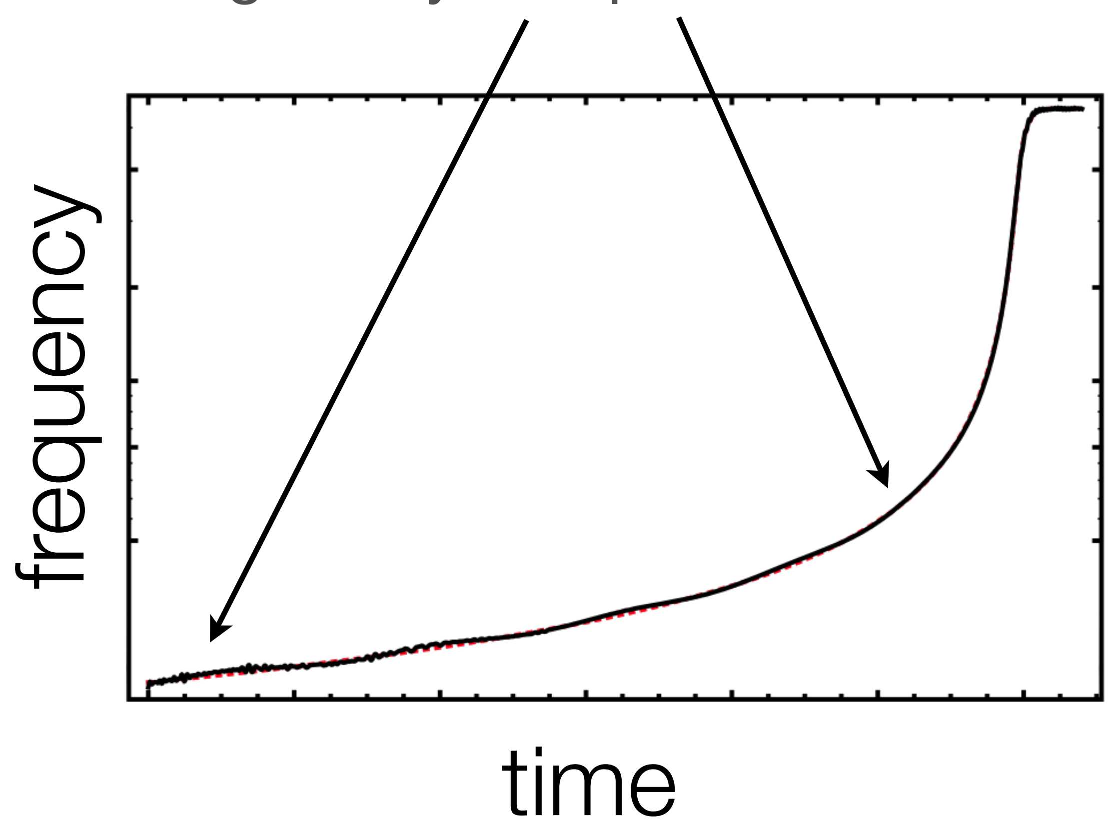
	$\mathcal{M}$ $(M_\odot)$			q			$\chi_{\rm eff}$		
	Median	Bias	90% CI	Median	Bias	90% CI	Median	Bias	90% CI
SXS:BBH:0049	$\mathcal{M} = 27.15\ M_\odot$			$q = 0.3$			$\chi_{\rm eff} = 0.13$		
$\iota = 163^\circ$	27.47	-0.32	4.92	0.31	0.02	0.18	0.14	-0.01	0.24
$\iota = 90^\circ$	20.28	6.87	3.44	0.28	0.05	0.12	-0.66	0.78	0.28
$\iota = 90^\circ$, $\psi = 120^\circ$	29.06	-1.92	6.28	0.33	0.01	0.14	0.19	-0.06	0.33
SXS:BBH:0522	$\mathcal{M} = 30.79\ M_\odot$			$q = 0.57$			$\chi_{\rm eff} = -0.65$		
$\iota = 163^\circ$	32.63	-1.84	5.21	0.79	-0.22	0.42	-0.56	-0.09	0.30
$\iota = 90^\circ$	30.26	0.53	9.46	0.46	0.11	0.58	-0.55	-0.11	0.46
$\iota = 90^\circ$, $\psi = 120^\circ$	31.06	-0.27	5.98	0.67	-0.10	0.49	-0.63	-0.03	0.35

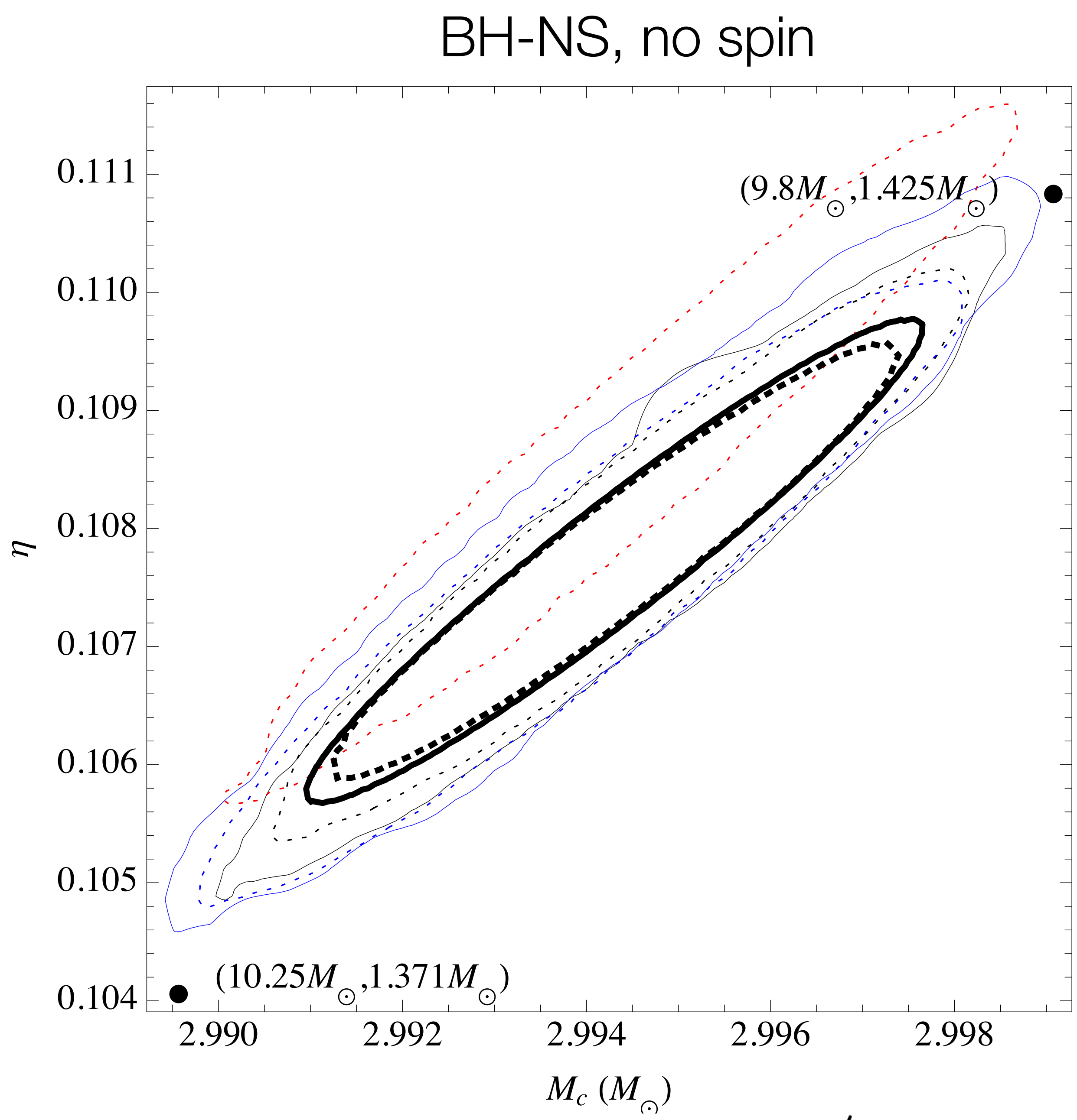
Abbott et al (1611.0753)

Reconstructions on this slide all done without higher modes

Long, low-mass signals can be very degenerate

- Shrinking binary "chirps"



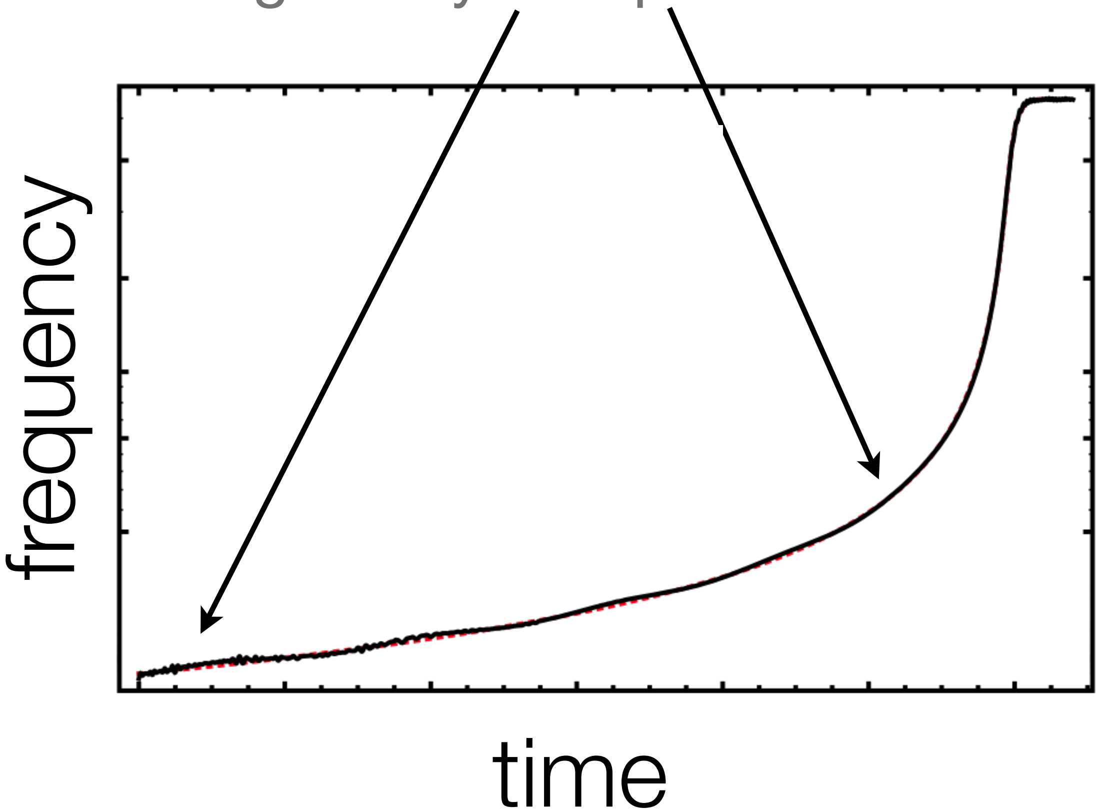


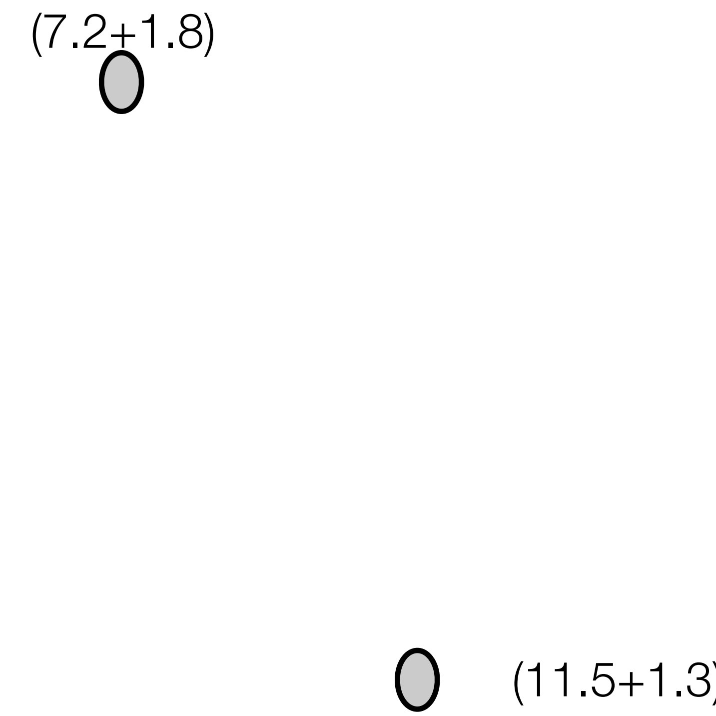
- Chirp rate (df/dt) set by "chirp mass"
  - "Exactly" measurable

$$\mathcal{M}_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

Weak limits without precession

- Shrinking binary “chirps”





- Measure masses, spins, tides, ...
  - Adding parameters (spin) degrades measurement accuracy
- Fisher matrix

$$\Gamma_{ab} = 2 \int_{-\infty}^{\infty} \frac{\partial_a h^* \partial_b h}{S_h} df$$

