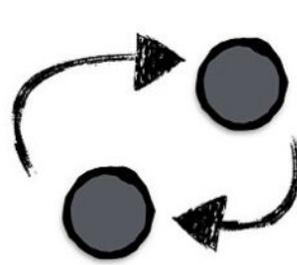


Kicked Waveforms

Observing Black Hole Recoils
in Gravitational Wave Signals



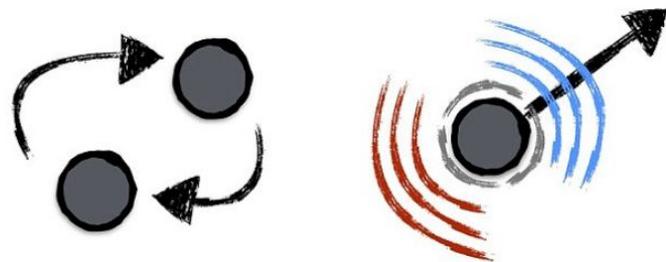
Christopher Moore, DAMTP, Cambridge, UK
StronG BaD, Mississippi

1st March 2017

Work done in collaboration with Davide Gerosa and Riccardo Barbieri

Outline

- Introduction to black hole kicks
- Modelling the kick in the GW signal
- Detecting the kick
- Parameter estimation for kicks
- Concluding remarks



Black hole kicks

Merging black holes recoil due to asymmetric GW emission in final stages of merger

Peres (1962)

Recoil, or ***kick***, velocities can be sufficient to eject the remnant from the galaxy

Redmount & Rees (1989)

Smaller kicks can also have important consequences as they can disrupt stars in the galactic nucleus

Gualandris & Merritt (2007)
Komossa & Merritt (2008)

Kicks leave an imprint on the GW signal via a Doppler shift of the final few wave cycles...

Gerosa & Moore (2016)

Black hole kicks

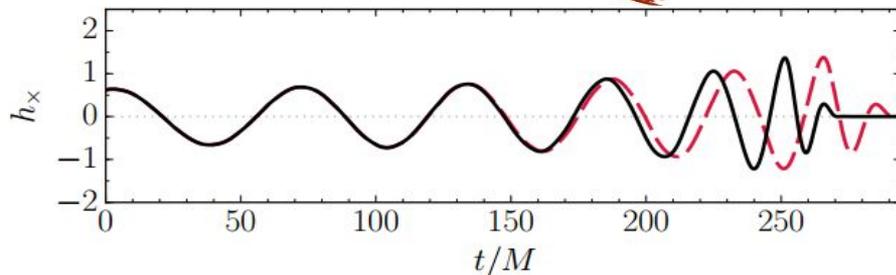
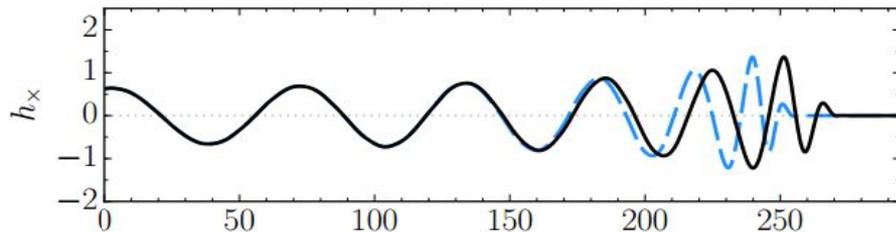
In a binary black hole merger the total mass and redshift are degenerate

A merger kick accelerates the system over a period of $\approx 20M$ around merger

The kick imprints itself on the GW signal similar to a ***time dependent mass***

This time dependence prevents any degeneracy with total mass or redshift

$$M \rightsquigarrow M(1+z)(1 + \vec{n} \cdot \vec{v}(t))$$



Black hole kicks

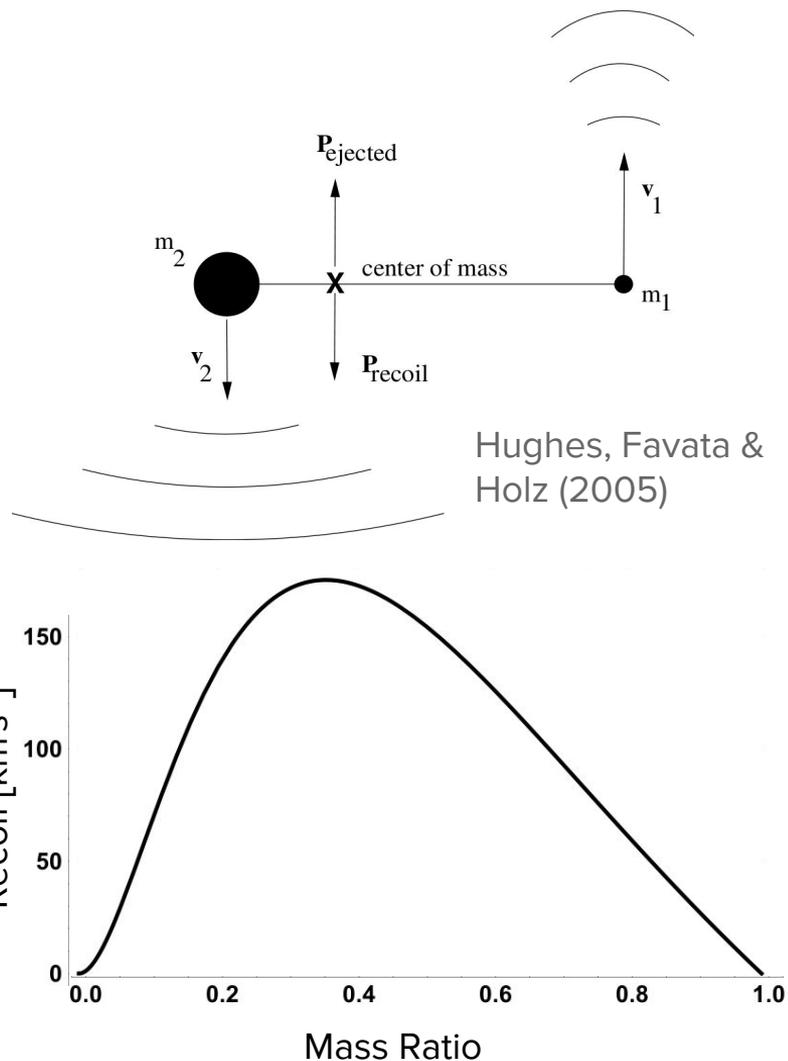
Equal mass, non-spinning binaries have no kicks... by symmetry!

