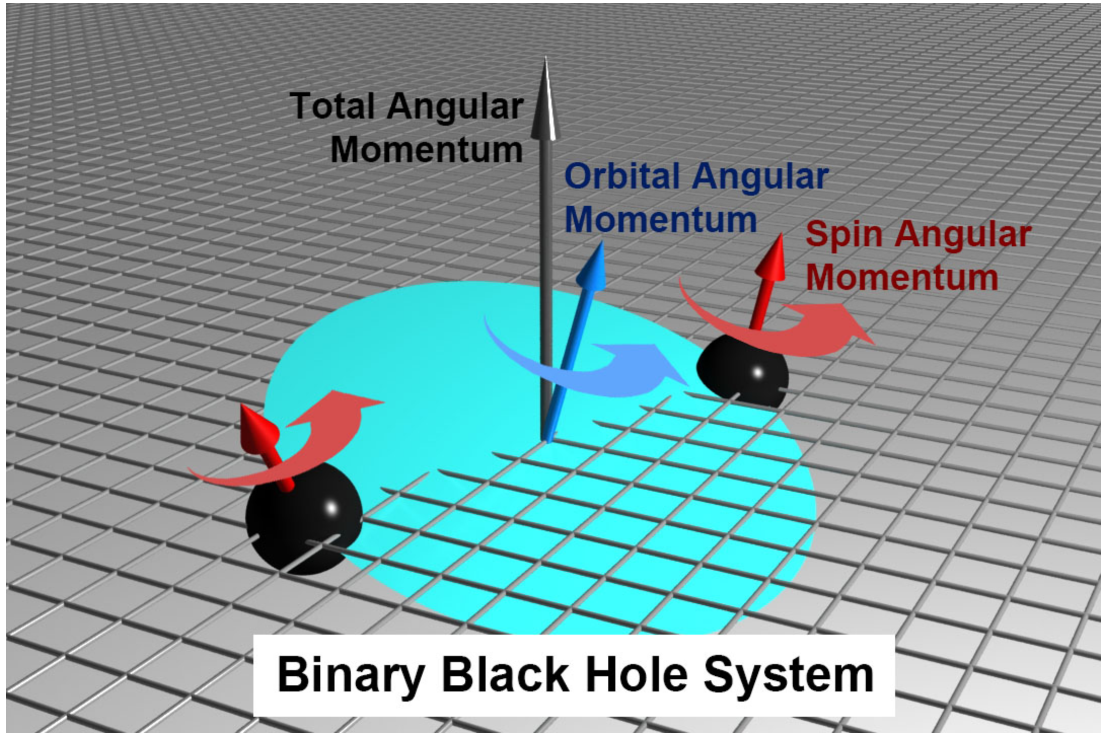
Precessional Dynamics: Insights and Questions



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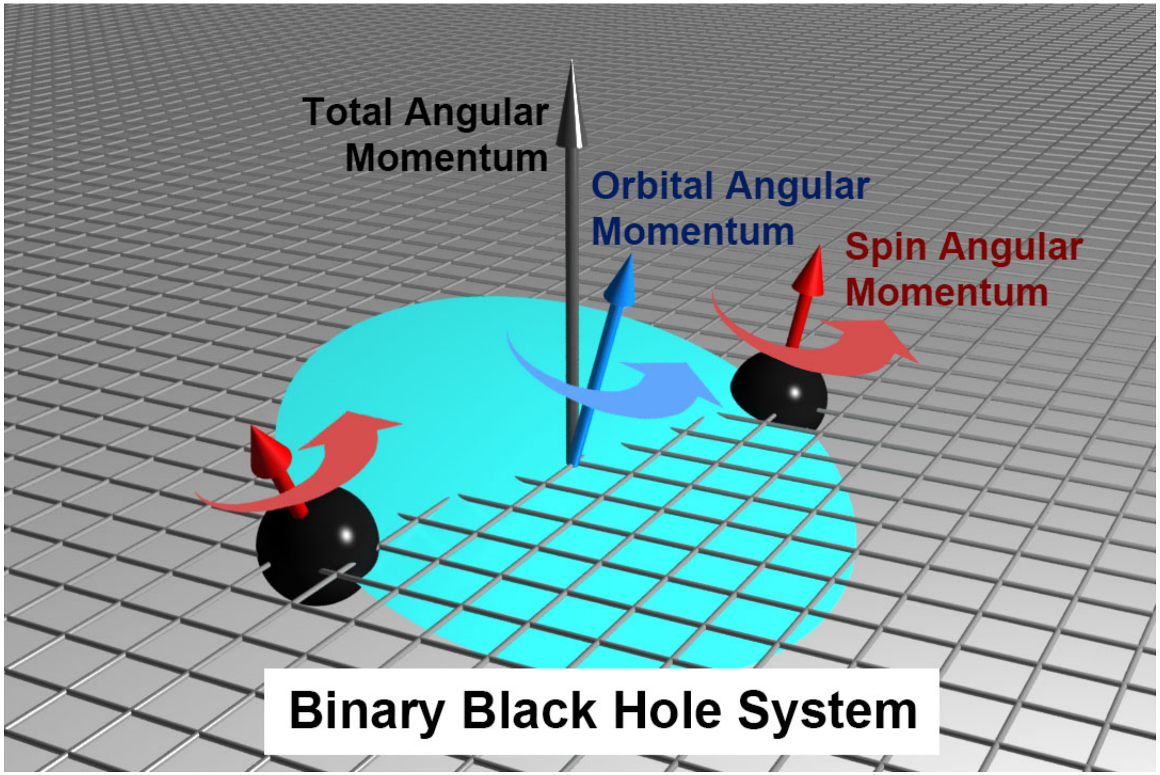
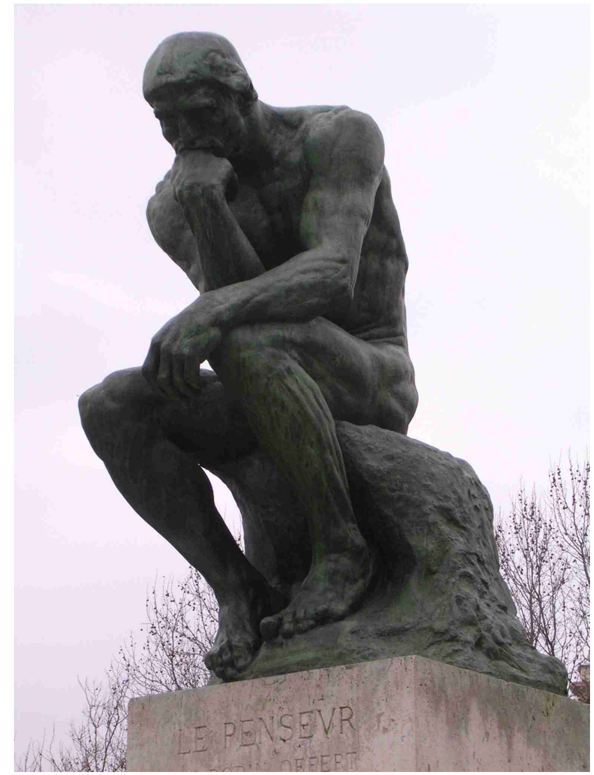
University of Mississippi

Strong BaD Workshop

Oxford, MS – Feb. 28, 2017

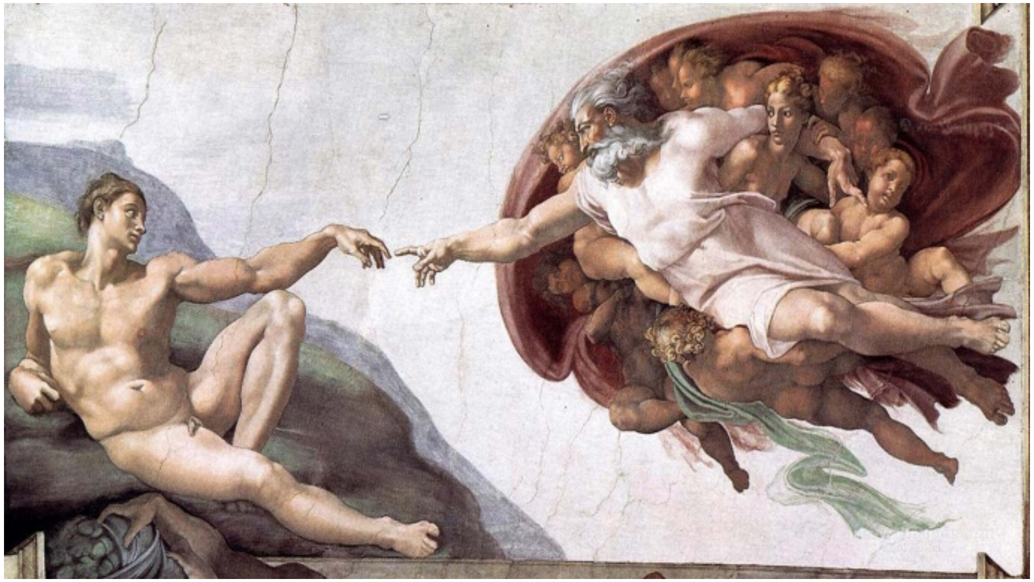
Why study spin precession?