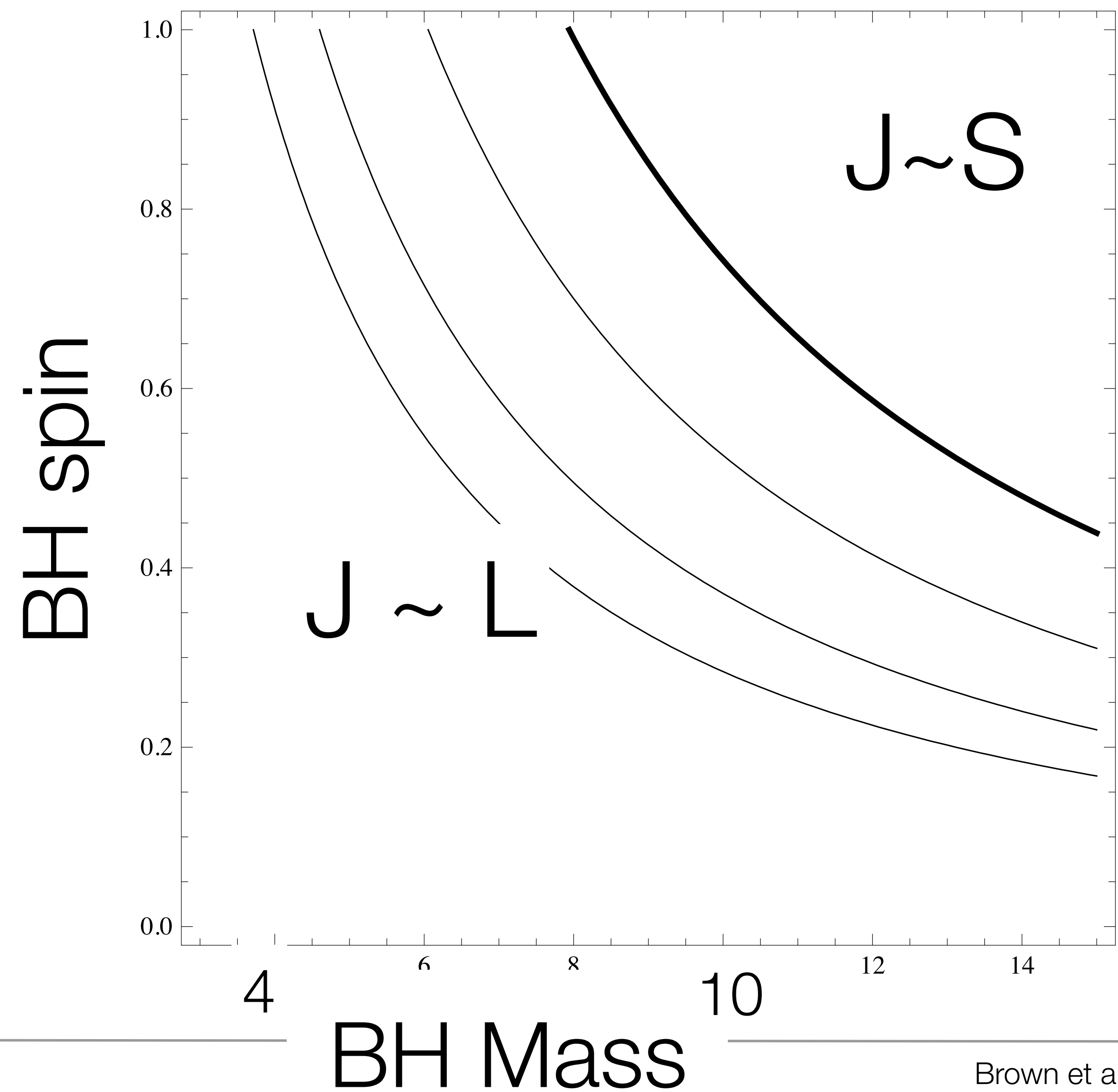
ROS et al 2013

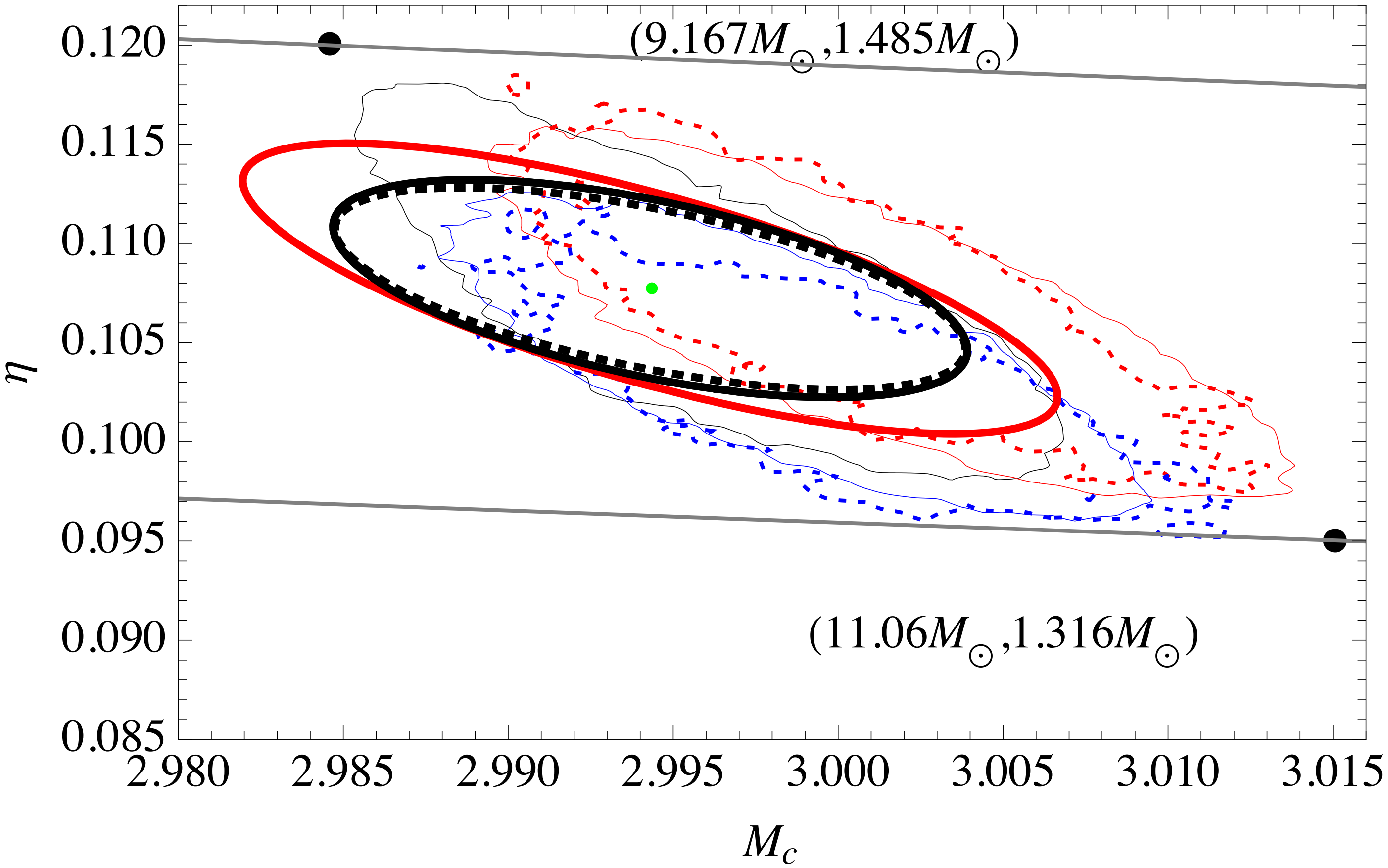
Precession breaks degeneracies 1: Single spin

12d MCMC vs 7d Fisher
ROS et al 2014 (PRD 89 102005)

- Many precession cycles

$$N_P \simeq \int_{\pi f_{min}}^{\pi f_{max}} df_{orb} \frac{dt}{df_{orb}} \Omega_p$$
$$= \frac{5}{96}(2 + 1.5\frac{m_2}{m_1})[(M\pi f_{min})^{-1} - (M\pi f_{max})^{-1}]$$
$$\approx \frac{27(1 + 0.75 m_2/m_1)}{M/10M_\odot}$$

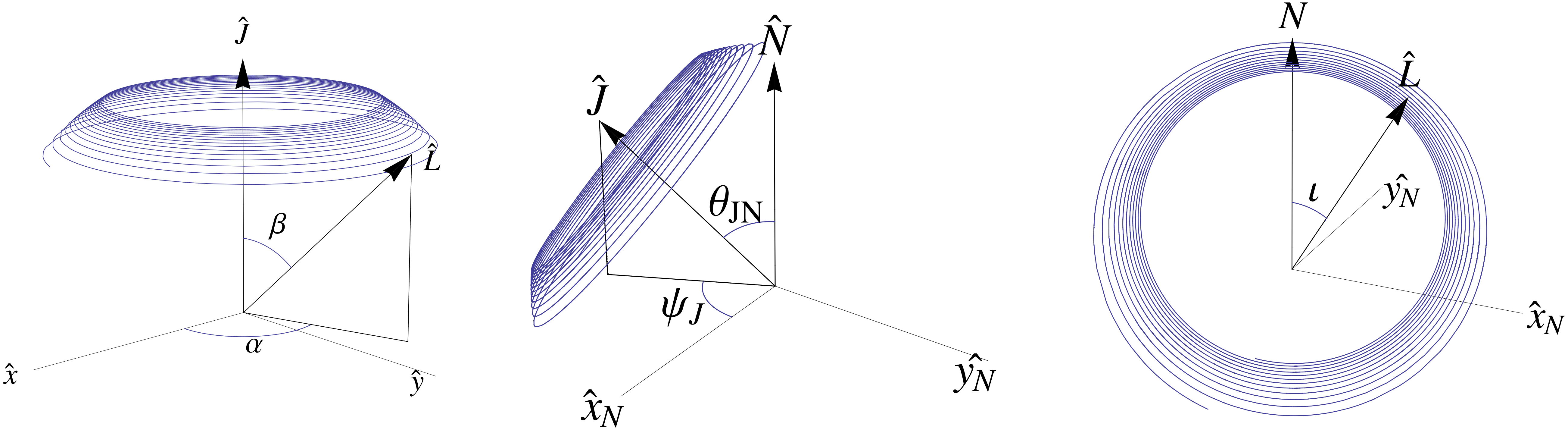


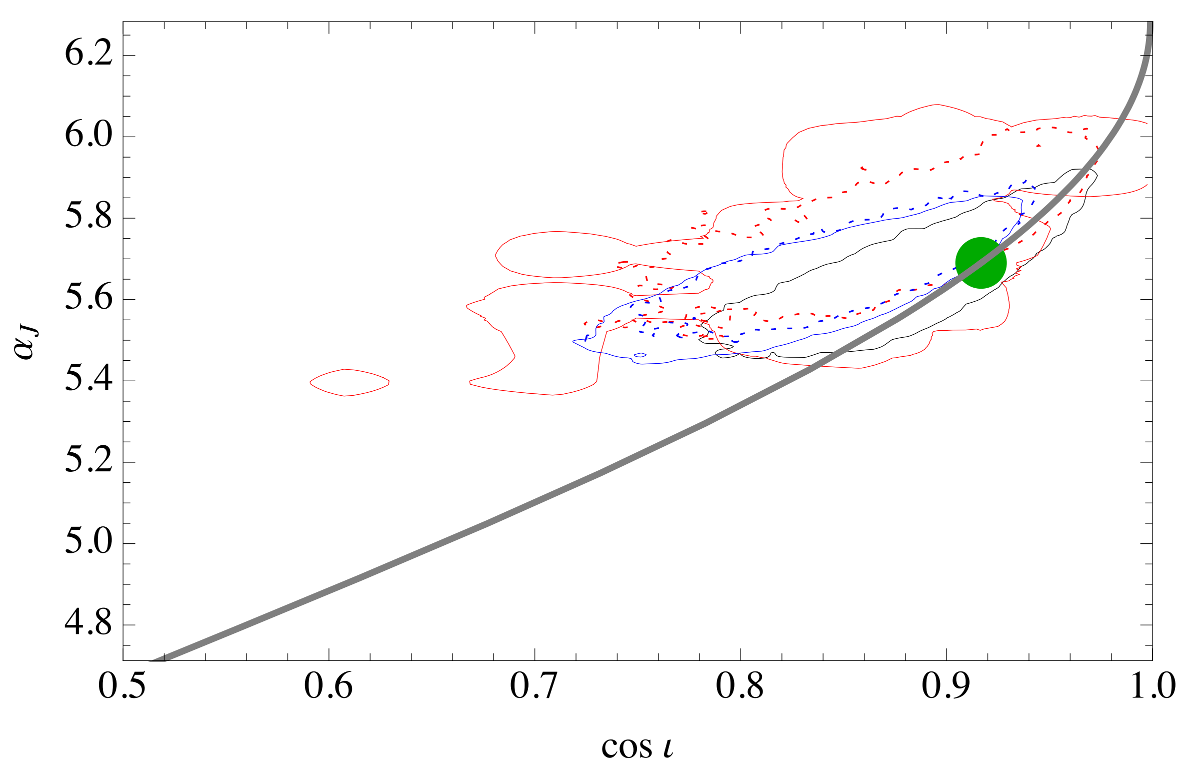


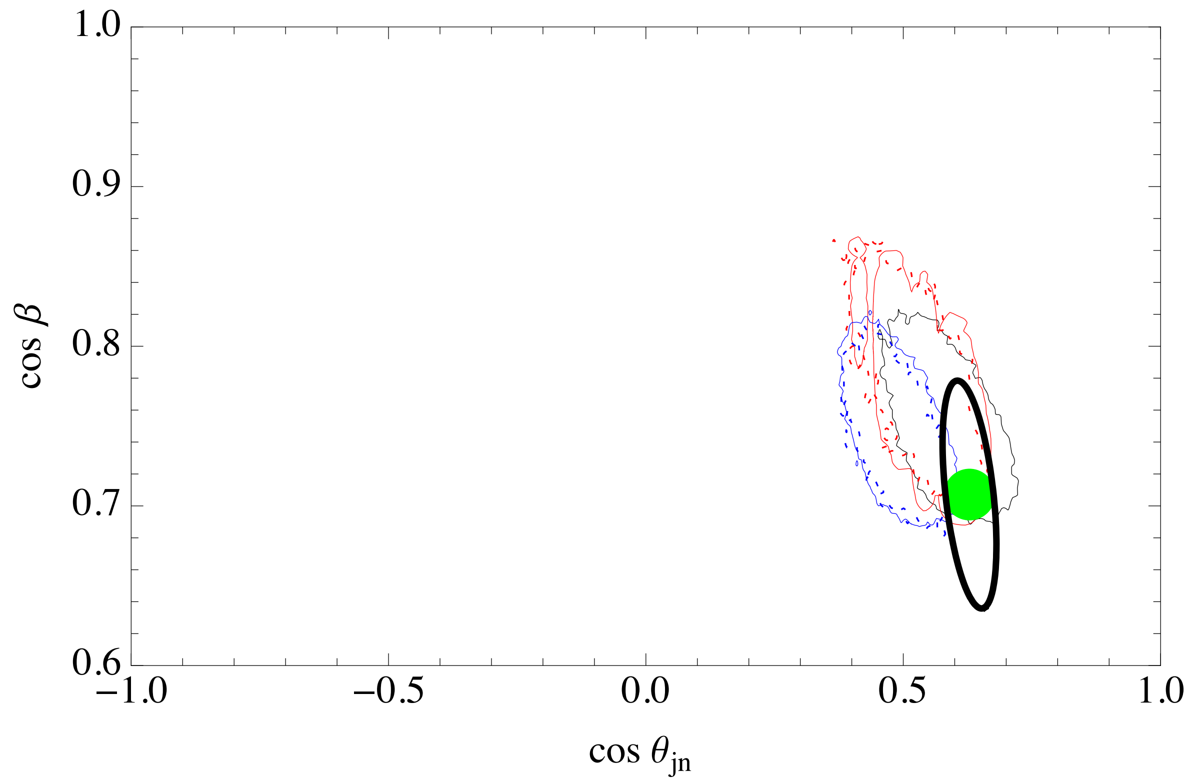
- Requires some misalignment versus line of sight
  - At low mass (but fixed ~ 40 Hz), L can be >>S
  - **Example**: BH-NS (left)
- Single spin well-studied, analytically calculable [ROS et al (1509.06581)]

