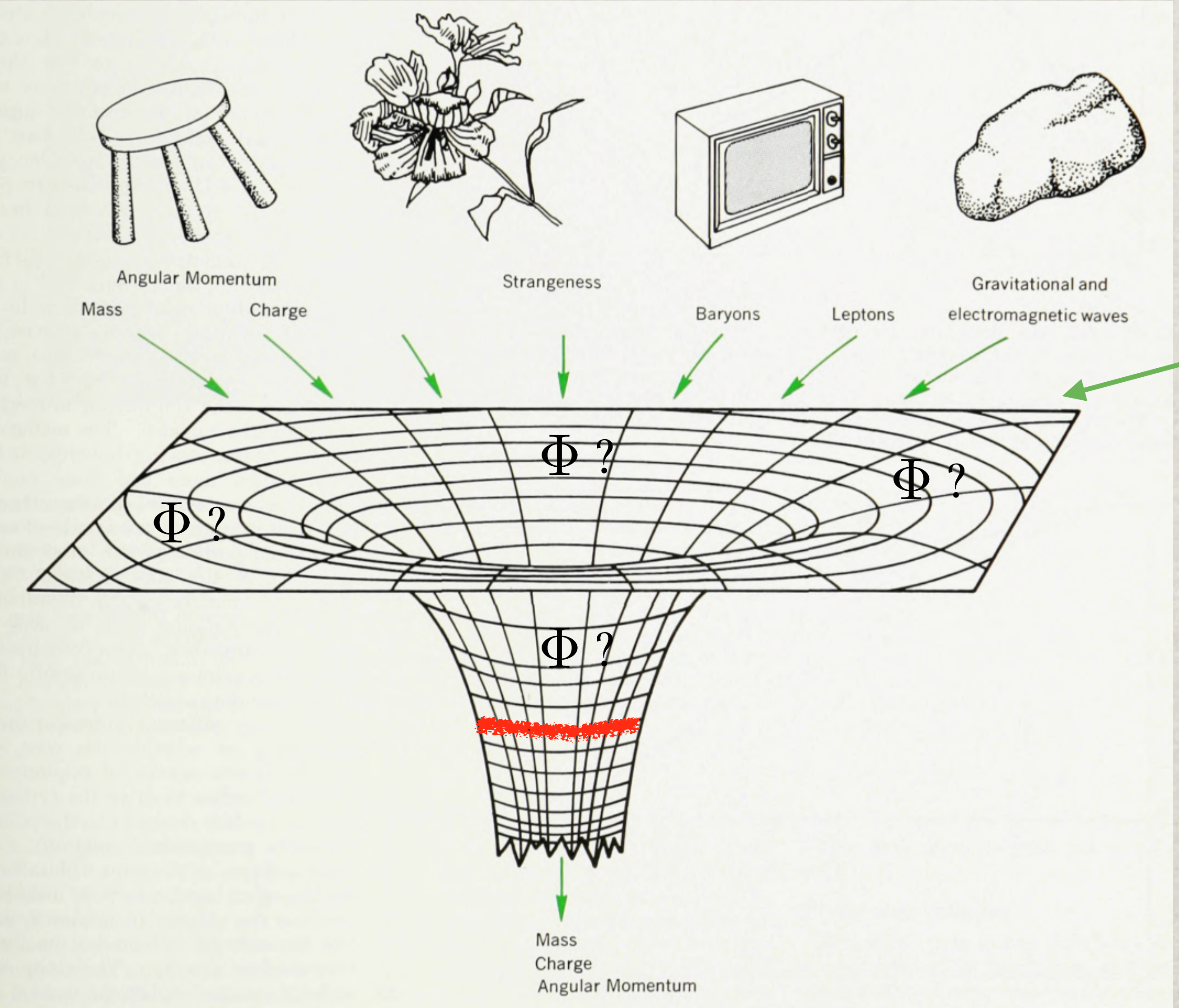
Kerr black holes can have hair.

So what?



Carlos Herdeiro
Departamento de Física da Universidade de Aveiro, Portugal

StronGBaD
Strong Gravity and Binary Dynamics with Gravitational Wave Observations
University of Mississippi, March 2nd 2017

The "no-hair" idea

The collapse leads to a black hole endowed with mass and charge and angular momentum but, so far as we can now judge, no other adjustable parameters: "a black hole has no hair." Make one black hole out of matter;

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Ruffini, Wheeler (1971)

Original idea:
collapse leads to equilibrium black holes uniquely determined by M,J,Q -
asymptotically measured quantities subject to a Gauss law
and no other independent characteristics (hair)

Motivated by uniqueness theorems

e.g: Israel 1967, 1968; Carter 1970; Hawking 1972; Robinson 1975, 1977; and many others
Overview: "Four decades of black hole uniqueness theorems" D. Robinson (2004, 2009)

D=4, asymptotically flat, regular (on and outside the event horizon) **black hole (BH) solutions of Einstein's gravity**

Vacuum:

$$\mathcal{S} = \frac{1}{16\pi}\int d^4x\sqrt{-g}R$$

Kerr Kerr 1963

Uniqueness Israel 1967; Carter 1970; Hawking 1972

No (independent-multipolar) hair

Electro-vacuum:

$$\mathcal{S} = \frac{1}{4\pi}\int d^4x\sqrt{-g}\left(\frac{R}{4} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)$$

Kerr-Newman Newman et al. 1965

Uniqueness Israel 1968; Robinson 1975, 1977

No (independent-multipolar) hair

Many no-scalar-hair theorems:

(only scalars, D=4, asymptotically flat)

Theory Lagrangian density $\mathcal{L}$	No-hair theorem	Known scalar hairy BHs with regular geometry on and outside $\mathcal{H}$ (primary or secondary hair; regularity)
Scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi$	Chase[22]	
Massive-scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - \frac{1}{2}\mu^2\Phi^2$	Bekenstein[11]	
Massive-complex-scalar-vacuum $\frac{1}{4}R - \nabla_\mu\Phi^*\nabla^\mu\Phi - \mu^2\Phi^*\Phi$	Pena– –Sudarsky[61]	Herdeiro–Radu[136, 137] (primary, regular); generalizations:[159]
Conformal-scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - \frac{1}{12}R\Phi^2$	Xanthopoulos– –Zannias[32] Zannias[33]	Bocharova–Bronnikov–Melnikov– –Bekenstein (BBMB)[16–18] (secondary, diverges at $\mathcal{H}$); generalizations:[87]
V-scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - V(\Phi)$	Heusler[46, 47, 50] Bekenstein[26] Sudarsky[51]	Many, with non-positive definite potentials:[71–75, 78–80] (typically secondary, regular)
P-scalar-vacuum $\frac{1}{4}R + P(\Phi, X)$	Graham– –Jha[62]	
Einstein-Skyrme $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi^a\nabla^\mu\Phi^a$ $-\kappa\lvert\nabla_{[\mu}\Phi^a\nabla_{\nu]}\Phi^b\rvert^2$		Droz–Heusler–Straumann[126] (primary but topological; regular); generalizations:[129, 131]
Scalar-tensor theories $\varphi\hat{R} - \frac{\omega(\varphi)}{\varphi}\hat{\nabla}_\mu\varphi\hat{\nabla}^\mu\varphi - U(\varphi)$	Hawking[27] Saa[34, 35] Sotiriou– –Faraoni[31]	
Horndeski/Galileon theories Full $\mathcal{L}$ in eq. (41)	Hui– –Nicolis[45]	Sotiriou-Zhou[43] (secondary; regular) Babichev–Charmousis[88, 90] (secondary[88] or primary,[90] diverges at $\mathcal{H}^+$ or $\mathcal{H}^-$); generalizations:[91–93]

Reviews hairy solutions

Sotiriou, 2015
Volkov, 2016

C.H., Radu, 2015

Goal: Present a simple model of hairy black holes

Massive-complex-scalar-vacuum:

$$\mathcal{S} = \frac{1}{4\pi}\int d^4x\sqrt{-g}\left(\frac{R}{4} - \nabla_\alpha\Phi^*\nabla^\alpha\Phi - \mu^2|\Phi|^2\right)$$

There are BH solutions:

- **within GR** (not alternative theories of gravity);
- with matter **obeying all energy conditions**;
- which can yield **distinct phenomenology**;

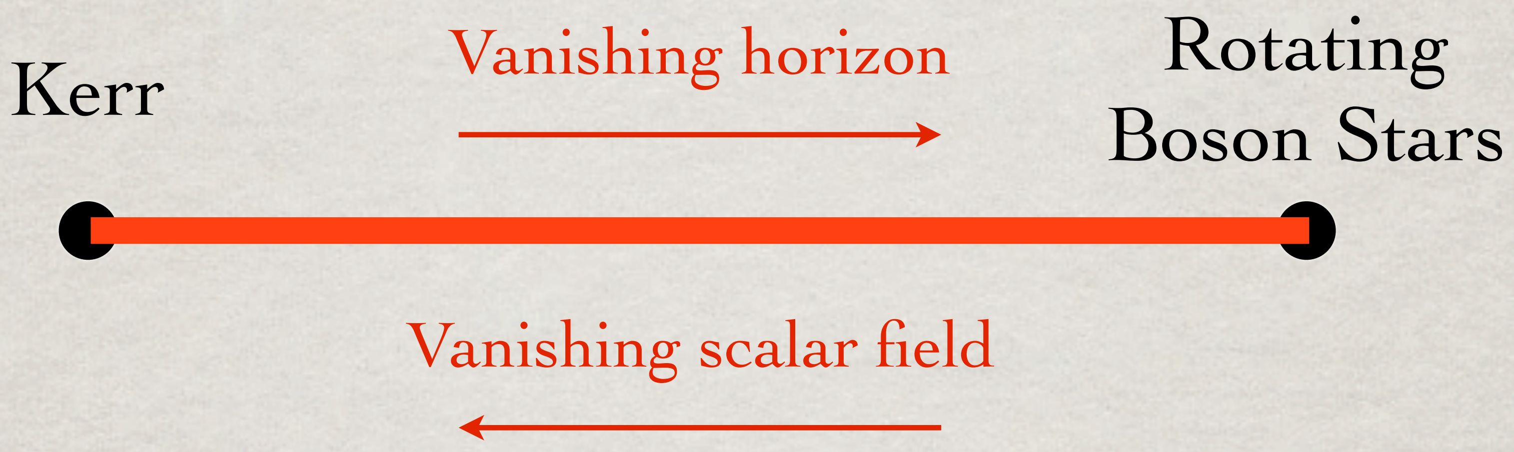
which are:

- asymptotically flat
- regular on and outside the horizon
- continuously connecting to the Kerr solution
- continuously connected to relativistic Bose-Einstein condensates (boson stars)
- with an independent scalar charge (primary hair)

Kerr Black Holes with scalar hair

C.H. and Radu, PRL 112 (2014) 221101

Continuum of hairy black hole solutions, interpolating between...



This model is an example of a more general construction. Generalizations include:

- other spin fields (Proca stars, Brito + 2016; ; C.H., Radu, Runarsson 2016), self-interactions,...
- scalarized black holes in scalar tensor theories (e.g. Kleihaus, Kunz, Yazadjiev 2015)
- higher dimensional models and different asymptotics

Plan:

1) What is the physics rationale for these solutions to exist ?
2) How could one distinguish them phenomenologically from Kerr ?

3) Open issues (dynamics, nature of the scalar field, ...)

1a) How can the scalar field be in equilibrium with a horizon? Superradiance (review) Brito, Cardoso, Pani 2015.

Linear analysis: Klein-Gordon equation on Kerr

$$\Box\Phi = \mu^2\Phi \qquad \Phi = e^{-iwt}e^{im\varphi}S_{\ell m}(\theta)R_{\ell m}(r)$$

Radial Teukolsky equation: Teukolsky (1972); Brill et al. (1972)

$$\frac{d}{dr}\left(\Delta\frac{dR_{\ell m}}{dr}\right) = \left(a^2w^2 - 2maw + \mu^2r^2 + A_{\ell m} - \frac{K^2}{\Delta}\right)R_{\ell m}$$

$$\Delta \equiv r^2 - 2Mr + a^2$$

$$K \equiv (r^2 + a^2)w - am$$

Generically one obtains *quasi*-bound states:

$$\omega = \omega_R + i\omega_I$$

critical frequency

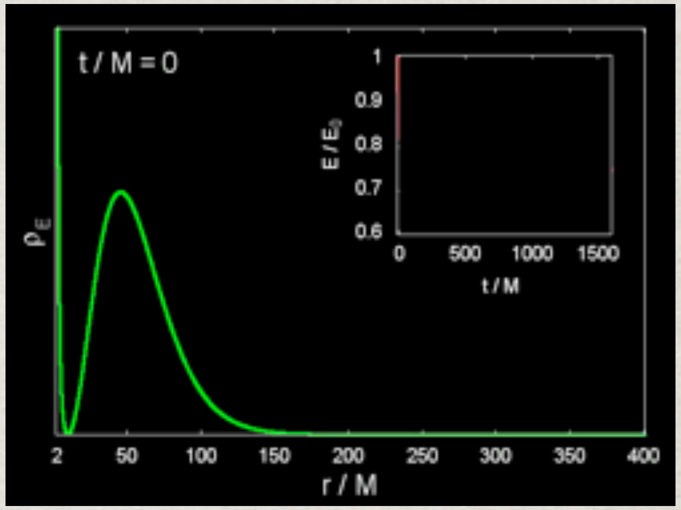
$$w_c = m\Omega_H$$

$$\omega = \omega_R + i\omega_I$$

critical frequency

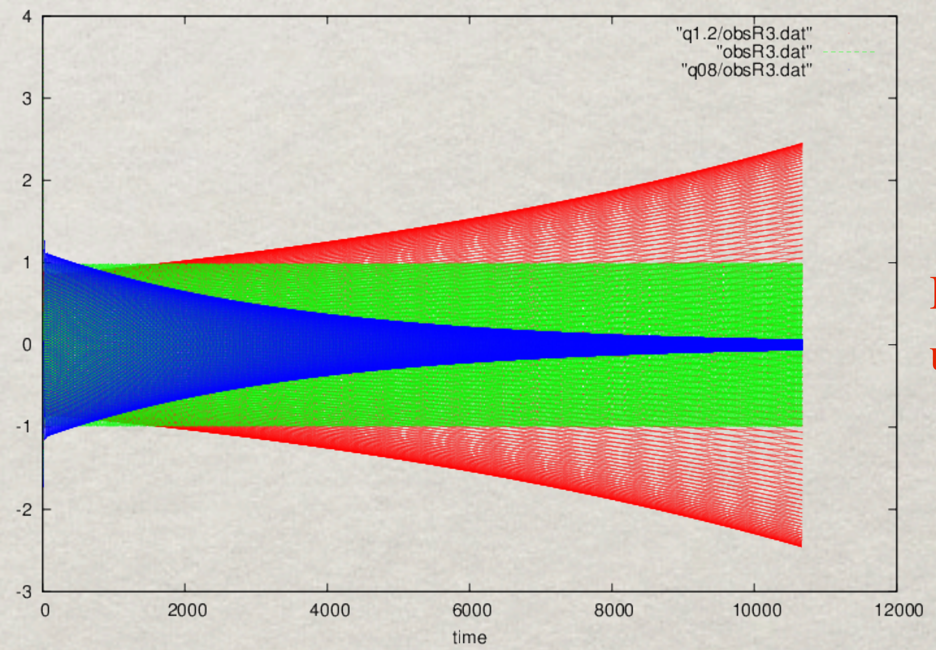
$$w_c = m\Omega_H$$

$w_I < 0$ if $w_R > w_c$ decay



Degollado et. al. 2012

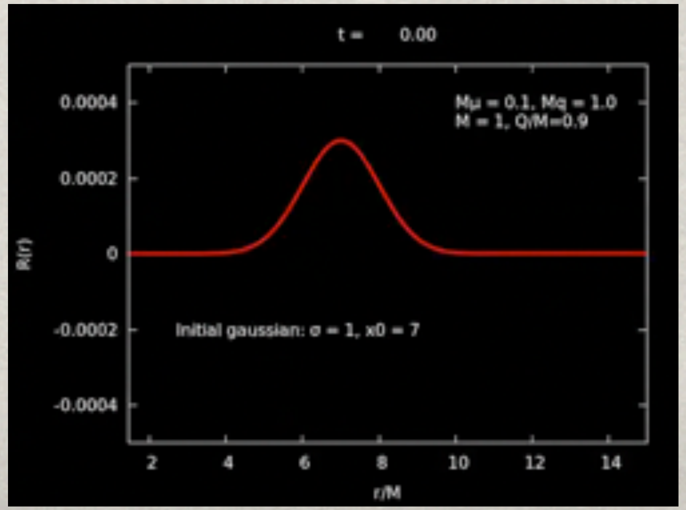
$w_I = 0$ if $w = w_c$ true bound states: *clouds*



Degollado, CH, unpublished

$w_I > 0$ if $w_R < w_c$ grow

Press and Teukolsky (1972)

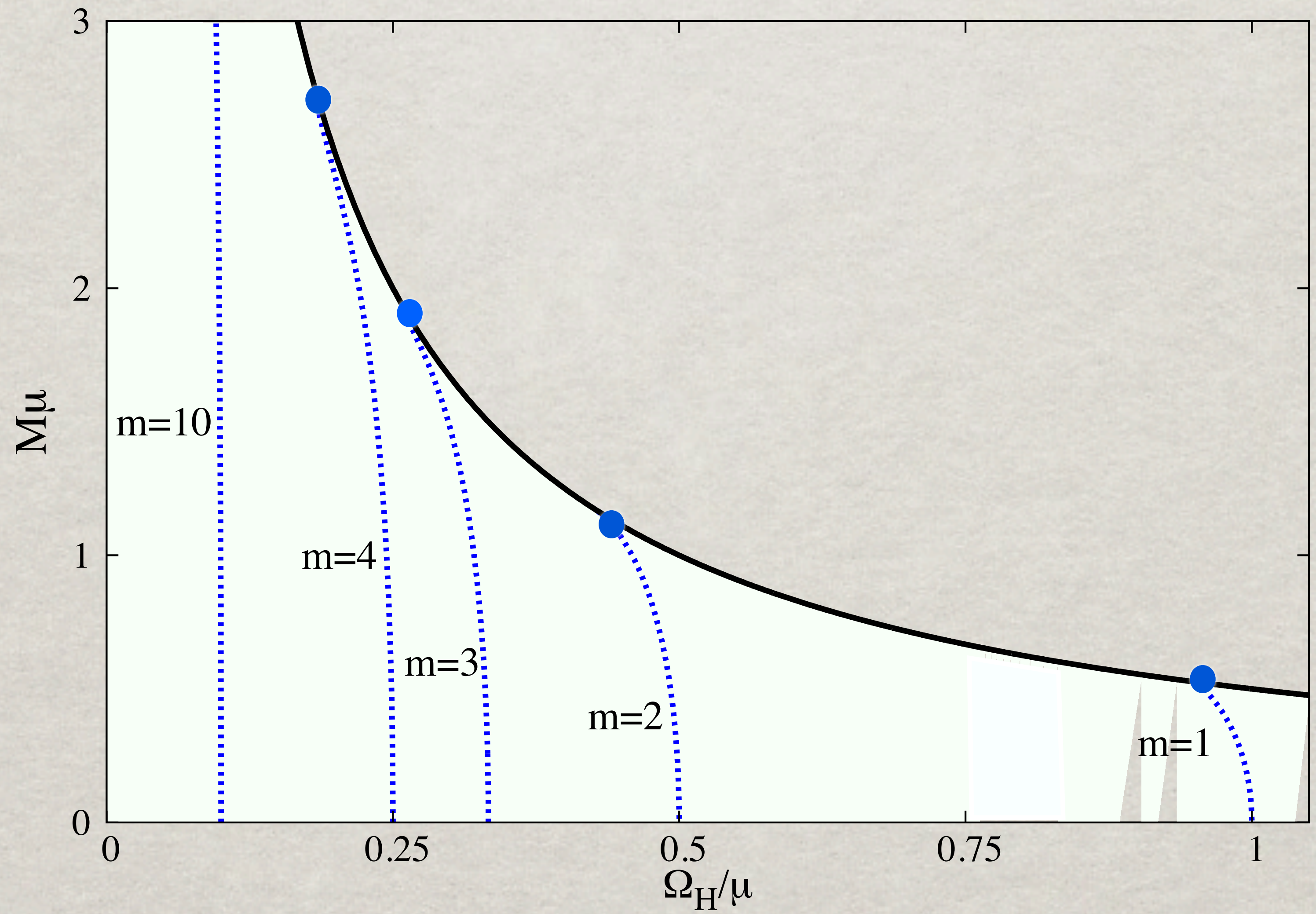


Degollado and C.H. 2014

Klein-Gordon (linear) stationary clouds around Kerr:

Damour, Deruelle and Ruffini (1976); Zouros and Eardley (1979); Detweiler (1980); Hod 2012; (...); Yakov Shilapentokh-Rothman (2014)

Clouds for Kerr: discrete set labelled by (n,l,m) subject to one quantization condition which yields BH mass,spin. Hod (2012)



Stationary clouds are **synchronized** “atomic orbitals” around a Kerr BH

1b) How do you construct them?

Scalar Boson stars

D. J. Kaup, Phys. Rev. 172 (1968) 1331; R. Ruffini and S. Bonazzola, Phys. Rev. 187 (1969) 1767;
Reviews: F. E. Schunck and E. W. Mielke, Class. Quant. Grav. 20 (2003) R301 [arXiv:0801.0307 [astro-ph]]
S. L. Liebling and C. Palenzuela, Living Rev. Rel. 15 (2012) 6 [arXiv:1202.5809 [gr-qc]]

$$\mathcal{S} = \frac{1}{4\pi}\int d^4x\sqrt{-g}\left(\frac{R}{4} - \nabla_\alpha\Phi^*\nabla^\alpha\Phi - \mu^2|\Phi|^2\right)$$

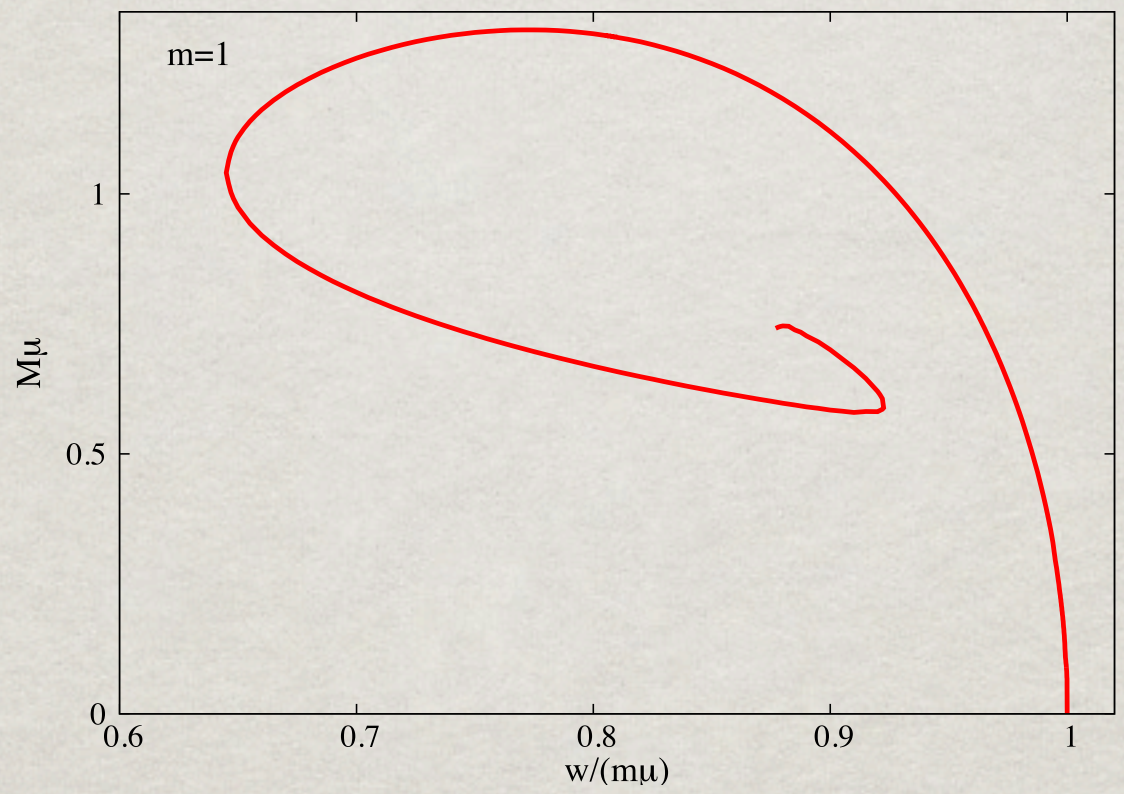
Rotating boson stars:
S.Yoshida and Y. Eriguchi, Phys. Rev. D 56 (1997) 762; F. E. Schunck and E. W. Mielke, Phys. Lett. A 249 (1998) 389

$$ds^2 = -e^{2F_0(r,\theta)}dt^2 + e^{2F_1(r,\theta)}\left(dr^2 + r^2 d\theta^2\right) + e^{2F_2(r,\theta)}r^2\sin^2\theta\,(d\varphi - W(r,\theta)dt)^2$$

$$\Phi = \phi(r,\theta)e^{i(m\varphi - wt)}$$

Three input parameters: (w,m,n)