1) Spins affect waveforms ⟹ detection and parameter estimation
2) Spins set by binary formation ⟹ astrophysical model selection
3) Spin evolution determined by GR ⟹ test new theories of gravity
4) Spin precession is beautiful and fun (it's why I got in this business)

In the beginning …



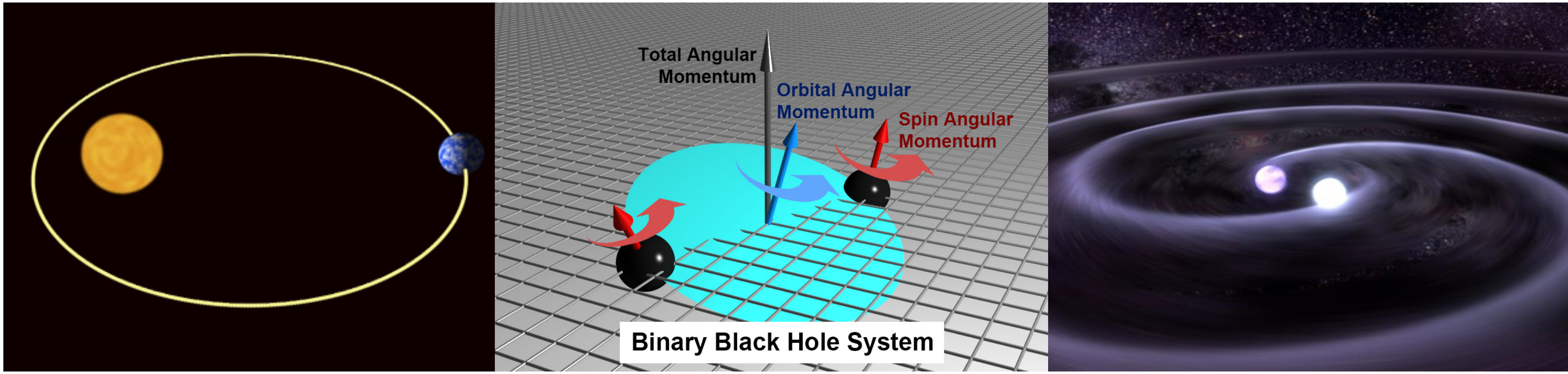
$$\dot{\mathbf{L}} = \frac{1}{r^3}\left[\frac{4M_1+3M_2}{2M_1}\mathbf{S}_1 + \frac{4M_2+3M_1}{2M_2}\mathbf{S}_2\right]\times\mathbf{L} - \frac{3}{2}\frac{1}{r^3}\left[(\mathbf{S}_2\cdot\hat{\mathbf{L}})\mathbf{S}_1 + (\mathbf{S}_1\cdot\hat{\mathbf{L}})\mathbf{S}_2\right]\times\hat{\mathbf{L}} - \frac{32}{5}\frac{\mu^2}{r}\left(\frac{M}{r}\right)^{5/2}\hat{\mathbf{L}}\,, \tag{11a}$$

$$\dot{\mathbf{S}}_1 = \frac{1}{r^3}\left[\frac{4M_1+3M_2}{2M_1}(\mu M^{1/2}r^{1/2})\hat{\mathbf{L}}\right]\times\mathbf{S}_1 + \frac{1}{r^3}\left[\frac{1}{2}\mathbf{S}_2 - \frac{3}{2}(\mathbf{S}_1\cdot\hat{\mathbf{L}})\hat{\mathbf{L}}\right]\times\mathbf{S}_1\,, \tag{11b}$$

$$\dot{\mathbf{S}}_2 = \frac{1}{r^3}\left[\frac{4M_2+3M_1}{2M_2}(\mu M^{1/2}r^{1/2})\hat{\mathbf{L}}\right]\times\mathbf{S}_2 + \frac{1}{r^3}\left[\frac{1}{2}\mathbf{S}_1 - \frac{3}{2}(\mathbf{S}_1\cdot\hat{\mathbf{L}})\hat{\mathbf{L}}\right]\times\mathbf{S}_2\,. \tag{11c}$$

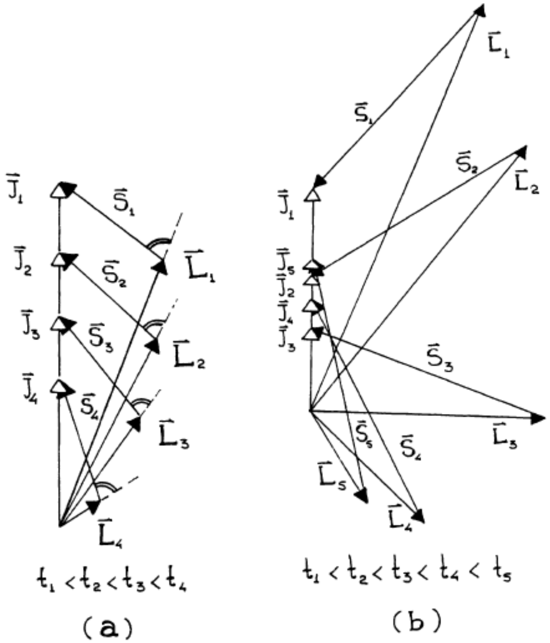
- Post-Newtonian (PN) spin precession (Apostolatos+ 1994)
- Each spin precession about ***L*** at different frequency at lowest order
- Spins are coupled to each other at +0.5PN
- Radiation reaction at 2.5PN

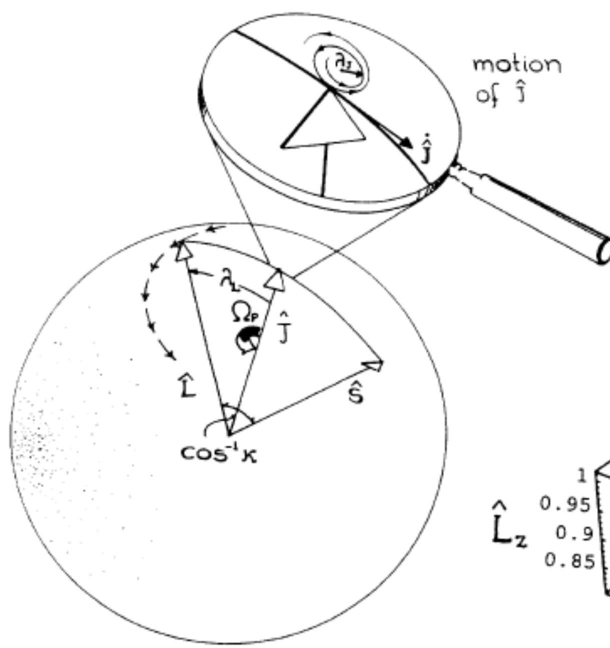
PN Timescale Hierarchy

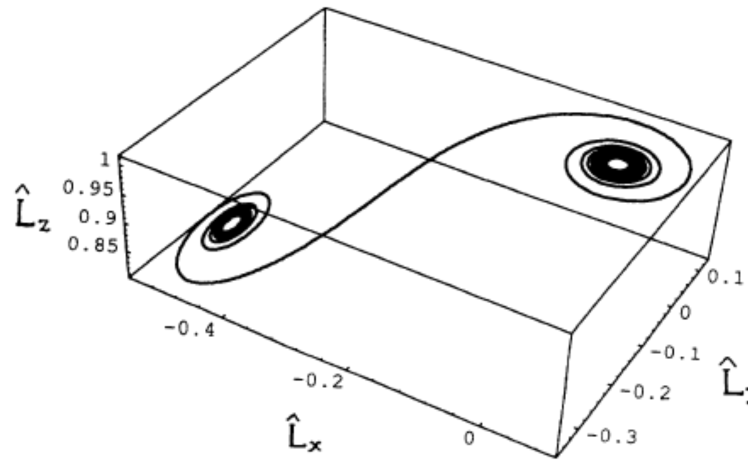


- 3 timescales for BBH evolution
  - Orbital time (0PN): $t_{\mathrm{orb}} \sim (r^3/GM)^{1/2}$
  - Precession time (1PN): $t_{\mathrm{pre}} \sim (t_{\mathrm{orb}}/\eta)(r/r_g) \gg t_{\mathrm{orb}}$
  - Radiation-reaction time (2.5PN): $t_{\mathrm{RR}} \sim (t_{\mathrm{orb}}/\eta)(r/r_g)^{5/2} \gg t_{\mathrm{pre}}$
- Hierarchy + analytic solutions to orbital motion (Keplerian ellipses) allows spin-precession equations to be *orbit-averaged*