Brown et al 2012

Sample precessing geometry: BH-NS

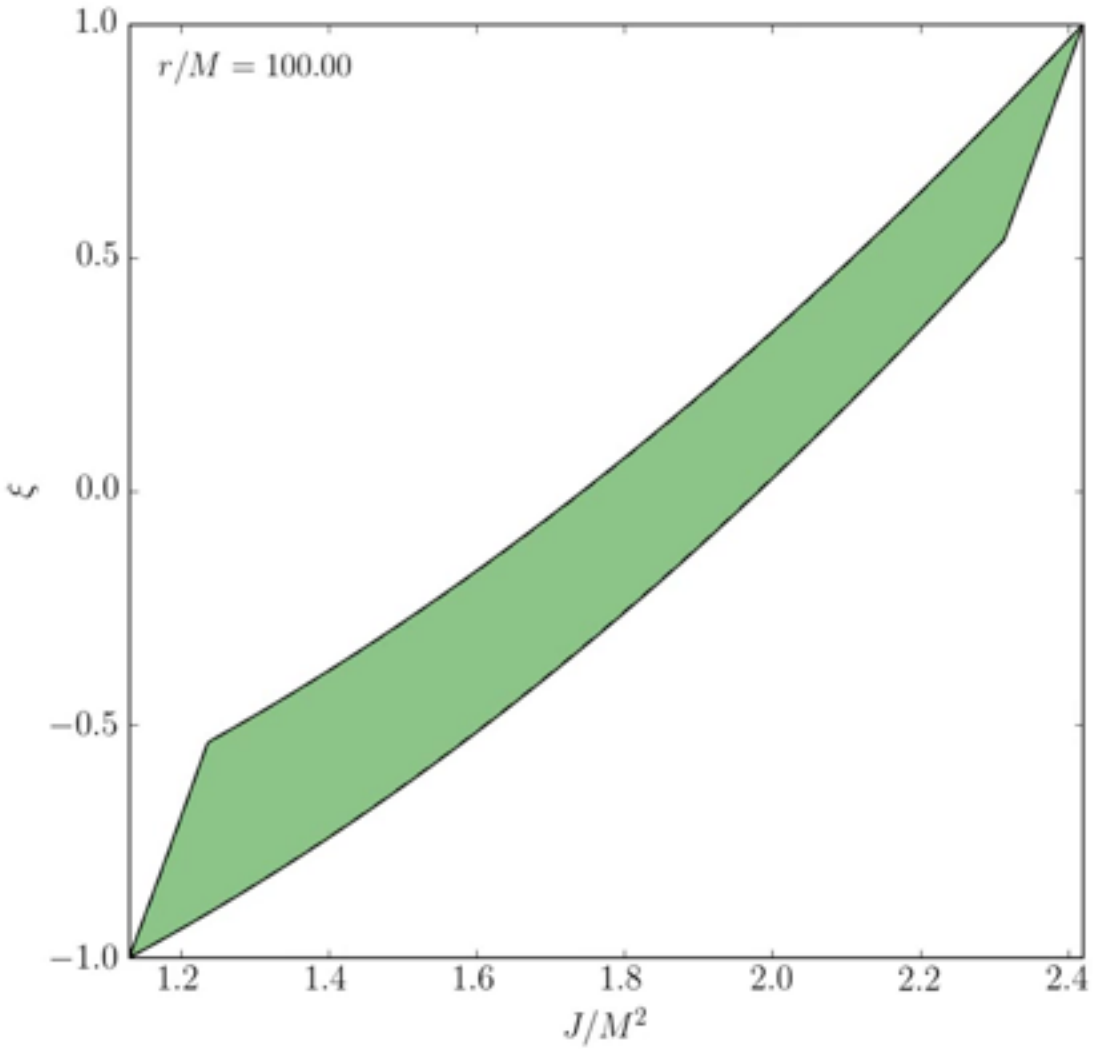


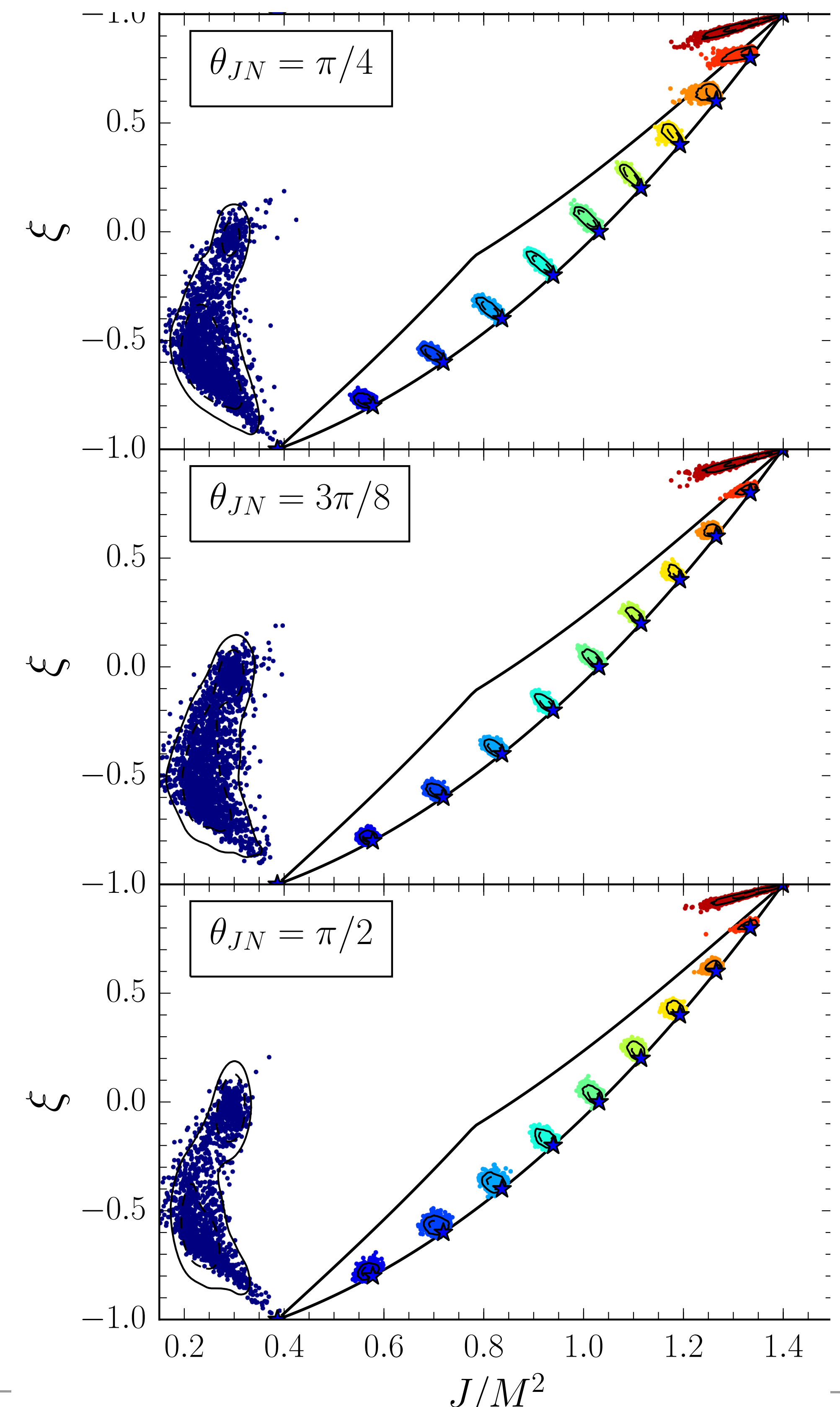




Long signals: Measuring conserved constants

- **Kesden/Berti talks**
  - Example: Both precessing spins measurable with PE



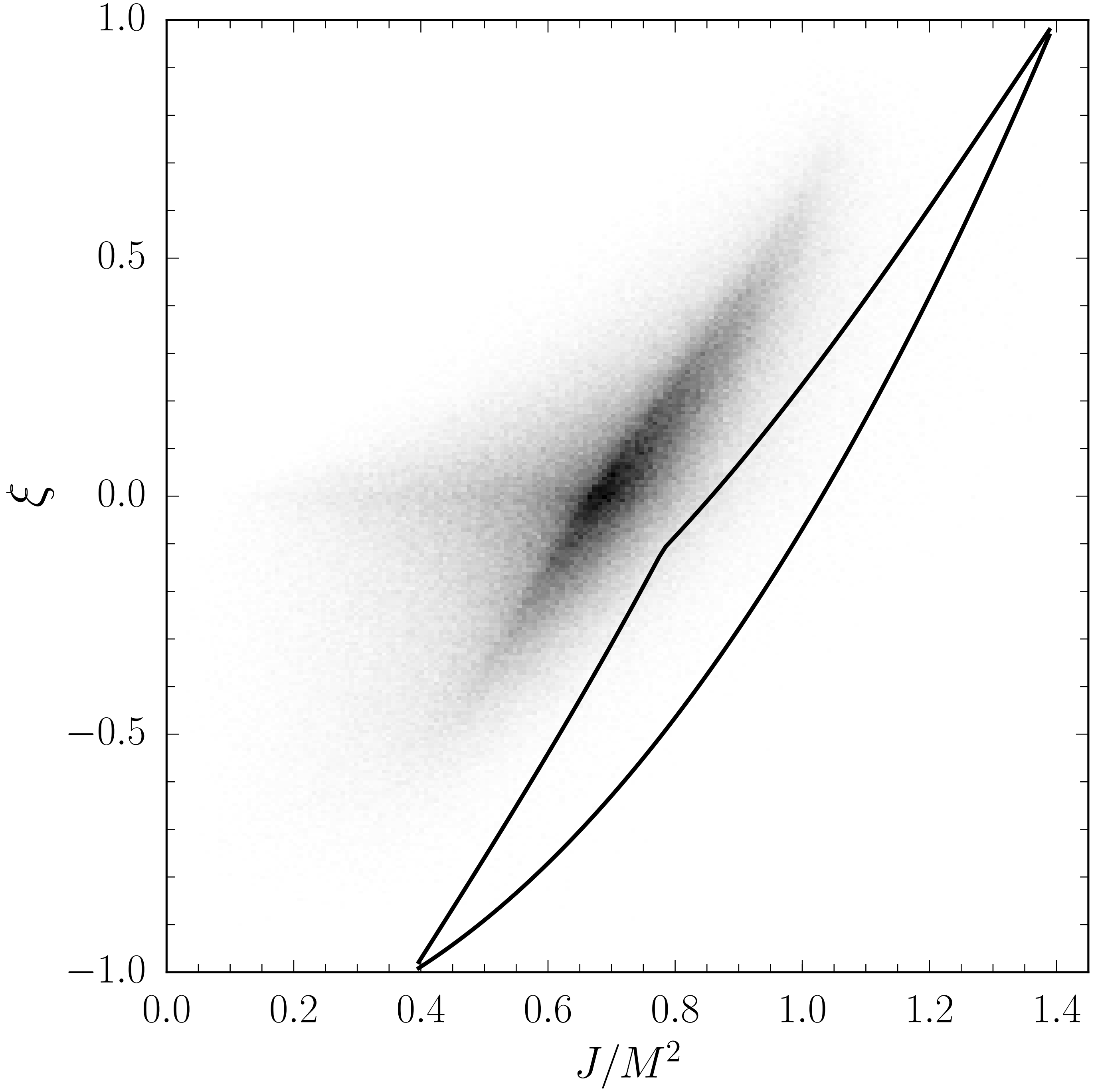


$$\mathbf{J}^2 = \mathbf{L}^2 + (\mathbf{S}_1 + \mathbf{S}_2)^2 + LS_1 \cos\theta_1 + LS_2 \cos\theta_2$$