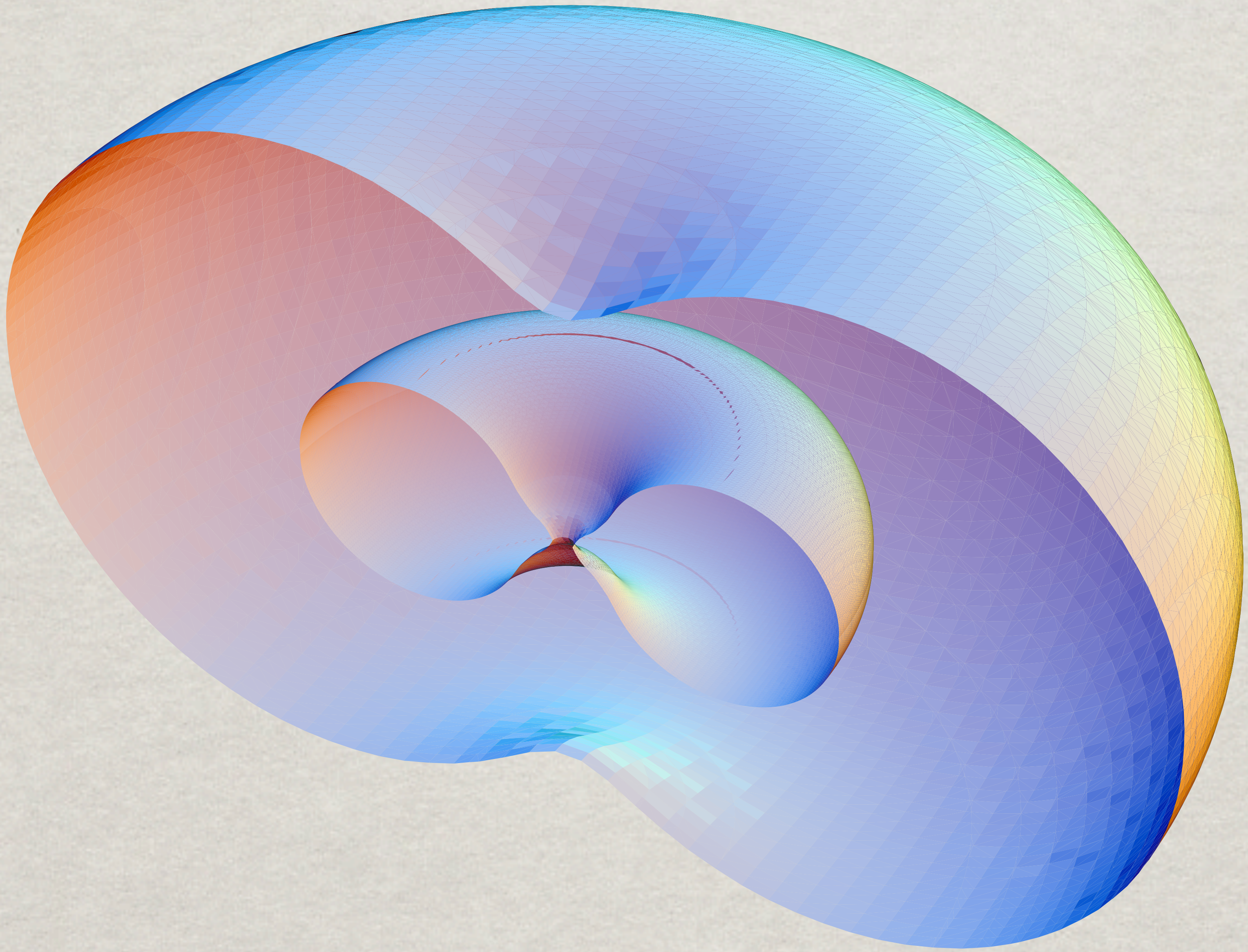
Boson stars phase space (nodeless):



Conserved Noether charge:

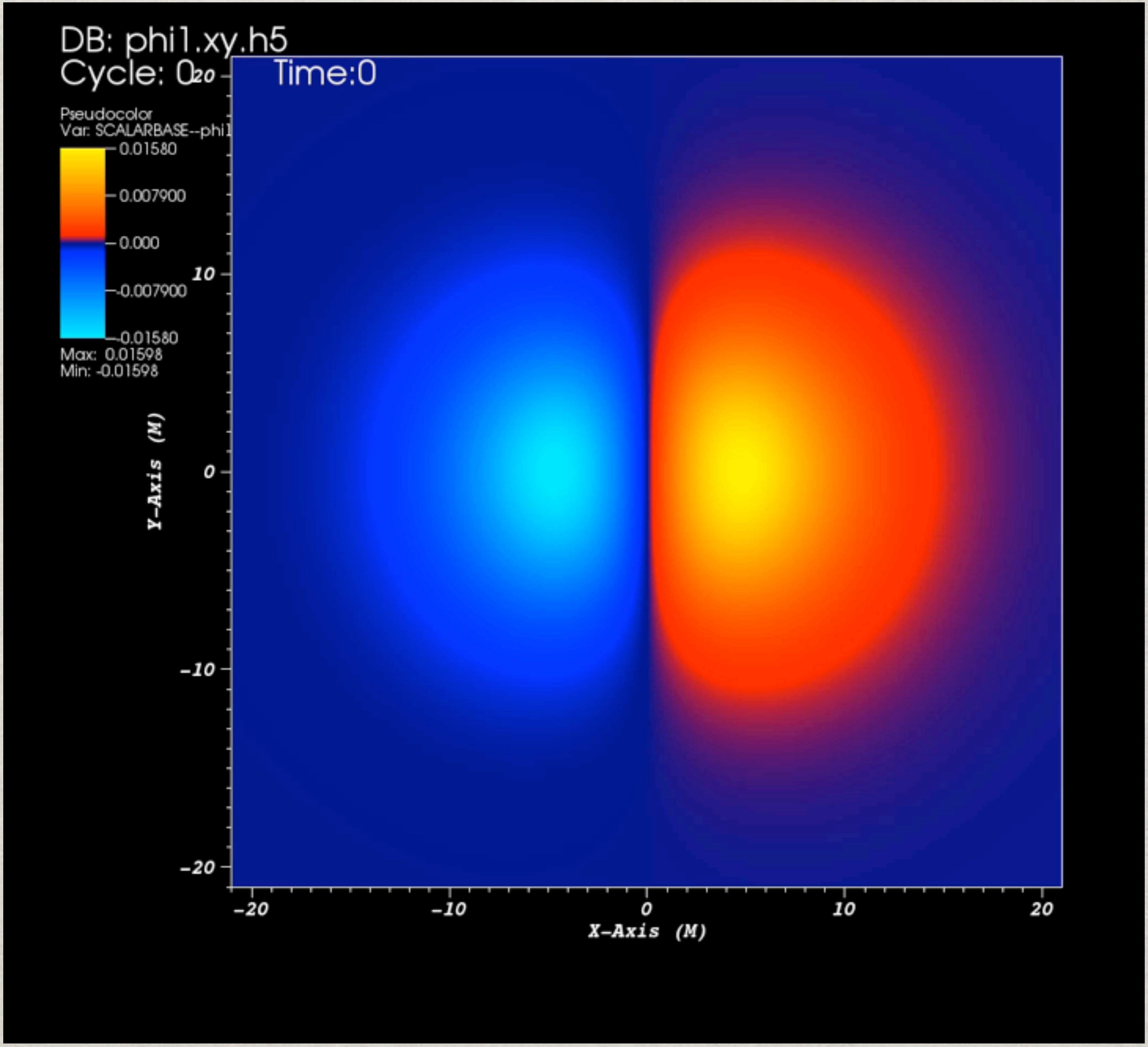
$$Q = \int_{\Sigma} dr d\theta d\varphi j^t \sqrt{-g}$$

Surfaces of constant scalar energy density



Rotating Boson Star

Time evolution of the real part of the scalar field



Simulation by M. Zilhão (in progress)

Using boson stars technology to compute
black holes surrounded by “heavy stationary clouds”

Einstein Klein-Gordon: non-linear setup

Ansatz:

$$ds^2 = -e^{2F_0(r,\theta)} N dt^2 + e^{2F_1(r,\theta)} \left(\frac{dr^2}{N} + r^2 d\theta^2 \right) + e^{2F_2(r,\theta)} r^2 \sin^2\theta \left(d\varphi - W(r,\theta) dt \right)^2 \qquad N = 1 - \frac{r_H}{r}$$

$$\Phi = \phi(r,\theta) e^{i(m\varphi - wt)}$$

Asymptotically:

$$g_{tt} = -1 + \frac{2M}{r} + \ldots, \quad g_{\varphi t} = -\frac{2J}{r} \sin^2\theta + \ldots$$

$$\phi = f(\theta) \frac{e^{-\sqrt{\mu^2 - w^2} r}}{r} + \ldots$$

take: $w < \mu$

Near the horizon:

$$x \equiv \sqrt{r^2 - r_H^2}$$

$$F_i = F_i^{(0)}(\theta) + x^2 F_i^{(2)}(\theta) + \mathcal{O}(x^4)$$

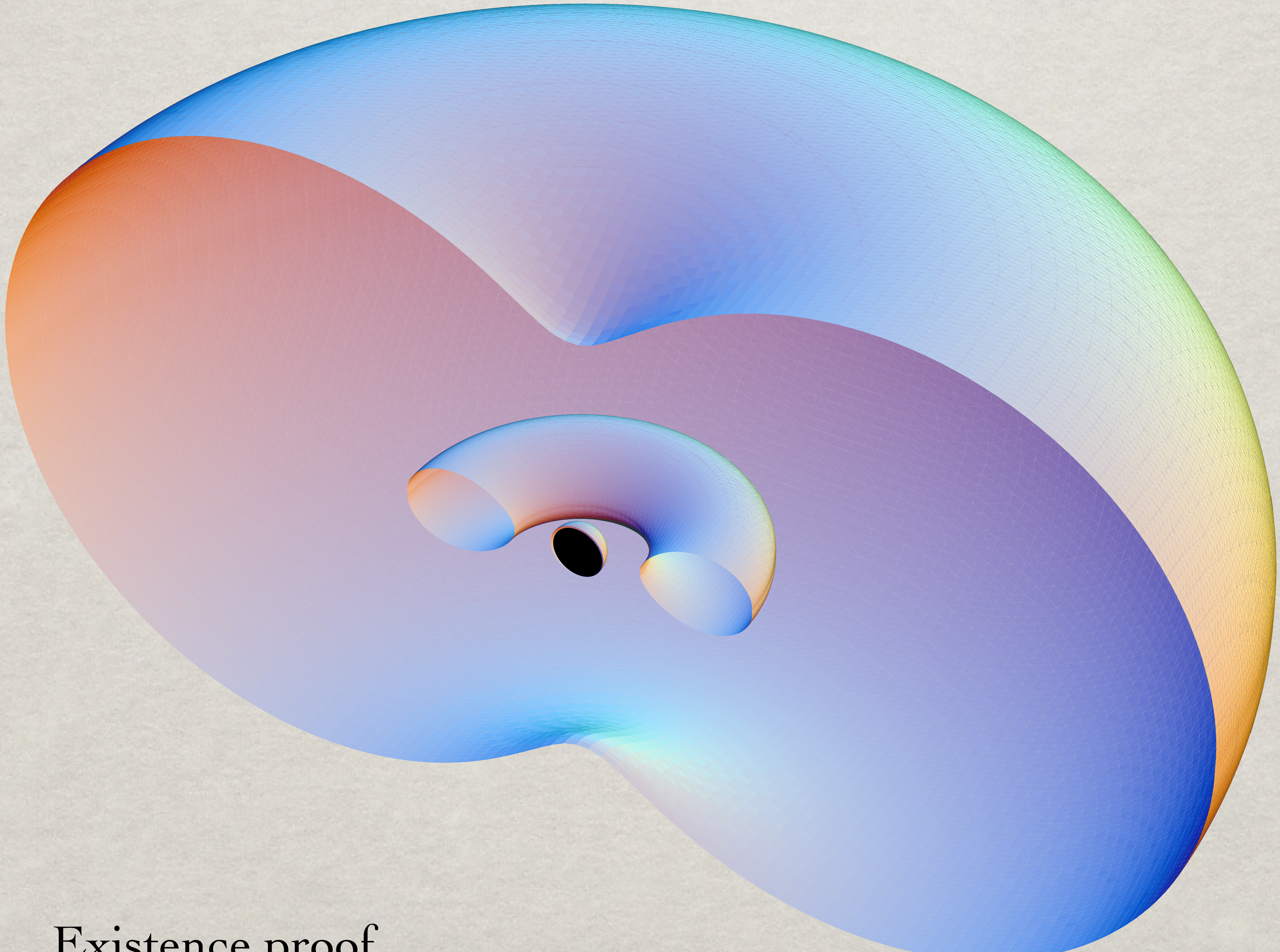
$$W = \Omega_H + \mathcal{O}(x^2)$$

$$\phi = \phi_0(\theta) + \mathcal{O}(x^2)$$

take: $\Omega_H = \dfrac{w}{m}$

Four input parameters: m, w, r_H, n

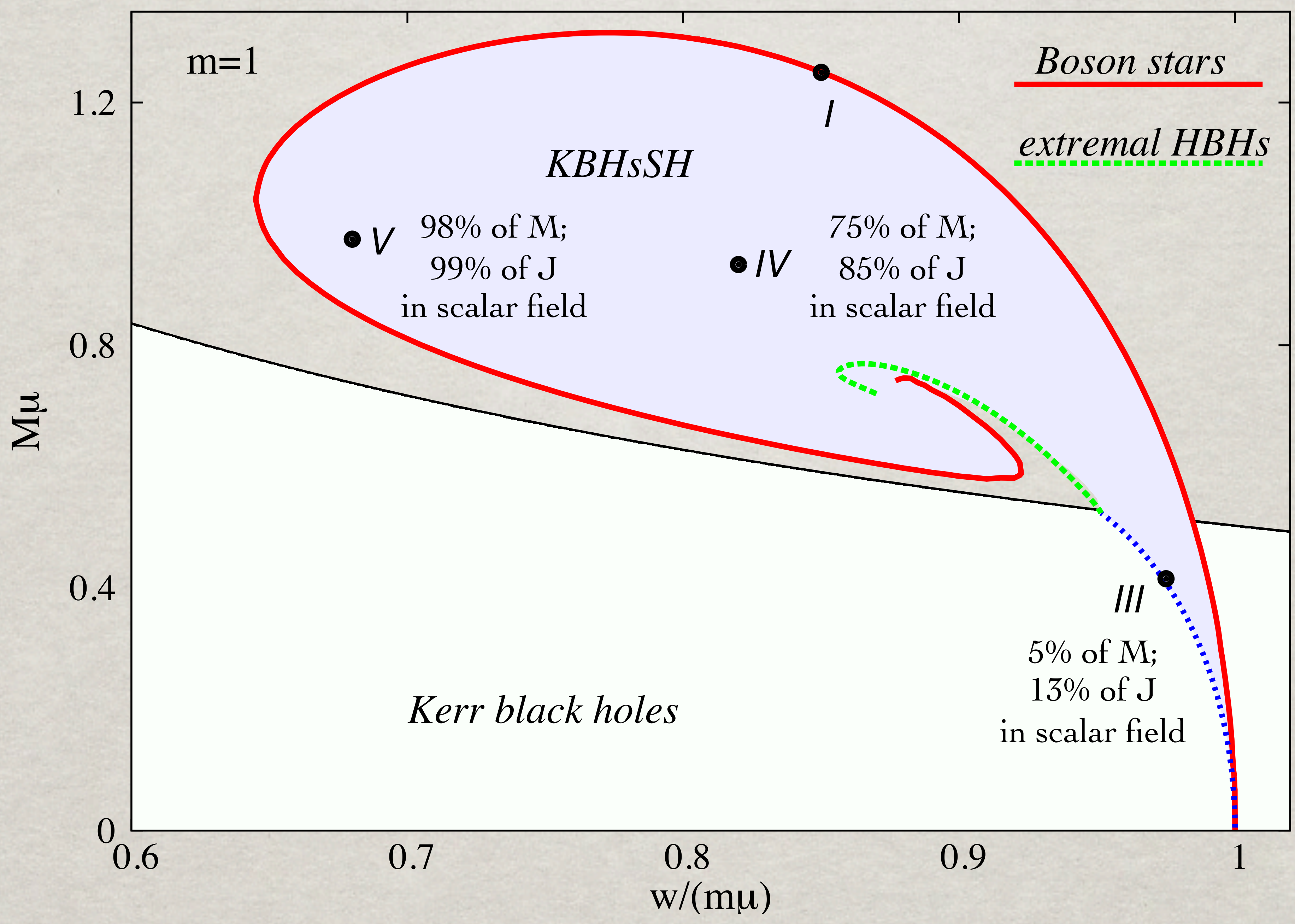
Kerr black holes with scalar hair



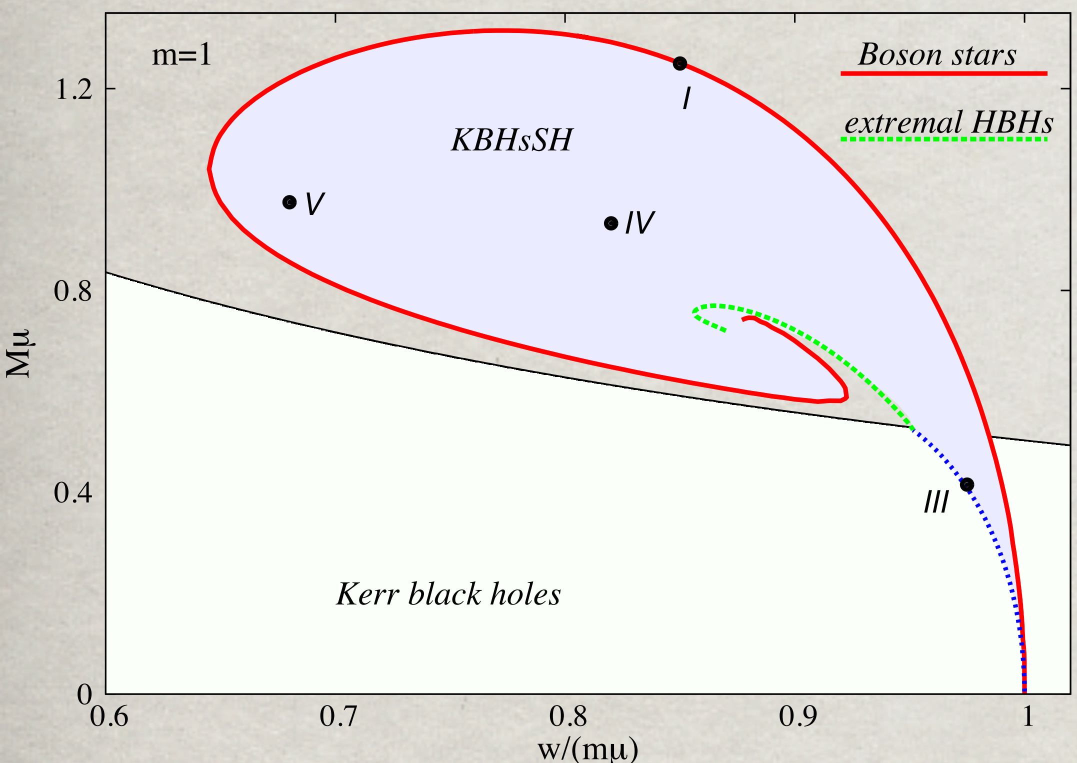
Existence proof

Chodosh and Shlapentokh-Rothman, arXiv: 1510.08025

Domain of existence:



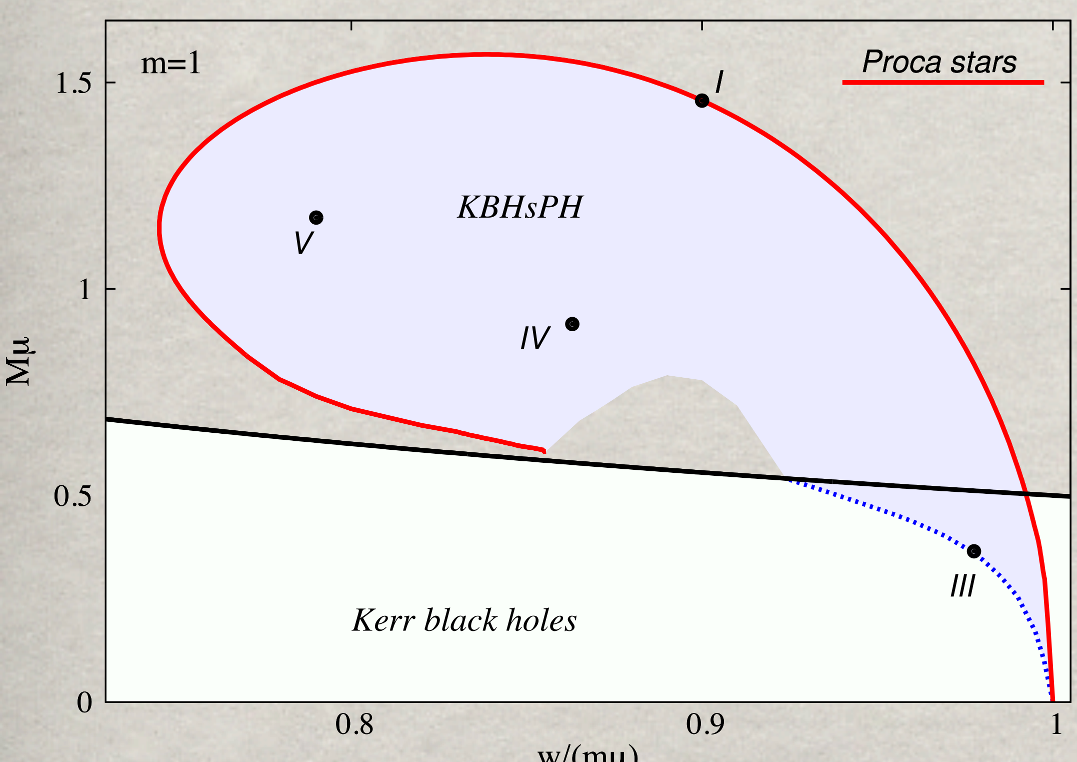
Domain of existence:



Scalar	w/μ	r_H/μ	μM_{ADM}	$\mu^2 J_{ADM}$	μM_H	$\mu^2 J_H$	$\mu M^{(\Psi)}$	$\mu^2 J^{(\Psi)}$
I - Scalar boson star	0.85	0	1.25	1.30	0	0	1.25	1.30
II - Vacuum Kerr	1.1112	0.0663	0.415	0.172	0.415	0.172	0	0
III - KBHSH	0.975	0.2	0.415	0.172	0.393	0.150	0.022	0.022
IV - KBHSH	0.82	0.1	0.933	0.739	0.234	0.114	0.699	0.625
V - KBHSH	0.68	0.04	0.975	0.850	0.018	0.002	0.957	0.848