Simple and transitional precession

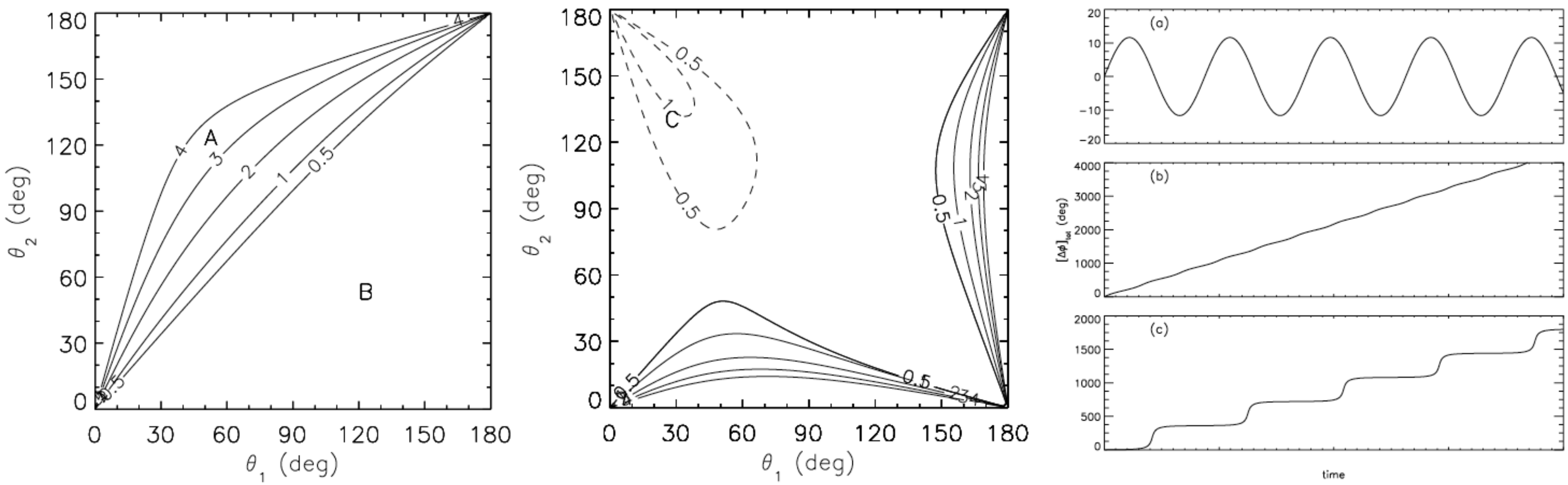






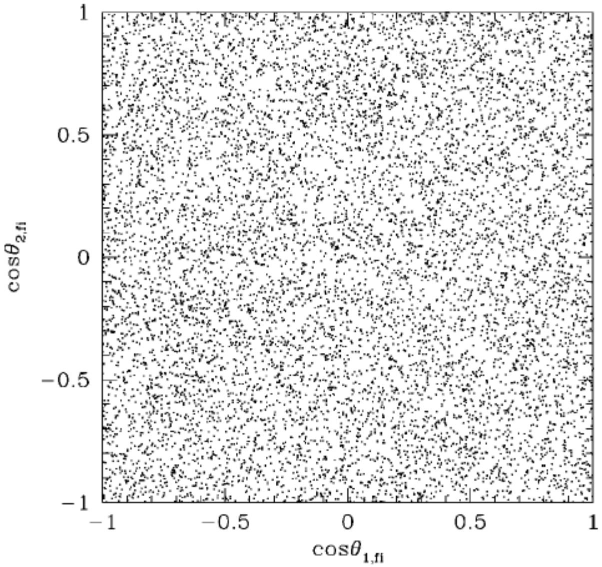
- Limited progress understanding generic spin precession analytically
- Precession simplifies for binaries with $m_1 \simeq m_2$ or $S_2 \simeq 0$
  - $\cos \theta_{LS}$ is constant
  - Simple precession: ***L***, ***J***, ***S*** undergo tight-spiral motion, $\lambda_J << \lambda_L$
  - Transitional precession: ***J*** tilts when ***J*** $\simeq 0$ (***L*** $\simeq$ -***S***)

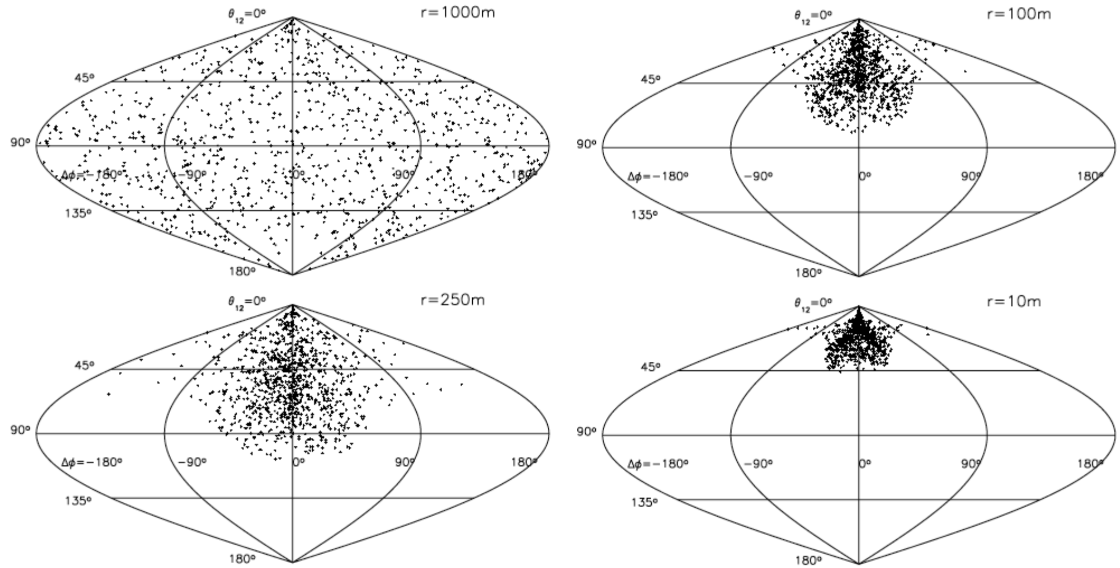
The Schnittman Revolution

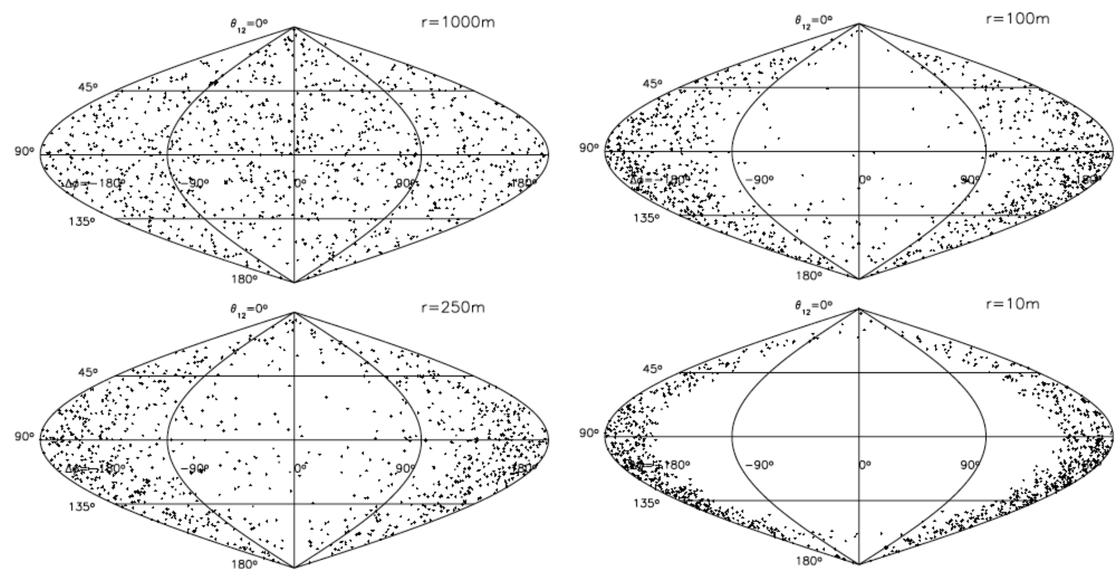


- Relative orientation of spins for generic binaries set by angles θ_i between $\boldsymbol{S}_i$ and $\boldsymbol{L}$, angle $\Delta\Phi$ between spin components $\perp$ to $\boldsymbol{L}$
- Schnittman (2004) discovered two families ($\Delta\phi$ = 0°, 180°) of "spin-orbit resonances" for which $\boldsymbol{L}$, $\boldsymbol{S}_1$, $\boldsymbol{S}_2$ jointly precess about $\boldsymbol{J}$
- "Quasi-stable" resonances near up-down configuration (more later)
- Three morphologies for generic binaries determined by behavior of $\Delta\Phi$: libration about 0°, circulation, libration about 180°

Does precession change spin distributions?