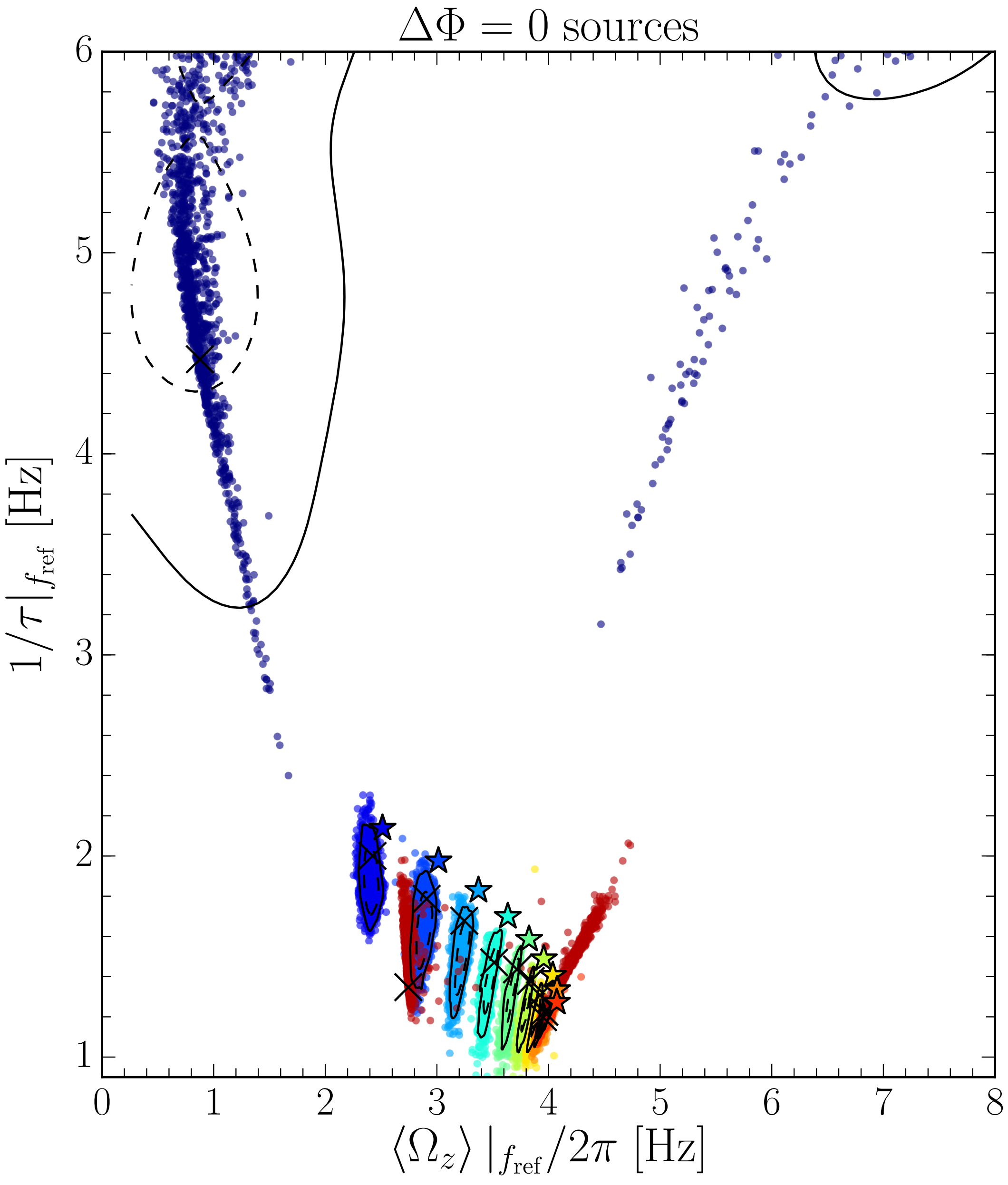
Gerosa et al PRL 2015, PRD 2015

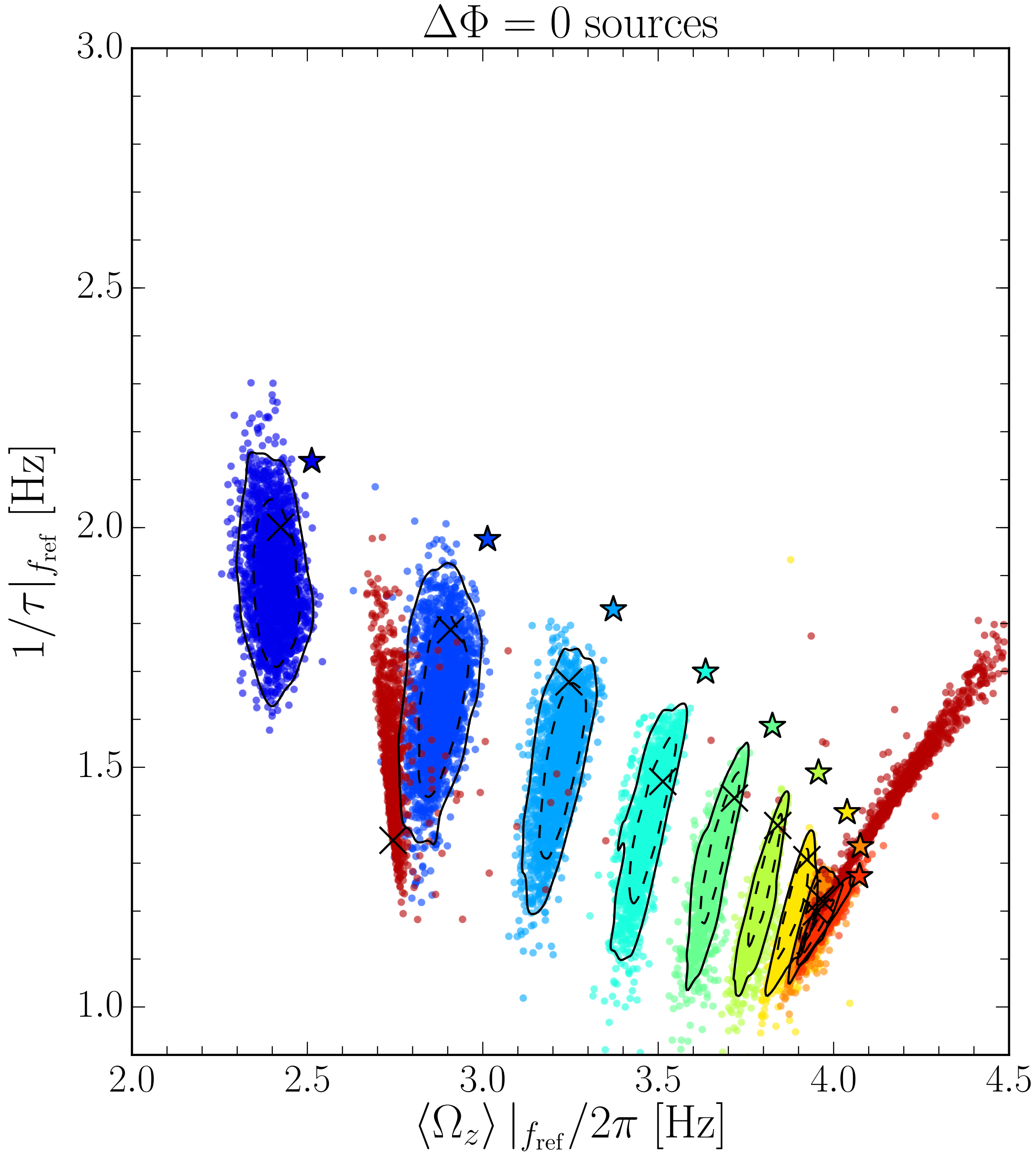
Trifiro et al PRD 93,4071

Priors, again



Timescales as observables?





Final remarks

- What is the imprint of spin on gravitational waves
  - Strong modulations through merger
  - Several effects encode effects of spin strongly, **not** all included in models
    - Importance in short term depends on what nature provides

- Parameter estimation: things to keep in mind
  - Prior matters: mass range & spin magnitude choices can hide spin effects
  - Coordinates matter: use something meaningful and conserved
  - PE methods ~ mature…progress needed in physics / systematics
    - NR available directly at very high mass … precessing hybrids on the way

Final remarks

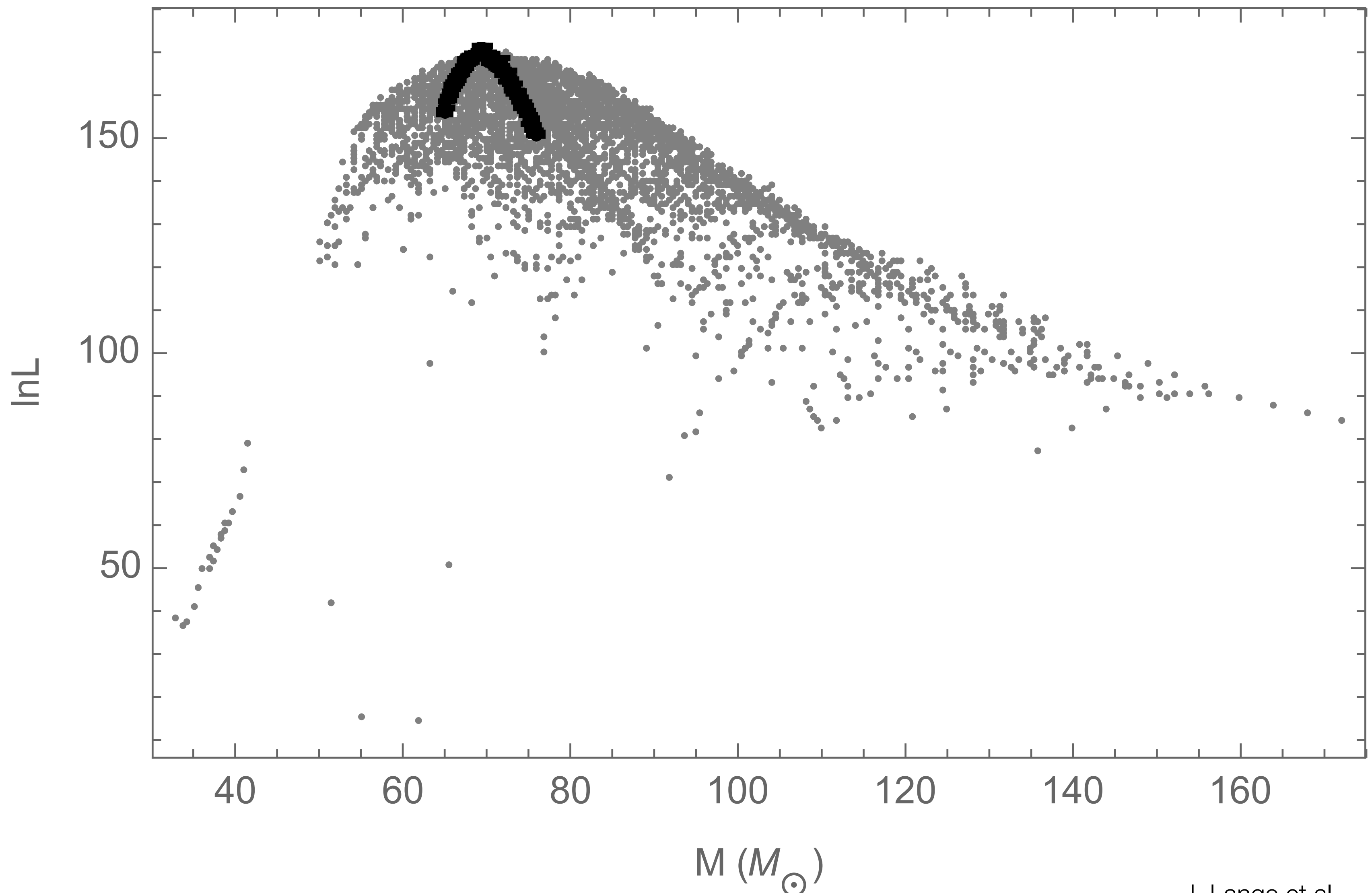
- High mass
  - Information content limited by duration (=low frequency sensitivity, mass)
    - A few precession cycles could help immensely
  - Strong-field physics matters (e.g., higher-order modes)
  - Models ~ adequate at very low SNR and for “typical” orientations
  - Parameter inferences improved by direct comparison to NR, or to NR surrogates

- Low mass
  - Long signals immensely informative, but have degeneracy (face on)
  - With precession, can measure both spins
  - Little SNR at merger, so details less important; end time/frequency most critical
  - Higher modes less critical. Systematics important. Prior matters

Can you tell if more information is available ?

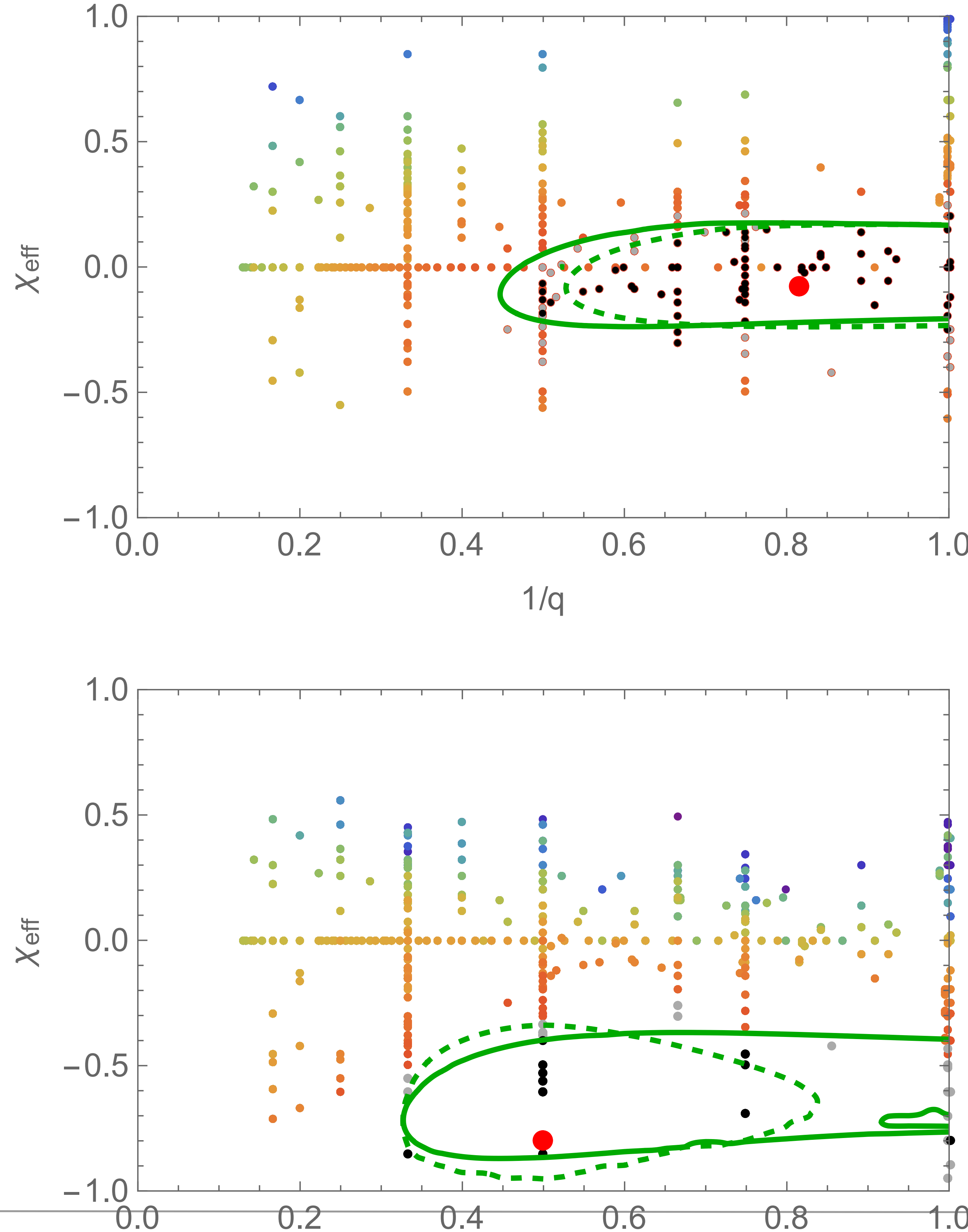
- What if you had a better model? Could you do better?

- **Check**: (Synthetic, known NR data): Is likelihood with full model better than aligned?