Asymmetry in the binary leads to large kicks

Mass asymmetry:

Mass ratios of $0.3 \lesssim q \lesssim 0.4$ produce the largest **mass kicks** of around 160 km s^{-1}

J. A. González et al. (2007)



Black hole kicks

Equal mass, non-spinning binaries have no kicks... by symmetry!

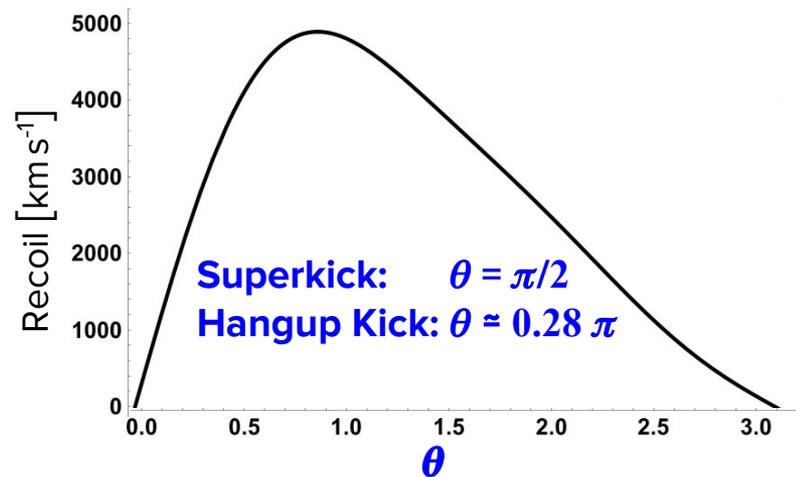
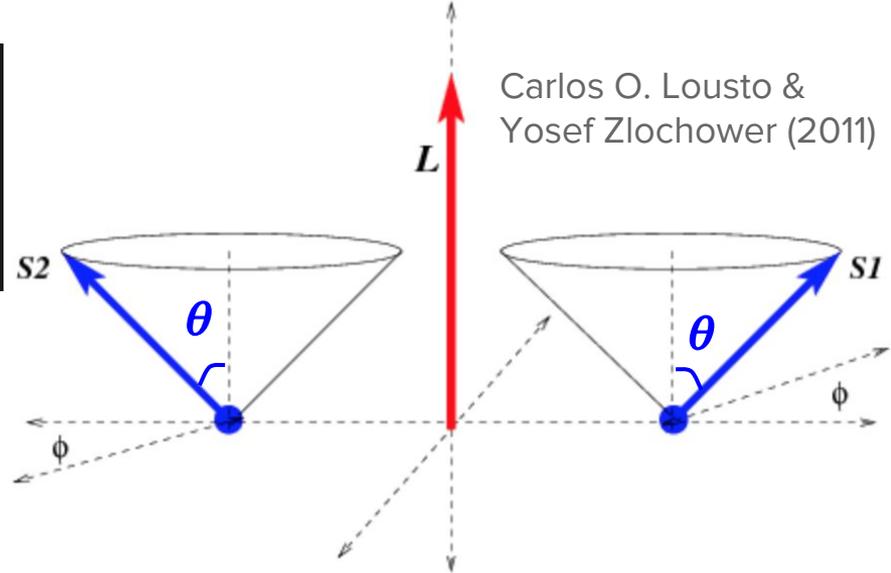
Asymmetry in the binary leads to large kicks

Spin asymmetry:

Spin asymmetry can lead to larger kicks... especially if the components of the two spins in the orbital plane are anti-parallel

The **superkick**, or **hangup kick**, spin configurations give kicks up to $\approx 5000 \text{ km s}^{-1}$

J. A. González et al. (2007) and M. Campanelli et al. (2007)
C. O. Lousto & Y. Zlochower (2011)



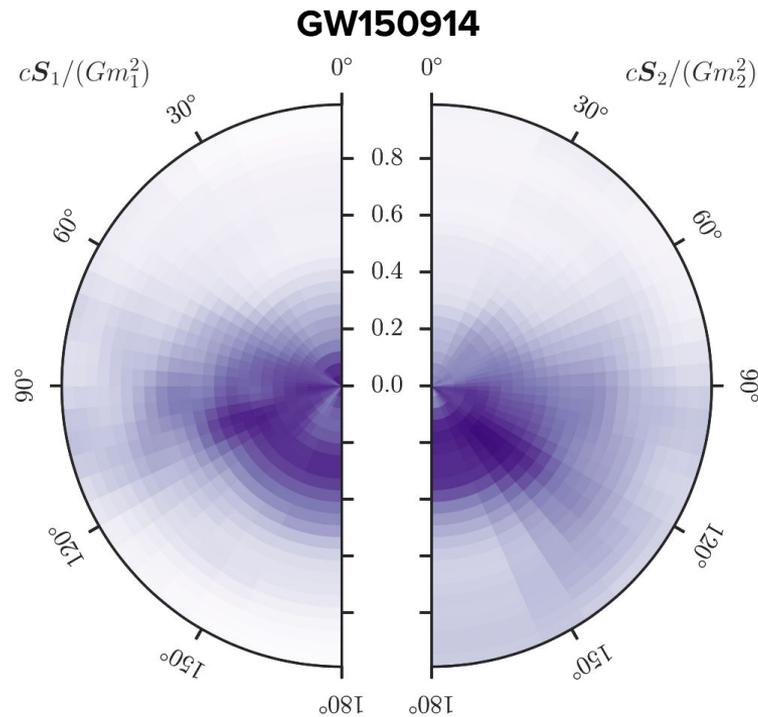
Difficulties of measuring kicks

The superkick configuration is an extreme case

Despite this it is remarkably difficult to distinguish a **superkick** binary from a **non-spinning** binary

When viewed face on an equal mass binary in the superkick configuration has no precession and and no effective spin...

... just like an equal mass non-spinning binary



Kick or push?

The kick is not instantaneous, it's more a push!