Data avaliable online at:

http://gravitation.web.ua.pt



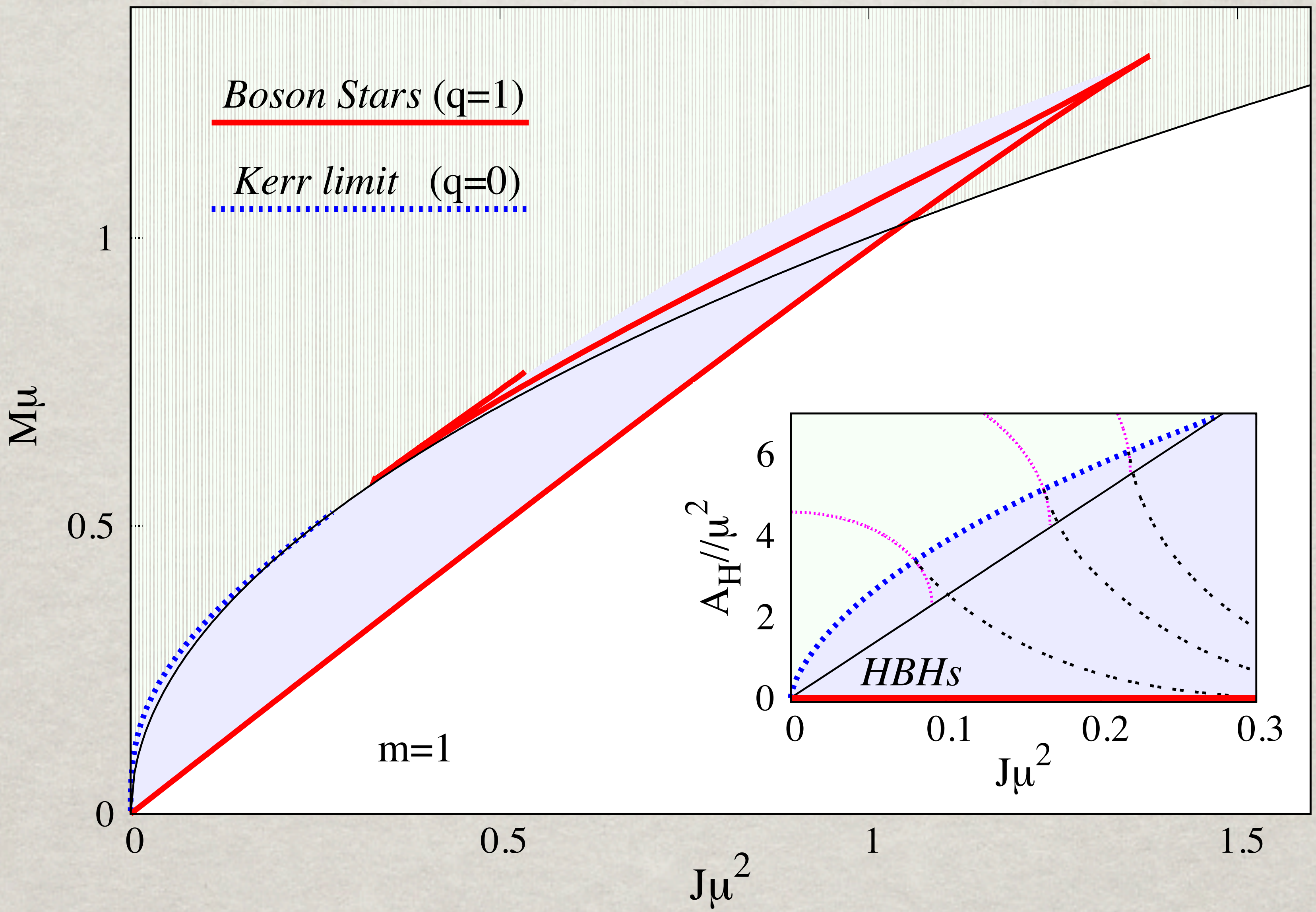
Vector	w/μ	r_H/μ	μM_{ADM}	$\mu^2 J_{ADM}$	μM_H	$\mu^2 J_H$	$\mu M^{(\mathcal{P})}$	$\mu^2 J^{(\mathcal{P})}$
I - Proca star	0.9	0	1.456	1.45	0	0	1.456	1.45
II - Vacuum Kerr	1.0432	0.1945	0.365	0.128	0.365	0.128	0	0
III - KBHPH	0.9775	0.2475	0.365	0.128	0.354	0.117	0.011	0.011
IV - KBHPH	0.863	0.09	0.915	0.732	0.164	0.070	0.751	0.662
V - KBHPH	0.79	0.06	1.173	1.079	0.035	0.006	1.138	1.073

2) Phenomenology:

(the little exotic and the very exotic)

There is a region of non uniqueness

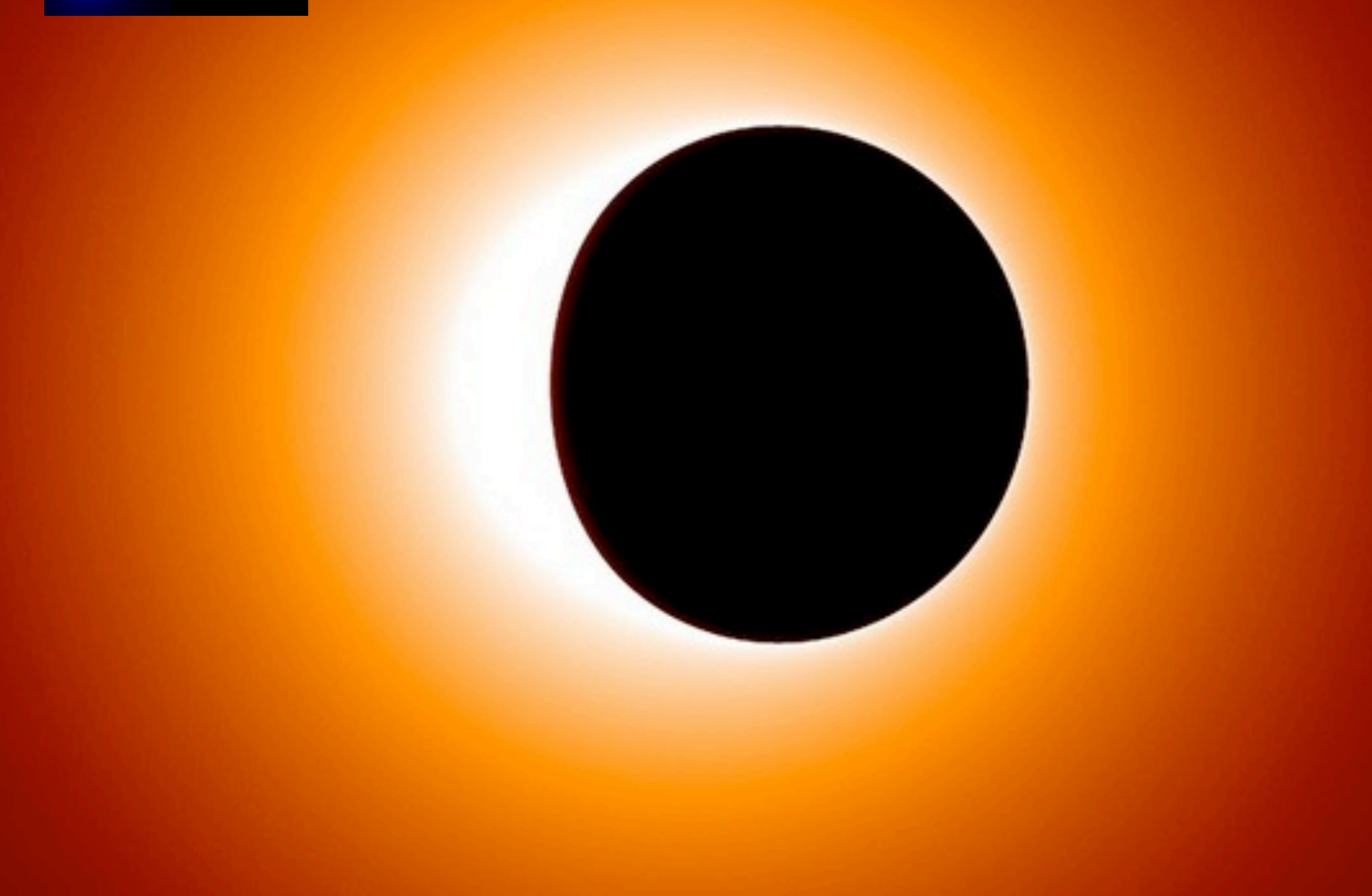
(different solutions for same M,J); but degeneracy raised with q



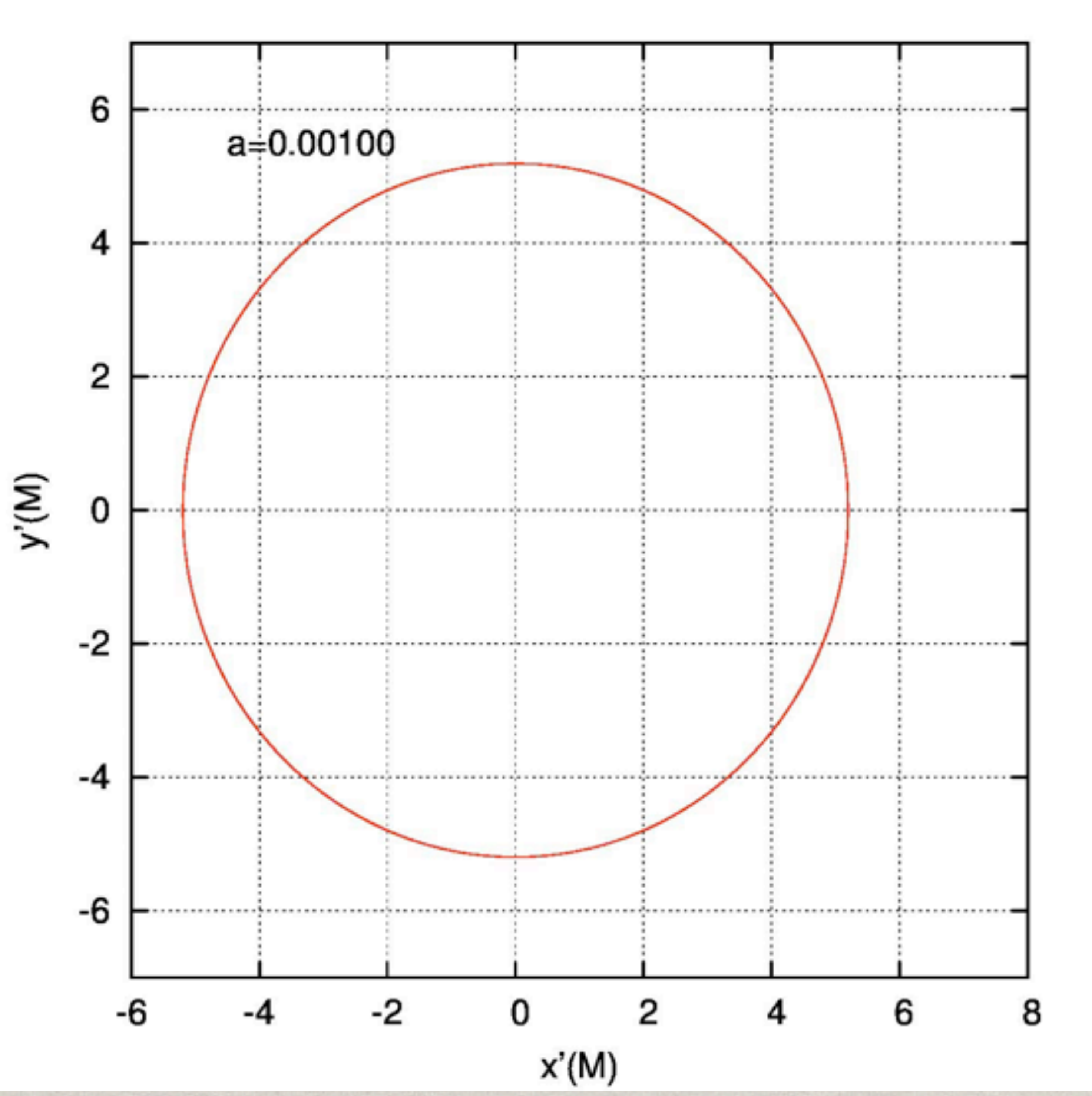
In this region, hairy black holes are entropically favoured

Can we distinguish by a local measurement degenerate configurations?

Black hole shadows



Shadow of a Kerr black hole:
(equatorial plane observation)



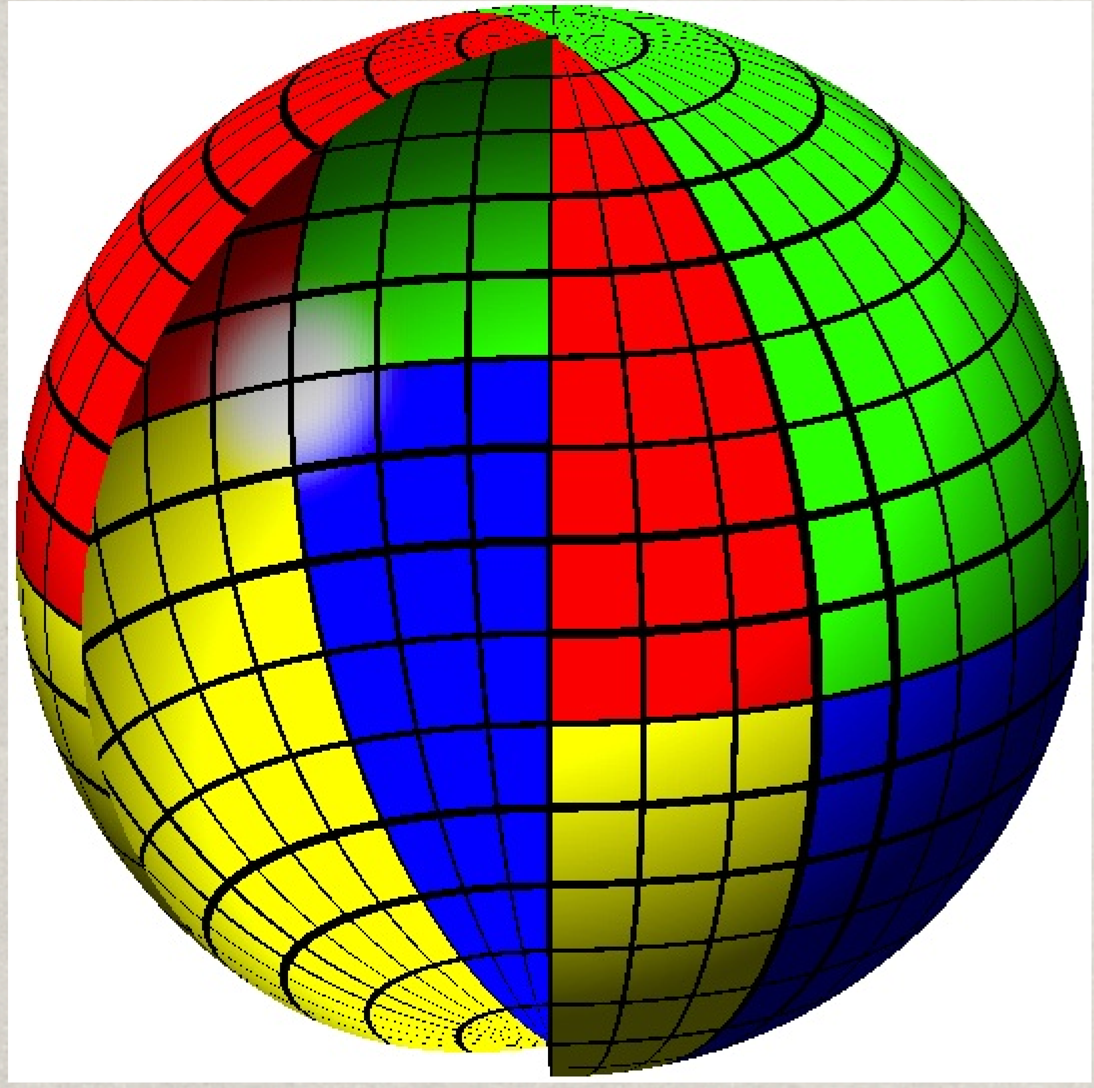
Cunha, M.Sc. Thesis

Technique: backwards ray-tracing

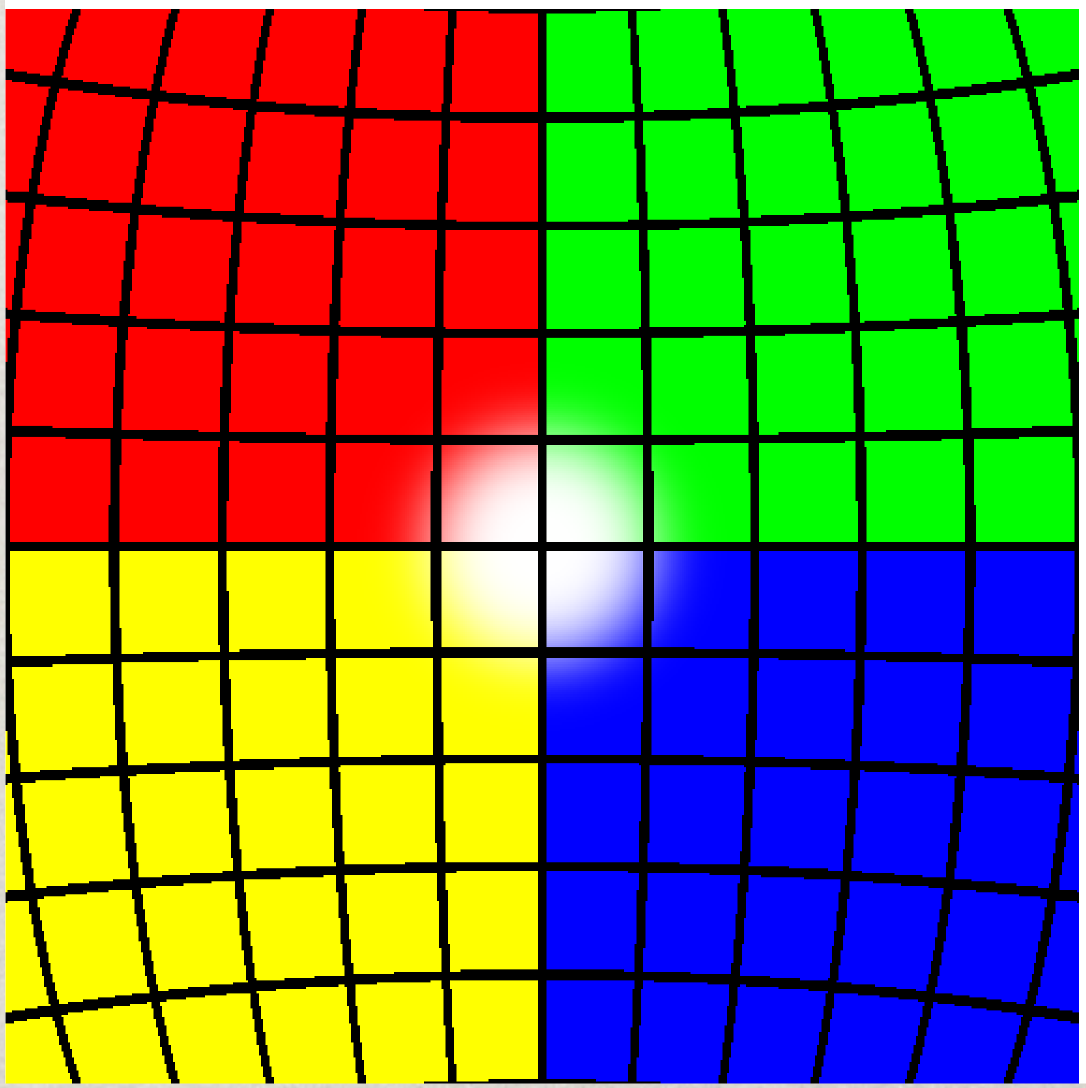
Cunha, M.Sc. Thesis

We have performed ray tracing to compute lensing and shadows.