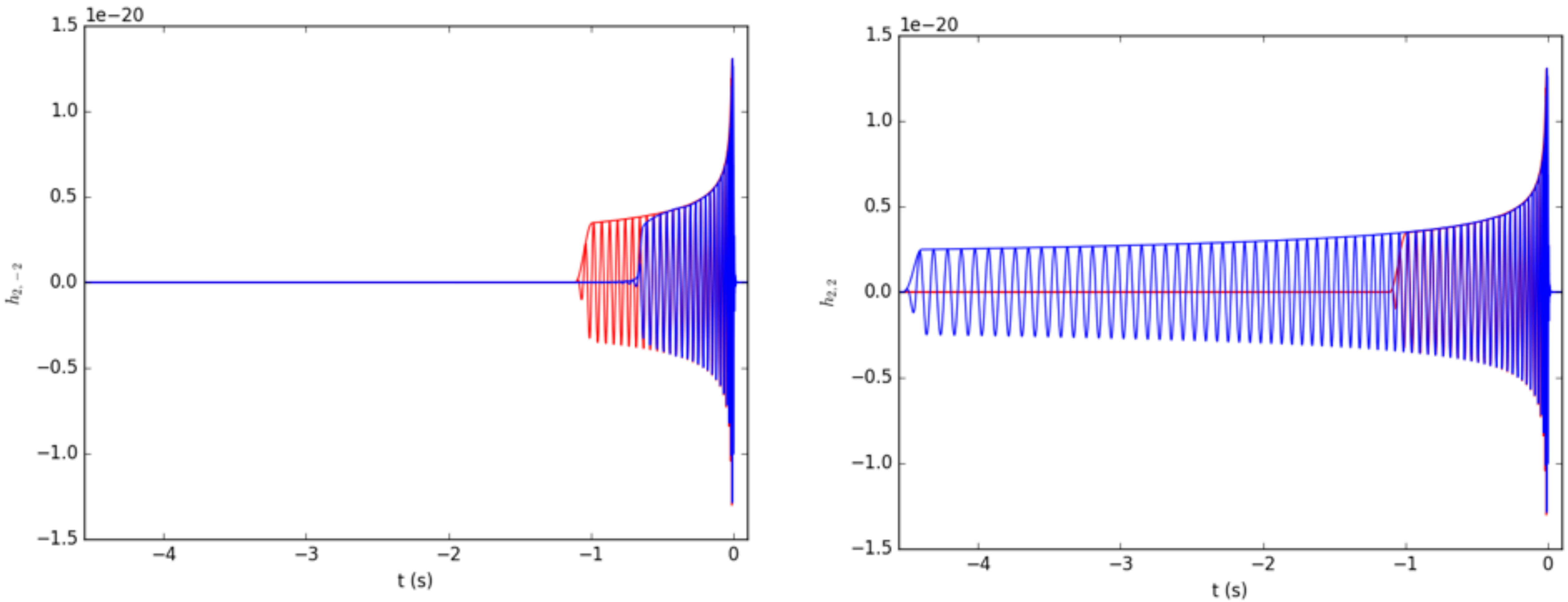


J. Lange et al,

Interpolation



Finite duration & Hybrids

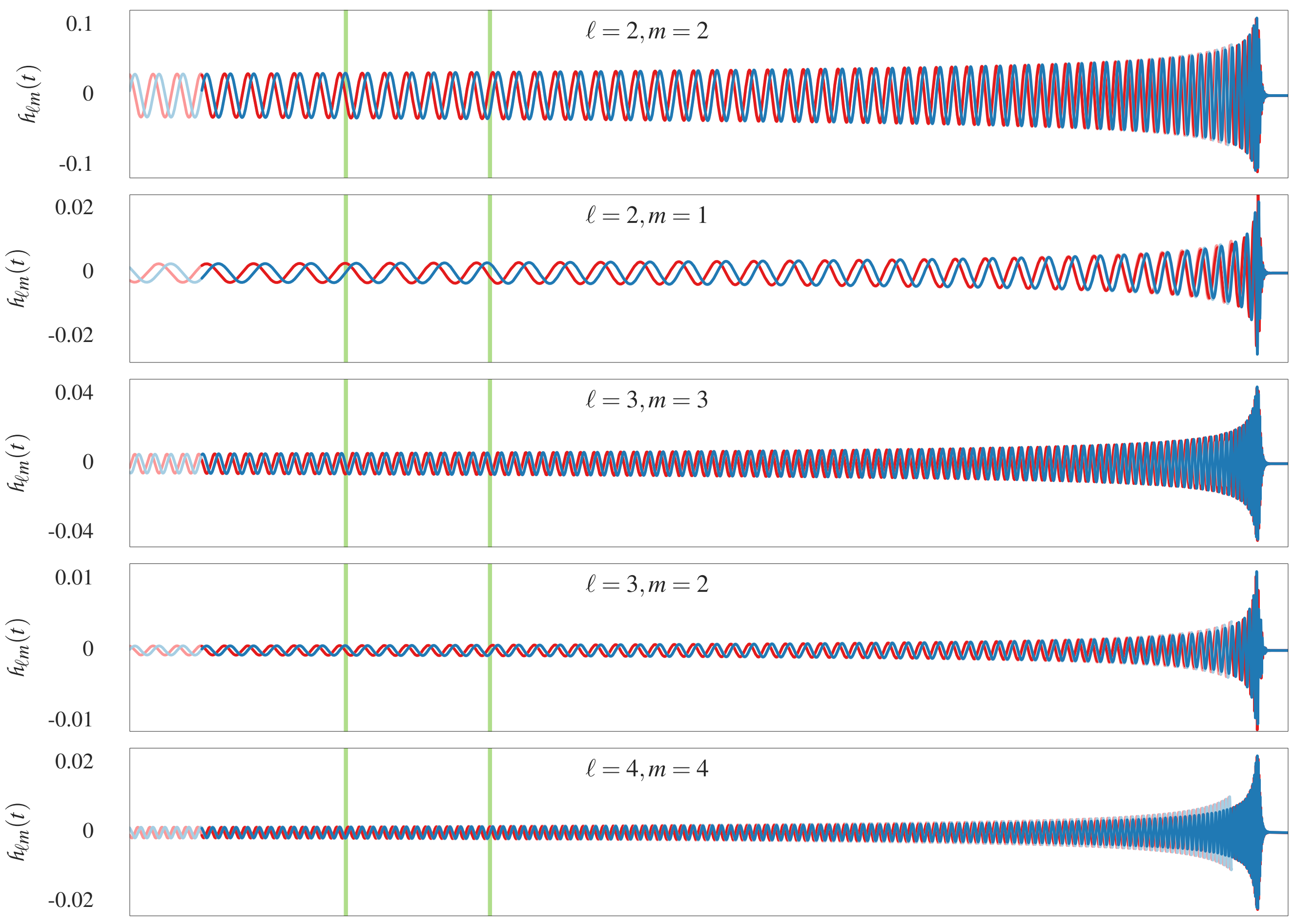


Original RIT GW150914-like

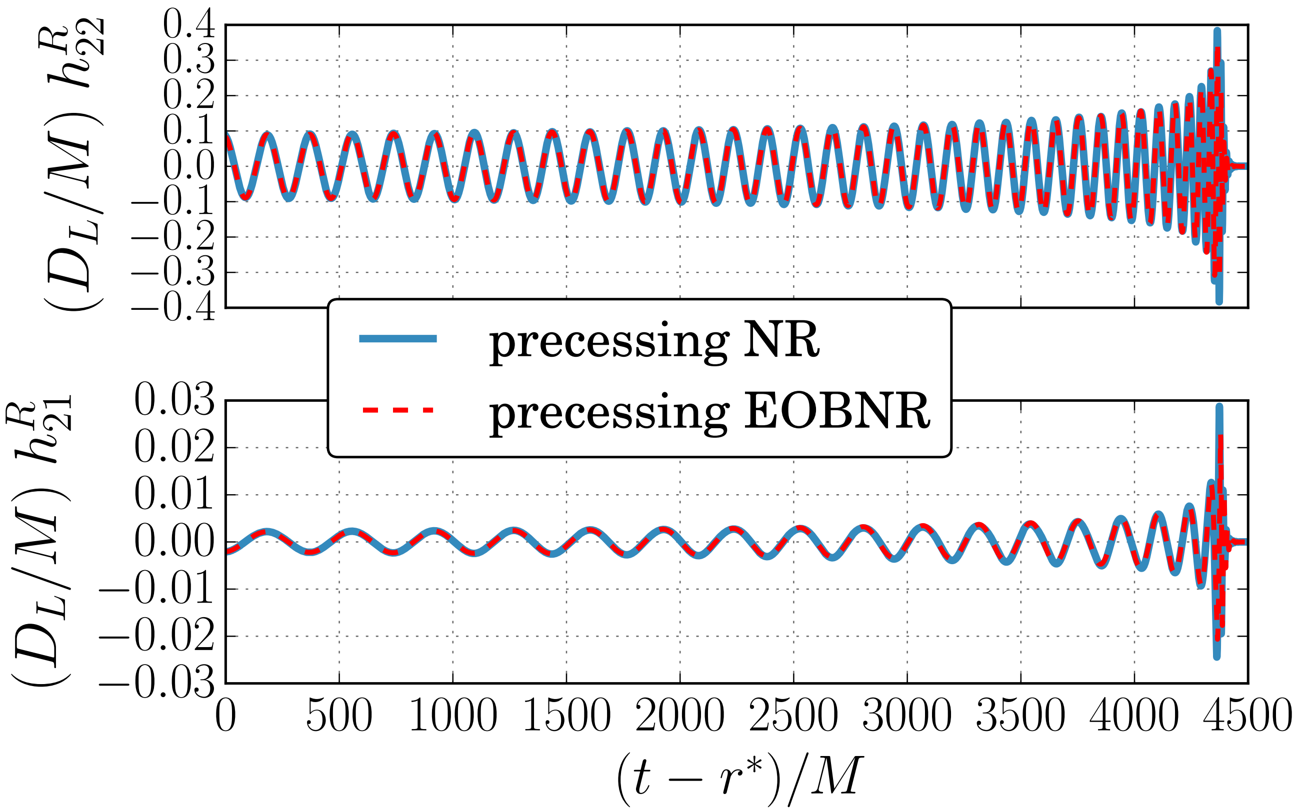
SXS event-like

Finite duration & Hybrids

- Familiar, well-used techniques for aligned (& precessing) spin



Varma and Ajith,1612.05608



Babak, Taracchini, Buonanno 1607.05661
[comparison paper, not a hybrid paper..same ideas]

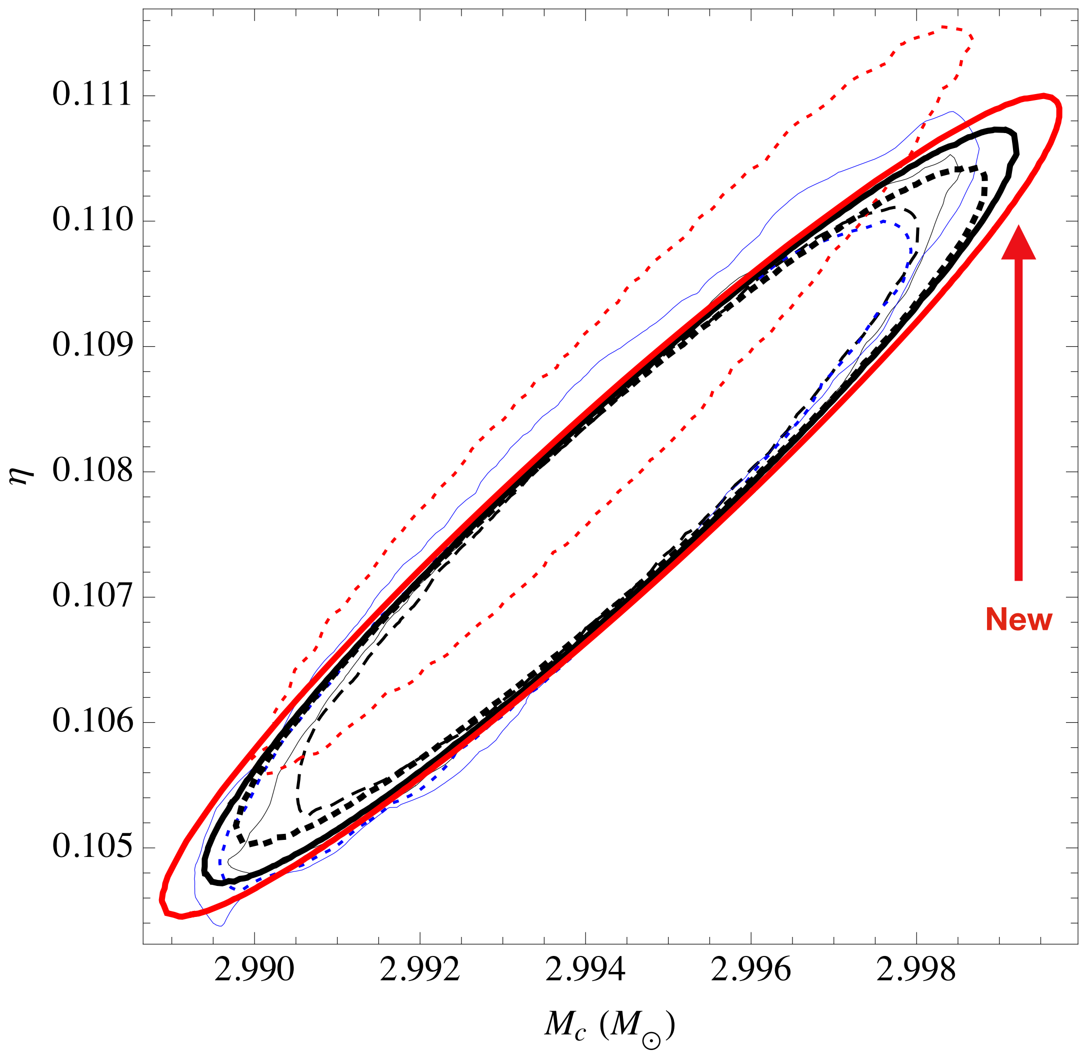
Simple approximate (intrinsic) Fisher matrix

$$\rho^2_{2ms} \equiv |{}_{-2}Y_{2m}(\theta_{JN}) d^2_{m,2s}(\beta)|^2 \int_0^\infty \frac{df}{S_h(f)} \frac{4(\pi \mathcal{M}_c{}^2)^2}{3 d_L^2} (\pi \mathcal{M}_c f)^{-7/3}$$

$$\hat{\Gamma}^{(ms)}_{ab} = \frac{\int_0^\infty \frac{df}{S_h(f)} (\pi \mathcal{M}_c f)^{-7/3} \partial_a(\Psi_2 - 2\zeta - ms\alpha)\partial_b(\Psi_2 - 2\zeta - ms\alpha)}{\int_0^\infty \frac{df}{S_h(f)} (\pi \mathcal{M}_c f)^{-7/3}}$$

- Amplitude
- Angular dependence
- Phase

- Good:
  - Easy to calculate
  - Similar to nonprecessing (weighted average)
  - Intuition about separating parameters
- “Bad”
  - Ansatz / approximation
  - At best, retains all degeneracies of full problem (phases, …)



9d MCMC vs 4d Fisher
ROS et al 2014 (PRD 89 064048)

Bonus slide group: Review of parameter estimation

Inferring source parameters

- Evidence for signal

$$Z(d|H_1) \equiv \frac{p(\{d\}|H_1)}{p(\{d\}|H_0)} = \int d\lambda p(\vec{\lambda}|H_1) \frac{p(\{d\}|\vec{\lambda}, H_1)}{p(\{d\}|H_0)}$$

posterior distribution

H_1 : with signal
H_0 : no signal

- Inputs:
  - Prior knowledge $p(\lambda|H_1)$ about distribution of λ
  - Signal model $h(\lambda)$
  - Noise model $p(\{d\}|H_0)$ $\quad p(\{d\}|\vec{\lambda}, H_1) = p(\{d - h(\vec{\lambda})\}|H_0)$
  - Algorithm for integral/exploration in many dimensions

- Noise model: Gaussian

$$\mathcal{L} \equiv p(\{d\}|\vec{\lambda}, H_1)/p(\{d\}|H_0)$$

$$= \frac{e^{-\langle h(\lambda)-d|h(\lambda)-d\rangle/2}}{e^{-\langle d|d\rangle/2}}$$

Measuring gravitational waves

Detector

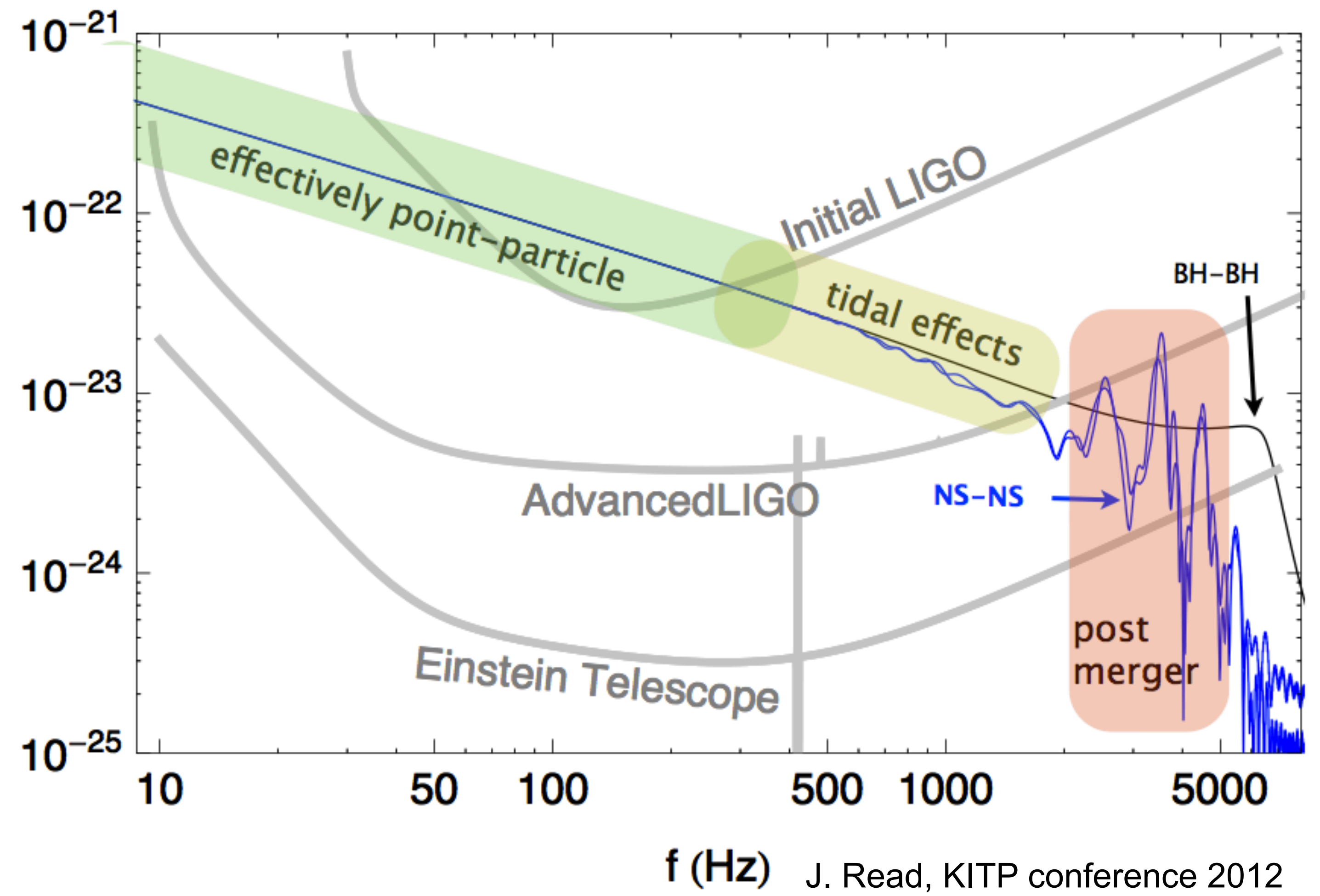
- Nearly gaussian, stationary

$$\langle n^*(f)n(f)\rangle = \frac{1}{2}S_h(|f|)\delta(f-f')$$

$$p(\{d\}|H_0) \propto \exp -\frac{\langle d|d\rangle}{2} dd_1 dd_2 \ldots dd_N$$

$$\langle a|b\rangle \equiv 2\int_{-\infty}^{\infty} df \frac{a^*(f)b(f)}{S_h(|f|)}$$

