Spin-flips in Black Hole Binaries

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What have we learn about the dynamics of orbiting spinning binary black holes?

- The spin *hang-up* effect (delaying) the merger of two black holes prevents the formation of naked singularities (Cosmic Censorship).

- The final larger black hole may acquire speeds up to 5000 km/s due to radiation of gravitational waves. This is the *hung-up recoil* effect.
Motivation: To further understand the dynamics of spinning (precessing) binary black holes

Equal mass binary with initial proper separation $d=25M$. Unequal spins $\alpha_1=0.2$ aligned with $L$ and $\alpha_2=0.8$ slightly misaligned with $L$ such that $S.L=0$.

Run lasts for $t=20,000M$ and makes 48.5 orbits before merger, 3 cycles of precession and one half of spin-flip.

After merger the final black hole acquires a recoil velocity of 1500 km/s.

FIG. 2. Precession of the orbital plane as displayed by the distance vector $\vec{d} = \vec{x}_1(t) - \vec{x}_2(t)$. 
FIG. 9. Evolution of the eccentricity versus coordinate separation of the BHB. Dots represent measurements of the eccentricity, $e_D$, and the continuous curve a fit to its decay with $e \sim r^{1.73486\pm0.1495}$. In comparison, the theoretical prediction is $e \sim r^{1.5833}$. 
FIG. 5. Change in the spin direction (in blue) of the BH1 (with the smaller spin magnitude) in the orbital frame, $\hat{L}$ (left), and the coordinate frame $\hat{z}$ (right). Plotted also the directions of $\vec{L}$ in red and $\vec{J}$ in green.
FIG. 3. The real part of the waveform strain for the modes $(\ell, m) = (2, 2)$ and $(\ell, m) = (2, 1)$. While the former (top) gives the leading chirping amplitude, the latter (bottom) clearly displays the precession effect, completing nearly three cycles during the $t = 20000M$ of the simulation.
From

\[
\frac{d\vec{S}_1}{dt} = \frac{1}{r^3} \left[ \left(2 + \frac{3}{2q}\right) \vec{L} - \vec{S}_2 + \frac{3(\vec{S}_0 \cdot \hat{n})}{1+q} \hat{n} \right] \times \vec{S}_1,
\]

\[
\frac{d\vec{S}_2}{dt} = \frac{1}{r^3} \left[ \left(2 + \frac{3q}{2}\right) \vec{L} - \vec{S}_1 + \frac{3q(\vec{S}_0 \cdot \hat{n})}{1+q} \hat{n} \right] \times \vec{S}_2,
\]

where \( \vec{n} = \vec{r}_1 - \vec{r}_2 \) and

\[
\vec{S}_0 = \left(1 + \frac{1}{q}\right) \vec{S}_1 + (1+q)\vec{S}_2.
\]

We obtain the polar and azimuthal frequencies with respect to

\[
\Omega_{ff} = 3 \frac{S}{r^3} \left[1 - \frac{2\vec{S} \cdot \hat{L}}{M^{3/2}r^{1/2}} \right],
\]

\[
\Omega_p = \frac{7L}{2r^3} + \frac{2}{r^3} (\vec{S} \cdot \hat{L}),
\]

The flip-flop happens when both spins are non-vanishing.
The flip-flopping frequency leading terms are now,

\[ M \Omega_{1,2}^{ff} \approx \frac{3}{2} \frac{|1-q|}{(1+q)} \left( \frac{M}{r} \right)^{5/2} + 3 \frac{S_1^L - S_2^L}{M^2} \left( \frac{M}{r} \right)^3 \text{sign}(1-q) \]

The origin of the additional term for unequal masses scaling with \( \sim r^{-5/2} \) is due to the non-conservation of the angle \( \beta \) between the two spins (as opposed to its conservation in the \( q = 1 \) case). These oscillations in \( \beta \) are due to the differential precessional angular velocity of \( \vec{S}_1 \) and \( \vec{S}_2 \) for \( q \neq 1 \) and hence provides the (precessional) scaling \( r^{-5/2} \).