The velocity accumulates in a period of $\approx 20M$

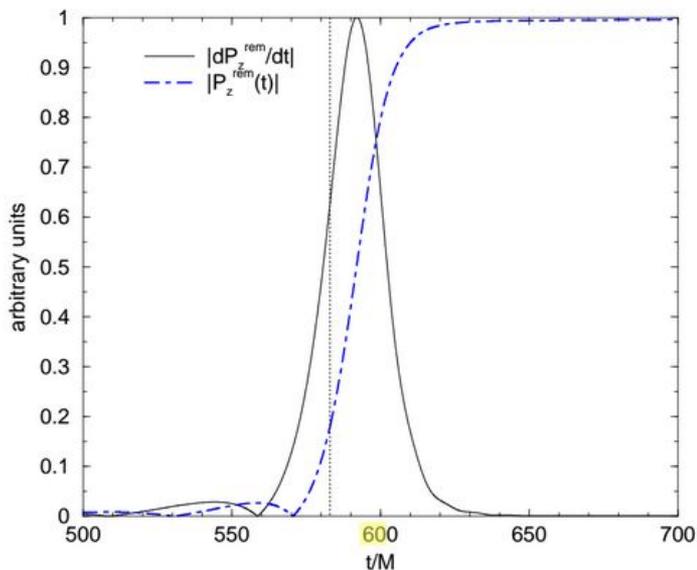
A lot is known about the total kick, much less is known about the kick **acceleration profile**

The small number of profiles reported in the literature are generally well approximated with **Gaussian** profiles

B. Brügmann et al. (2008) and C. O. Lousto & Y. Zlochower (2008)

Exception is the **antikick**, which decelerates post merger

M. Koppitz et al. (2007) and L. Rezzolla et al. (2010)



Lousto & Zlochower (2014)

SXS kicks

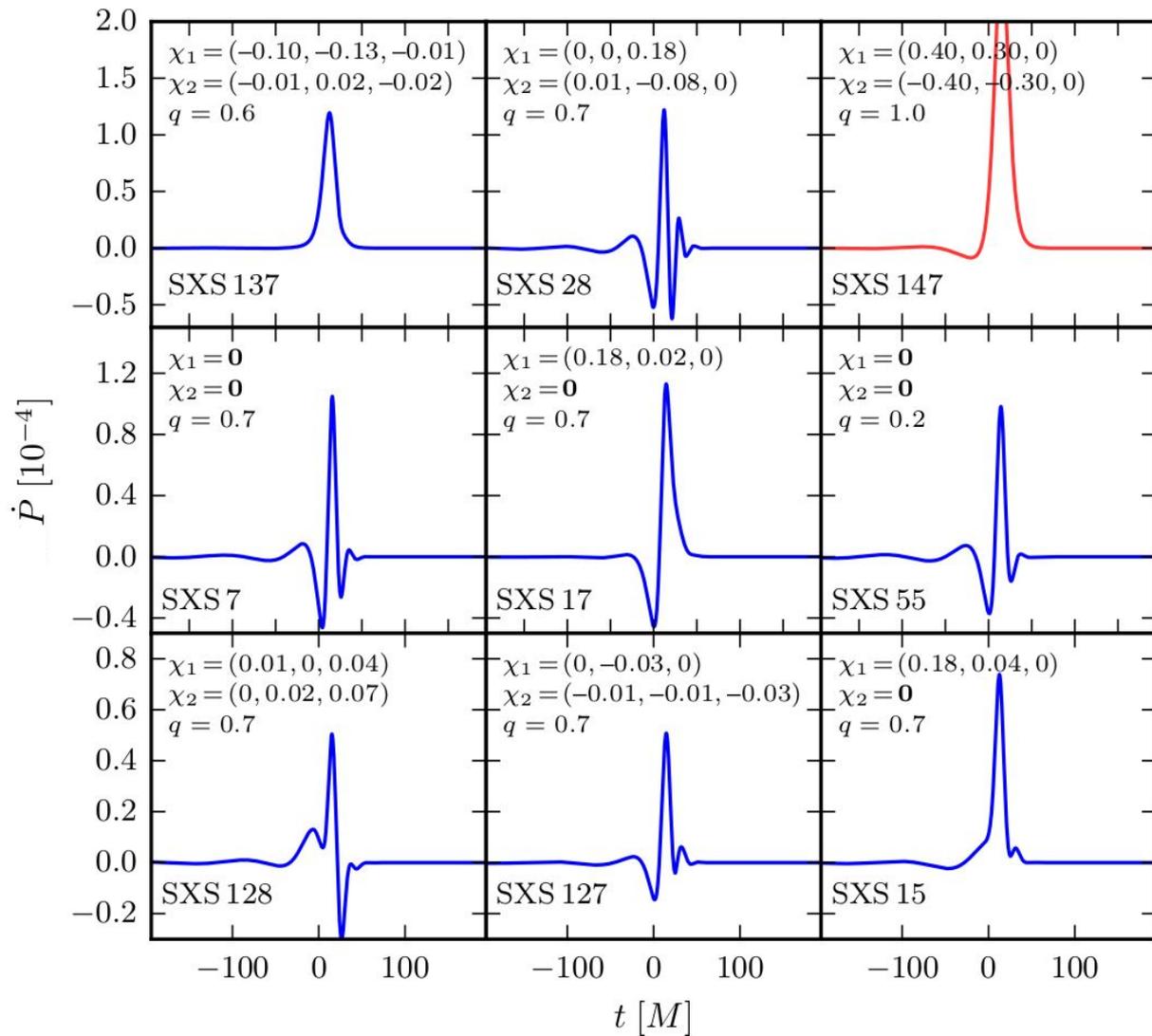
A selection of kick profiles from the public **SXS catalog**

The sources are observed along the kick direction

There is a lot of diversity in kick profiles

Shown in red is the largest kick in the catalog, **1550 km s⁻¹**

This binary is equal mass, with dimensionless spins of 0.5 arranged in the **superkick** configuration