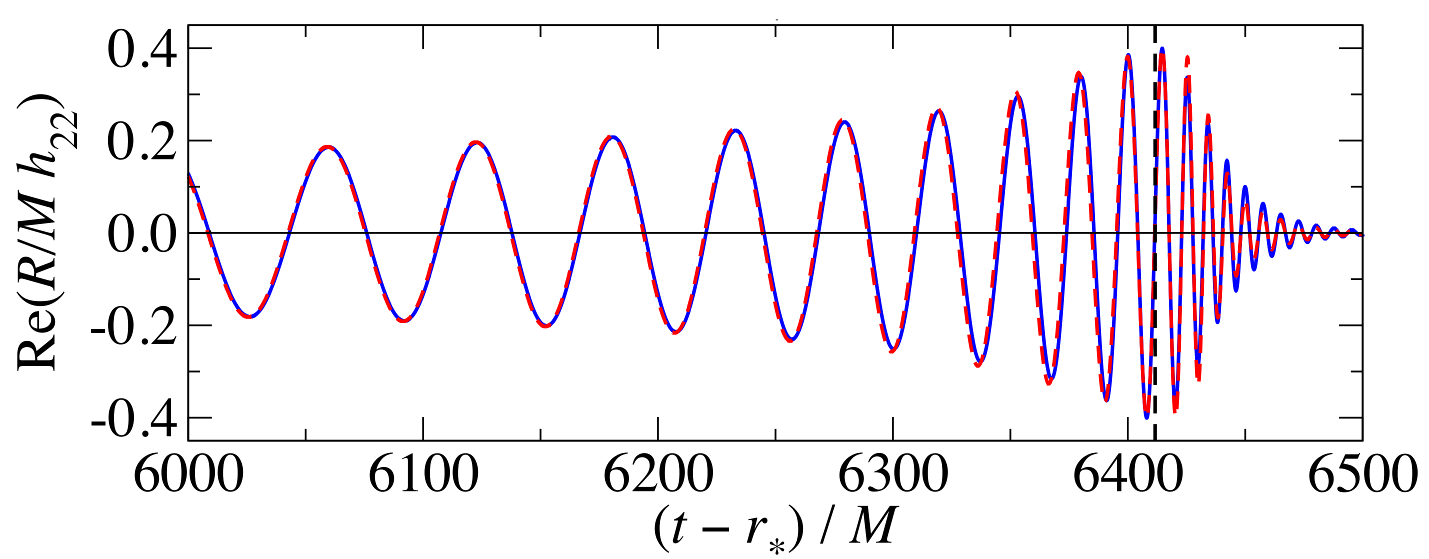
- Band limited



J. Read, KITP conference 2012

Signal

- More cycles at low frequency
- “Typical” merger physics not in band
  - “Input” binary dominates
- Orbital phase: degenerate evolution

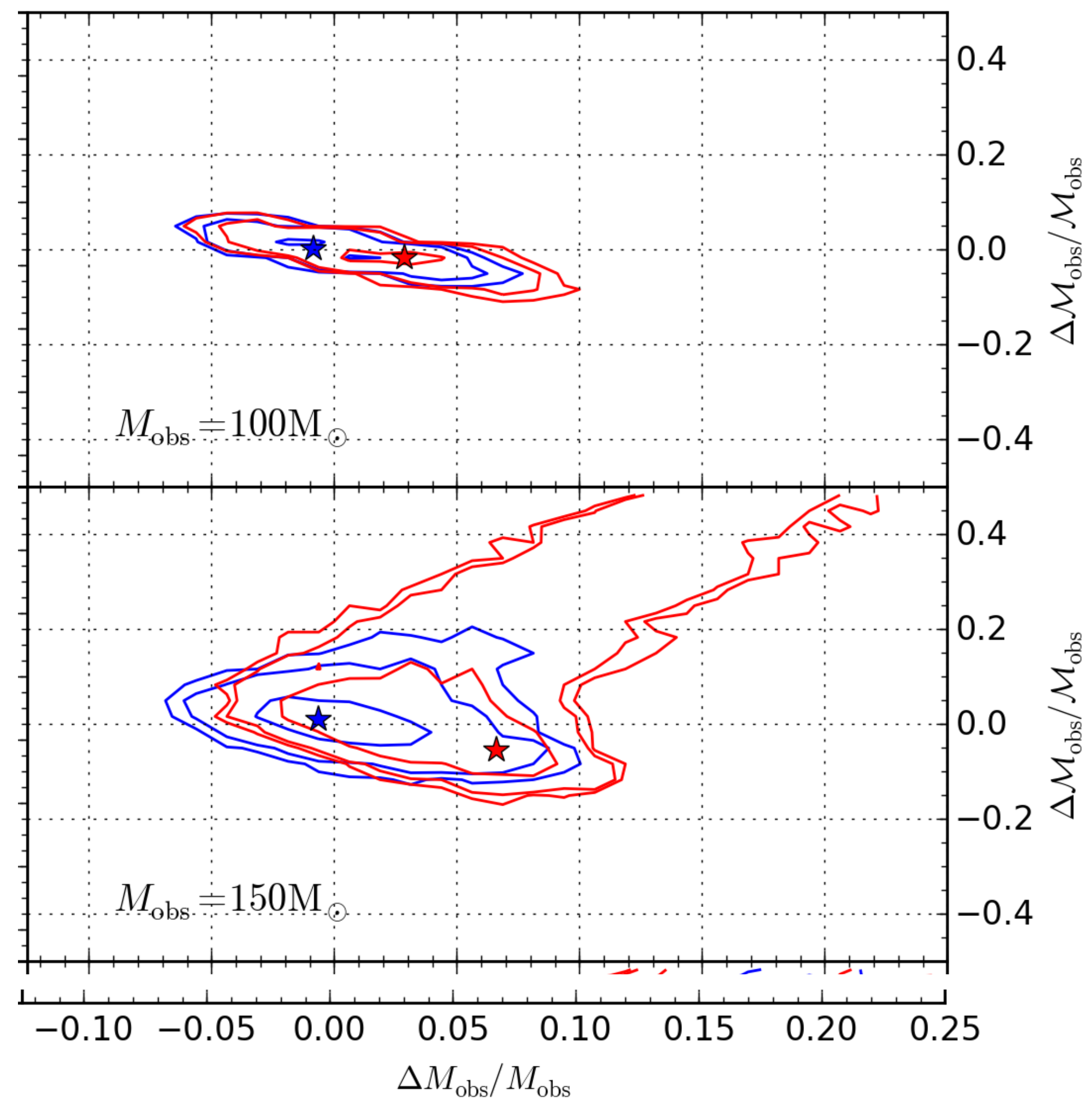


Taracchini et al 2013 (1311.2544)

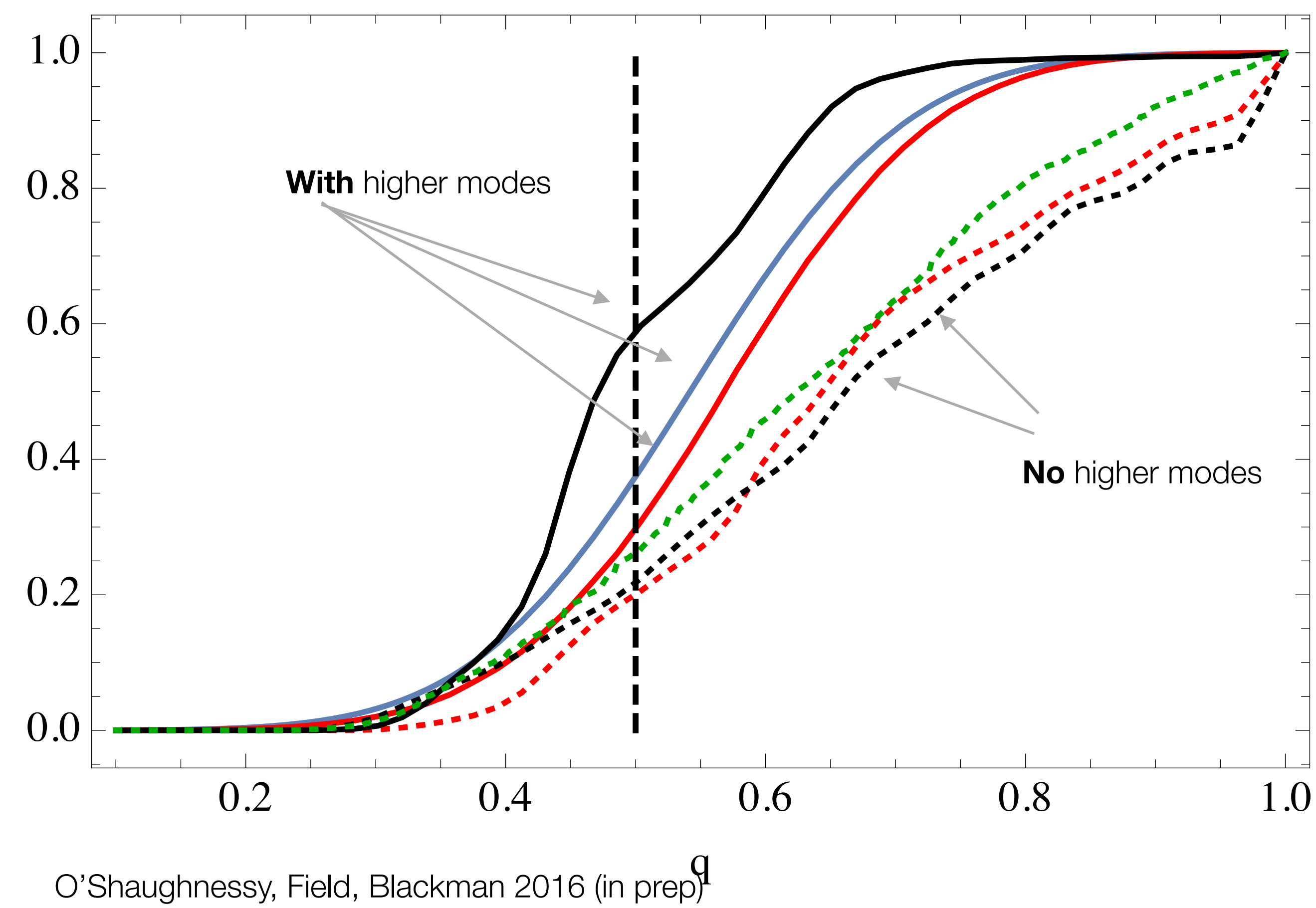
Bonus slide group 1: Impact of higher modes

Higher modes have an impact (relative to mod-GR)

- **More important at high (observed) mass**

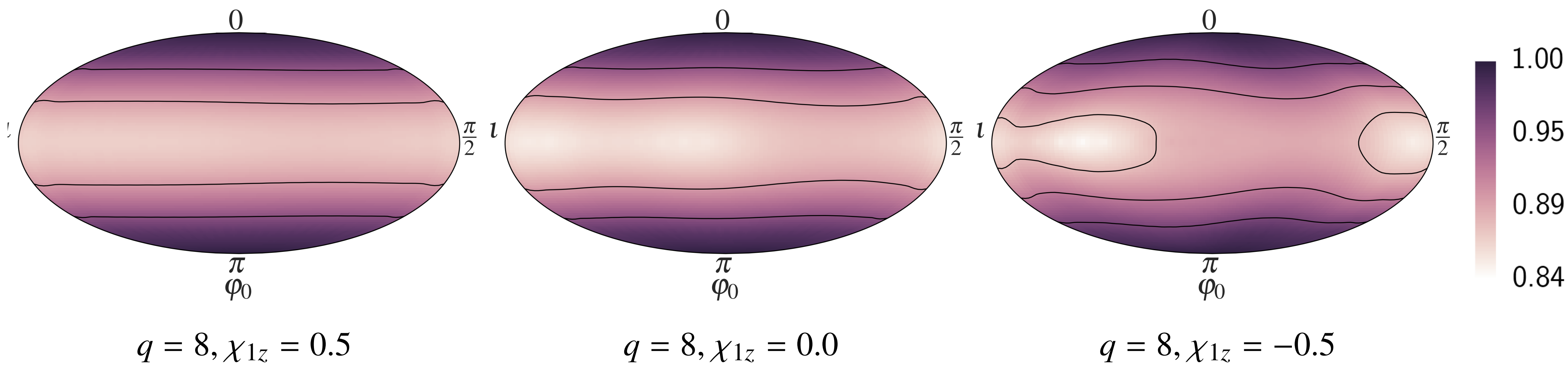


Graff et al 2015
q=4, SNR=12, zero spinn

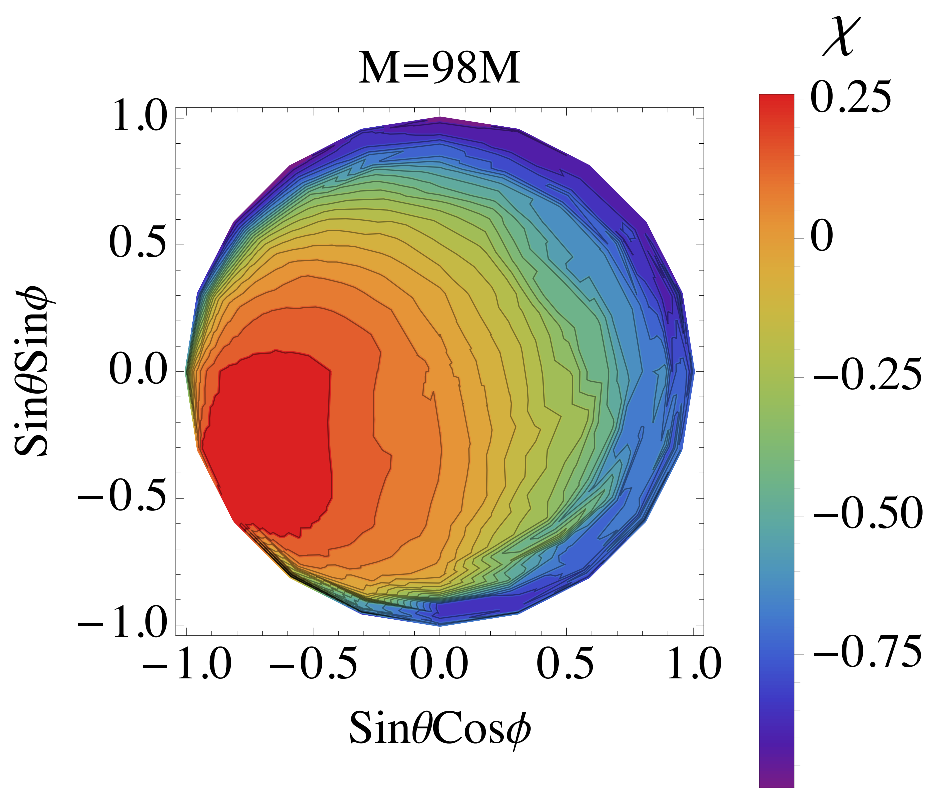


O'Shaughnessy, Field, Blackman 2016 (in prep)
M=150, q=2, aLIGO SNR=25, zero spin