The maximum flip-flop angle is now,

\[ 1 - \cos(\Delta \theta_{1L}^{ff}) \approx \frac{2\alpha_2^2}{(1-q)^2} \left( \frac{M}{r} \right) + \frac{4\alpha_1 \alpha_2^2 q}{(1-q)^3} \left( \frac{M}{r} \right)^{3/2} \]
Unequal mass spinning binaries: 3.5PN Evolution Probabilities

below. These results are for initial random spin distributions evolved with 3.5PN form 100$M$ of initial separation down to 5$M$ (approximating the merger).
In Fig. 22 we display the differences for a few selected mass ratios. We observe in general that larger angles are seen in the $J$-frame. Particularly for $q = 0.1$, due to the effects of transitional precession [63], that may reverse the orientation of $\vec{J}$. But we also observe larger angles for $q > 1/4$, 
Accretion disk internal rim will change location with spin orientation. This changes
• Efficiency of the EM radiation
• Spectrum of EM radiation (hard part)
• Cutting frequency of oscillations
Proper modeling using GRMHD simulations

X-shaped galaxies should show ‘orange peeling’ jets when they were about to merge
• For our simulation this corresponds to 1.2 seconds for 10Msun and 142 days for $10^8$Msun
• The effect is still present in unequal mass binaries, (and BH-NS and NS-NS).

We need full numerical GRMHD simulations

Simulation by RIT group
Bonus Track: BBHs Last tango

t=87 M

5x
Flip-flops and alignment instability

Flip-Flopping Black Holes: Study of polar oscillations of BH spins


Up-down spin configurations can be unstable (using low averaged PN)


in between $r_{ud^\pm} = (\sqrt{\alpha_2} \pm \sqrt{q\alpha_1})^4 M/(1 - q)^2$.

We study the instability here by direct integration of the 3.5PN equations of motion and 2.5PN spins evolutions

FIG. 1. Snapshots of the spin components along the orbital angular momentum at a binary separation $r/M = 11$. The integration of the PN evolution equations for each binary mass ratio $q$, started at $r/M > R_c$ with a uniform distribution of spins in the range $0 \leq \alpha_{2L} \leq 1$ for the large BH and $-1 \leq \alpha_{1L} \leq 0$ for the small BH, which was antialigned with the orbital angular momentum by 179-degrees. The color indicates the original value of the spins. The black curve models the depopulation region as given in Eq. (4).
FIG. 2. Evolution of a binary with mass ratio $q = 0.75$, large BH spin $\alpha_{2L} = 1$ initially aligned and small BH spin $\alpha_{1L} = -1$ antialigned with the orbital angular momentum by 179 degrees. The upper panels display the onset of the instability from stable flip-flop oscillations. The middle panels display the development of the instability as the binary separation shrinks. Here $\cos \theta_{iL} = S_{iL}/S_i$ with $i = 1, 2$ for the small, large holes. The lower panels display a polar view of the onset of the misalignment instability.
Flip-flop instability

2 PN Analytic study

\[
d^2 (\vec{S}_i \cdot \hat{L}) / dt^2 = -\Omega_{ff}^2 \vec{S}_i \cdot \hat{L} + \cdots
\]

\[
\begin{align*}
\Omega_{ff}^2 &= \frac{9}{4} \frac{(1-q)^2 M^3}{(1+q)^2 r^5} + \frac{9}{2} \frac{(1-q) (S^{1\hat{L}} - S^{2\hat{L}}) M^{3/2}}{(1+q) r^{11/2}} \\
&\quad - \frac{9}{4} \frac{(1-q) (3+5q) S^{1\hat{L}}}{q^2 r^6} + \frac{9}{2} \frac{(1-q)^2 S^{1\hat{L}} S^{2\hat{L}}}{qr^6} \\
&\quad + \frac{9}{4} \frac{(1-q) (5+3q) S^{2\hat{L}}}{r^6} + \frac{9}{4} \frac{S^0}{r^6} + 9 \frac{(1-q)^2 M^4}{(1+q)^2 r^6},
\end{align*}
\]

where \( \vec{S}_0 / M^2 = (1+q) \left[ \vec{S}_1 / q + \vec{S}_2 \right] \).

\[
\Omega_{ff}(q, \vec{\alpha}_1, \vec{\alpha}_2, R_c) = 0.
\]  

The solution of this quadratic equation for antialigned spins leads to two roots \( R_c^\pm \).