Cunha et al. PRL115(2015)211102



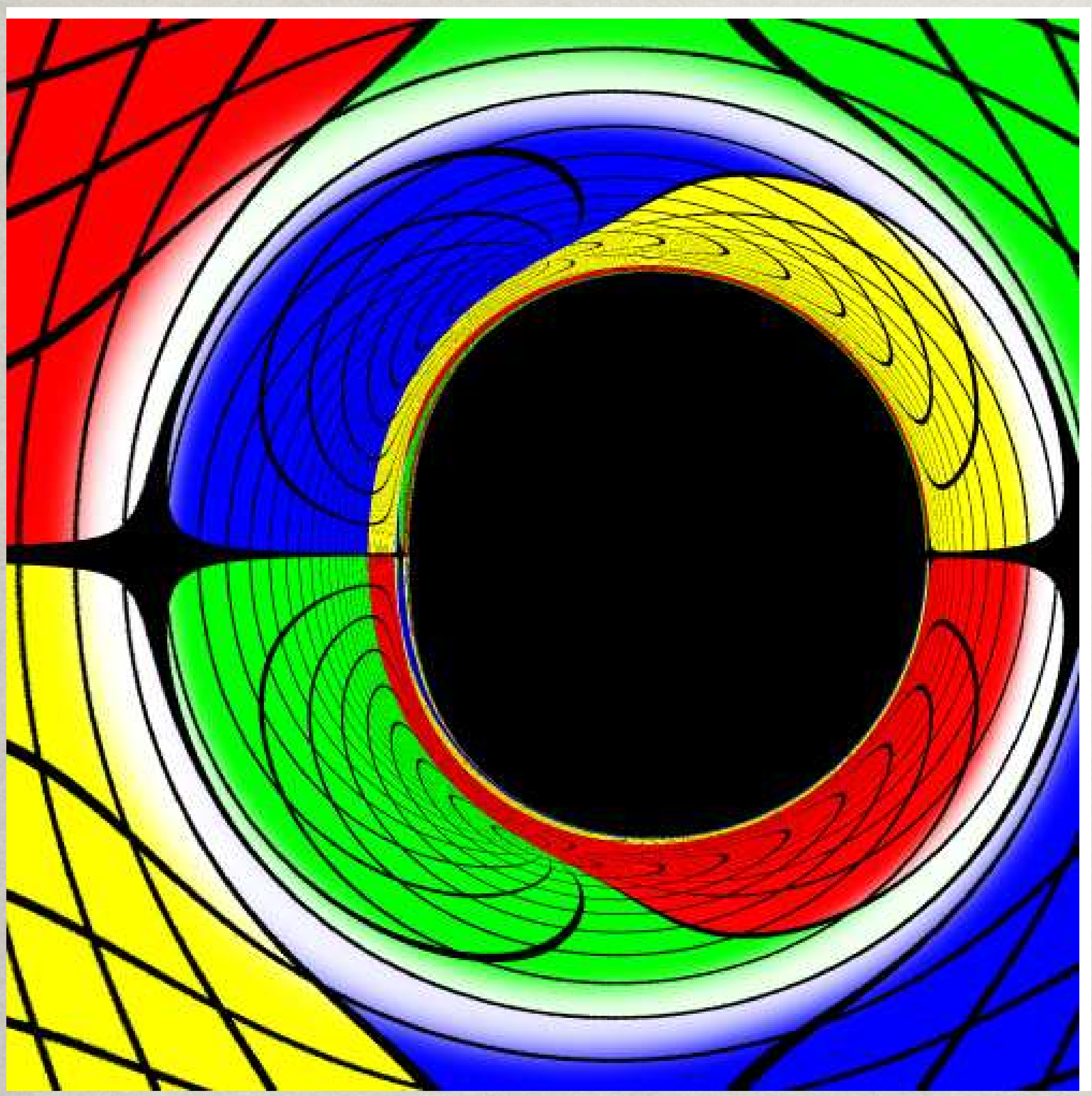
The full celestial sphere

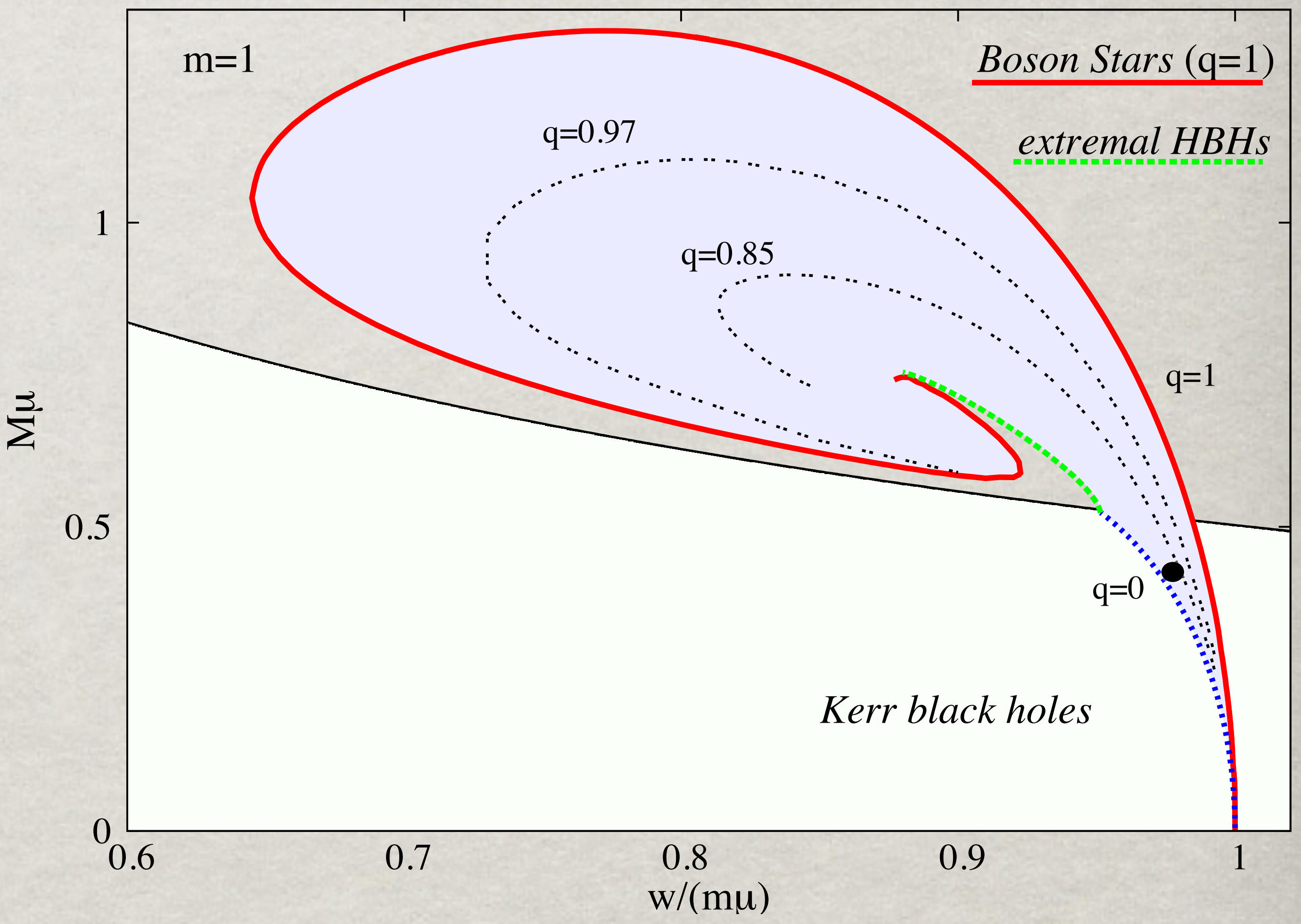


The "camera" opening angle

Following A. Bohn et al. arXiv:1410.7775

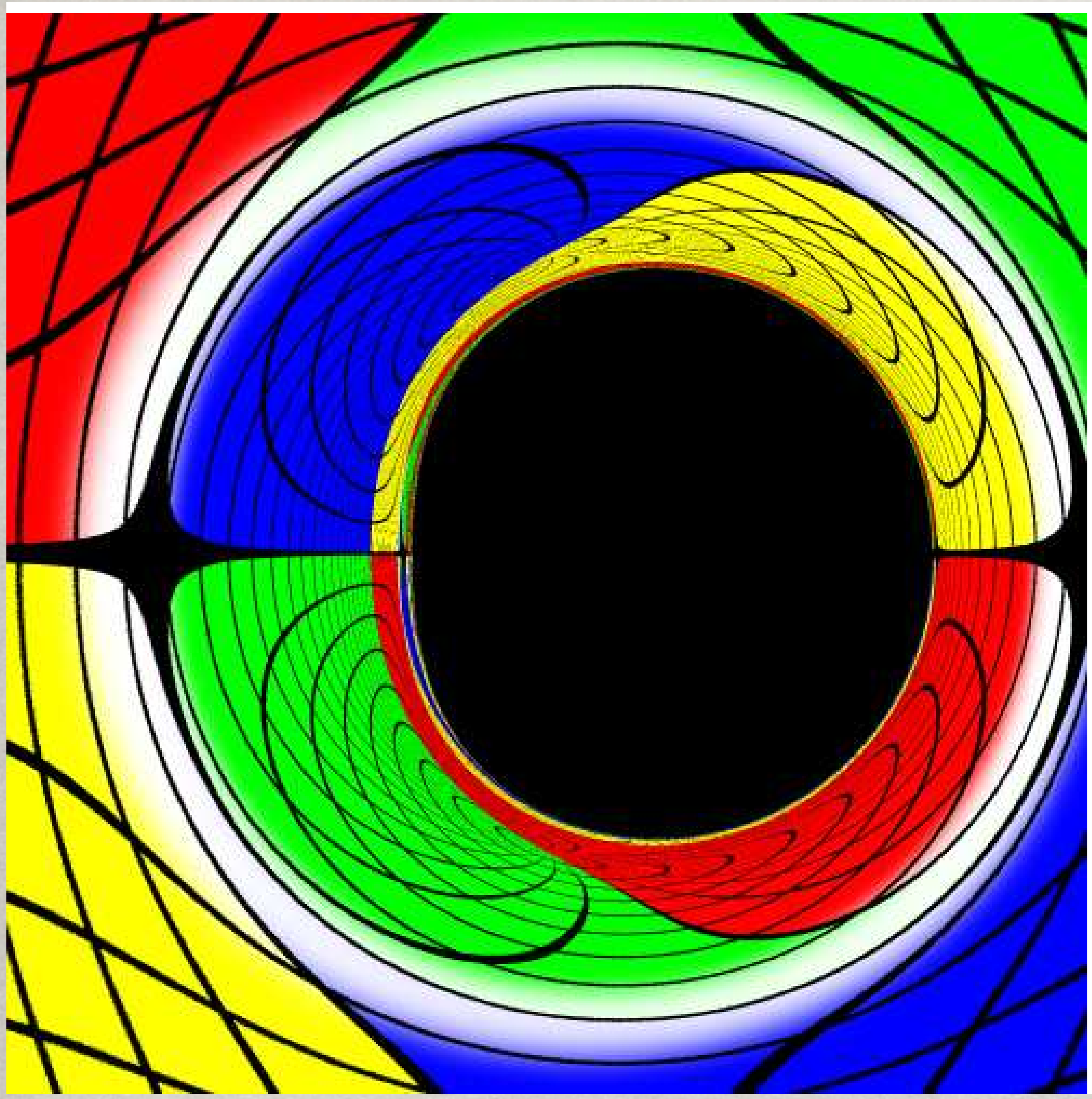
Config III - the “not so hairy” BH



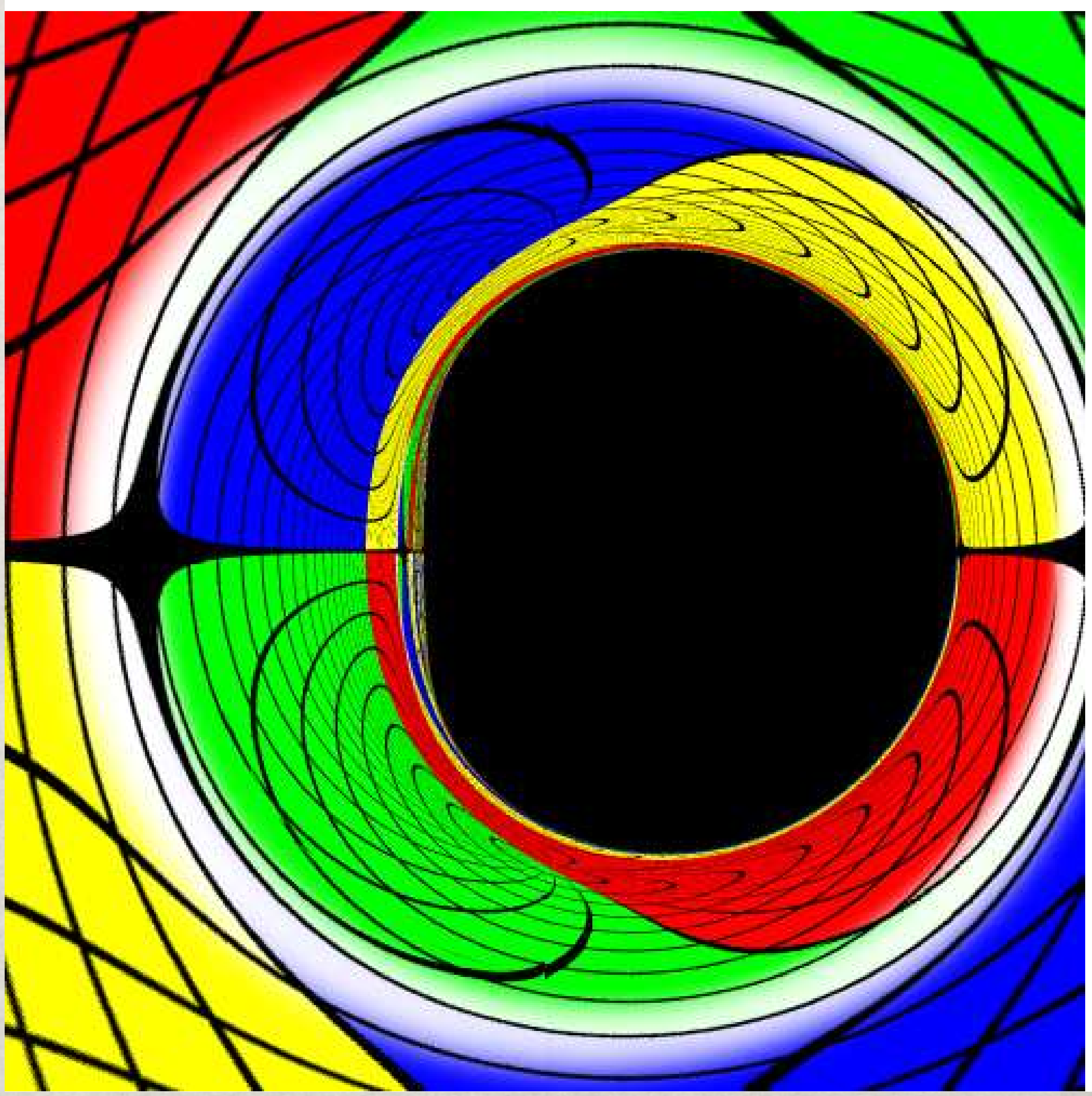


5% of mass;
13% of angular momentum
is stored in the scalar field

Config III - the “not so hairy” BH

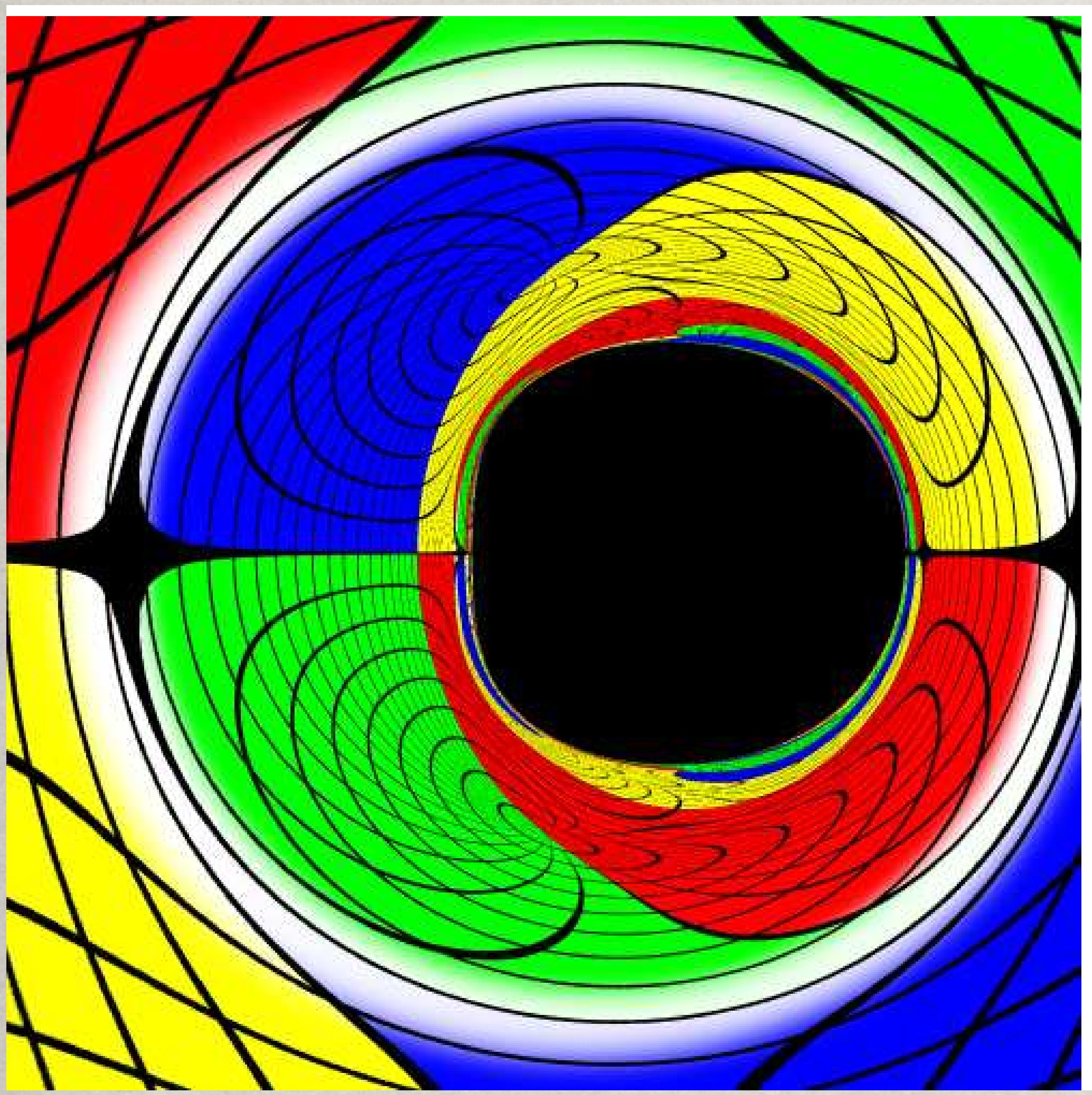


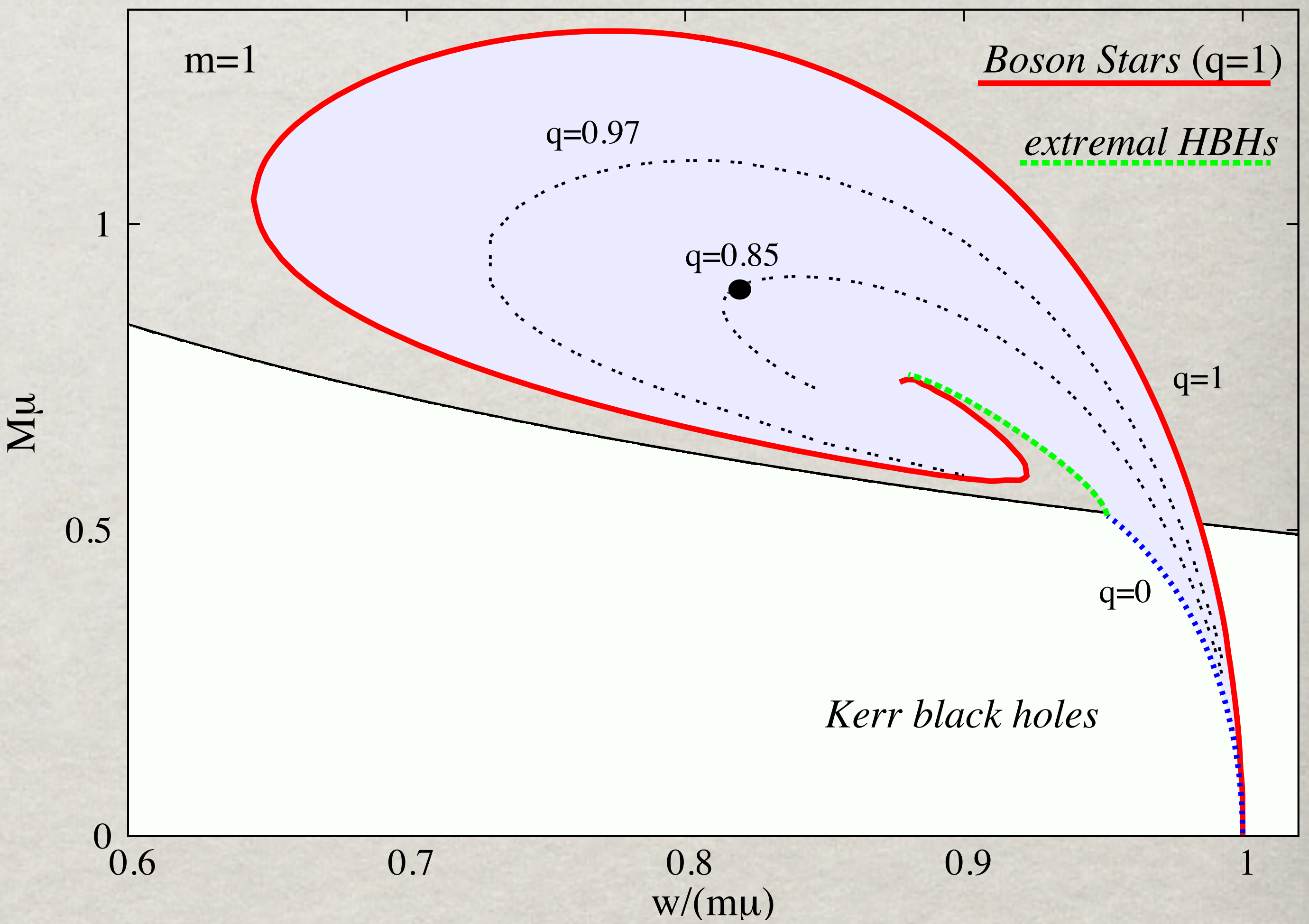
Kerr BH with scalar hair
M=0.393; J=0.15 (horizon)
M=0.022; J=0.022 (scalar field)