Omission introduces orientation-dependent error



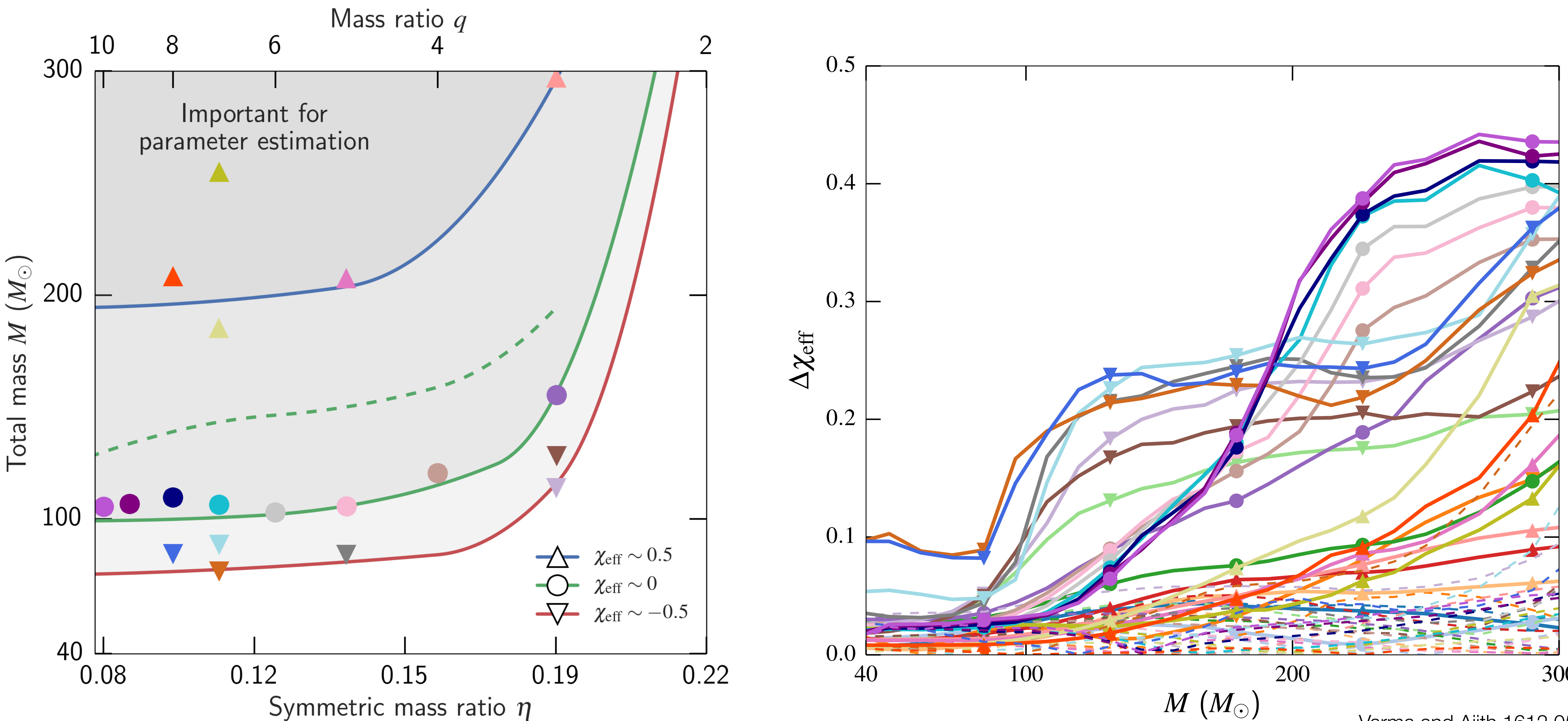
Varma and Ajith,1612.05608



J. Calderon-Bustillo et al 1511.02060
(early aLIGO)

Literature review I: Varma et al

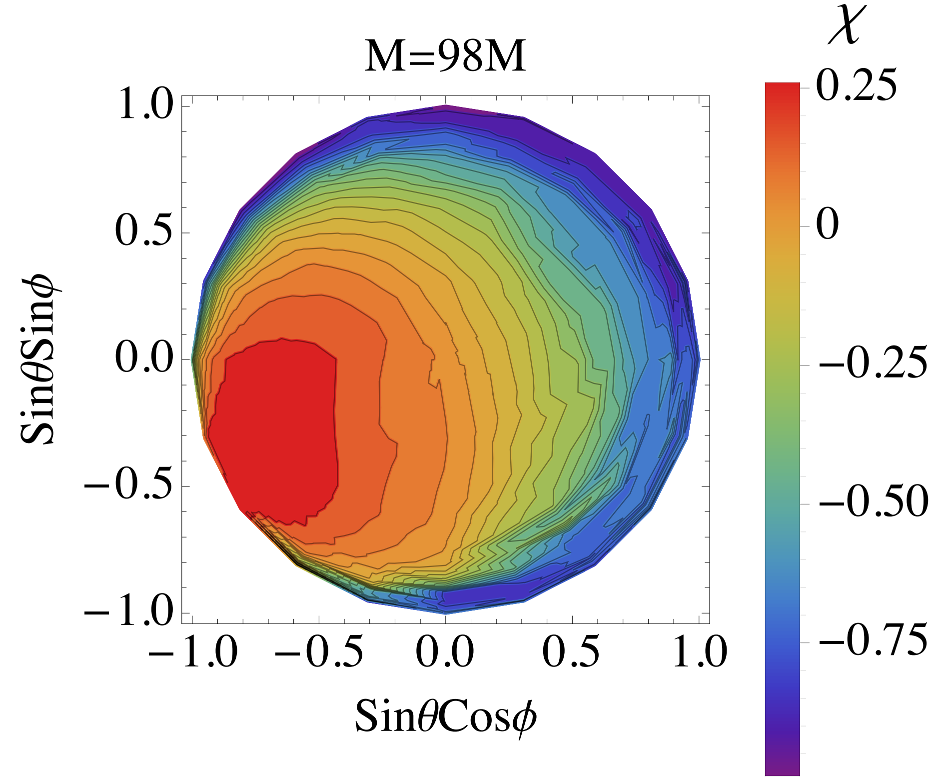
- Aligned-spin hybrid match-based calculation, to estimate PE biases
- Result: Higher modes matter
- **MLE** estimator bias with just 22 is modest [offset >= statistical error]
    - Figures illustrate it is **significant**, & **MLE is not posterior**



Varma and Ajith,1612.05608

Literature review 2: JCB

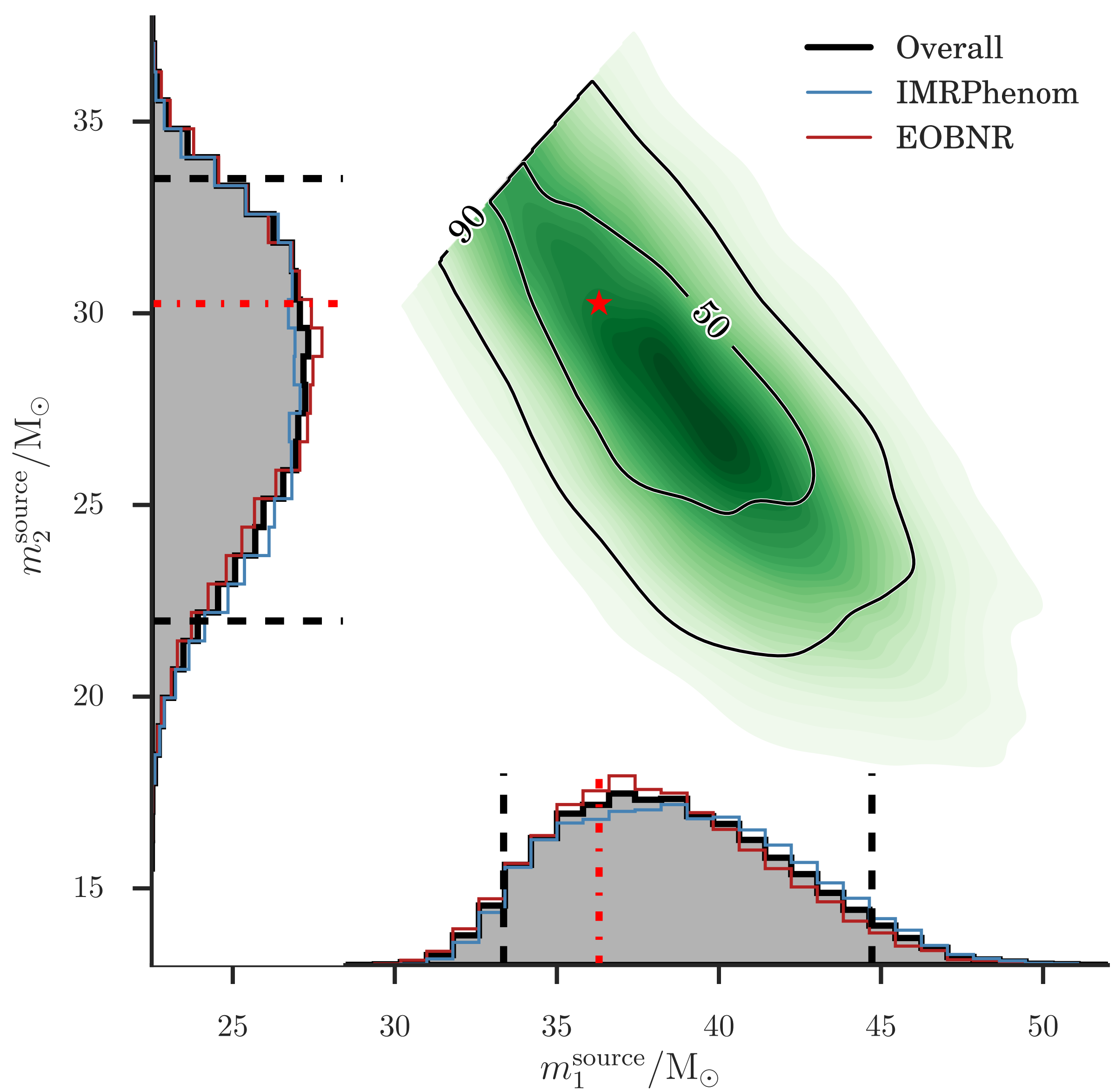
- Orientation-dependent biases

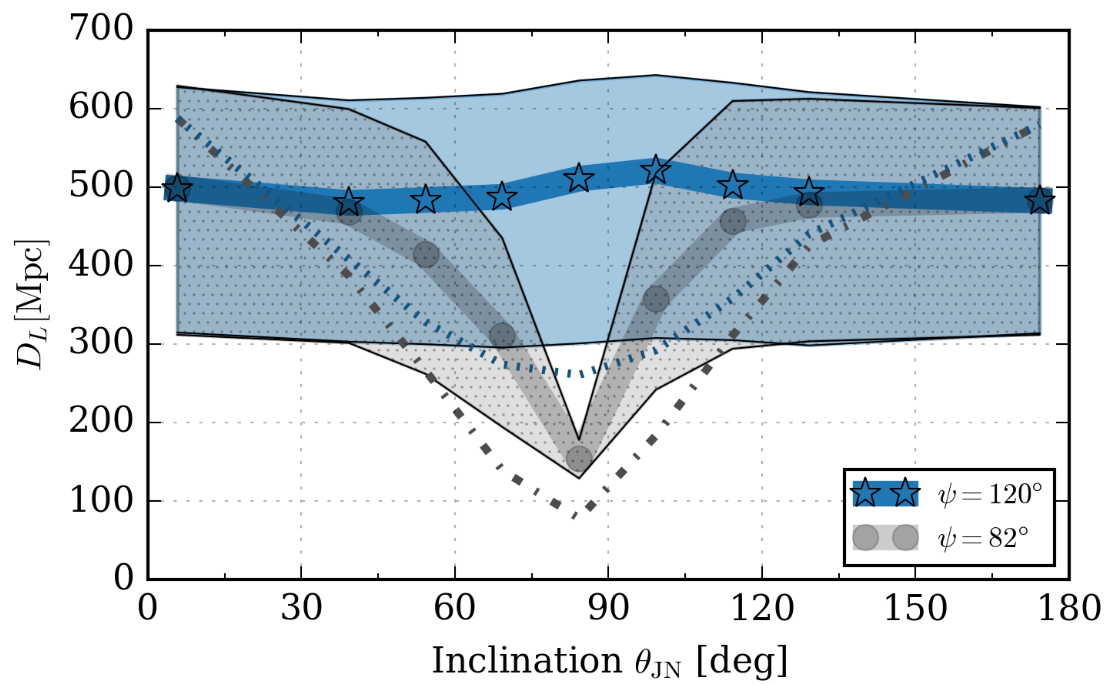


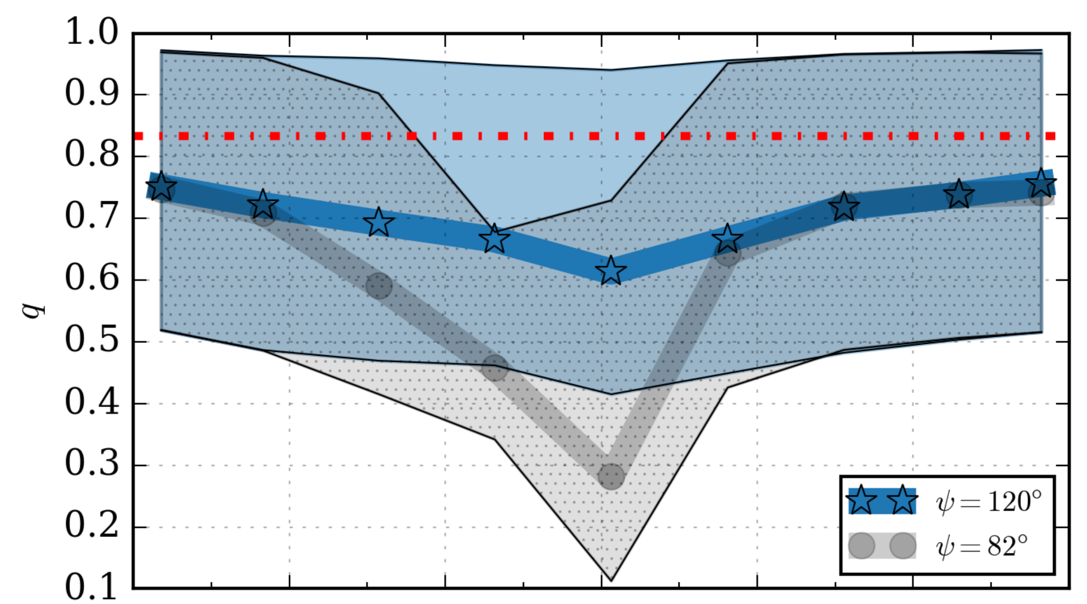
J. Calderon-Bustillo et al 1511.02060
(early aLIGO)

Literature review 3: LVC NR systematics paper

- NR injection study, but recovery with existing models
  - Orientation-dependent biases using quadrupole-only templates
  - What would the posterior be, with a better model?