Modelling the Kick

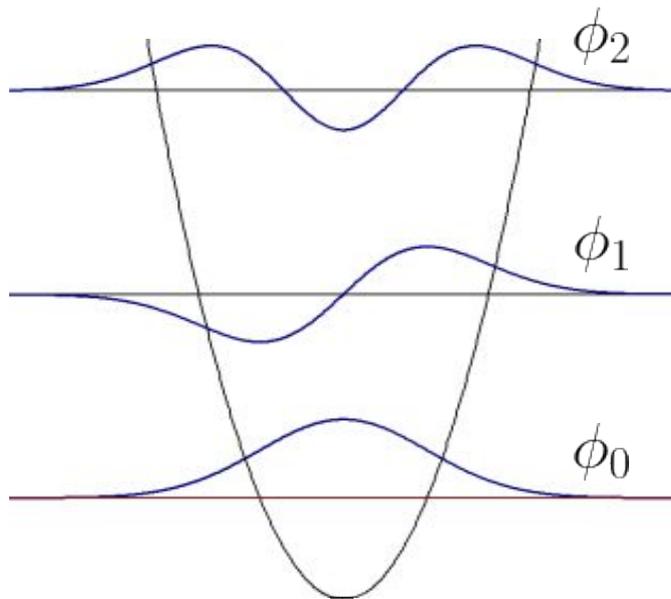
We adopt an agnostic approach to modelling the kick

Expanding the acceleration profile as a combination of a simple set of basis functions

$$a(t) = \sum_{n=0}^N a_n \phi_n \left(\frac{t - t_c}{\sigma} \right) \quad \phi_n(x) = \frac{\exp\left(-\frac{x^2}{2}\right) H_n(x)}{\sqrt{2^n n!} \sqrt{\sigma^2 \pi}}$$

Basis has the property that first term models a **Gaussian** kick, and the second models an **antikick**

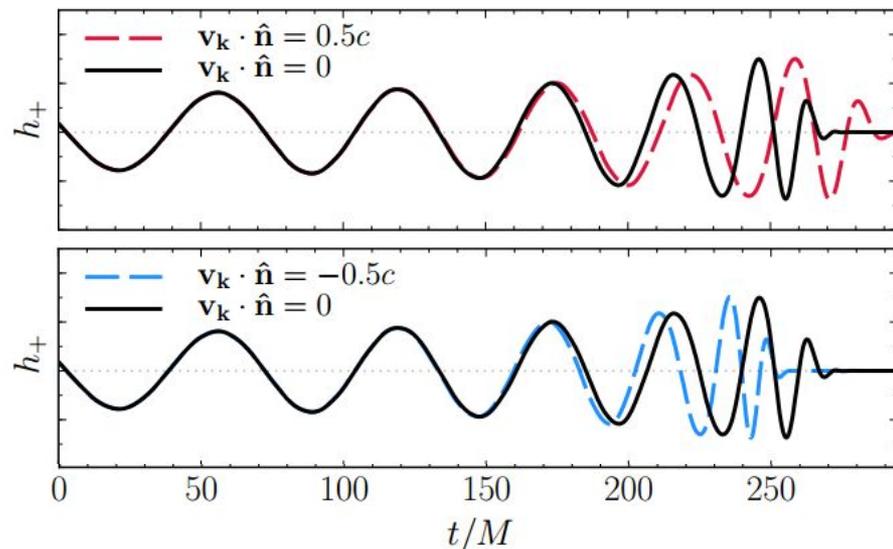
Treat $\{\sigma, t_c, a_0, a_1, \dots, a_n\}$ as free parameters, this prescription has been tested against the SXS catalog



These are familiar as solutions for the quantum harmonic oscillator.

Kicked waveforms

- Start with existing model *IMRPhenomP*
Hannam *et al.* (2014)
- Spin configuration given at large separation
- Spin configuration is evolved to small separation
- NR fitting formulae used to estimate kick speed
Gerosa & Kesden (2016) and references therein
- The system is evolved from the start again to get the GW signal. In this evolution the kick is included via a time-dependent Doppler shift
Gerosa & Moore (2016)



Kick, with speed $v_k = 0.5c$, over duration $\sigma = 60M$
(Unphysically large for clarity)

Kicked waveforms

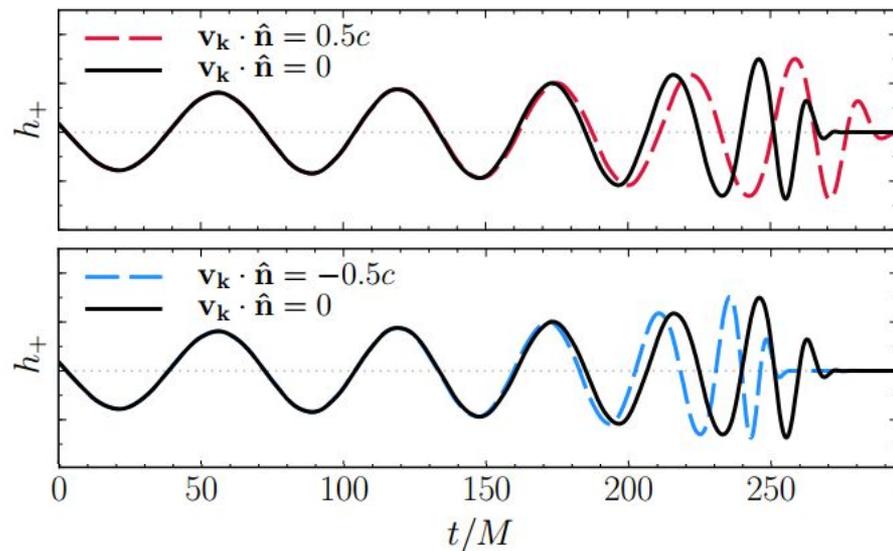
Our approach takes an existing model and adds a kick described by a few extra parameters

Pros:

- Easy to implement for best available models
- Can use to assess detectability of the kick
- Can inform requirements for how accurately future models must capture kick

Cons:

- Approach will likely not be accurate enough for analysing the high SNR signals where kicks can be detected



Kick, with speed $v_k = 0.5c$, over duration $\sigma = 60M$
(Unphysically large for clarity)