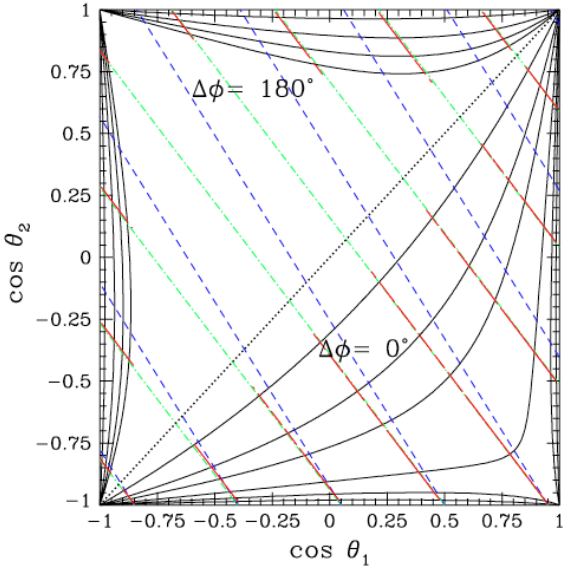


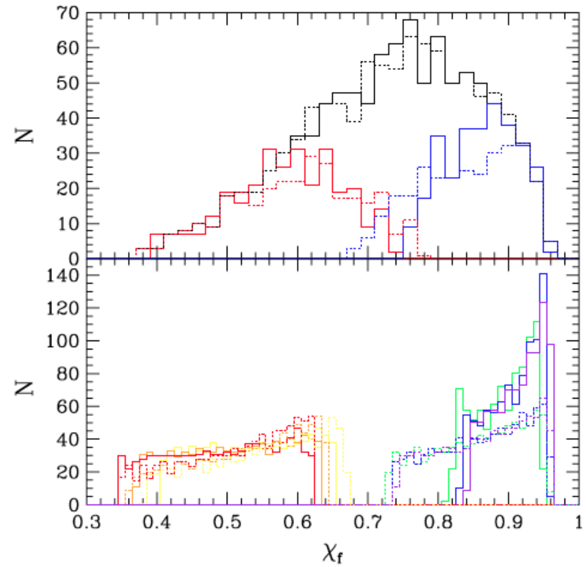
- Generic binaries become "locked into Schnittman resonances

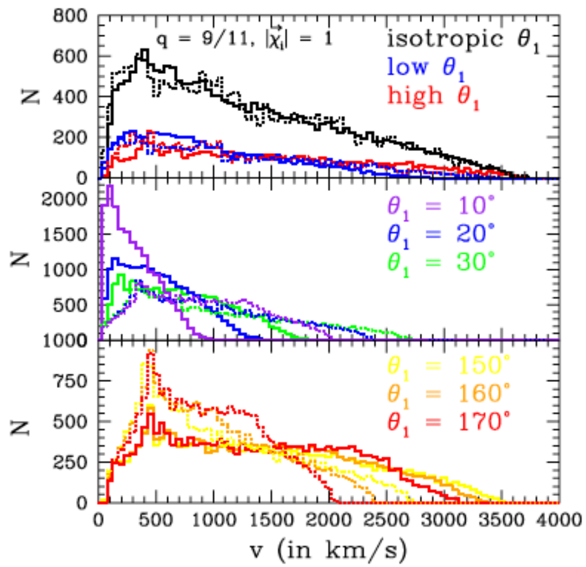
but

- Isotropic distributions stay isotropic (Bogdanović+ 2007)
- ***Why!?!***

A New Hope (conserved quantity)





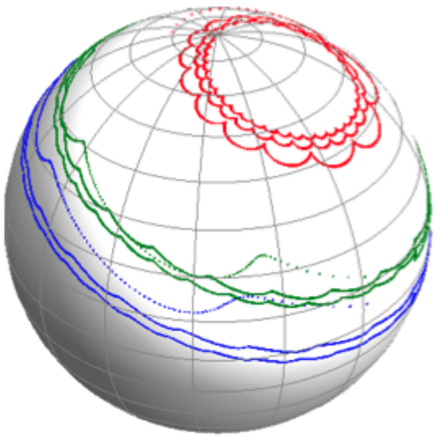


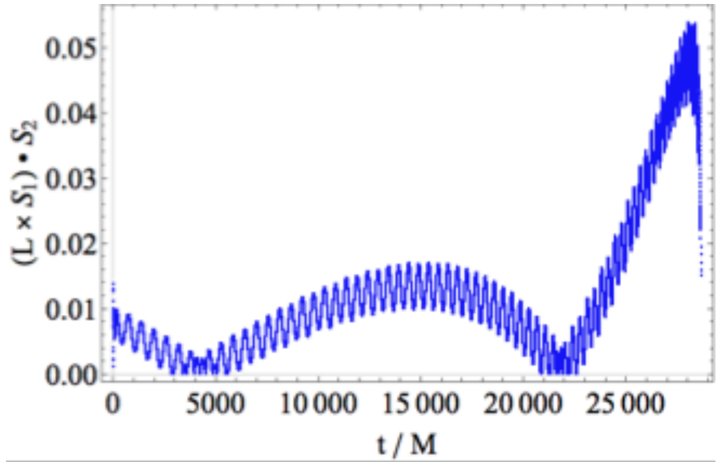
- Racine (2008) added the quadrupole-monopole term to spin-precession equations and discovered a new conserved quantity:

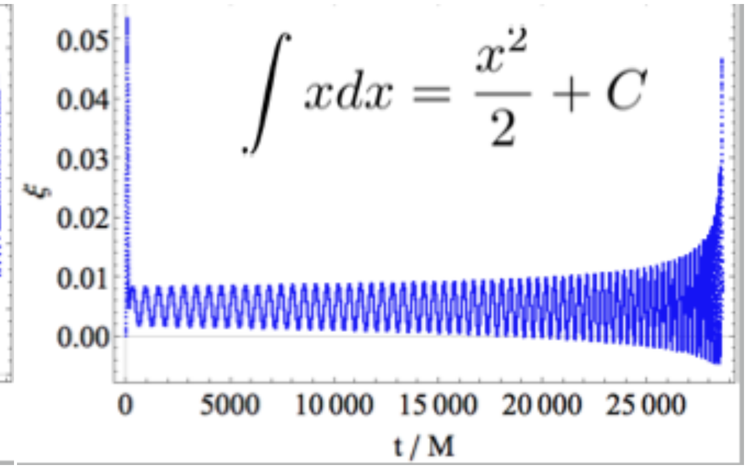
$$\xi = \chi_{\mathrm{eff}} = M^{-2}\,[(1+q)\mathbf{S}_1 + (1+q^{-1})\mathbf{S}_2]\bullet\mathbf{L}/L$$

- New term didn't seem to change qualitative behavior
- Oscillation in θ_1-θ_2 plane with fixed ξ yields flip-flops (Lousto+ 2015)
- Migration of Schnittman resonances on t_{RR} affects final spins and kicks (Kesden+ 2010a, 2010b)

Does PN intuition extend to NR regime?