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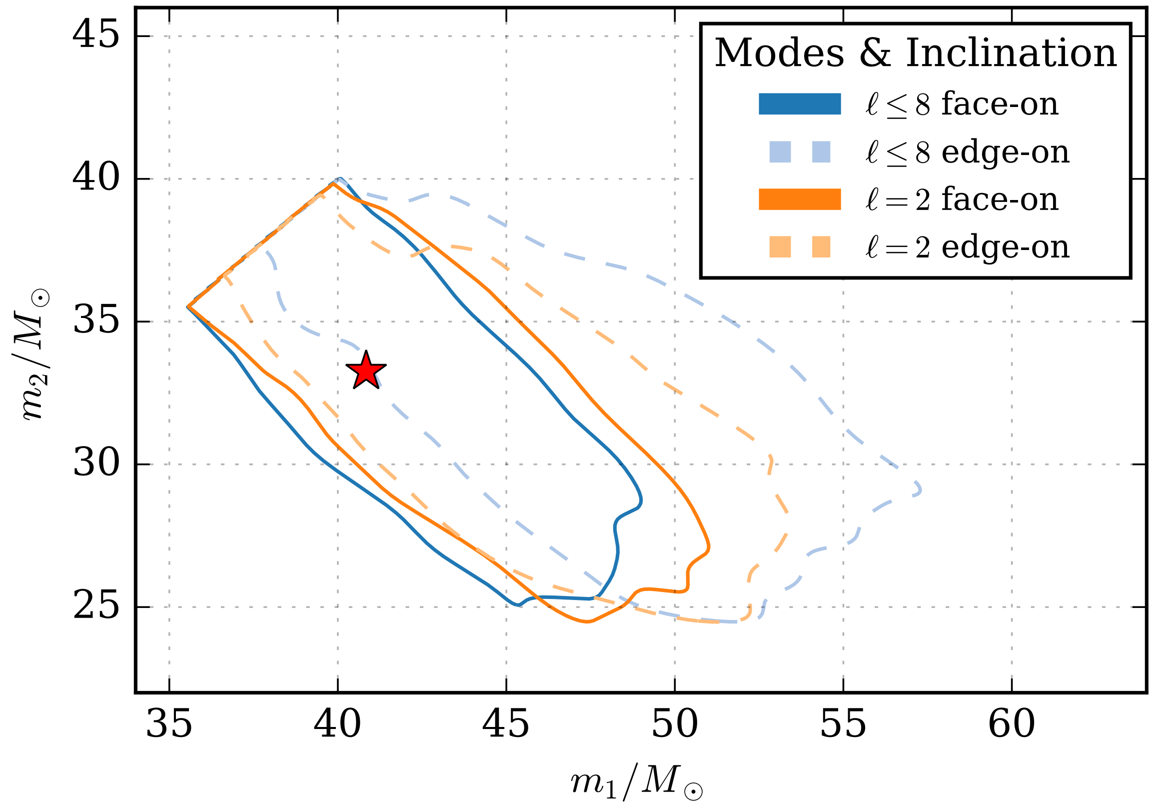
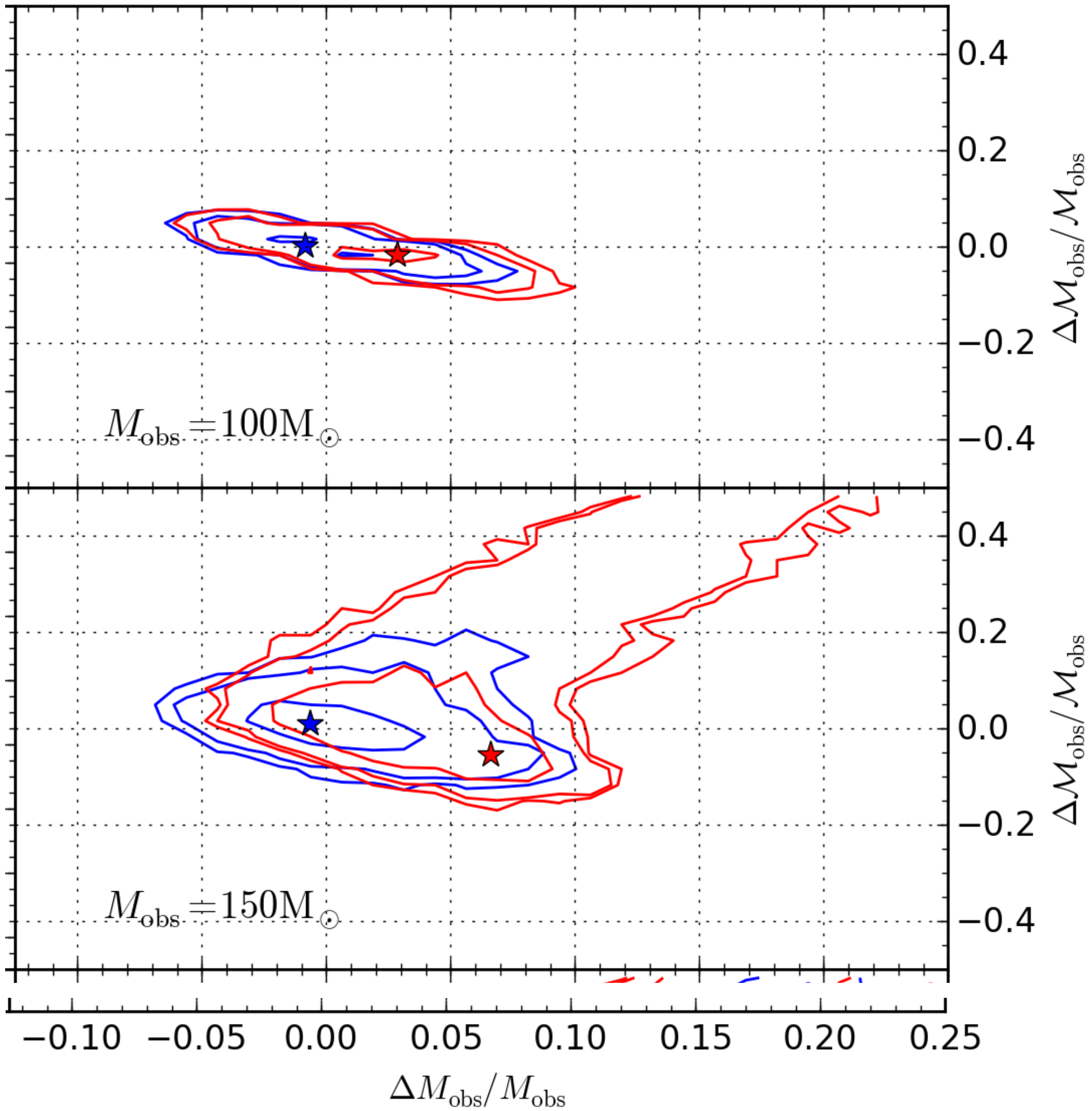
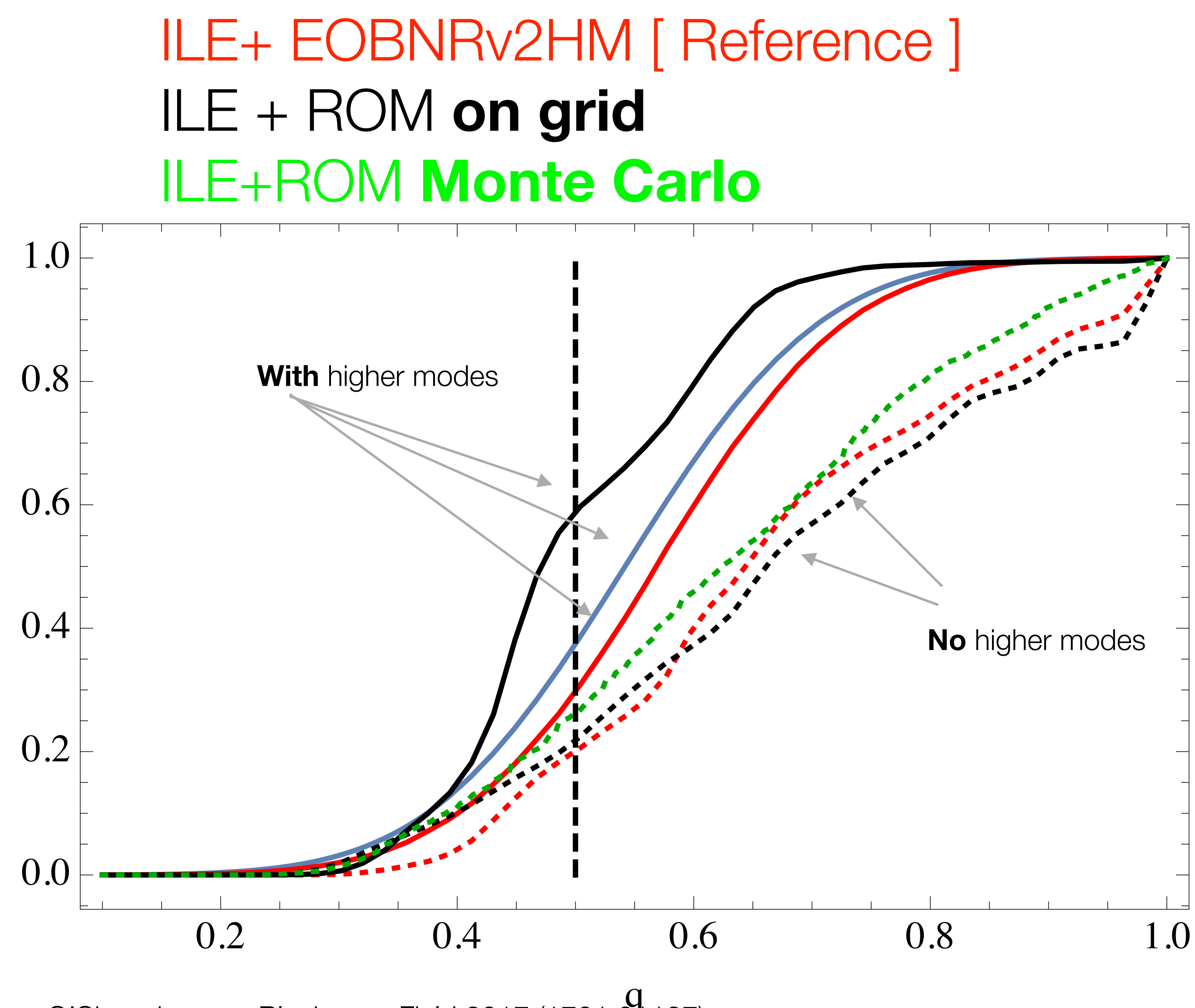


Literature review 4: Graff et al / ROS, JB, Field

- Zero-spin PE calculations with higher modes (EOB; NR surrogate)
- Higher modes matter.   NR surrogate differs from EOB



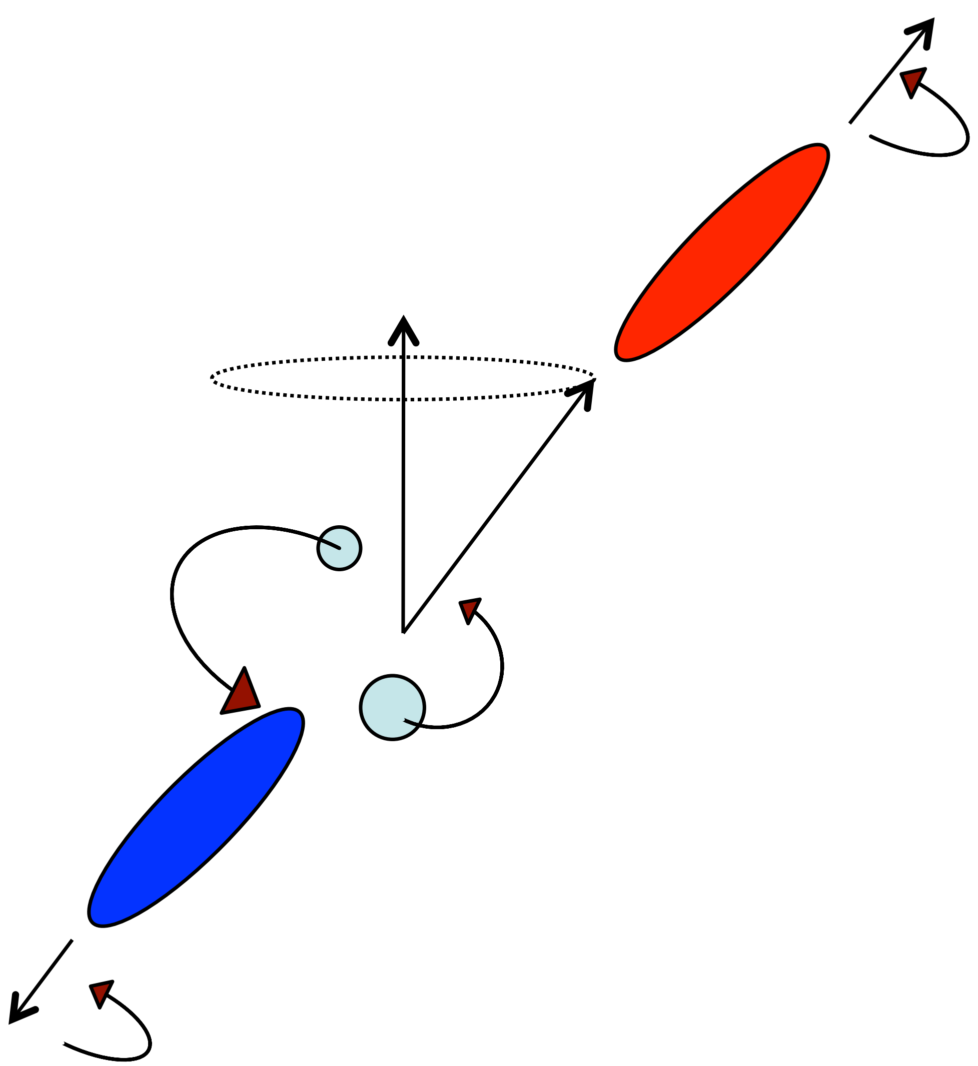
Graff et al 2015
q=4, SNR=12, zero spinn



O'Shaughnessy, Blackman, Field 2017 (1701.01137)
M=150, q=2, aLIGO SNR=25, zero spin

Polarization (versus time)

- Left- and right-handed radiation **easy** to distinguish
  - Constrains opening angle of precession
  - Sets lower bound on (transverse) spin
  - Often: separation of timescales



R

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Polarization for alignment and precession

- Polarization **easy** to measure
  - "Only see what we see" = at 100 Hz !
- Measure **spin-orbit misalignment**
  - via simple geometry + polarization
  - Traces strength whatever misaligns them
    - SN kicks
    - Stellar dynamics [binary collisions]
  - Measure BH spin
    - Insight into SN, massive star physics