Detecting the kick

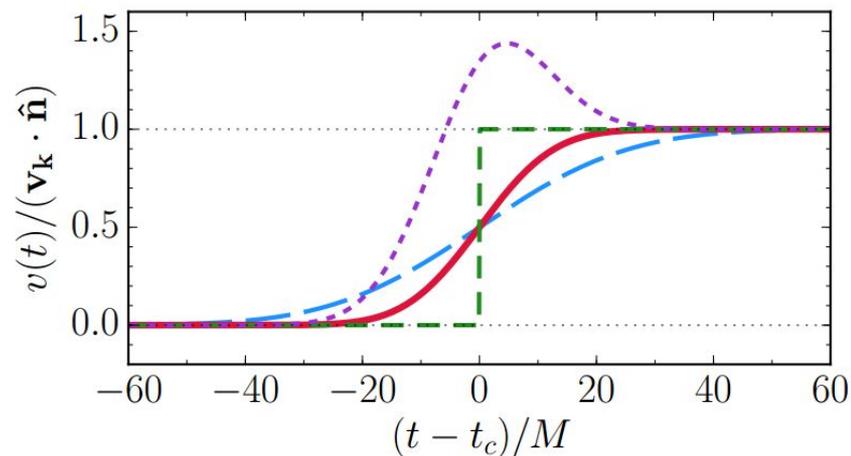
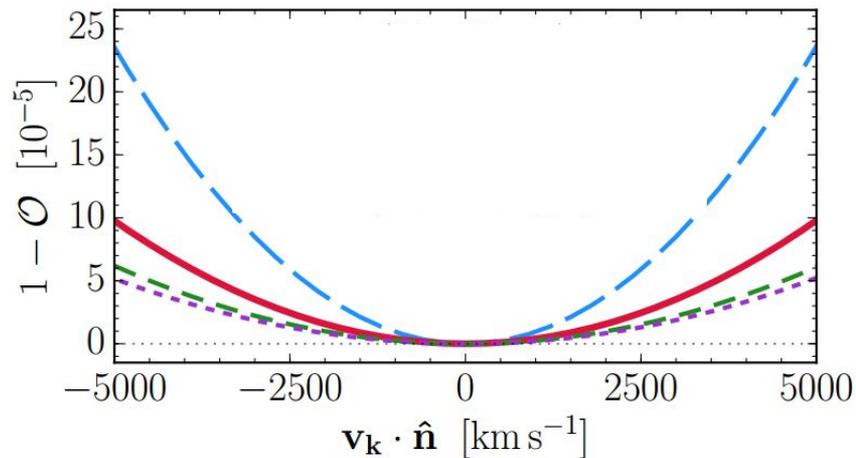
The mismatch between the **un-kicked** (h_0) and **kicked** (h_k) waveforms increases with kick speed

$$\text{Mismatch: } 1 - \mathcal{O} = 1 - \max_{t_c, \phi} \left\{ \frac{\langle h_0 | h_k \rangle}{\sqrt{\langle h_0 | h_0 \rangle \langle h_k | h_k \rangle}} \right\}$$

The signal-to-noise ratio in the **ringdown** (ρ_r) governs the detectability of the kick

A back-of-the-envelope estimate yields the following criteria for the kick to be detectable

$$\rho_r \geq \frac{0.322}{v_k/c}$$



Detecting the kick with LIGO

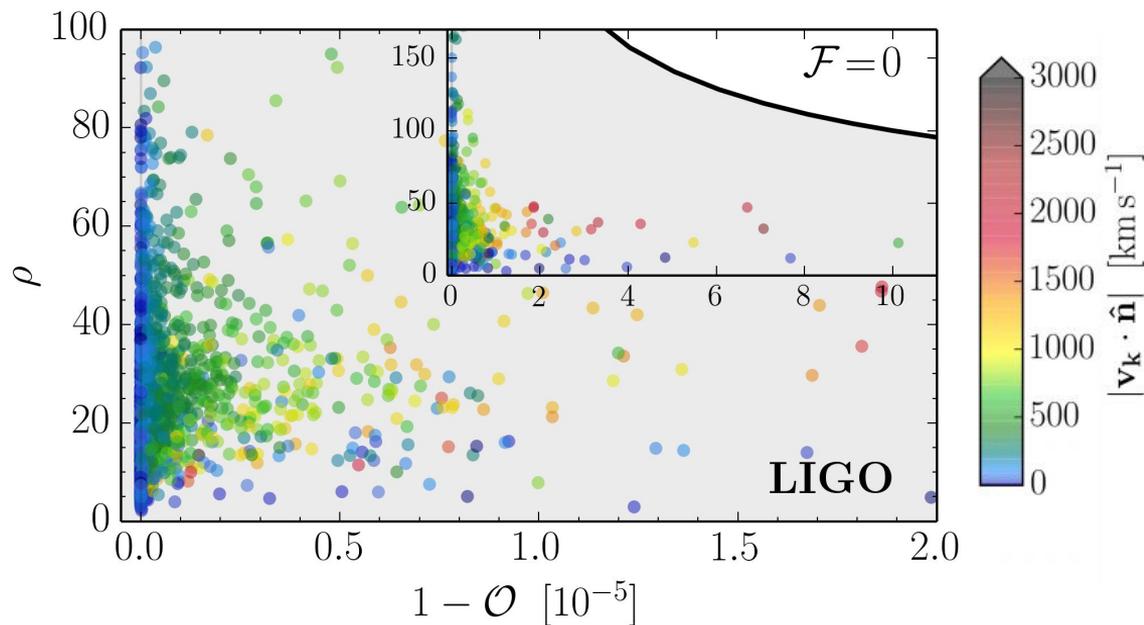
Simulate population of aLIGO binary BHs with $m_1, m_2 \in [10, 100] M_\odot$

Compare waveforms with and without kicks

Kick detectability assessed using a simple mismatch criteria

$$1 - \mathcal{O} \geq \rho^{-2}$$

Shaded region indicates systems with **undetectable** kicks



Using “Zero-Det-High_P” sensitivity from <https://dcc.ligo.org/LIGO-T0900288/public>