Vacuum Kerr BH
M=0.415; J=0.172

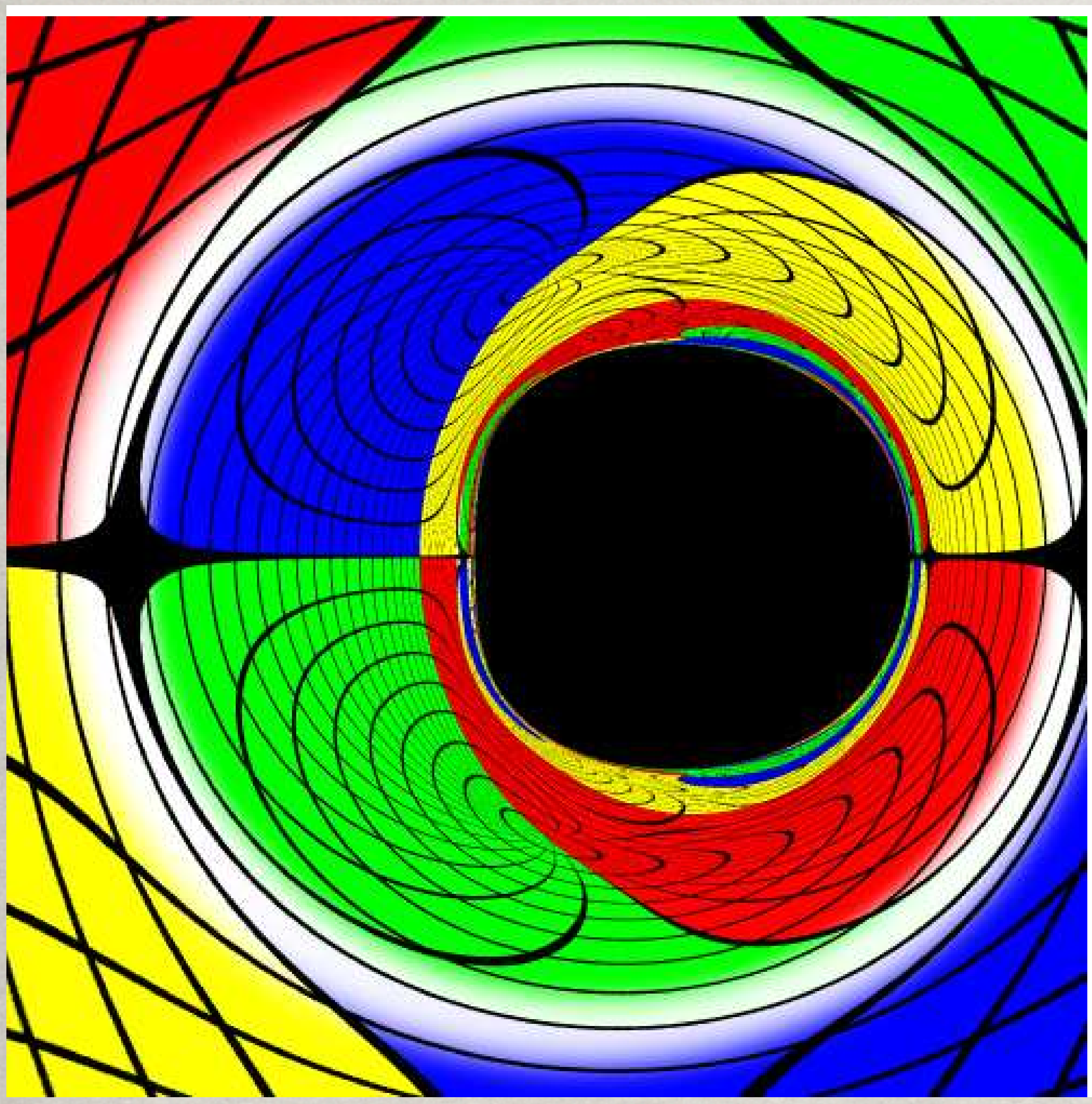
Config IV - the "very hairy" BH



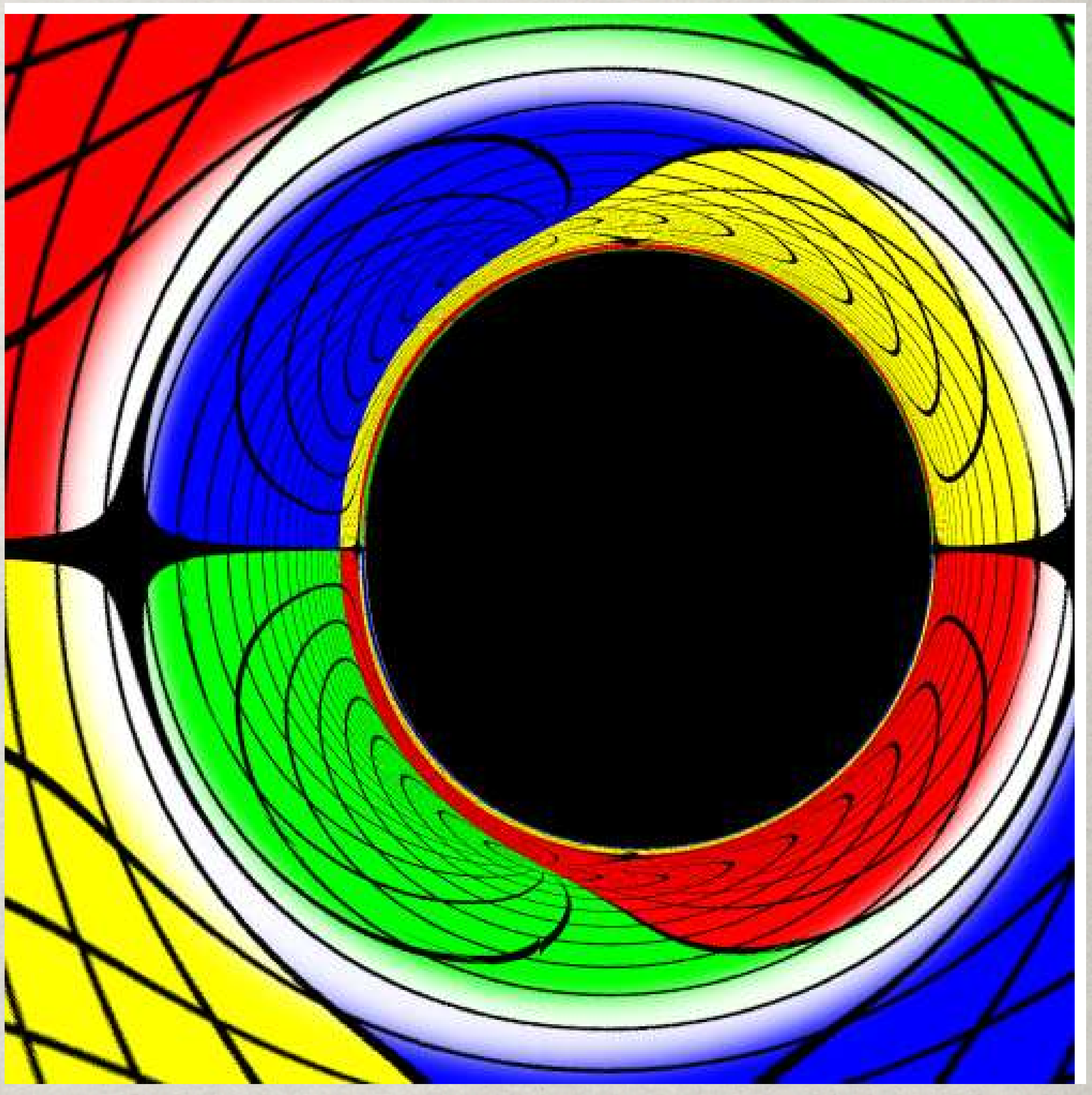


75% of mass;
85% of angular momentum
is stored in the scalar field

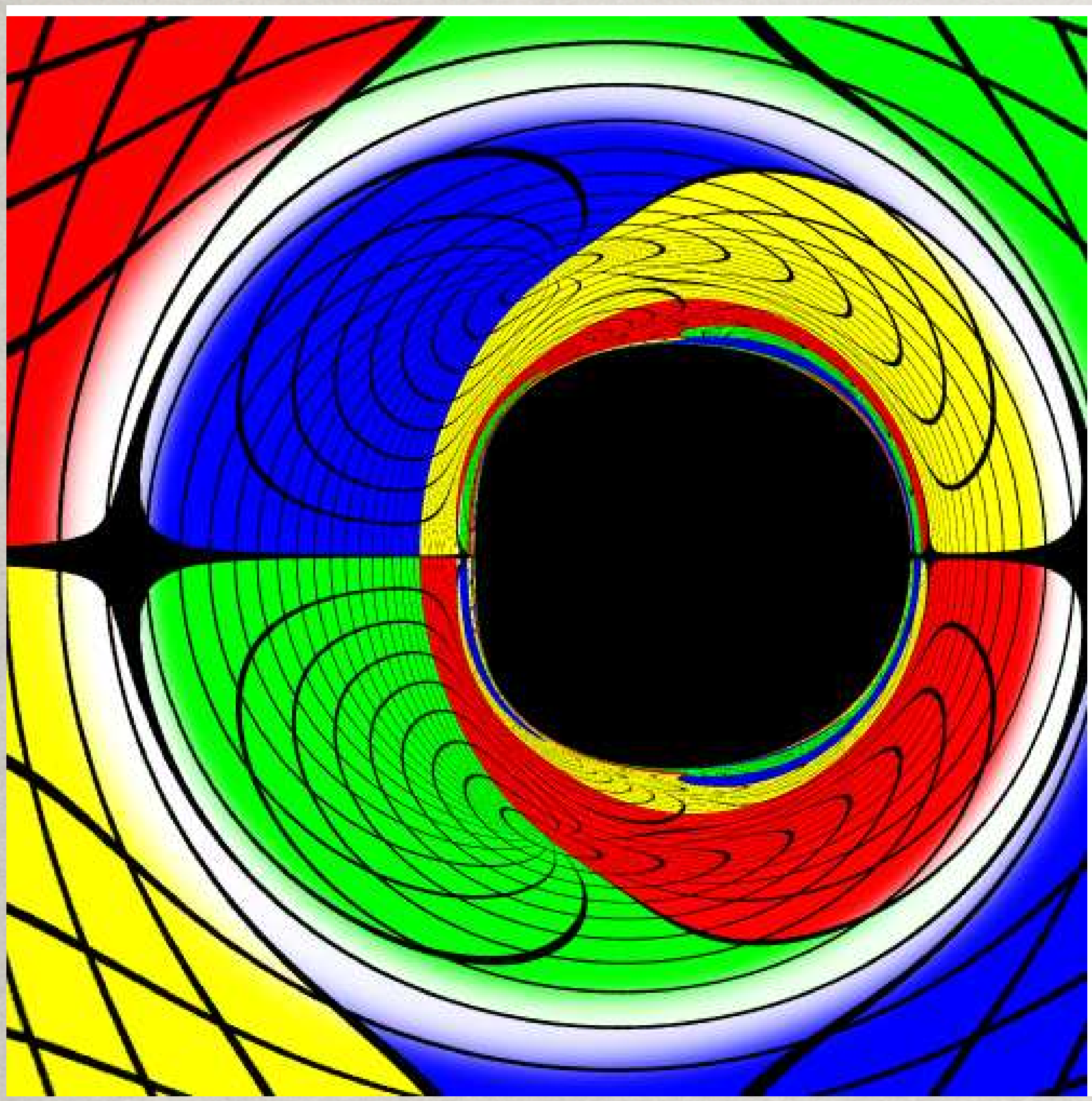
Config IV - the "very hairy" BH

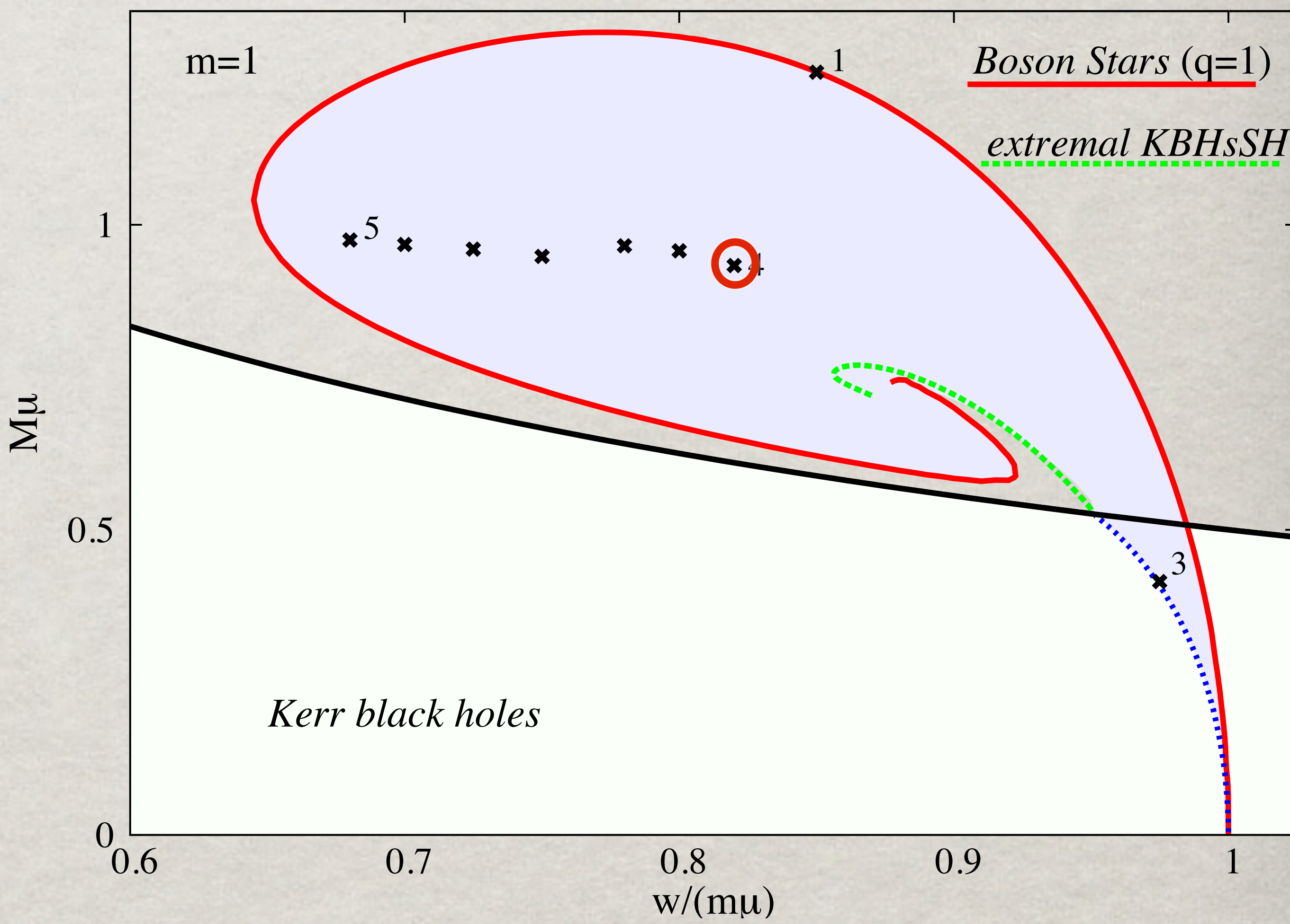


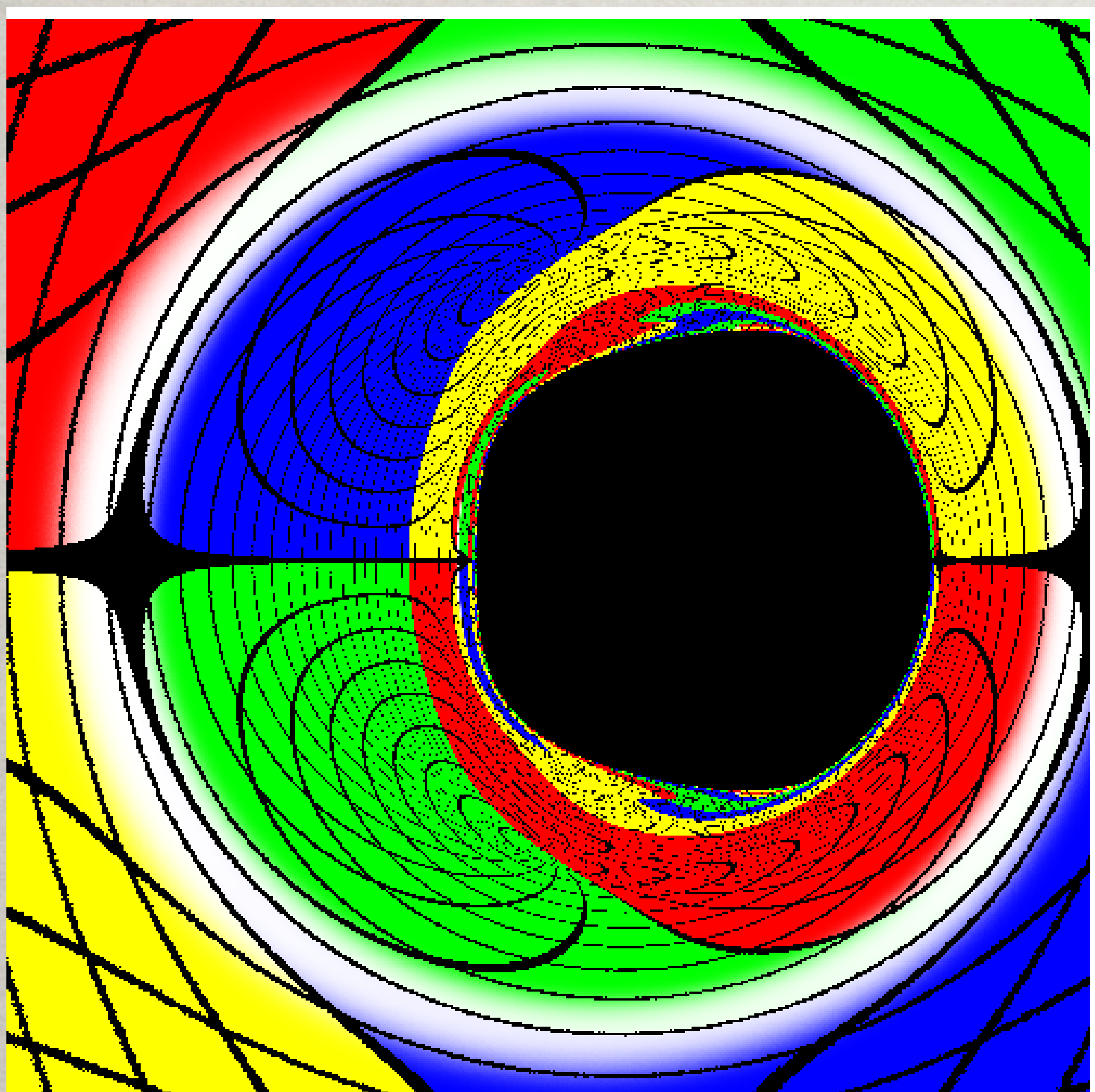
Kerr BH with scalar hair
M=0.234; J=0.114 (horizon)
M=0.699; J=0.625 (scalar field)

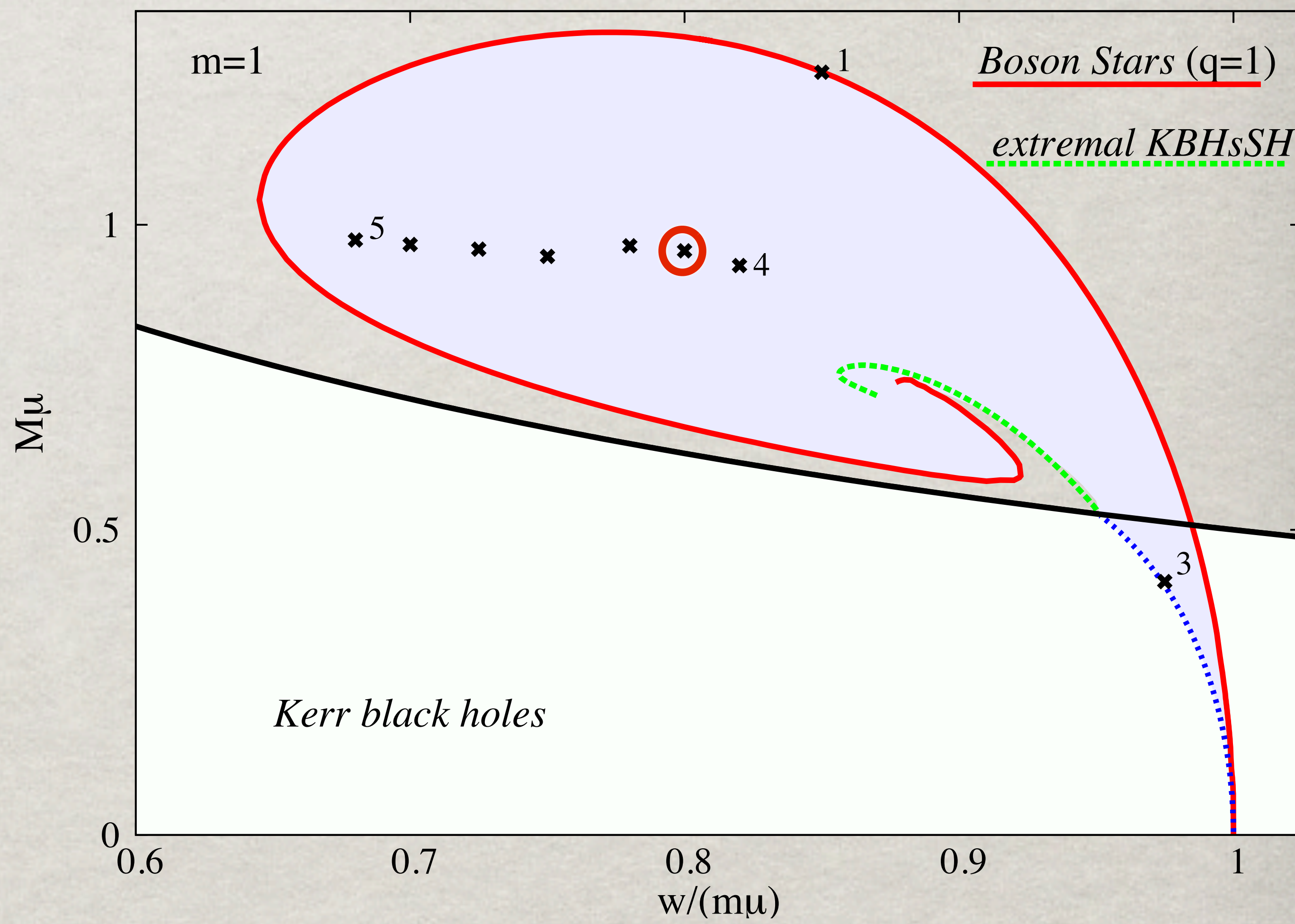


Vacuum Kerr BH
M=0.933; J=0.739

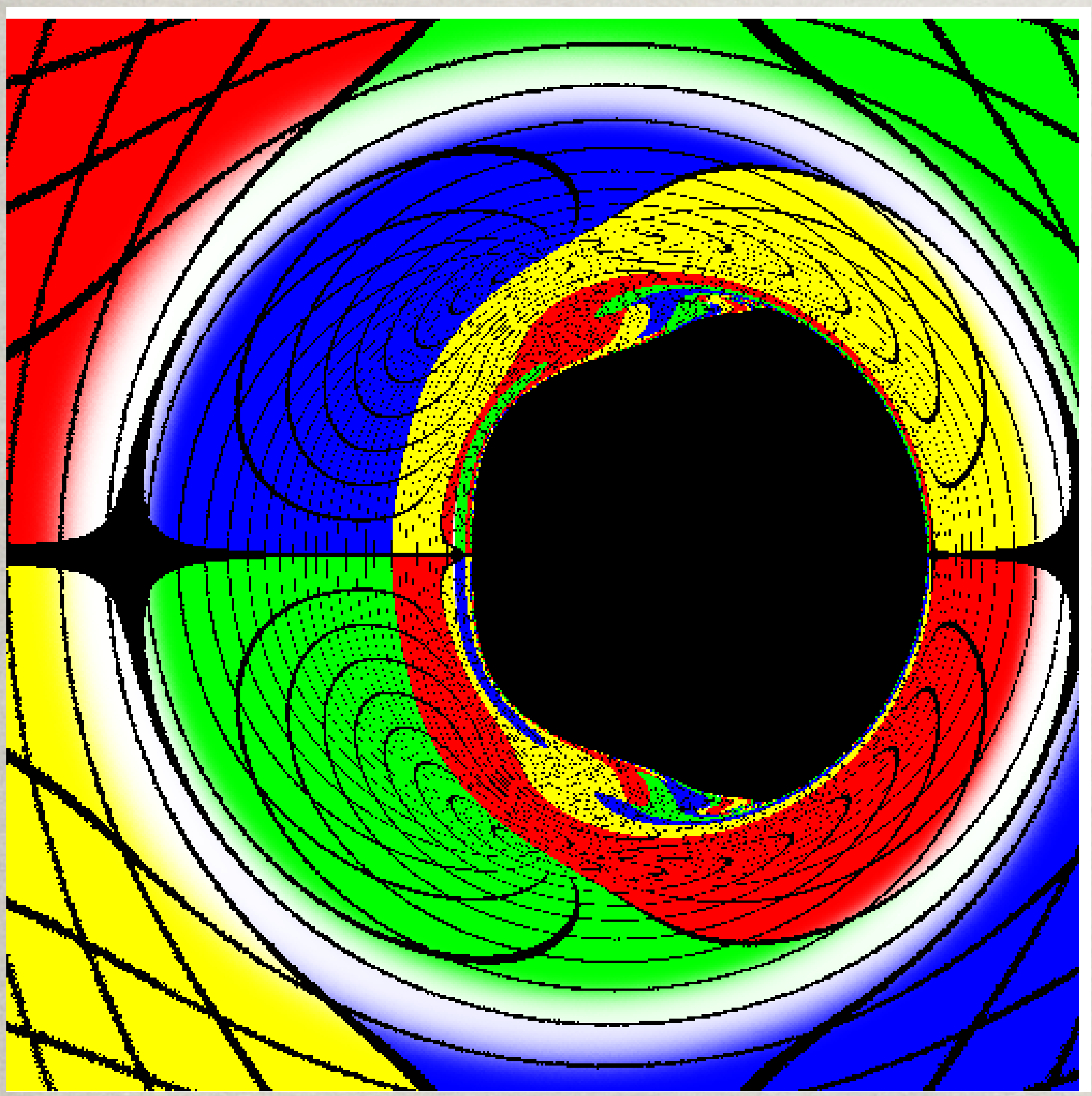


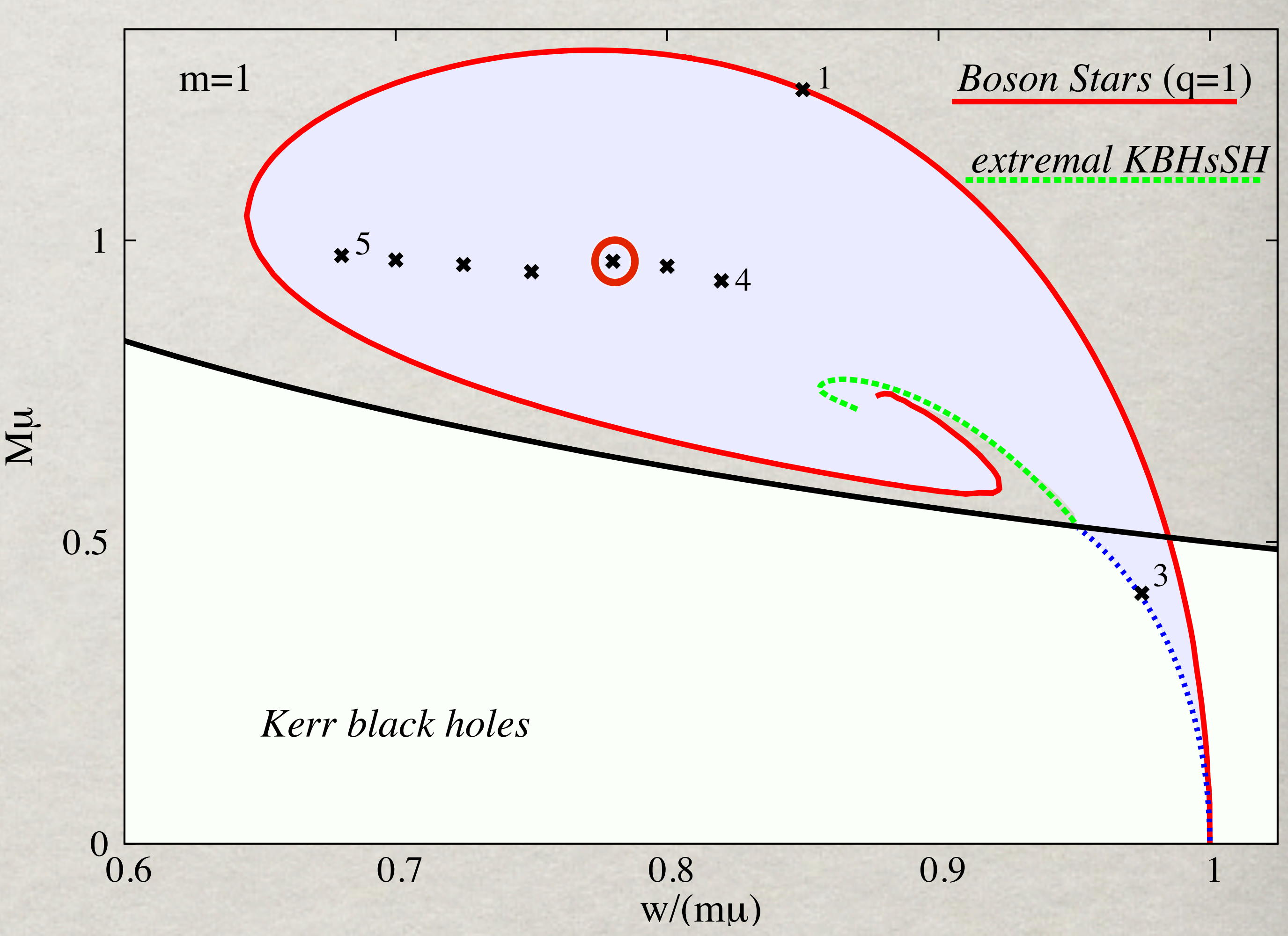

m=1
Boson Stars (q=1)
extremal KBHsSH
5
1
4
3
Kerr black holes
Mμ
w/(mμ)
0
0.5
1
0.6
0.7
0.8
0.9
1





m=1
Boson Stars (q=1)
extremal KBHsSH
Kerr black holes
Mμ
w/(mμ)
0
0.5
1
0.6
0.7
0.8
0.9
1
1
3
4
5