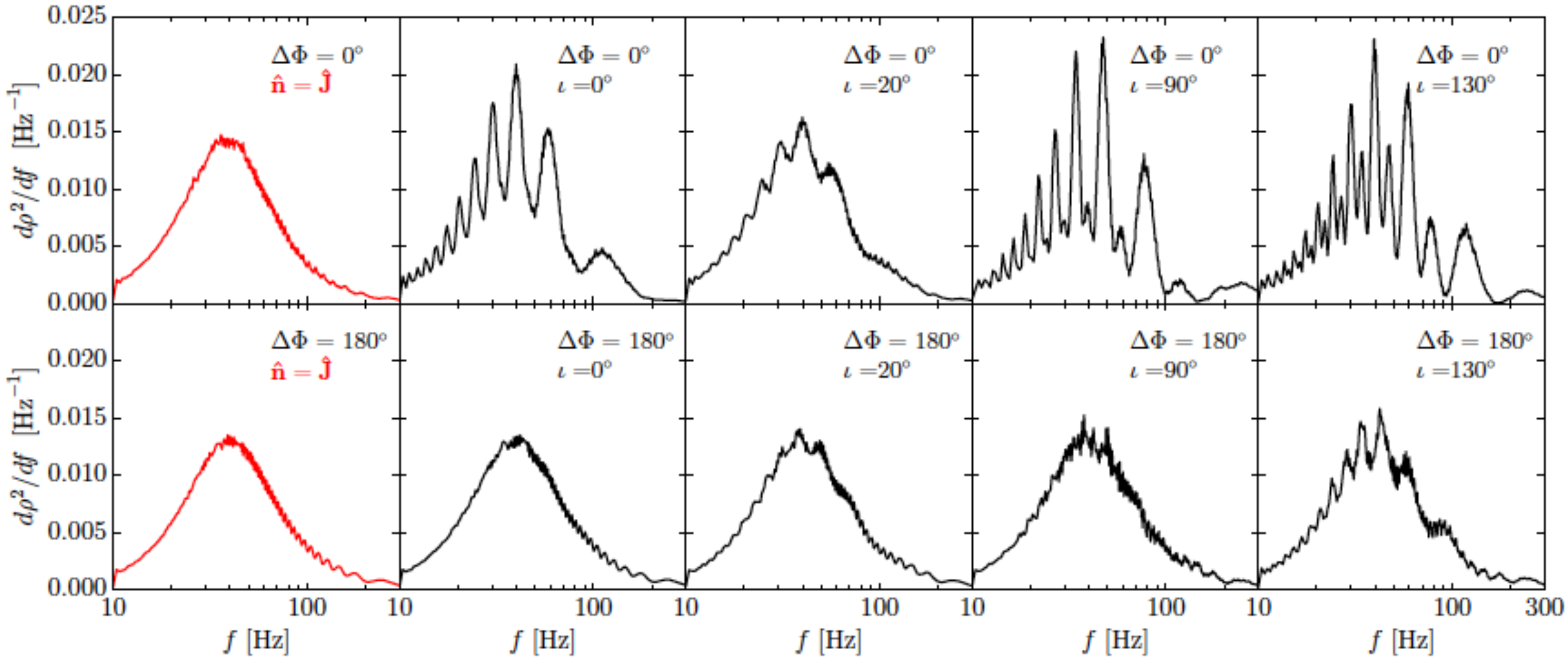




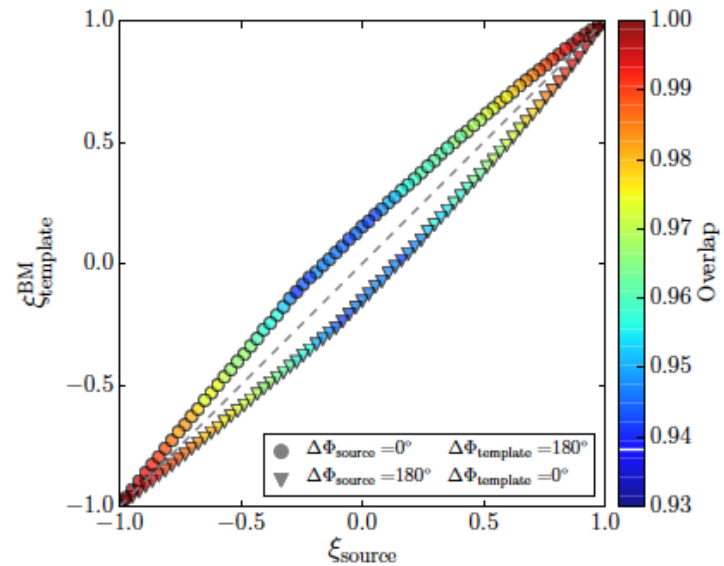


- Comparison between PN spin precession and SXS NR catalog suggests spins agree within 1° (5°) during inspiral (merger) (Ossokine+ 2015)
- Dedicated NR simulations suggest Schnittman resonances and conservation of ξ persist near merger (Demos and Lovelace 2015)
- ***More work needed studying PN-NR discrepancies***

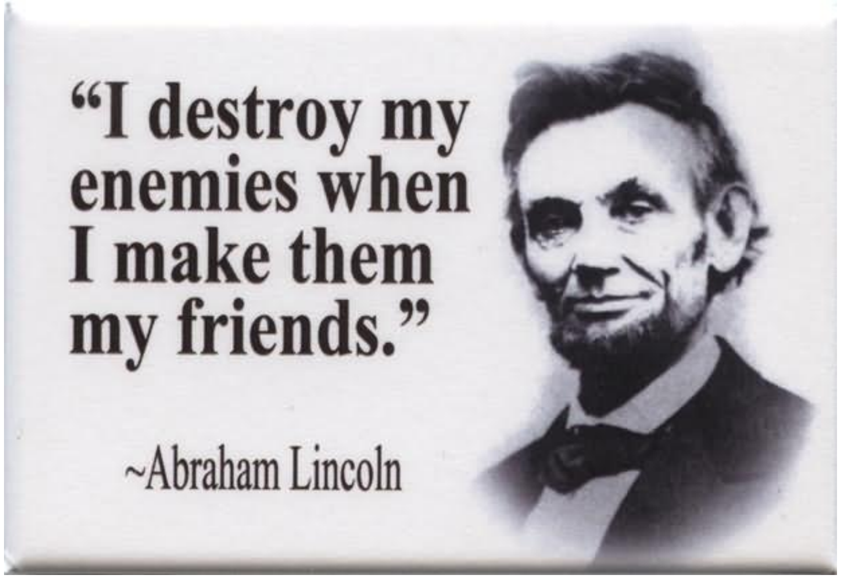
Are Schnittman resonances distinguishable?



- $\Delta\Phi = 0°$ has greater misalignment between ***J*** and ***L*** for *same* value of ξ, $\Longrightarrow$ greater modulation (Gerosa+ 2014).
- The perpendicular spin components also matter!!!

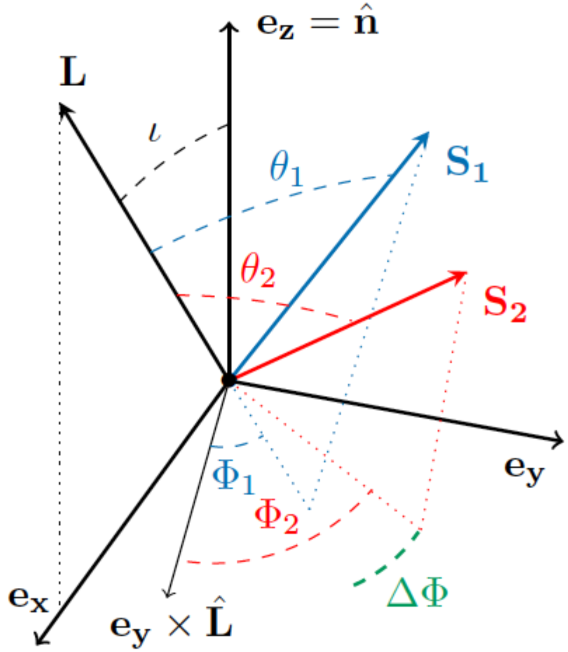


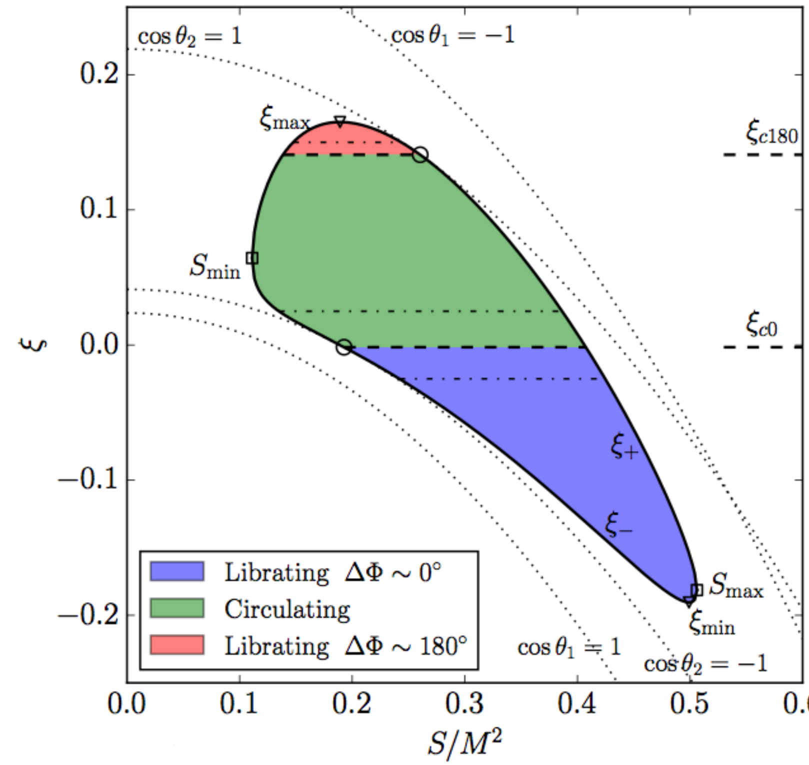
Analytic spin precession for generic spins



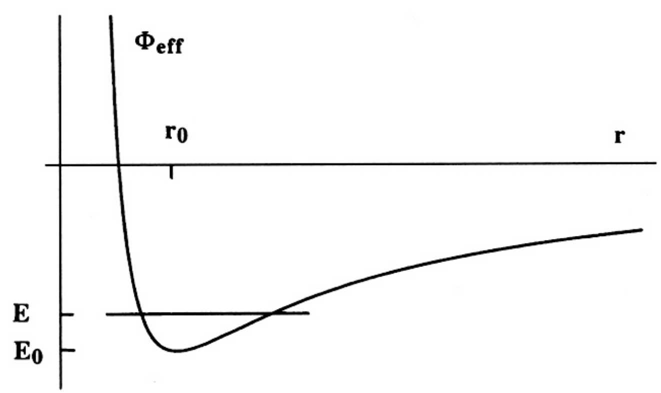
- Isolate variables that evolve on t_{pre} $\Longrightarrow$ average over variables that evolve on t_{orb}, hold constant variables that evolve on t_{RR}
- Count remaining degrees of freedom:
  - Magnitudes L, S_1, S_2 conserved: 9 – 3 = 6.
  - Total angular momentum $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}_1 + \boldsymbol{S}_2$ conserved: 6 – 3 = 3.
  - Projected effective spin ξ is conserved: 3 – 1 = 2.
- 2 degrees of freedom remain: S (magnitude of total spin $\boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2$) and Φ_L (azimuthal angle of **L** in frame with **J** on z axis).