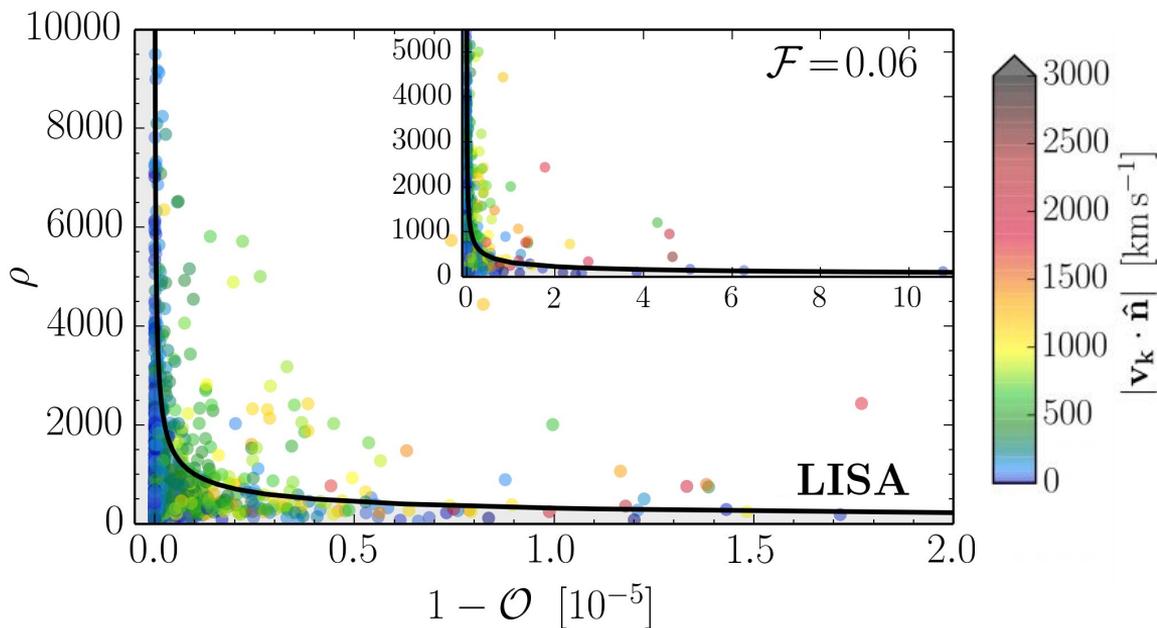
Detecting the kick with LISA

Simulate population of LISA binaries with $m_1, m_2 \in [10^5, 10^6] M_\odot$

Use the same detectability criteria

A significant fraction of binaries have detectable kicks (6% for this population)

LISA may detect kicks in a few tens of systems



Using “N2A5L6” sensitivity from
A. Klein et al. Phys. Rev. D 93, 024003 (2016)

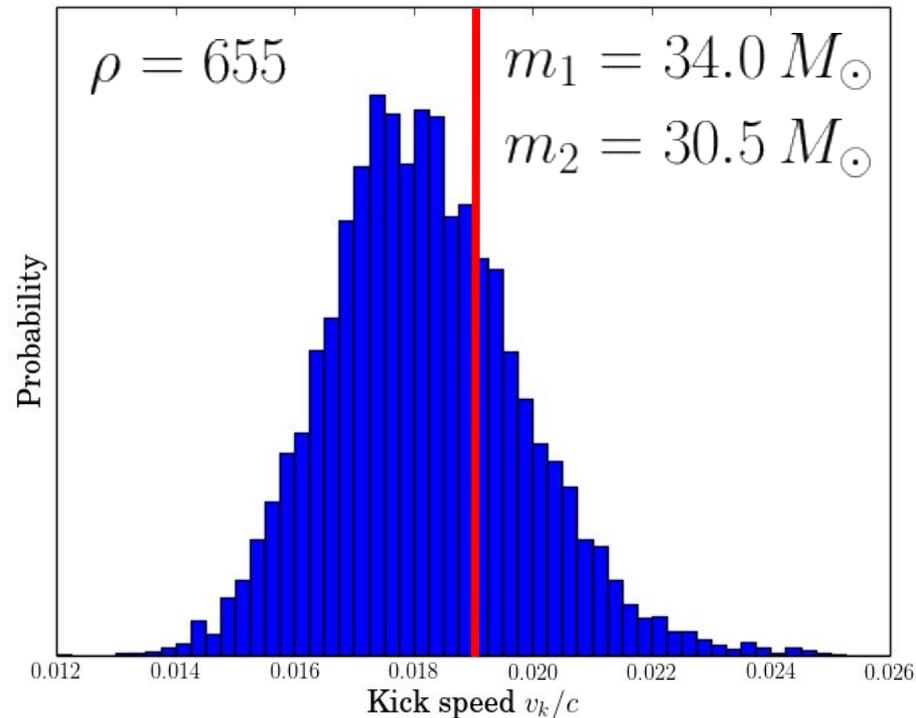
Parameter estimation for kicks

Using waveform mismatch to assess detectability of the kick assumes no degeneracy between the kick and other parameters

Don't expect degeneracies because other system parameters affect inspiral and merger, the kick only affects the merger

Previously we checked for degeneracies using Fisher matrices, now do full parameter estimation on the kicks

Now do full parameter estimation including kick



Parameter estimation for kicks

Inject very high SNR source into L1-H1 network

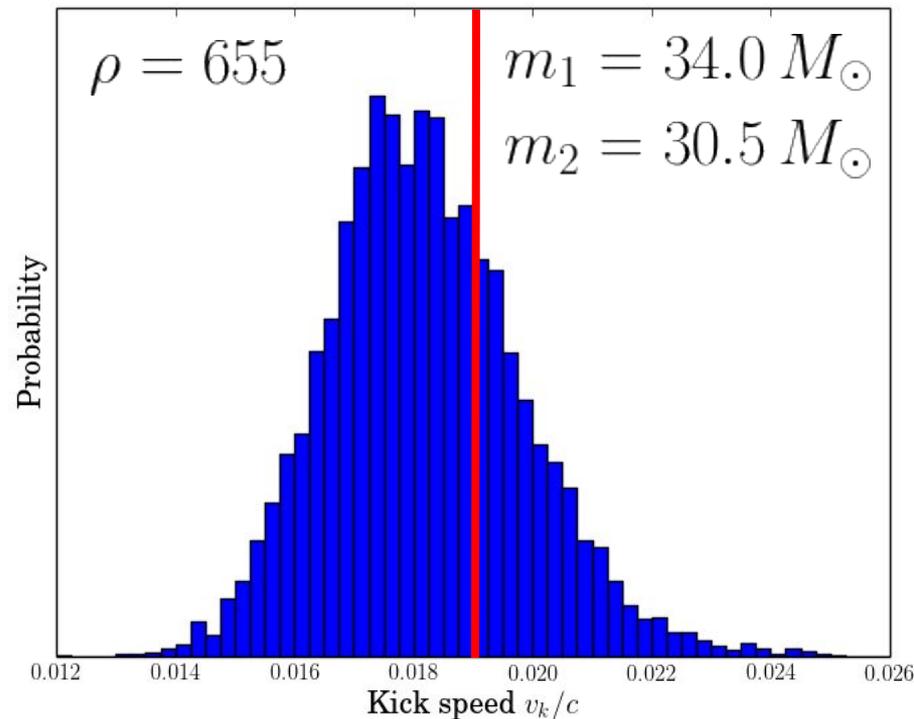
Injected signal has a superkick directed away from Earth

$$\dot{P}(t) = a_0 \exp\left(-\frac{(t - t_c)^2}{2\sigma^2}\right)$$

$t_c = 0$
$\sigma = 7.0 M$
$a_0 = 0.0015$

Can measure total kick to accuracy of 10%

No strong degeneracies between the standard parameters and the extra ***kick parameters***