m=1
Boson Stars (q=1)
extremal KBHsSH
Kerr black holes
Mμ
w/(mμ)
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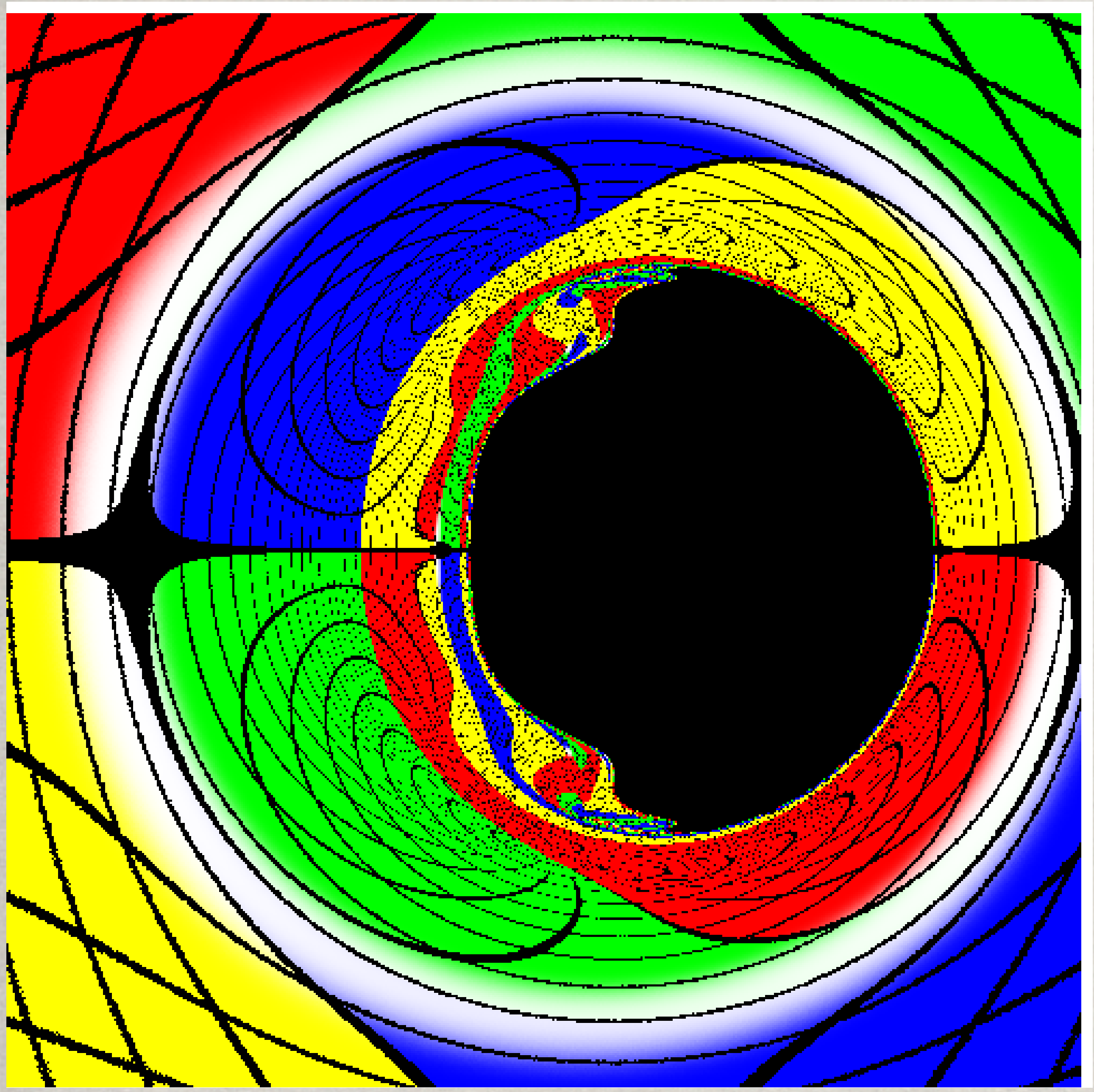
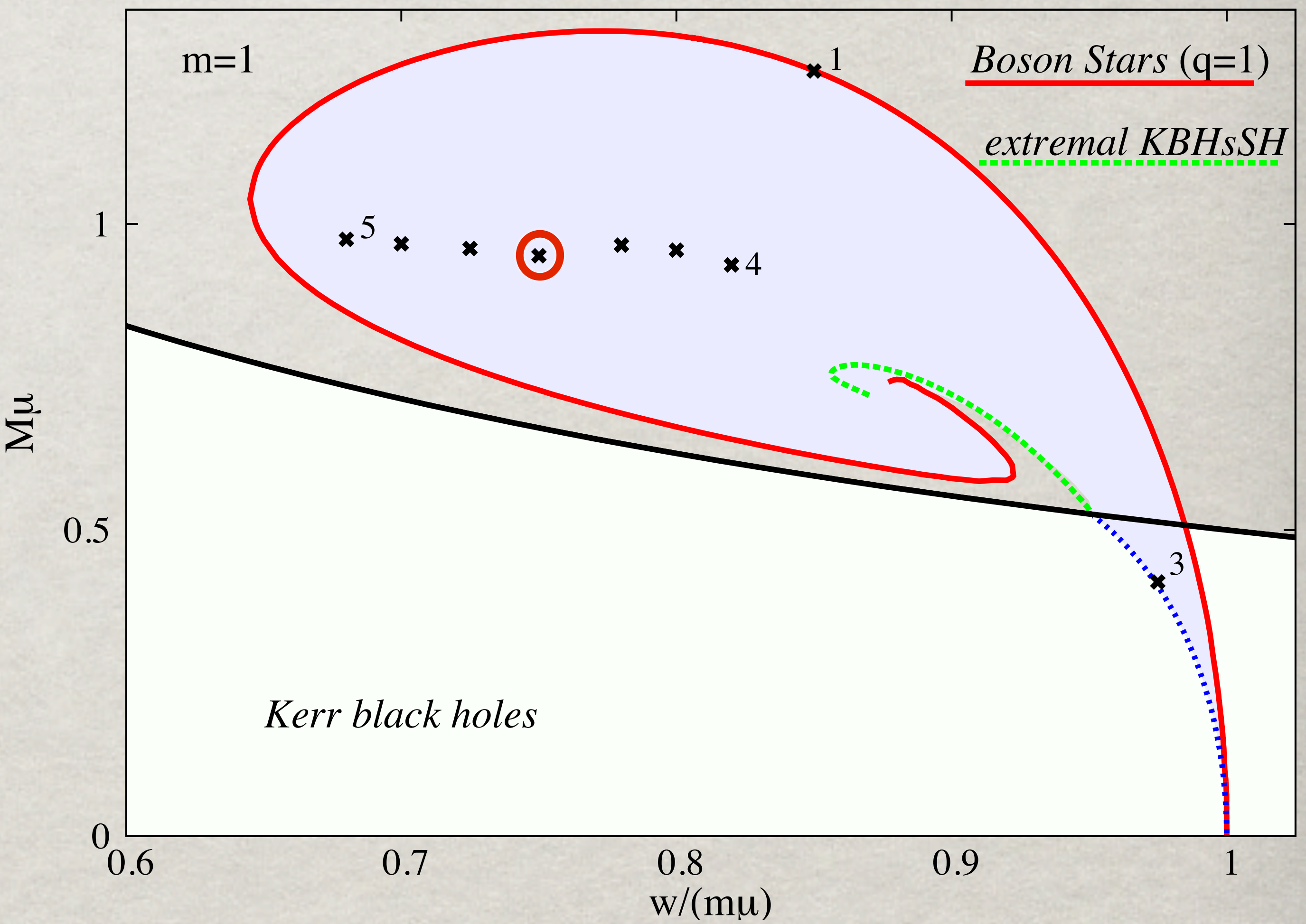
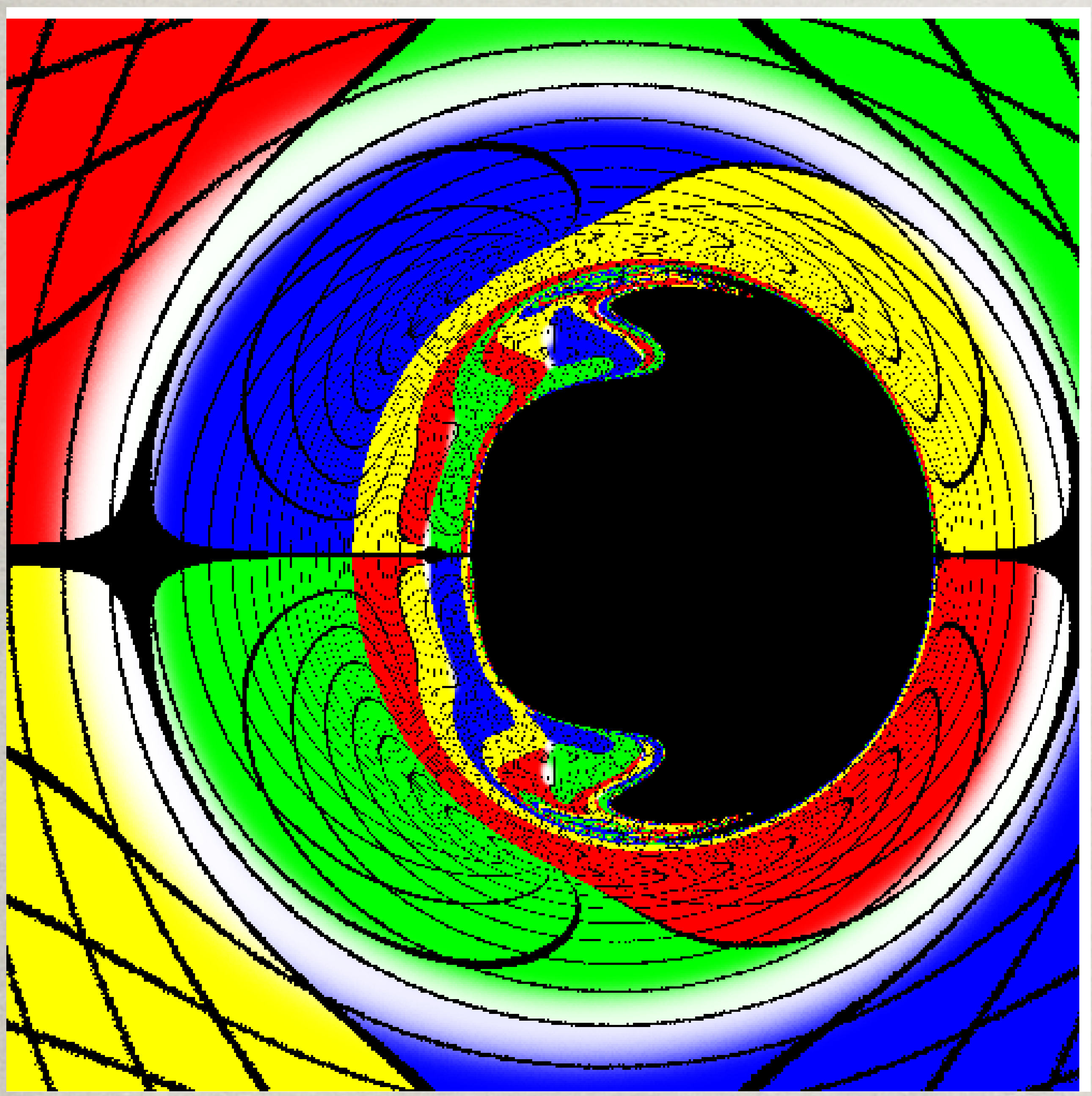
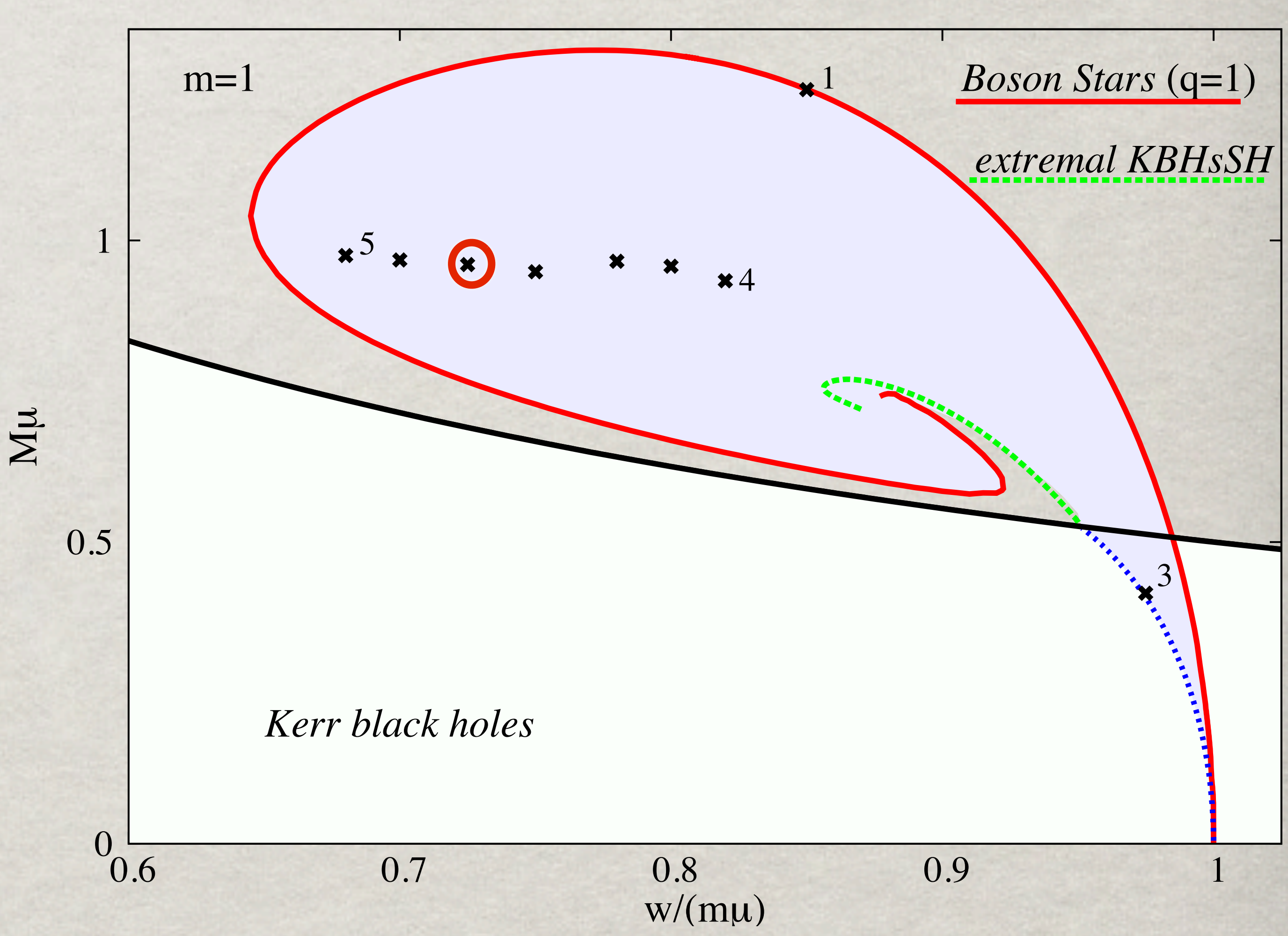


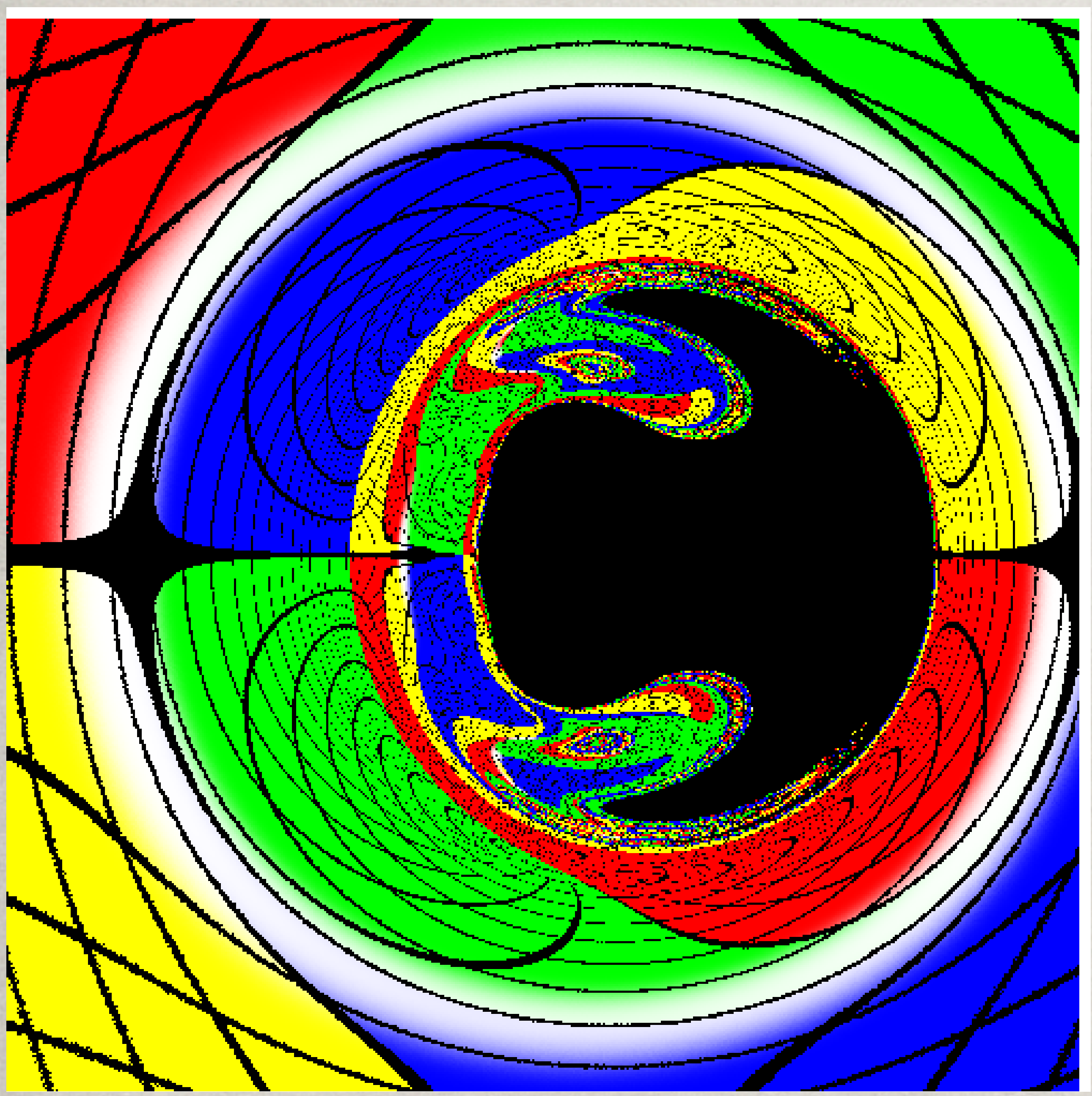


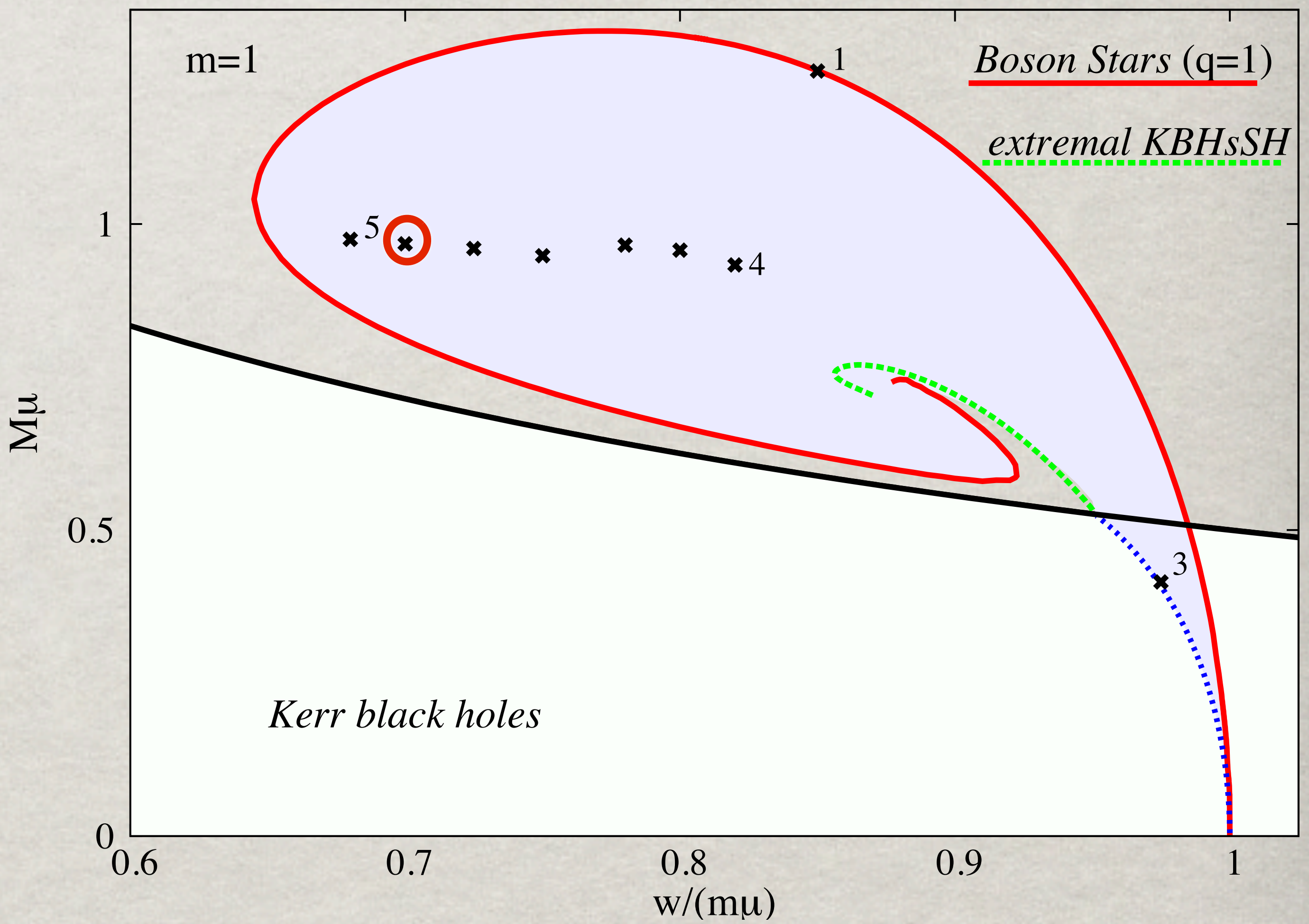
m=1
Boson Stars (q=1)
extremal KBHsSH
Kerr black holes
Mμ
w/(mμ)
1
3
4
5
0
0.5
1
0.6
0.7
0.8
0.9
1

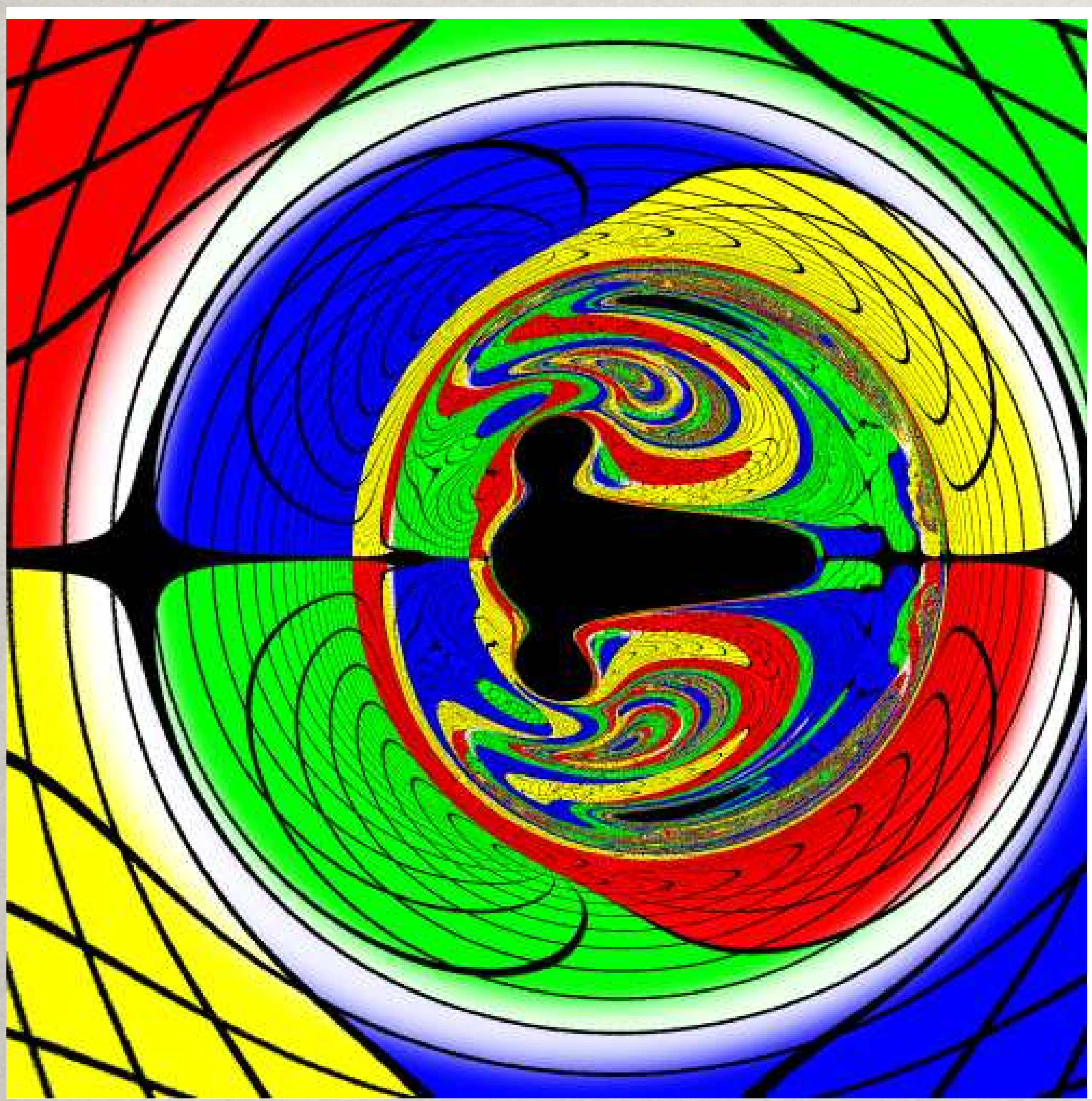


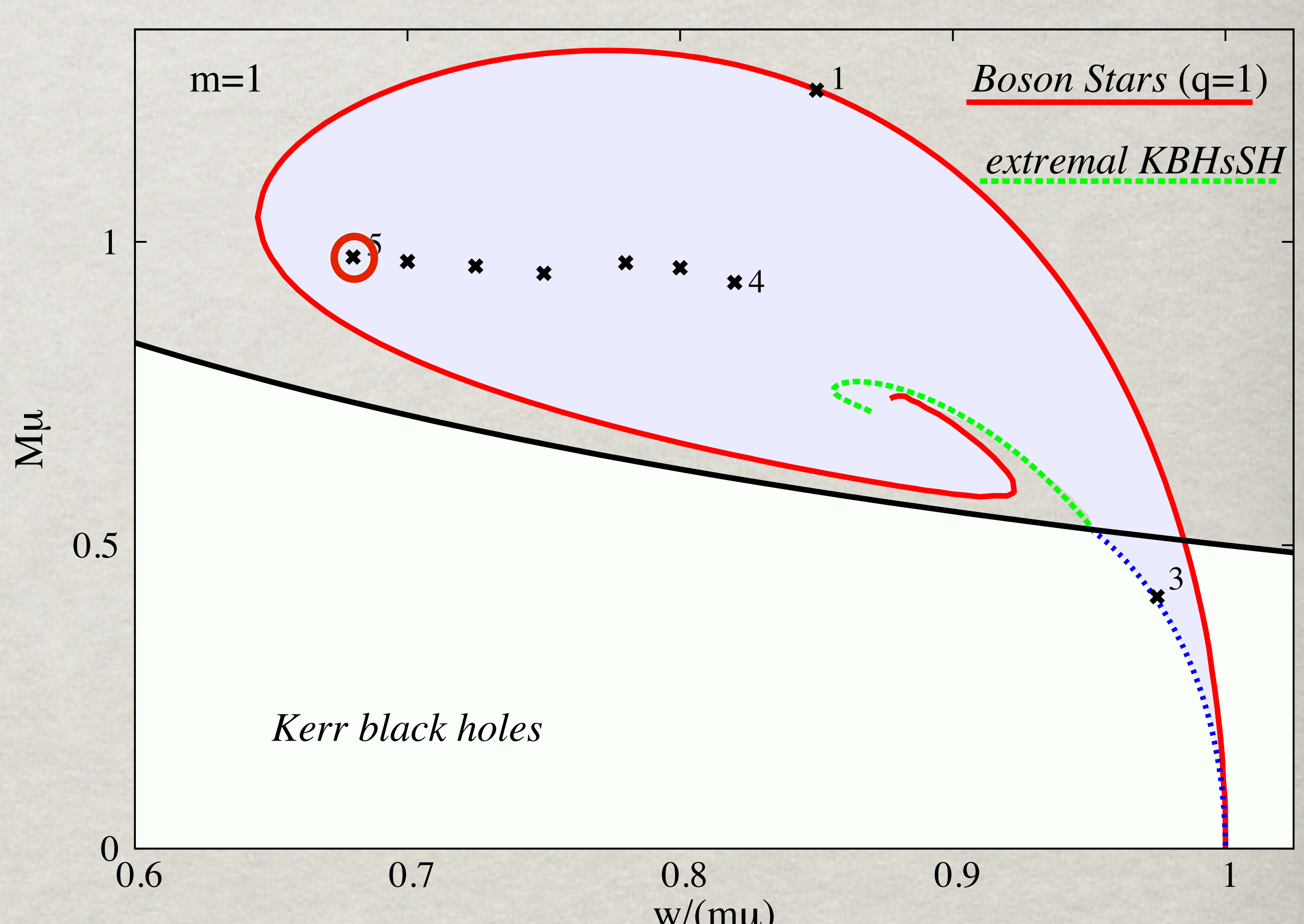


m=1
Boson Stars (q=1)
extremal KBHsSH
Kerr black holes
Mμ
w/(mμ)
1
3
4
5
0
0.5
1
0.6
0.7
0.8
0.9
1