Effective potential for spin precession





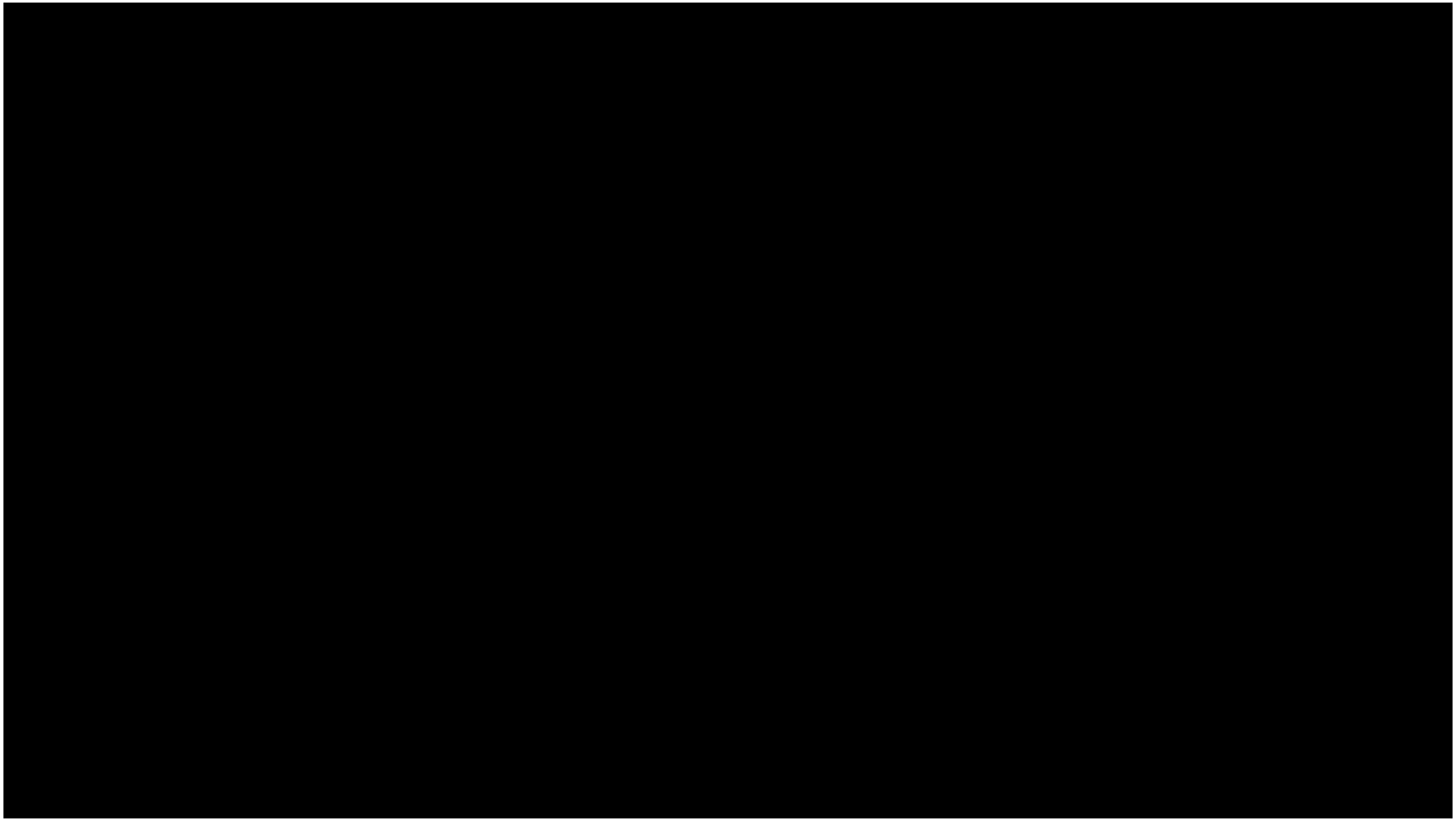
- *S* specifies *relative* orientation
- $E = \Phi_{eff}(r_\pm)$ sets limits of radial motion
- $\xi = \xi_\pm(S_\pm)$ sets limits on *S* (Kesden+ 2015)



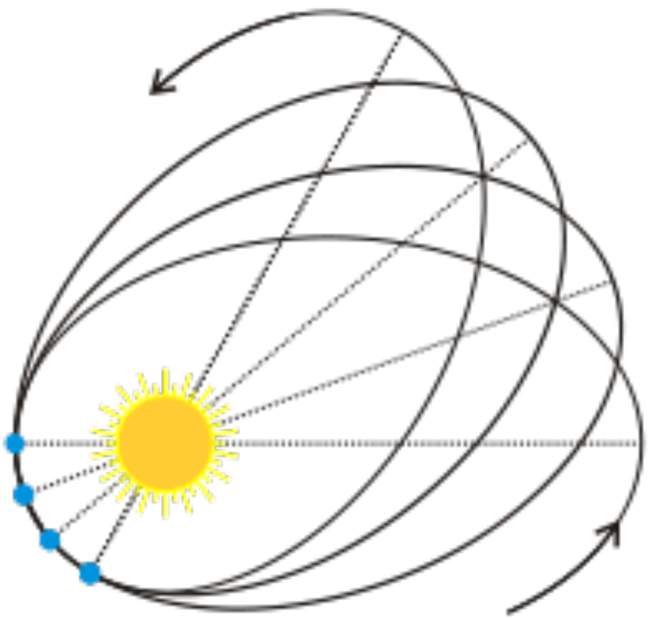
$$A_1 \equiv [J^2 - (L-S)^2]^{1/2},$$
$$A_2 \equiv [(L+S)^2 - J^2]^{1/2},$$
$$A_3 \equiv [S^2 - (S_1 - S_2)^2]^{1/2},$$
$$A_4 \equiv [(S_1 + S_2)^2 - S^2]^{1/2}.$$

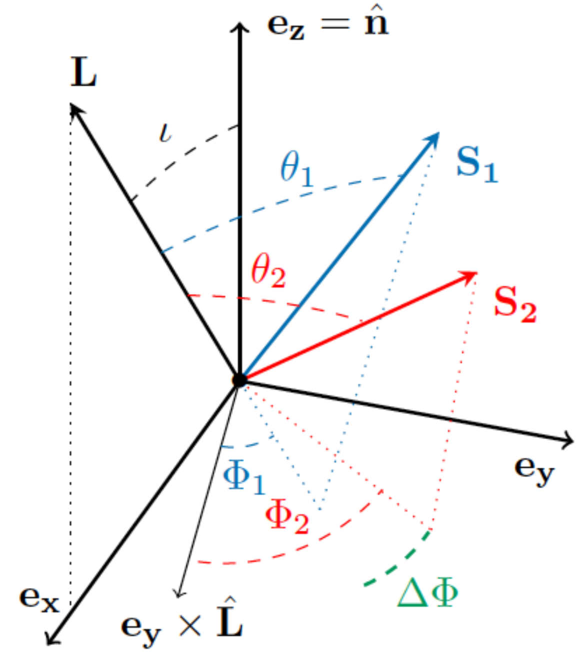
$$\xi_\pm(S) = \frac{(J^2 - L^2 - S^2)[S^2(1+q)^2 - (S_1^2 - S_2^2)(1-q^2)] \pm (1-q^2)A_1A_2A_3A_4}{4qM^2S^2L}$$

BBH Precession Animation



BBH spin precession is *quasi-periodic*





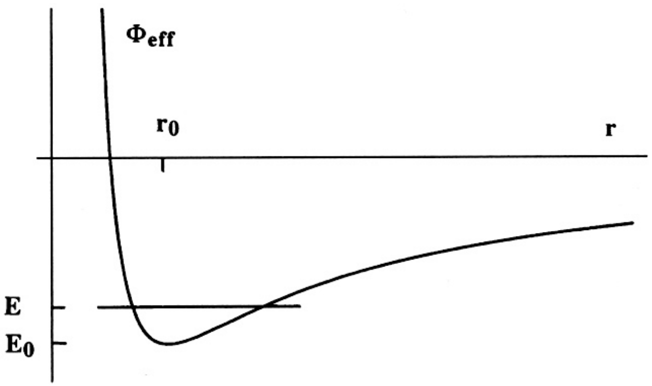
- Orbits in a spherical potential are regular (quasi-periodic)

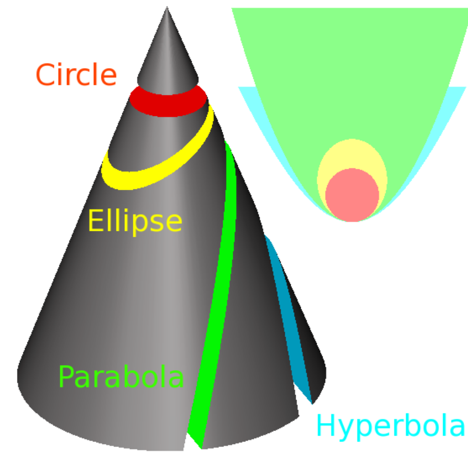
$$\tau(E, L) = 2\int_{r_-}^{r_+} \frac{dr}{dr/dt} \qquad \alpha(E, L) = 2\int_{r_-}^{r_+} \frac{\Omega_z \, dr}{dr/dt}$$