Parameter estimation for kicks

Inject very high SNR source into L1-H1 network

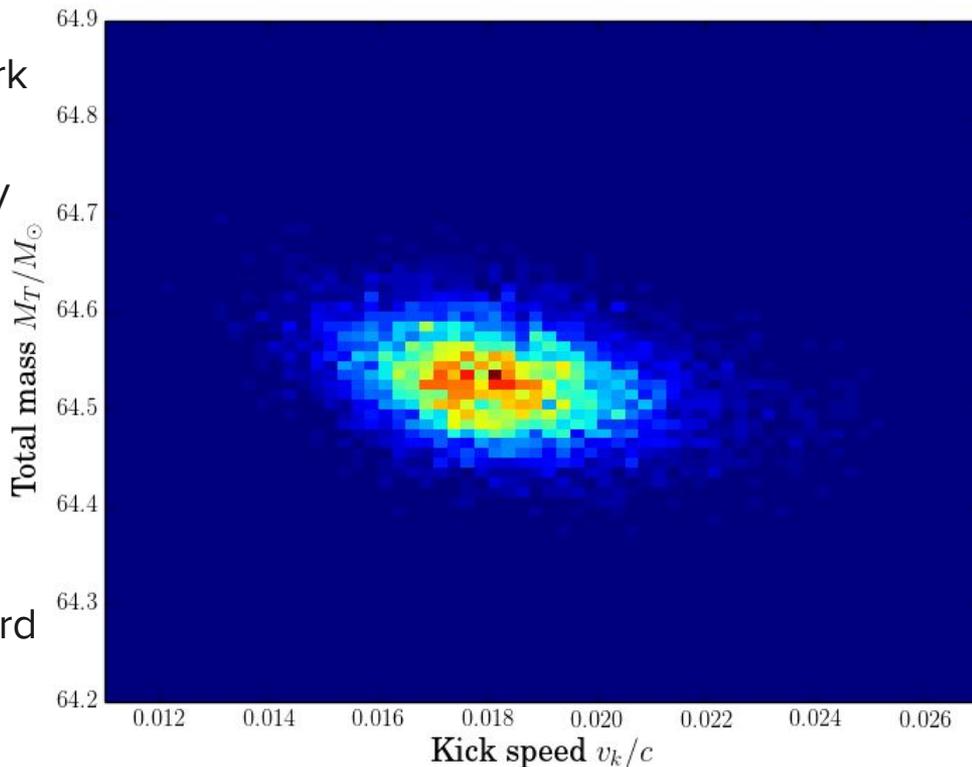
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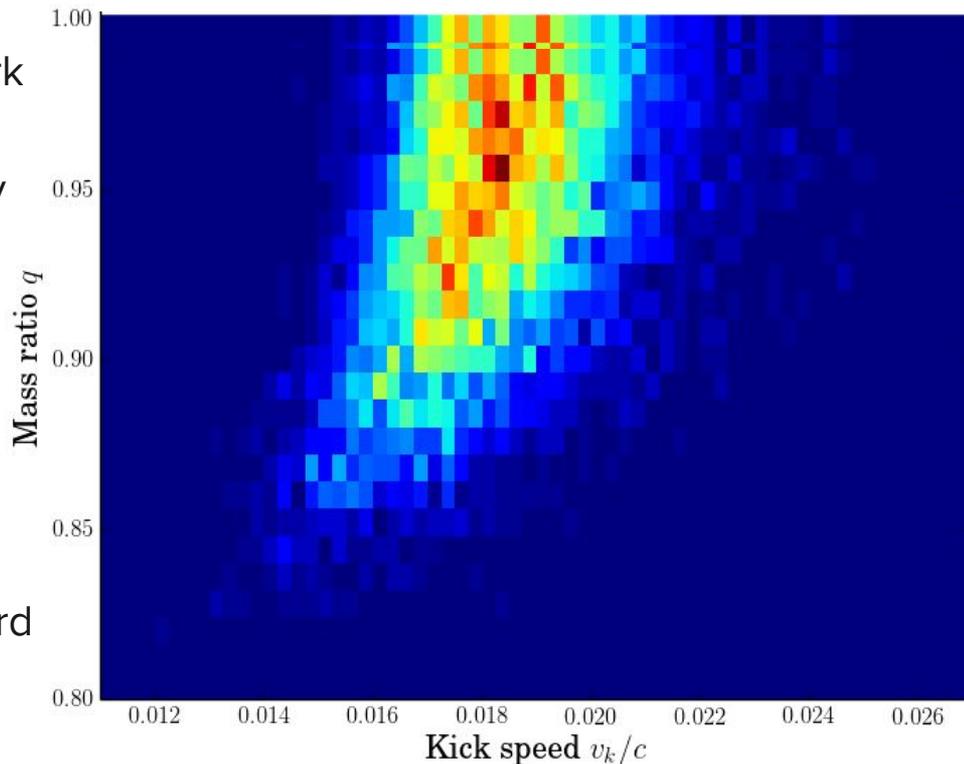
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Parameter estimation for kicks