m=1
Boson Stars (q=1)
extremal KBHsSH
Kerr black holes
1
3
4
5
Mμ
w/(mμ)
0
0.5
1
0.6
0.7
0.8
0.9
1





m=1
Boson Stars (q=1)
extremal KBHsSH
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1
3
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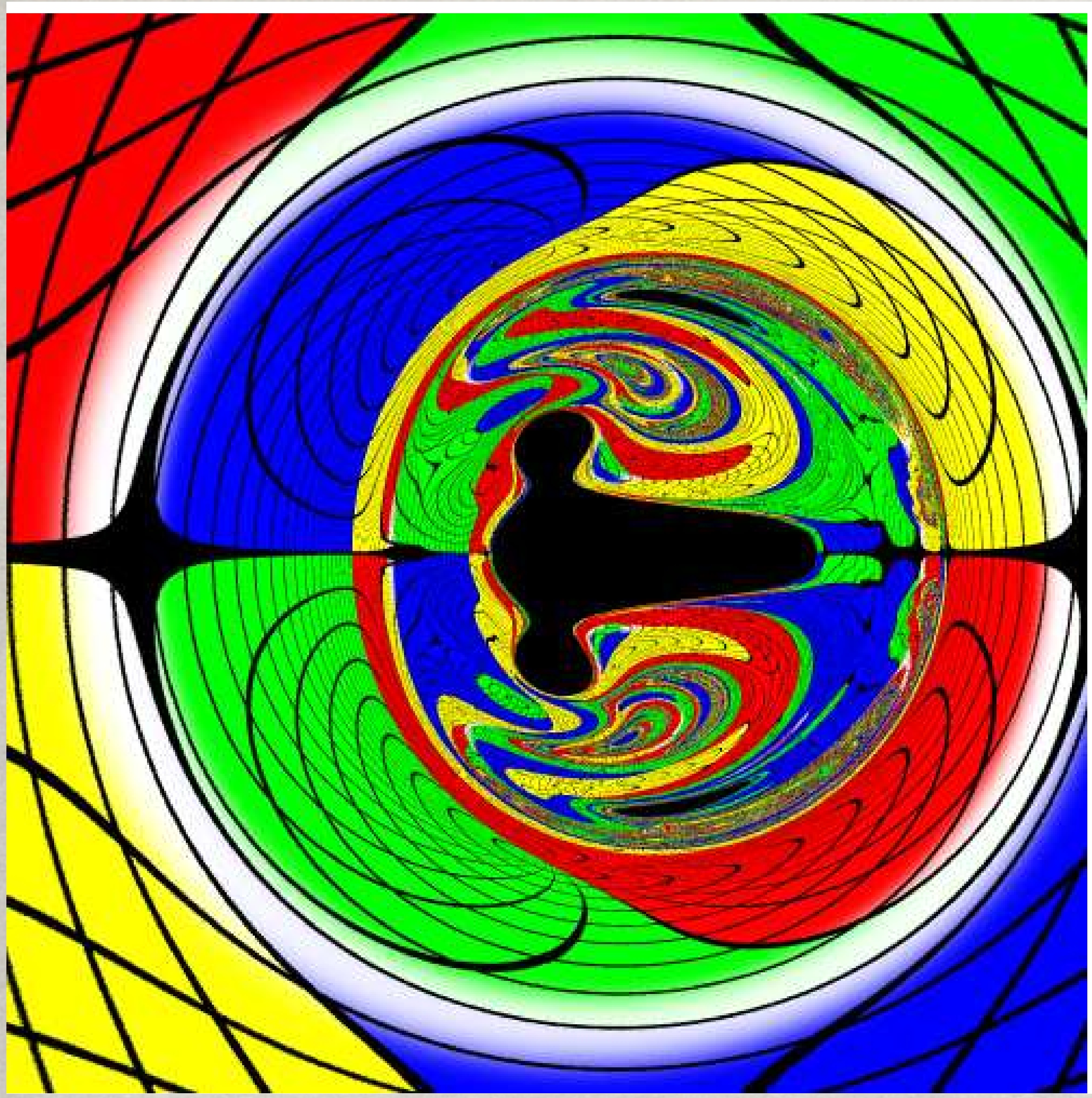
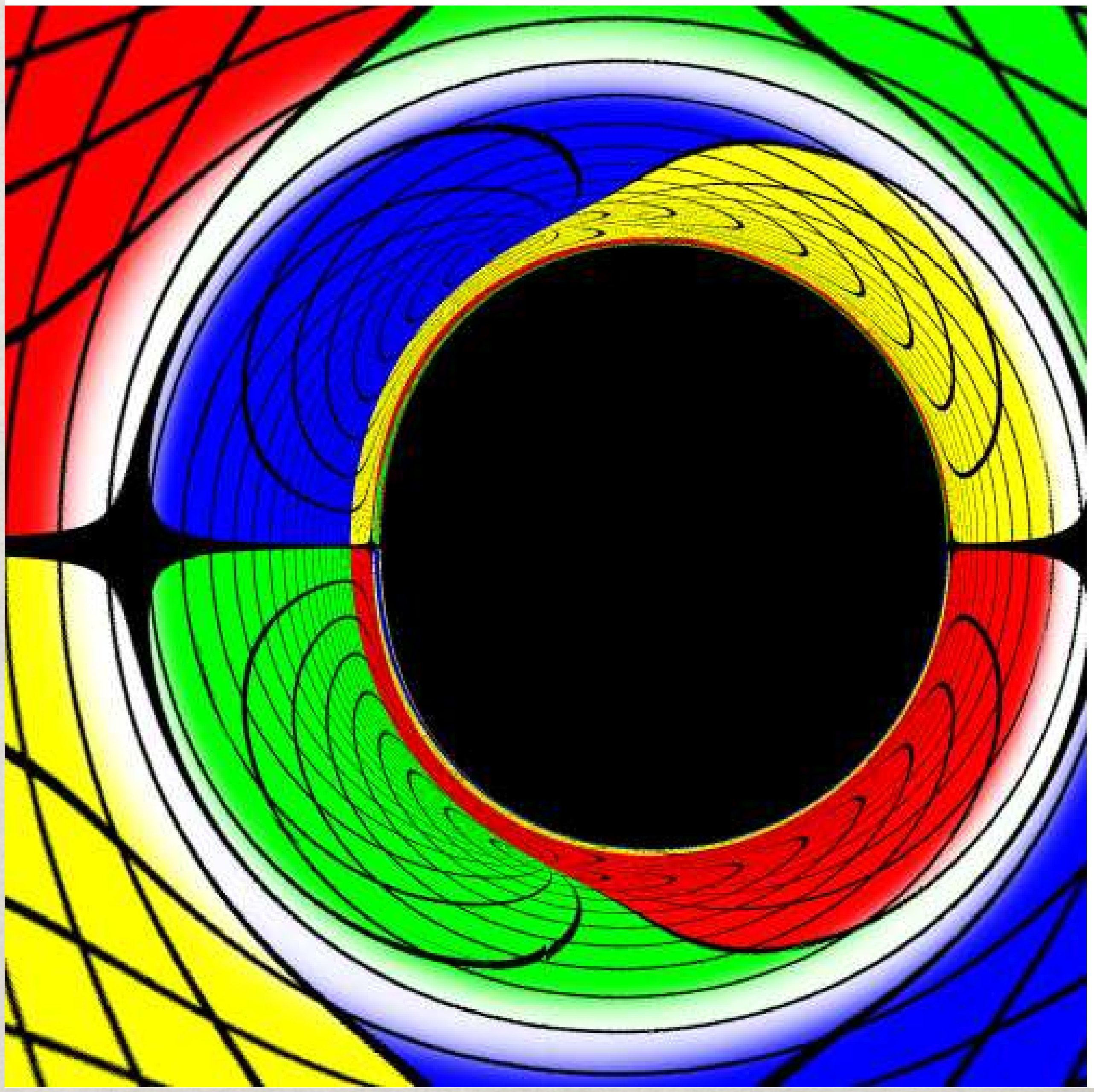
Mμ
w/(mμ)
0
0.5
1
0.6
0.7
0.8
0.9
1

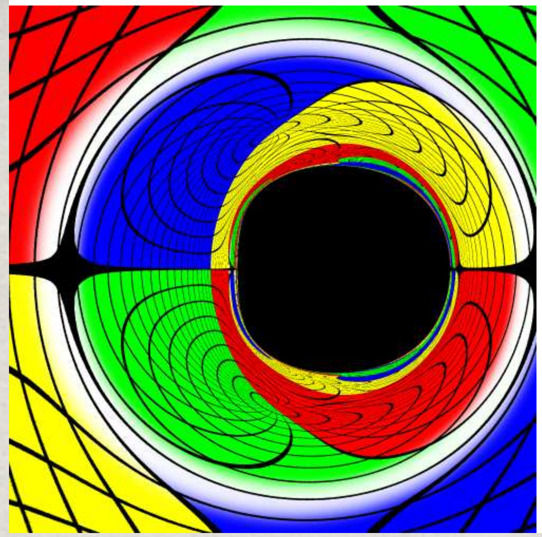
Config V - the “extremely hairy” BH

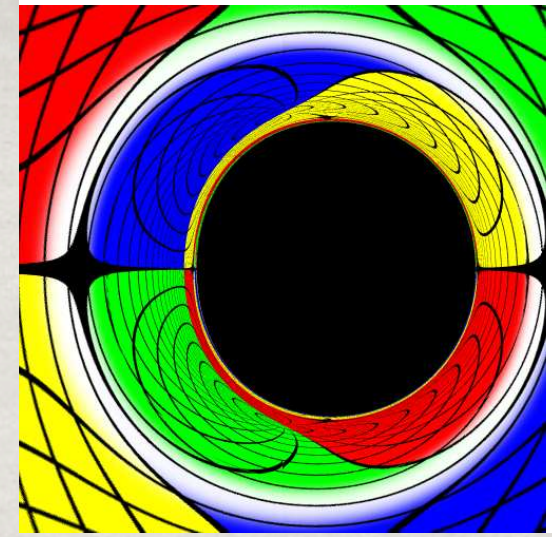


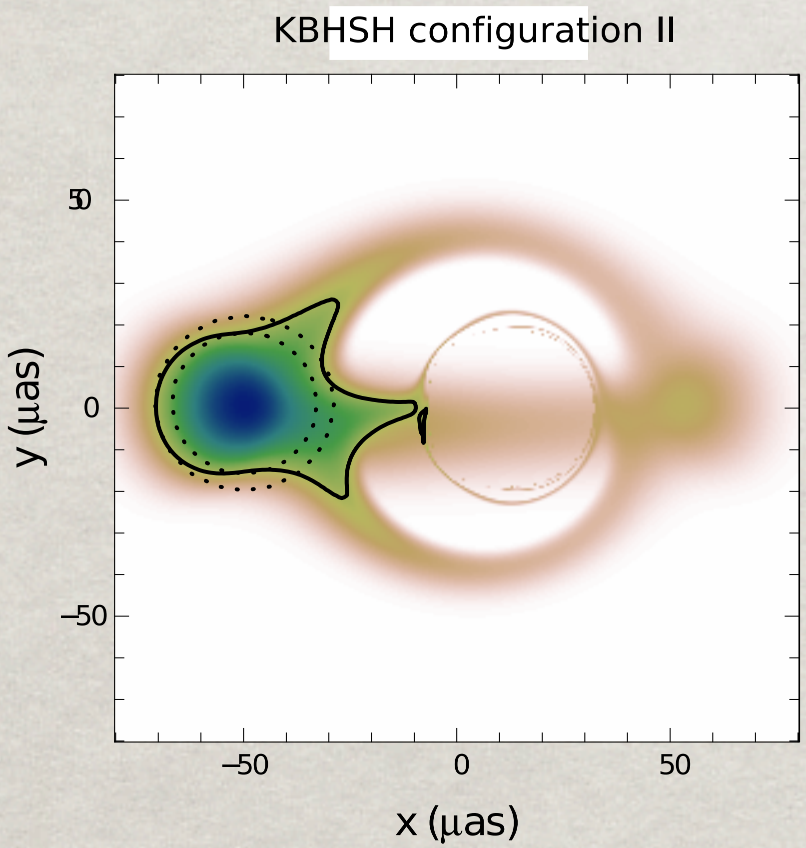


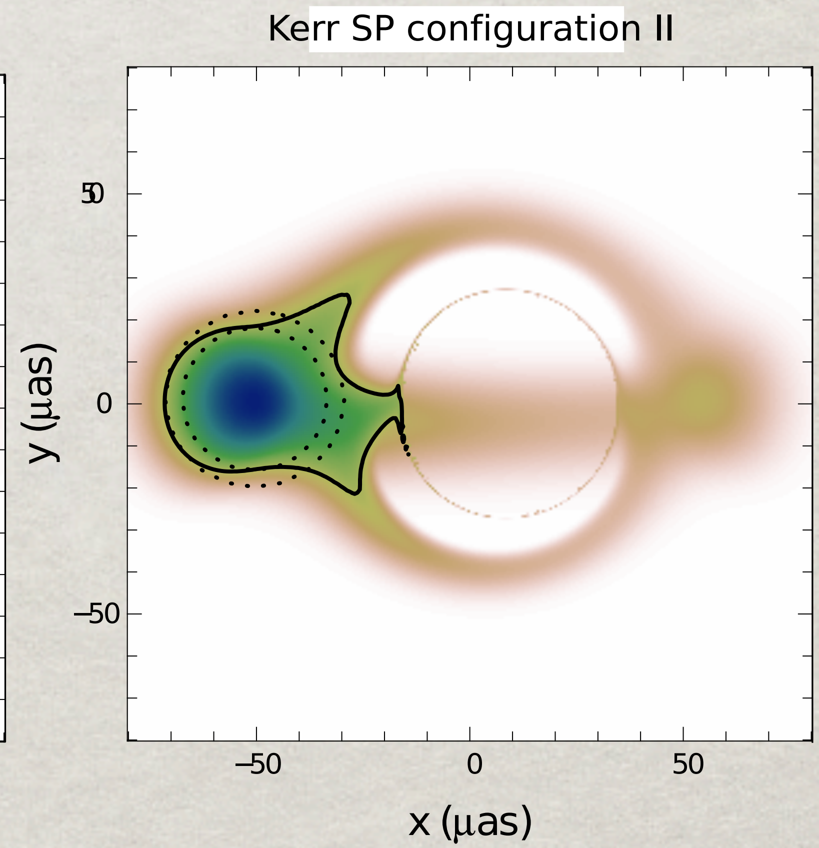
Qualitatively new feature:
multiple shadows of a single black hole

“Academic Setup”









Differences remain in an astrophysically more realistic setup

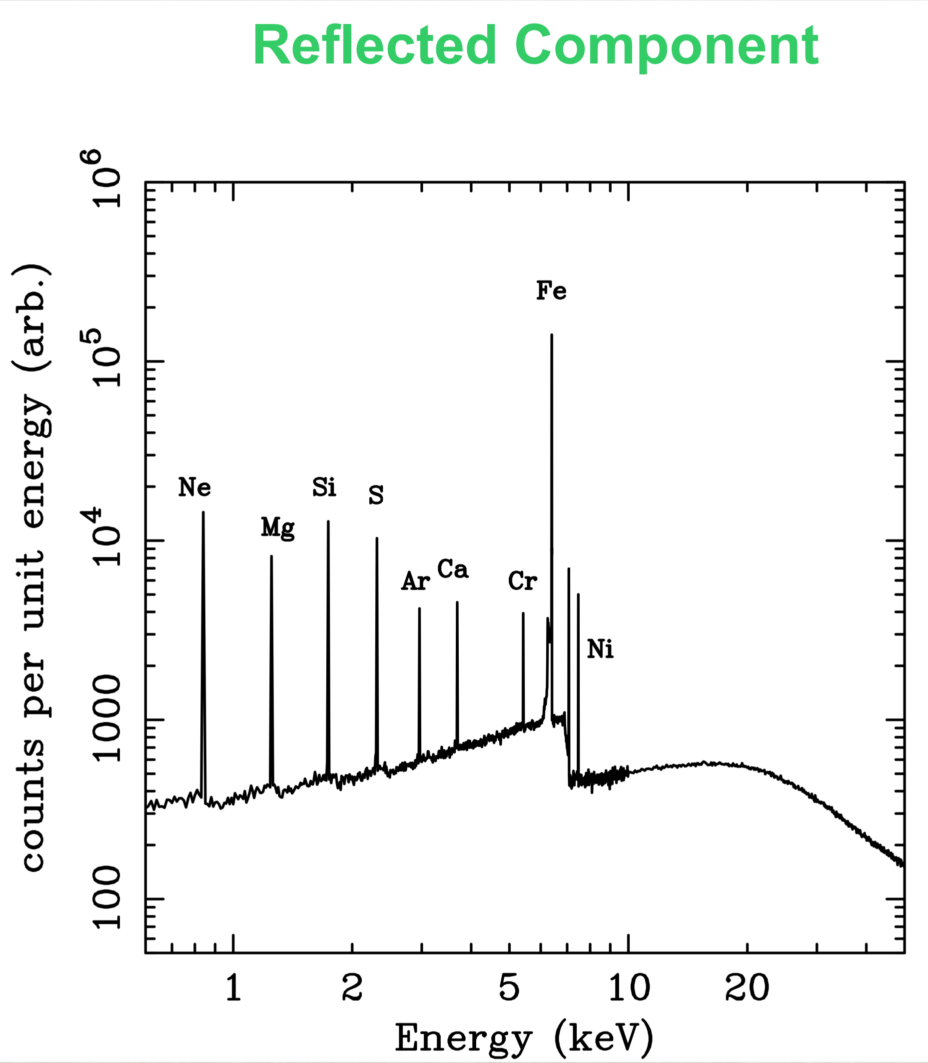
Vincent et al., PRD 94 (2016) 084045

Similar story for other observables such as the:

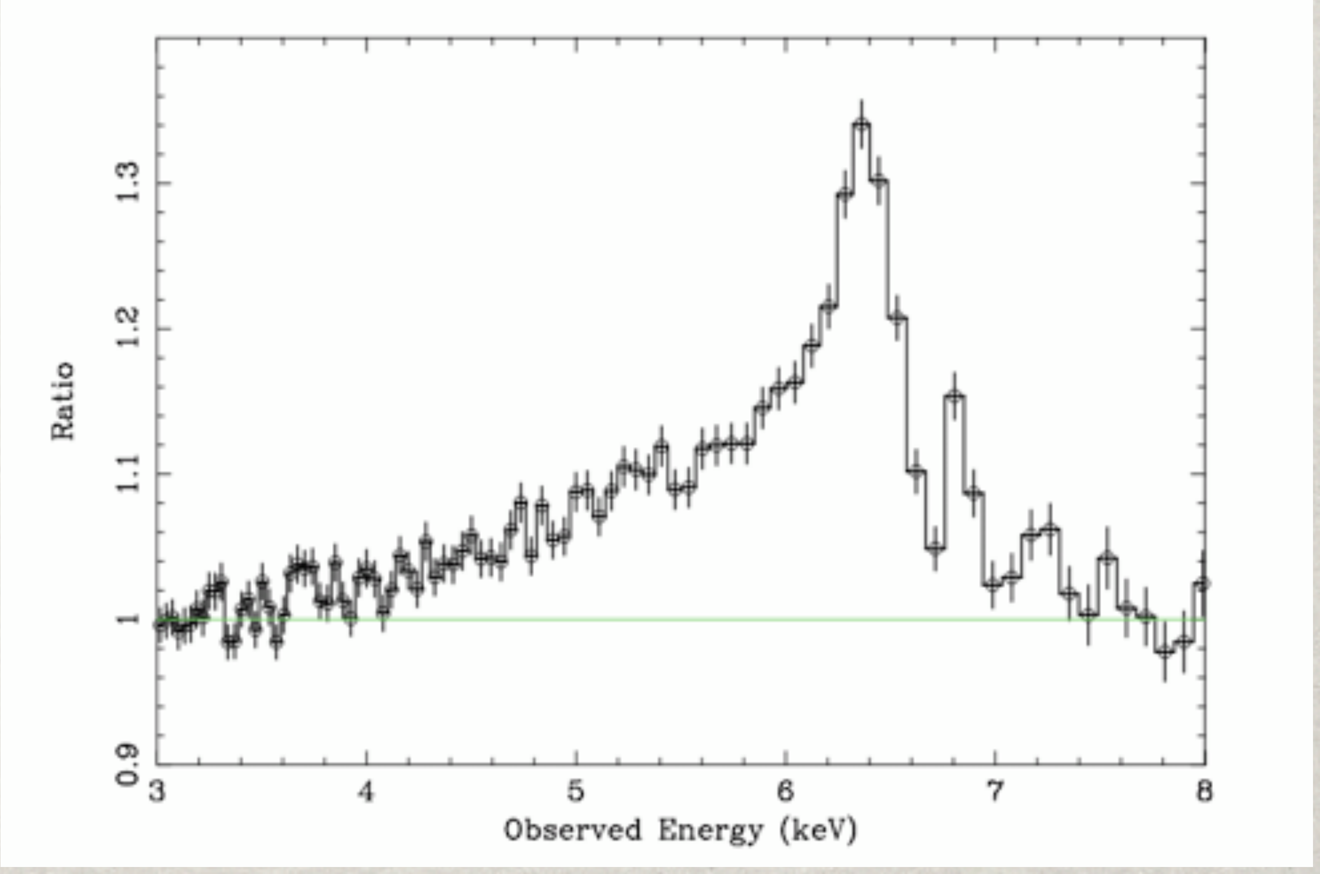
- iron Kα-line in the reflexion spectrum Ni et al., JCAP1610(2016)003
- QPOs Franchini, Pani, Maselli, Gualtieri, C.H., Radu, Ferrari arXiv:1612.00038

The iron line method:

Propagation in strong gravity makes the **locally** Dirac delta-like line...



... broad and skew at the **observation** point...