\[
R_c^\pm = 2M \frac{A \pm 2(\alpha_{2L} - q^2 \alpha_{1L}) \sqrt{B}}{(1-q^2)^2},
\]

\[
A = (1+q^2) (\alpha_{2L}^2 + q^2 \alpha_{1L}^2),
\]

\[
B = 2(1+q) \left[ (1-q)^2 \alpha_{1L}^2 - (1-q) \alpha_{2L}^2 \\
- 2q(1+q) \alpha_{1L} \alpha_{2L} - 2(1-q)^2 (1+q) \right].
\]
FIG. 3. The instability region, between $R_c^\pm$, as a function of the mass ratio, $q$, as the binary transitions from real to imaginary flip-flop frequencies (blue curve) for maximal spins $\alpha_{1L} = -1$ and $\alpha_{2L} = +1$. For comparison also plotted are $r_{ud\pm}$ from [8] (red curve). The dots correspond to 3.5PN evolutions.
In which sense aligned spin templates are representative of more generic systems?

$q = m_1/m_2 = 0.82$

$a_1^L = -0.44$

$a_2^L = +0.33$
Depletion effective for up to 35, 45, 50 degs. misalign. For \(q=0.6, 0.75, 0.90\) respectively.

**FIG. 4.** Upper: Initial configuration of a binary with mass ratio \(q = 0.75\) at \(r = 500M\) of separation. Color labels angular deviations of the spins from the orbital angular momentum direction \(\hat{L}\). Lower: The spin orientations near merger at \(r = 11M\) displaying a replenish of the unstable region from the highly misaligned spins.
• Flip-flop and instabilities can be seen as a single phenomena of “real” and “imaginary” oscillation frequencies
• Instabilities introduce a notable “dent” into the spins distribution of quasi-aligned binaries
• The instabilities lead to notable changes in the predicted recoil and merger scenario

### TABLE I. Initial data parameters and system details for full numerical evolutions. The initial coordinate separation is $D = 11M$ and the intrinsic spins are $\alpha^{x,y,z}_{1,2}$. The eccentricity measured at the end of the inspiral is $e_f$, and the number of orbits just before merger $N$. # labels the PN runs that started at binary separation $r = 500M$ with normalized spins $(a_1^z, a_2^z)$.

<table>
<thead>
<tr>
<th>#</th>
<th>$(a_1^z, a_2^z)$</th>
<th>$q$</th>
<th>$\alpha^x_1$</th>
<th>$\alpha^y_1$</th>
<th>$\alpha^z_1$</th>
<th>$\alpha^x_2$</th>
<th>$\alpha^y_2$</th>
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<th>$N$</th>
<th>$e_f$</th>
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<td>(-0.8, 0.8)</td>
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<td>0.7738</td>
<td>0.1876</td>
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<td>0.4183</td>
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<td>0.0037</td>
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<td>0.75</td>
<td>-0.3205</td>
<td>0.2392</td>
<td>0.0070</td>
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<td>-0.2040</td>
<td>0.4971</td>
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<td>0.0009</td>
</tr>
<tr>
<td>3</td>
<td>(-0.6, 0.6)</td>
<td>0.75</td>
<td>0.5467</td>
<td>0.2462</td>
<td>-0.0223</td>
<td>0.4724</td>
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<td>8.4</td>
<td>0.0024</td>
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<tr>
<td>4</td>
<td>(-0.8, 0.8)</td>
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<td>0.0559</td>
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<td>8.6</td>
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### TABLE II. Remnant properties of the merged black hole. The final mass $m_{rem}$ and spin $\alpha_{rem}$ (normalized to total initial mass) are measured from the horizon, and the recoil velocity (in km/s) is calculated from the gravitational waveforms. Comparison with predicted aligned spins values $m_{pre}$, $\alpha_{pre}^{x,y,z}$, $V_{pre}^{x,y,z}$, is based on [13].

<table>
<thead>
<tr>
<th>#</th>
<th>$m_{rem}$</th>
<th>$m_{pre}$</th>
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<th>$\alpha^y_{rem}$</th>
<th>$\alpha^z_{rem}$</th>
<th>$\alpha^x_{pre}$</th>
<th>$\alpha^y_{pre}$</th>
<th>$\alpha^z_{pre}$</th>
<th>$V_{rem}^x$</th>
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<td>282.9</td>
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