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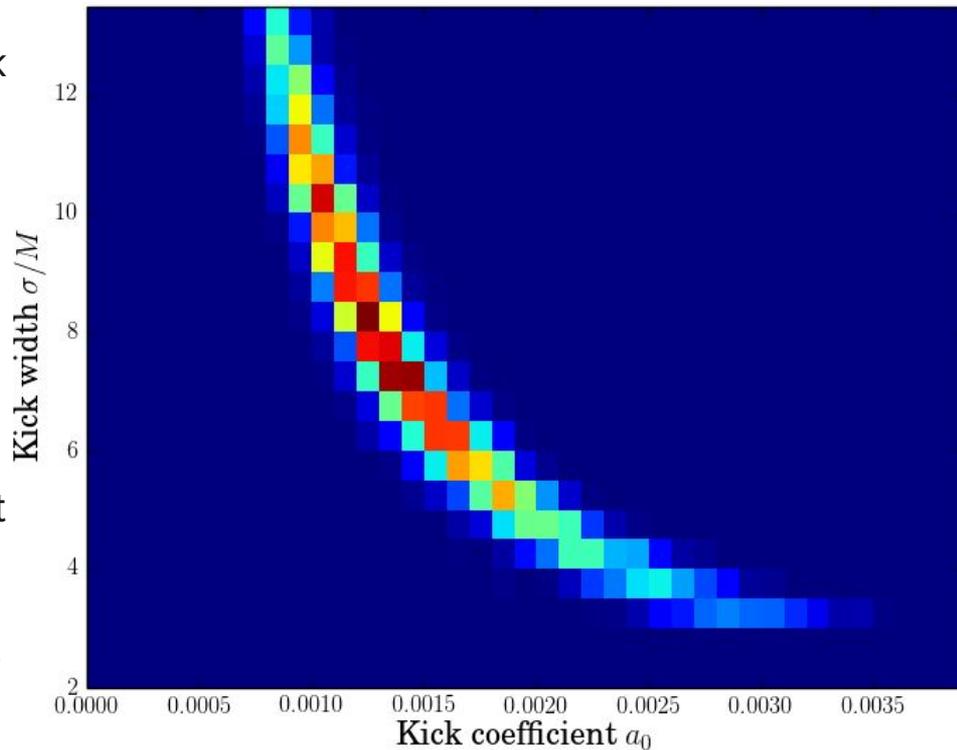
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$a_0 = 0.0015$

There are degeneracies between the different kick parameters

E.g. final kick speed $v_k = \sqrt{\pi}\sigma a_0 = 0.019c$



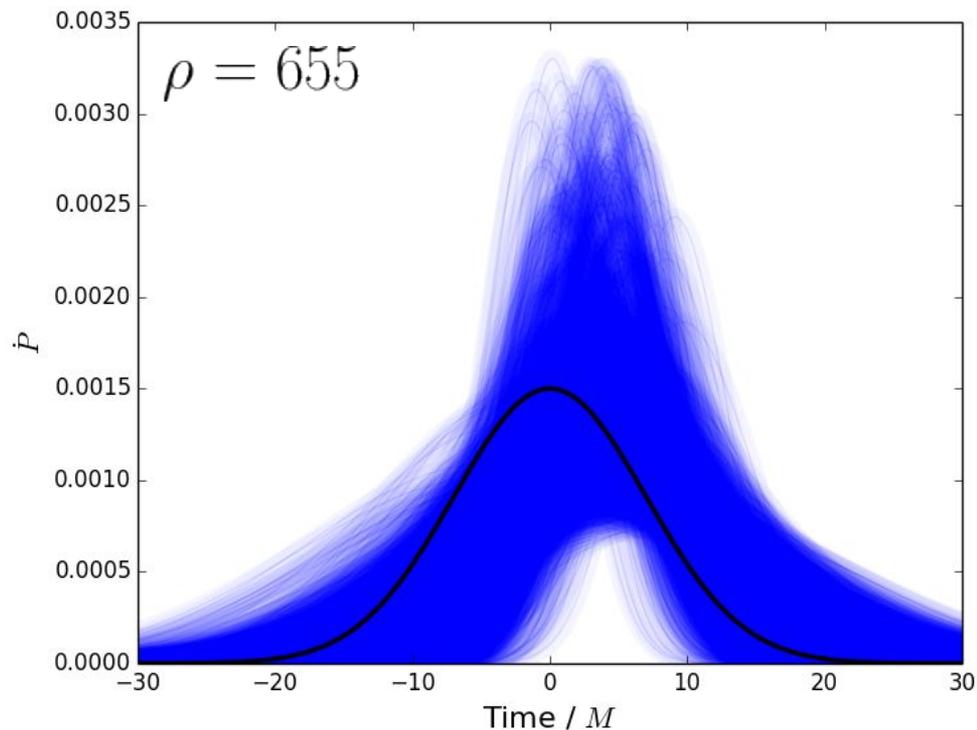
Parameter estimation for kicks

Can also measure the the acceleration profile for the kick

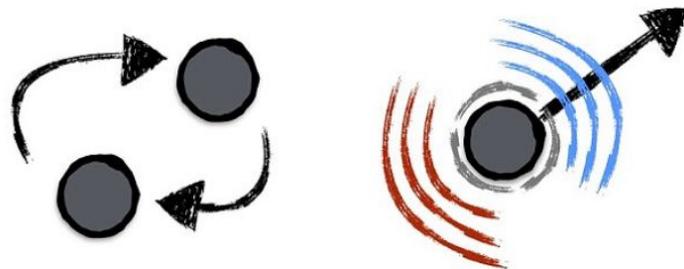
LISA can expect sources with even higher SNRs in the ringdown, and will be capable of detecting **antikicks**

Fisher matrix calculations suggest for the highest SNR LISA sources errors on the total kick speed can be as low as

$$\Delta v_k \approx 200 \text{ km s}^{-1}$$



Conclusions



BH merger kicks may be included into existing GW waveform models

Simple kicked waveform model used to assess the detectability of kicks, the signal-to-noise ratio in the ringdown governs the detectability of the kick

A few percent of supermassive binary BHs observed with LISA can be expected to have detectable kicks, may also be accessible with 3rd generation ground based detectors

Can measure the kick speed and detailed shape of acceleration profile

Modelling the kick in greater detail (both kick speed and profile shape) will be necessary for analysing loud black hole mergers

Thanks for listening!

Detecting the kick with ET

Simulate population of ET binary
BHs with $m_1, m_2 \in [10, 100] M_\odot$

Use the same detectability criteria

A small fraction of binaries have
detectable kicks

ET may detect a small number of
BH kicks