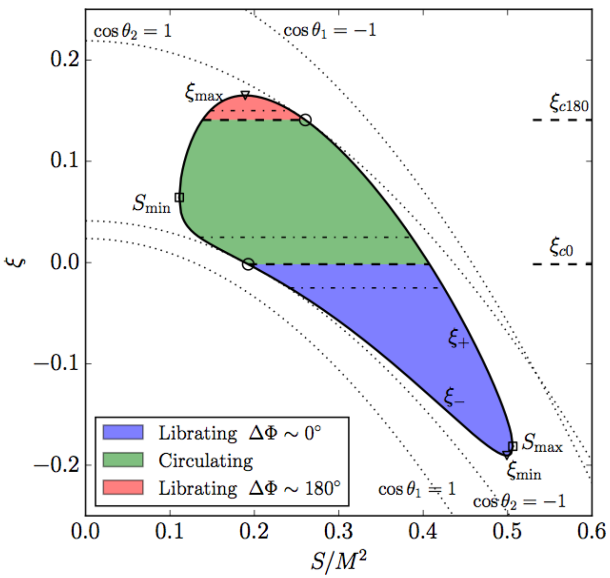
- BBH spin precession is regular (quasi-periodic)

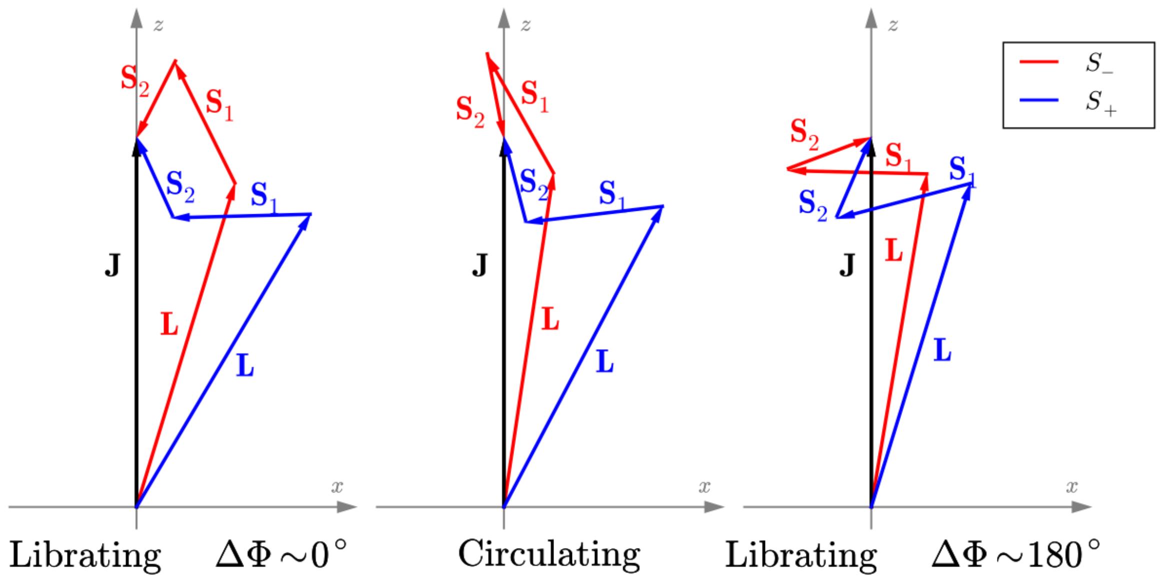
$$\tau(L, J, \xi) = 2\int_{S_-}^{S_+} \frac{dS}{dS/dt} \qquad \alpha(L, J, \xi) = 2\int_{S_-}^{S_+} \frac{\Omega_z \, dS}{dS/dt}$$

Spin morphology



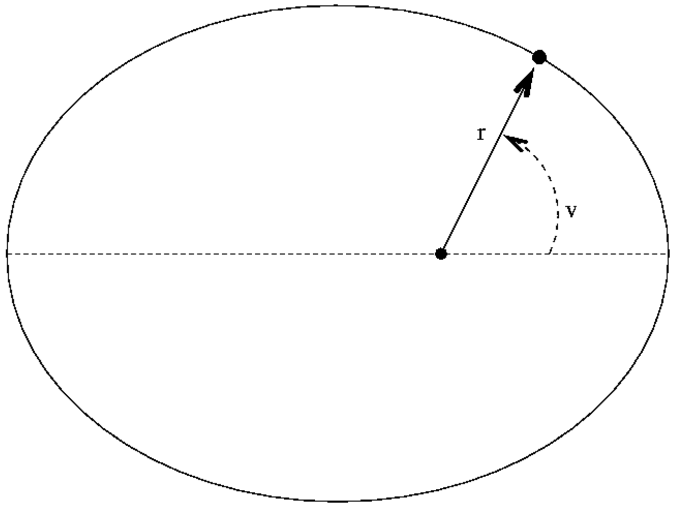


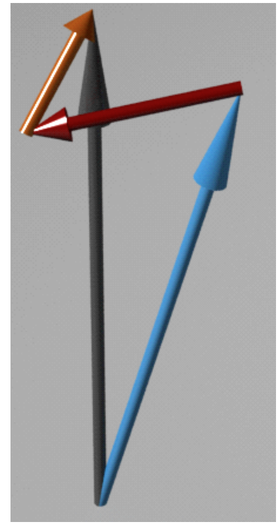


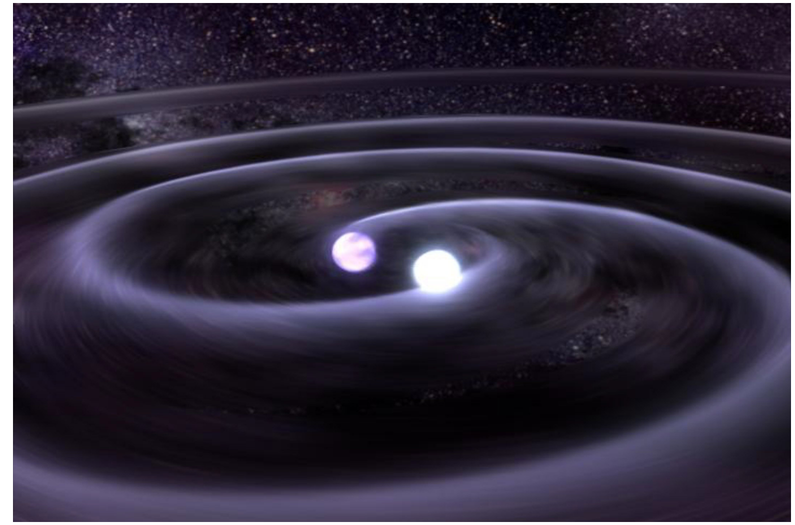


- Keplerian orbital shape determined by *E*, *L* ⟺ *a*, *e*
- Spin precession morphology determined by *L*, *J*, *ξ*
- Spin-orbit resonances analogue of circular orbits.

Radiation reaction





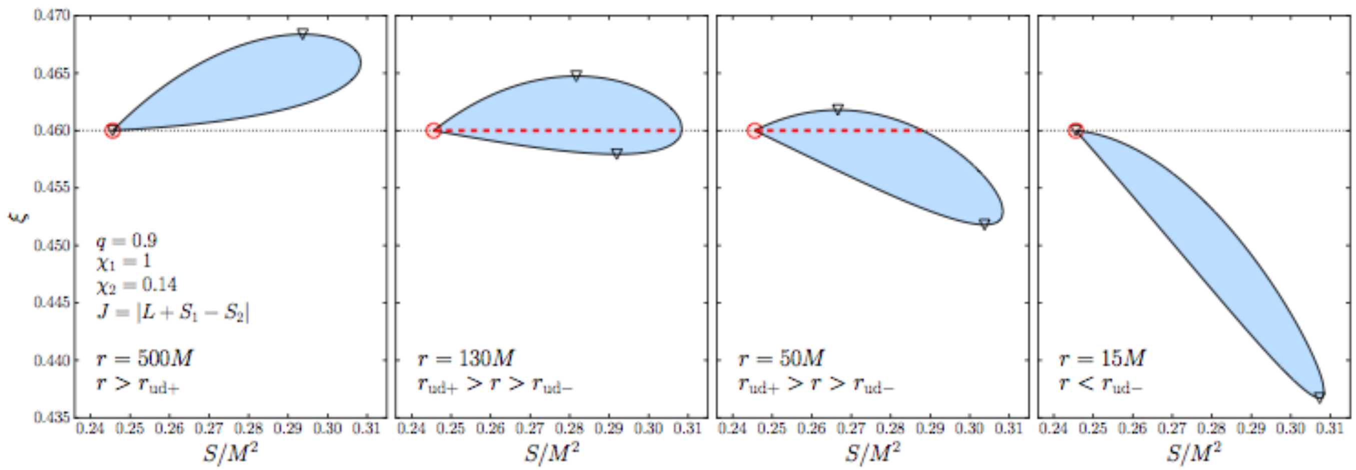


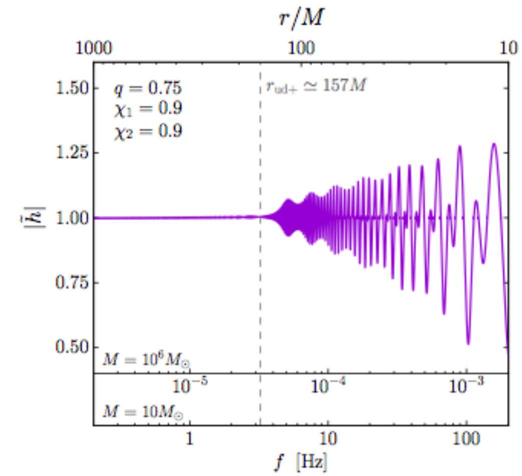
- Analytic Keplerian orbits and hierarchy $t_{\text{orb}} \ll t_{\text{pre}}$ allow us to *orbit-average* spin-precession equations
- Analytic precession solutions and hierarchy $t_{\text{pre}} \ll t_{\text{RR}}$ allow us to *precession-average* GW flux:

$$\left\langle \frac{dJ}{dL} \right\rangle_{\text{pre}} = \frac{2}{\tau} \int_{S_-}^{S_+} \frac{\cos\theta_L \; dS}{dS/dt}$$

$$= \frac{1}{2LJ} \left[J^2 + L^2 - \frac{2}{\tau} \int_{S_-}^{S_+} \frac{S^2 \; dS}{dS/dt} \right]$$

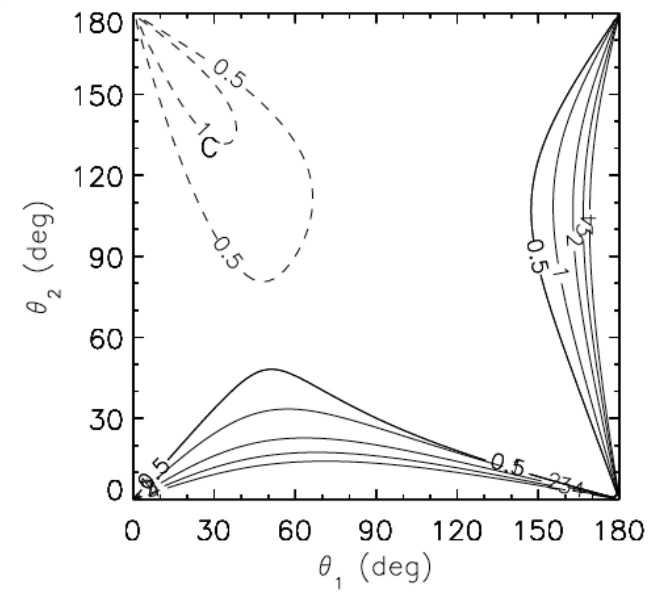
Up-down instability

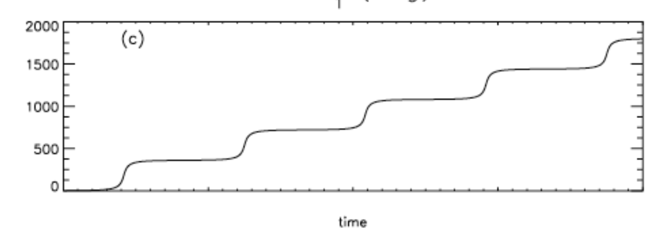




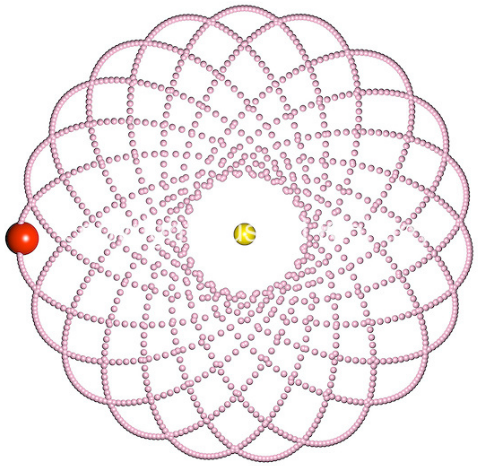
- BHs with $\mathbf{S}_1$ aligned and $\mathbf{S}_2$ anti-aligned with **L** (up-down) unstable to spin precession (Gerosa+ 2015)
- Instability for $r_- < r < r_+$ where:

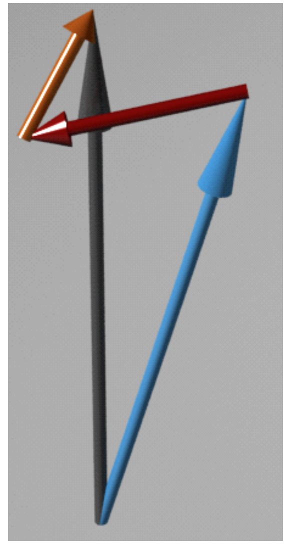
  $r_\pm = [\surd\chi_1 \pm \surd(q\chi_2)]^4(1 - q)^{-2}M$
- BHs in neighborhood of up-down in *J*-*ξ* plane are quasi-stable Schnittman resonances!

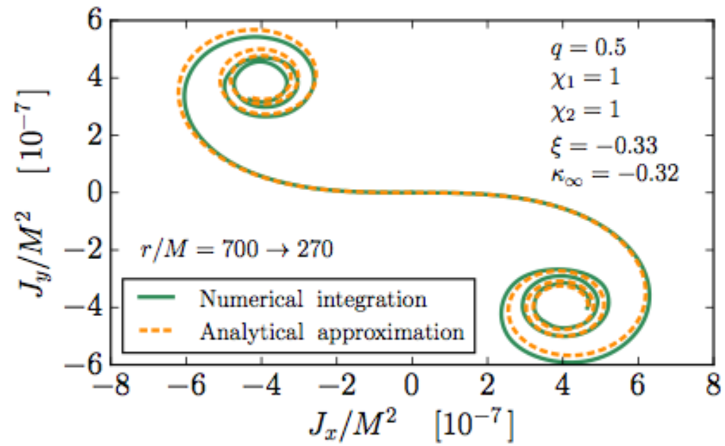


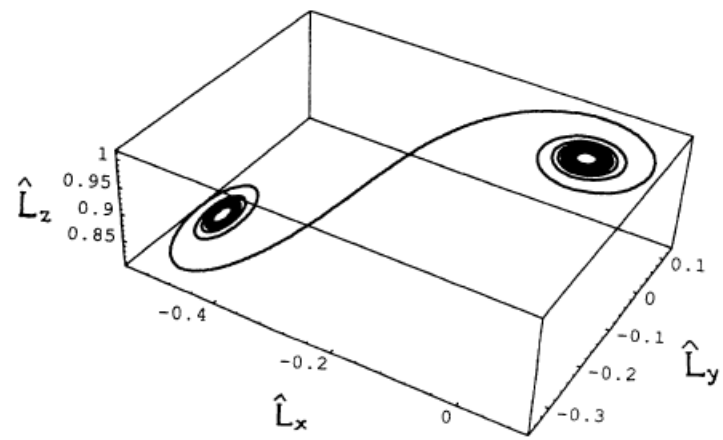


Nutational resonances









$$\mathbf{L}_\perp(t) = L \sum^{+\infty} \theta_{Ln}\{\cos[(\Omega - n\omega)t]\hat{\mathbf{x}} + \sin[(\Omega - n\omega)t]\hat{\mathbf{y}}\}$$

$$\frac{d\mathbf{J}_\perp}{dt} = -\frac{32}{5}\left(\frac{r}{M}\right)^{-4}\frac{\eta \mathbf{L}_\perp}{M}$$
$$= \frac{dL}{dt}\sum_{n=-\infty}^{+\infty} \theta_{Ln}\{\cos[(\Omega - n\omega)t]\hat{\mathbf{x}} + \sin[(\Omega - n\omega)t]\hat{\mathbf{y}}\}$$

- BBH spin precession *resonant* (periodic) if $\alpha = n\pi/m \Longrightarrow \Omega = n\omega$ where $\Omega = \alpha/\tau$ and $\omega = 2\pi/\tau$
- Resonances with $m = 1$, $n = 2j$ have $d\mathbf{J}/dt \nparallel \mathbf{J} \Longrightarrow$ Direction of **J** is kicked as BBHs inspiral through resonance (Zhao+ 2017 in prep)

Speculation and Questions

- Much progress made in understanding generic spin precession
- More progress needed on constraining generic spin precession with GW observations
- Waveforms based on "single effective spin" by design do not include effects of generic precession
- Simple precession: ξ, χ_p conserved, single precession frequency Ω
- Generic precession: ***L*** precesses with frequency Ω, but also nutates with frequency ω in range $\theta_{L,min} < \theta_L < \theta_{L,max}$
- Wise choice of parameters will facilitate parameter estimation:
  - θ_i, ϕ_i, χ_p bad because they vary on t_{pre}
  - ξ, J better because they vary on t_{RR}
  - Ω, ω, $\theta_{L,min}$, $\theta_{L,max}$ best because they directly affect waveforms
- Precessional and nutational phases "at coalesnce" as nuisance parameters that affect detectability and parameter estimation