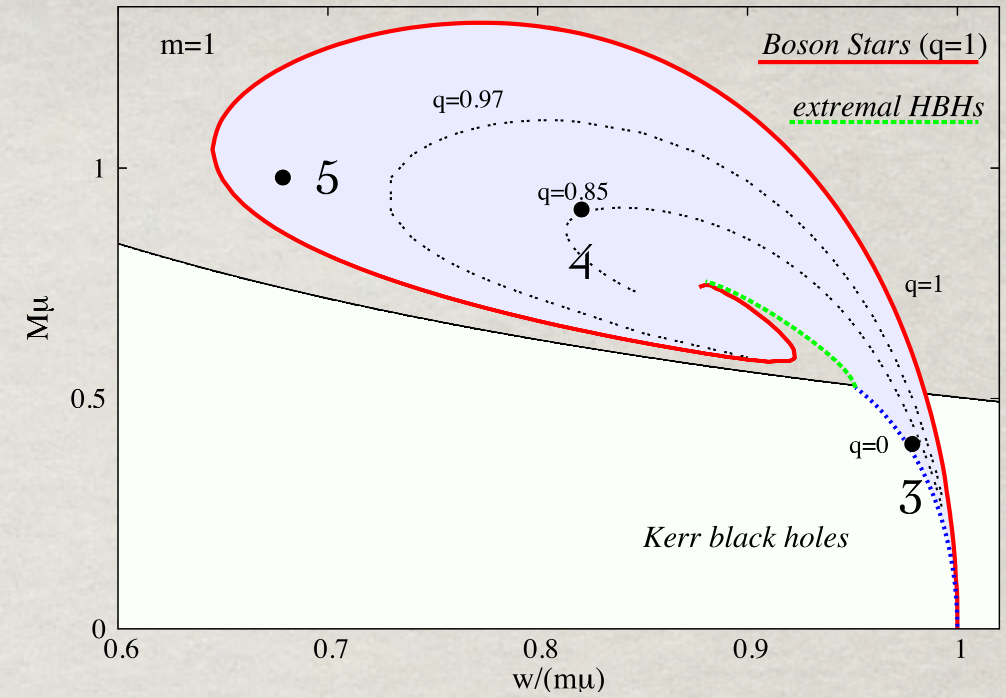


Guainazzi, Ap&SS 320 (2009) 129

Iron Kα-line:

For our three solutions:

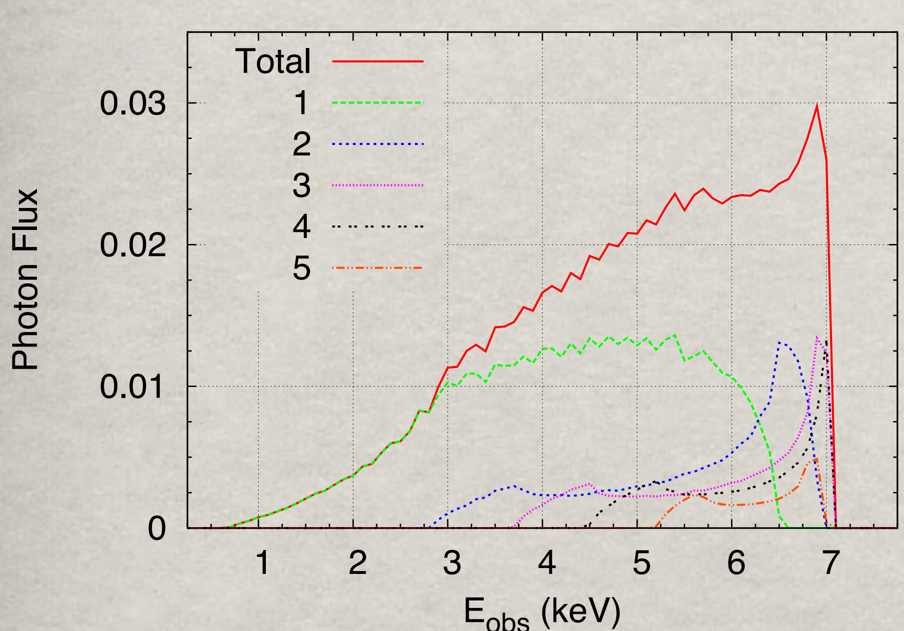
Annulus 1:	$r_{in} = r_{ISCO}$	$r_{out} = r_{ISCO} + 1$
Annulus 2:	$r_{in} = r_{ISCO} + 1$	$r_{out} = r_{ISCO} + 2$
Annulus 3:	$r_{in} = r_{ISCO} + 2$	$r_{out} = r_{ISCO} + 4$
Annulus 4:	$r_{in} = r_{ISCO} + 4$	$r_{out} = r_{ISCO} + 10$
Annulus 5:	$r_{in} = r_{ISCO} + 10$	$r_{out} = r_{ISCO} + 25$

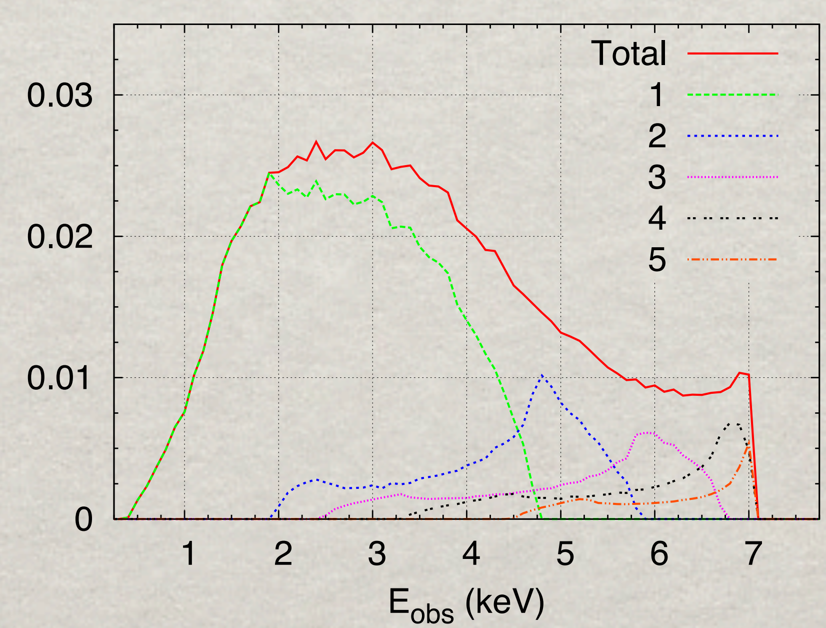


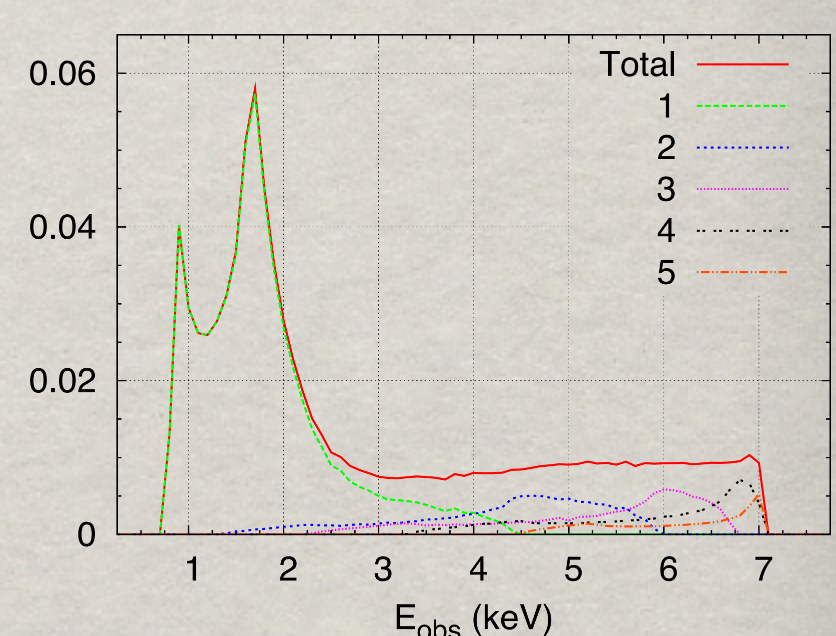
Config 3
5% of M; 13% of J in scalar field

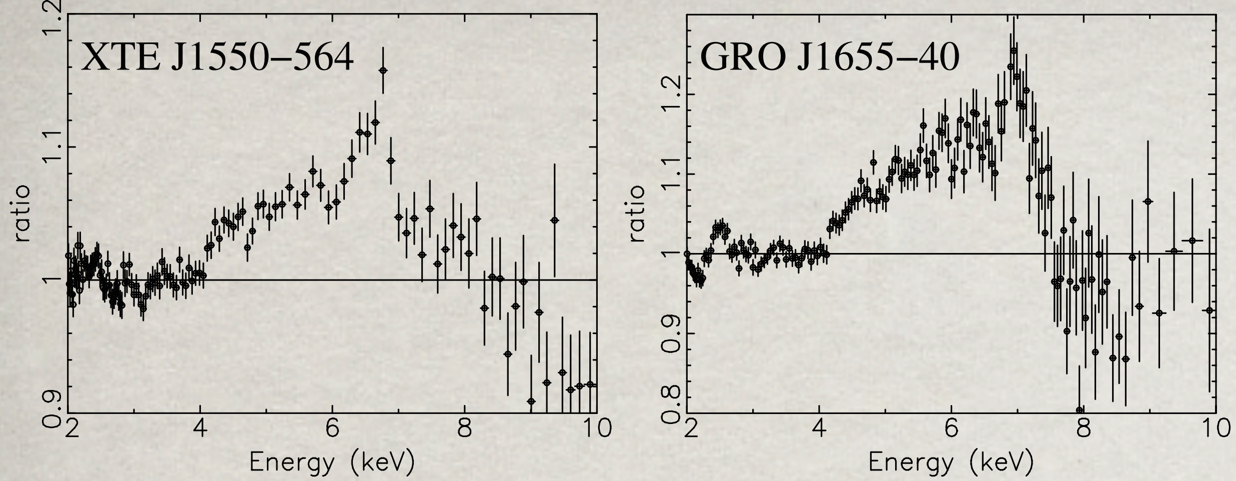
Config 4
75% of M; 85% of J in scalar field

Config 5
98% of M; 98% of J in scalar field

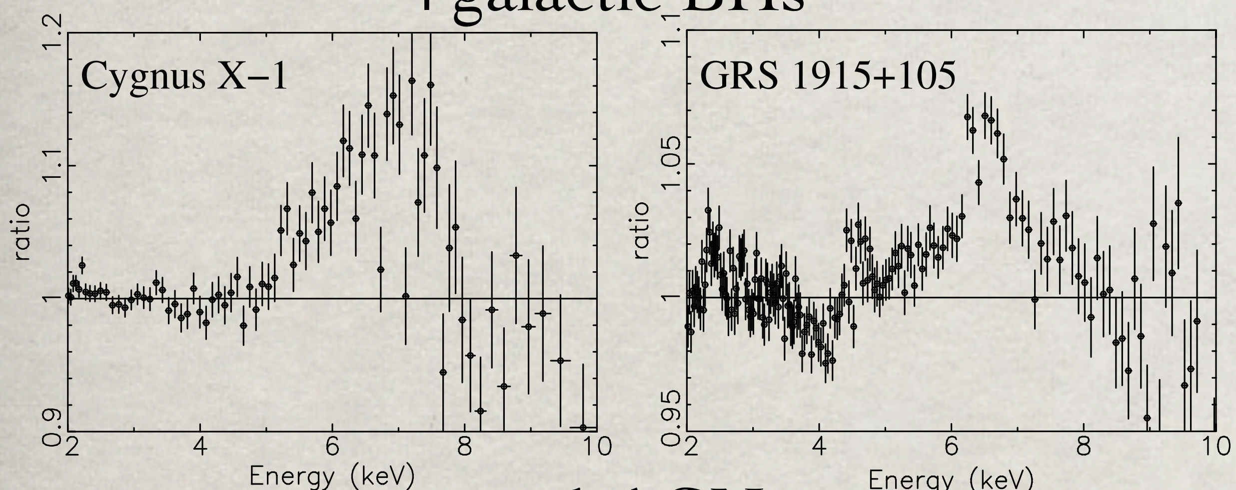




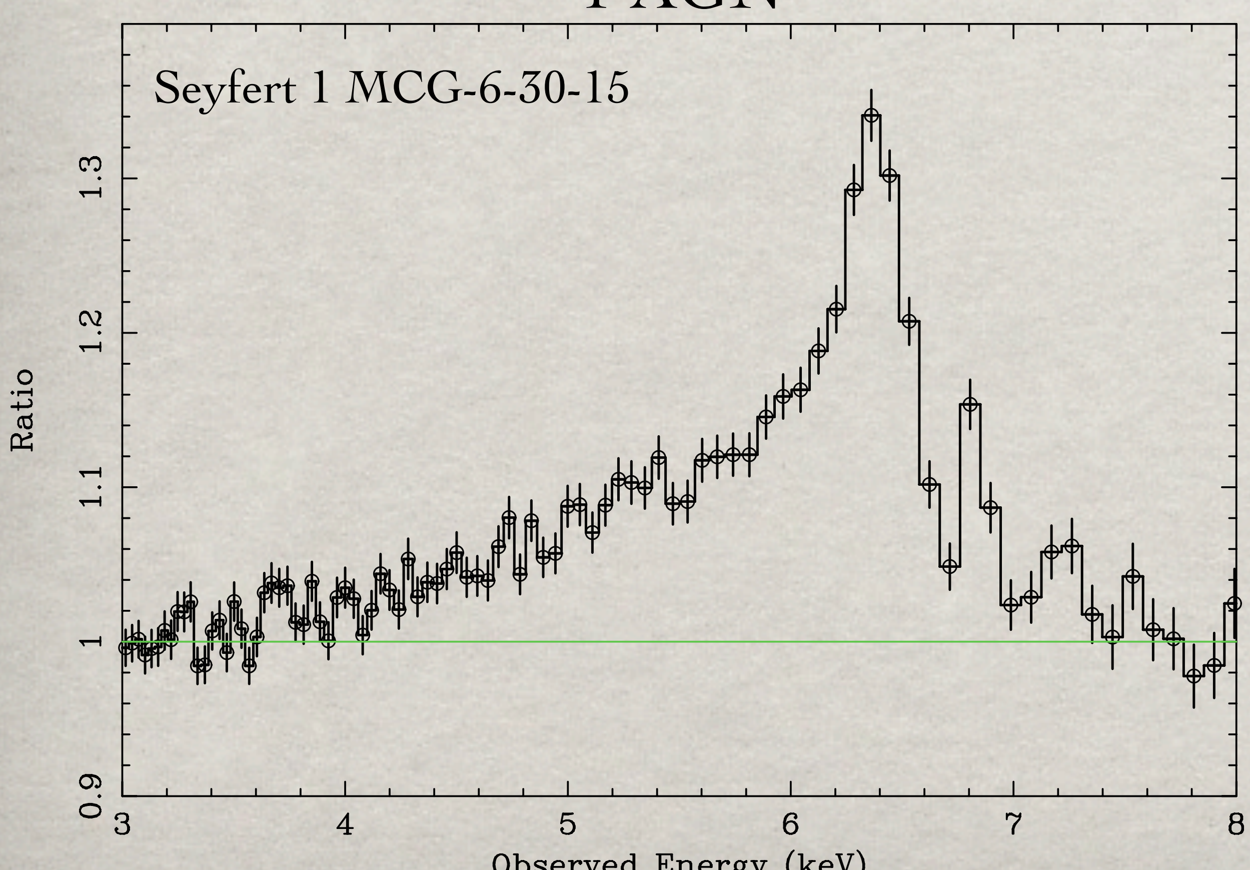


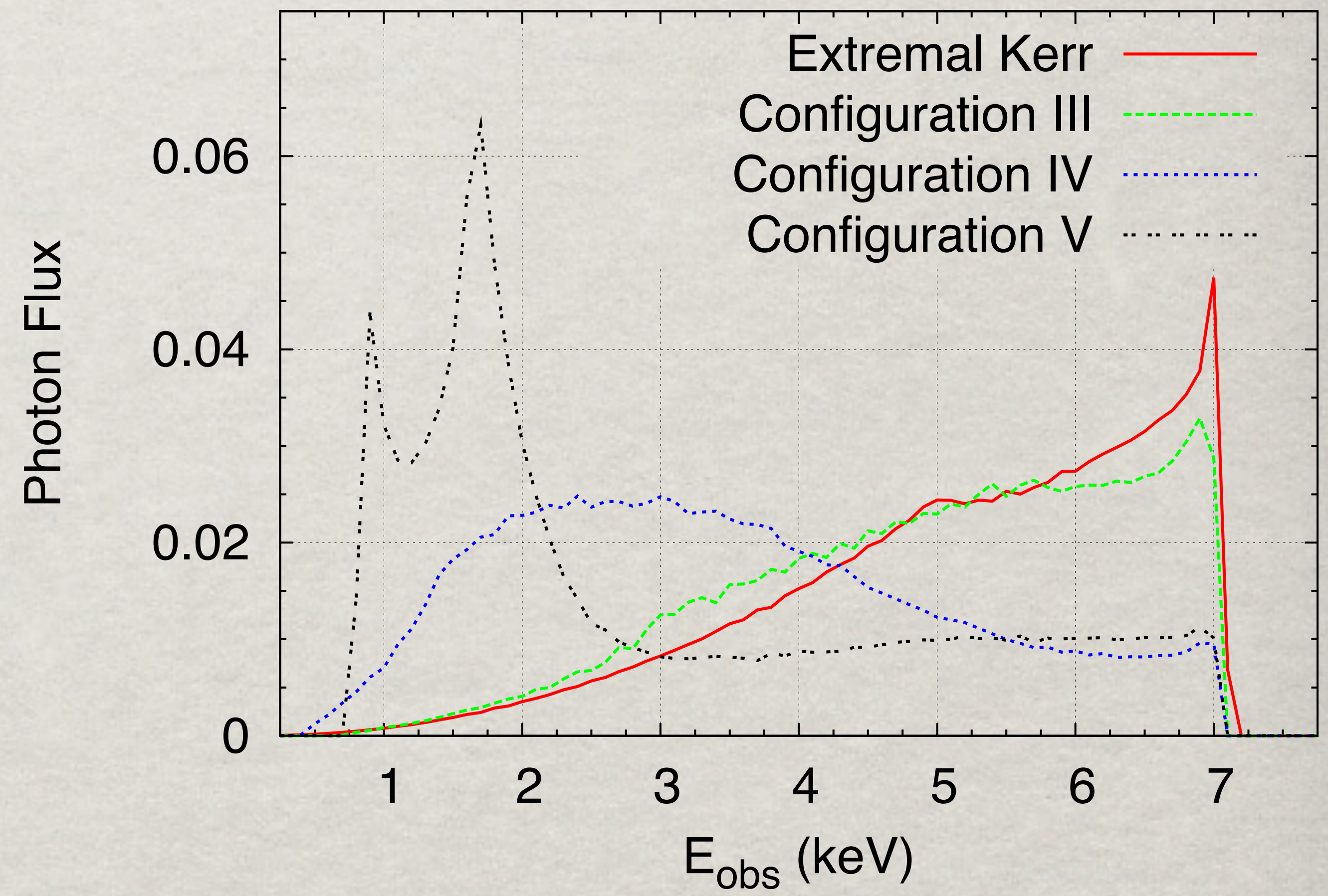


4 galactic BHs



1 AGN





Guainazzi, Ap&SS 320 (2009) 129

3) Open issues/opportunities

What is the scalar field?
(the dark matter connection)

Ultra-light bosonic fields have been suggested as dark matter candidates (“fuzzy dark matter”); they gravitationally clump into boson stars // Bose-Einstein condensates

see e.g. recent discussion Hui, Ostriker, Tremaine, Witten, arXiv:1610.08297

Massive, complex, scalar field, minimally coupled to gravity (no self-interactions)

$$M_{\rm max} \simeq 1.315\frac{M_{Pl}^2}{\mu} = 1.315 \times 10^{-19} M_\odot \left(\frac{\rm GeV}{\mu}\right)$$

Introducing a quartic self-coupling

$$M_{\rm max} \overset{\lambda \gg 1}{\simeq} 0.208\sqrt{\lambda}\frac{M_{Pl}^3}{\mu^2} = 0.208\sqrt{\lambda} M_\odot \left(\frac{\rm GeV}{\mu}\right)^2$$

First observed by Colpi, Shapiro, Wasserman PRL57(1986)2485,
see e.g. for a discussion C. H., Radu, Rúnarsson PRD92(2015)084059

What is the scalar field?
(the high energy physics connection)

In some HEP models it is natural to have bosonic particles with very low mass (QCD axion, Axiverse Arvanitaki, Dimopoulos, Dubovsky, Kaloper and March-Russell PRD81(2010)123530)

These could have astrophysical impact and **convert black holes into (new) particle detectors**.
Arvanitaki and Dubovsky, 1004.3558

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Arvanitaki and Dubovsky, 1004.3558

If

$$M\mu \sim 1$$

the existence of a scalar field efficiently triggers the
superradiant instability
of a “bald” BH and can grow hair around the BH
that saturates due to non-linear phenomena and forms a “hairy” BH

Question:

In these models
vacuum Kerr black holes are **unstable**
(against superradiance).

What is the endpoint of the instability?

In a toy model it is a hairy black hole of this sort:
Sanchis-Gual et al. , PRL 116 (2016)141101

Simulations (under approximations) suggest the hairy BHs formed are never very hairy
Brito, Cardoso, Pani, CQG 32 (2015) 134001

Other issues:

- Dynamics: stability, formation or quasi-formation are open issues for both rotating boson stars or hairy black holes;

- This relates to gravitational wave signals:
ringdown and possibility of echos ? binaries ?

- Relation to dark matter (halos) ?

- Natural embeddings in HEP models ?

- More detailed astrophysical constraints (Shadows, Kα-line, QPOs) ?

- Approximate parameterizations of solutions ?

- Uniqueness theorems ?

- Differences/similarities with the real bosonic field case ?

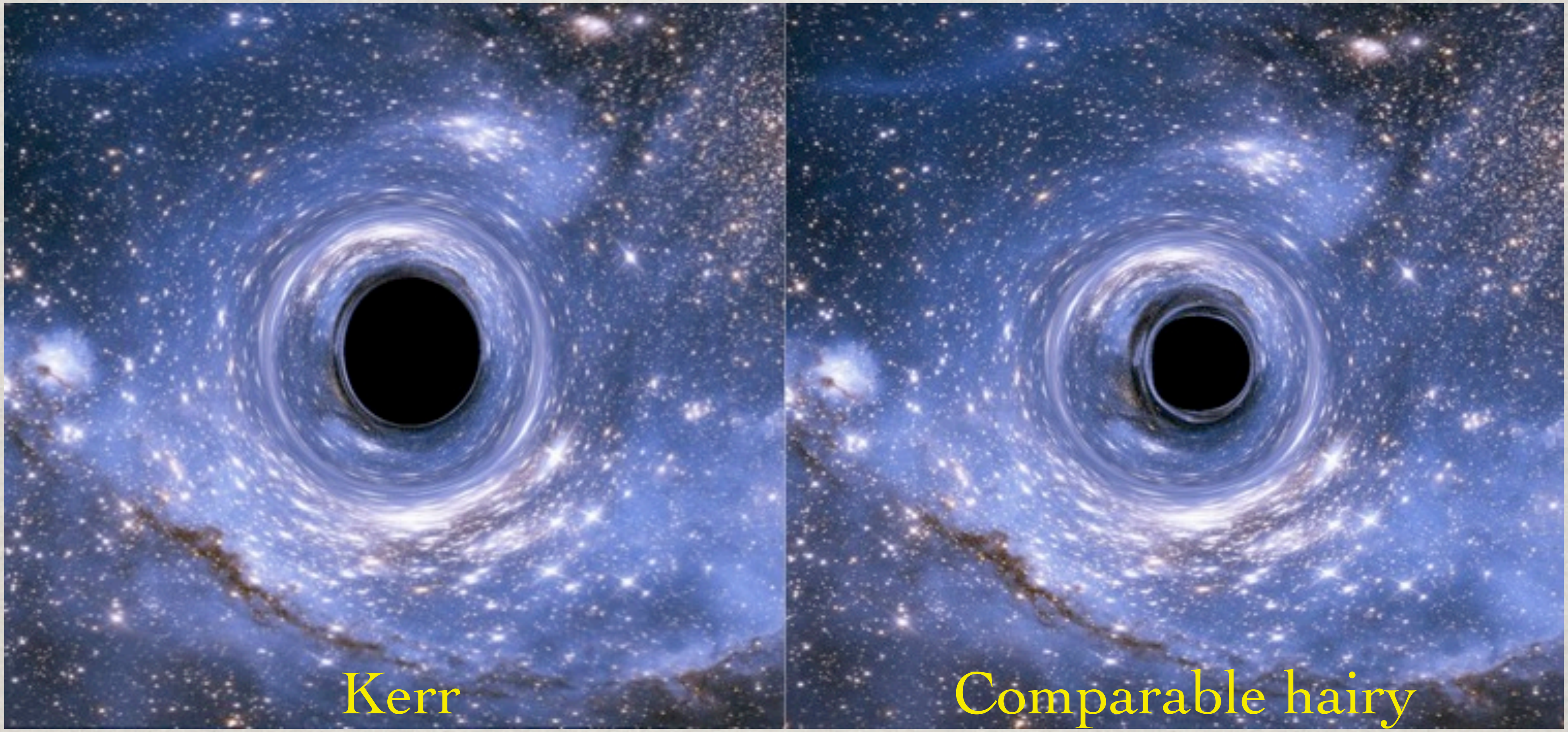

Image:
P. Cunha

Thank you for Your Attention!