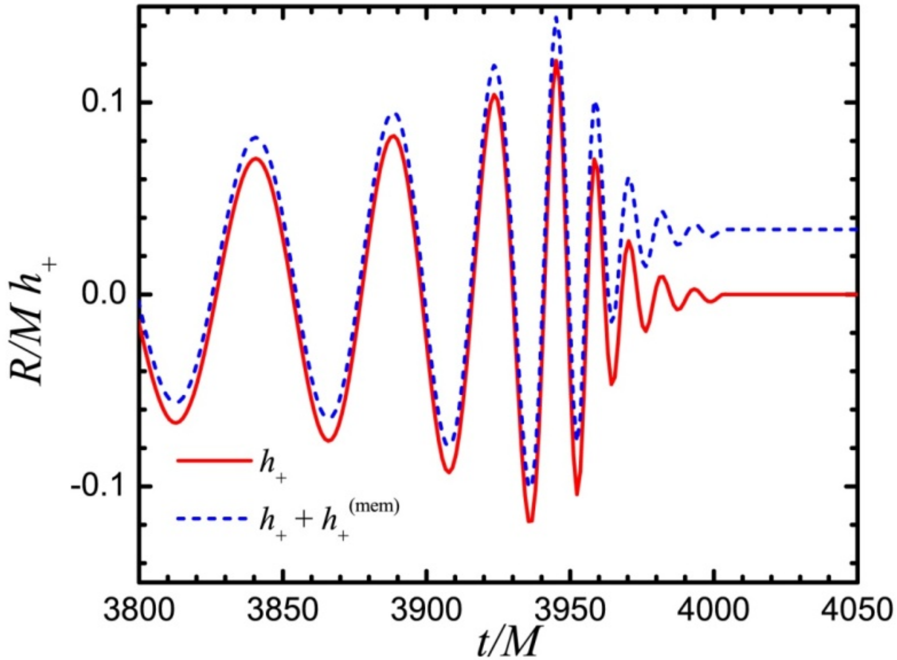
Gravitational-wave memory: an overview



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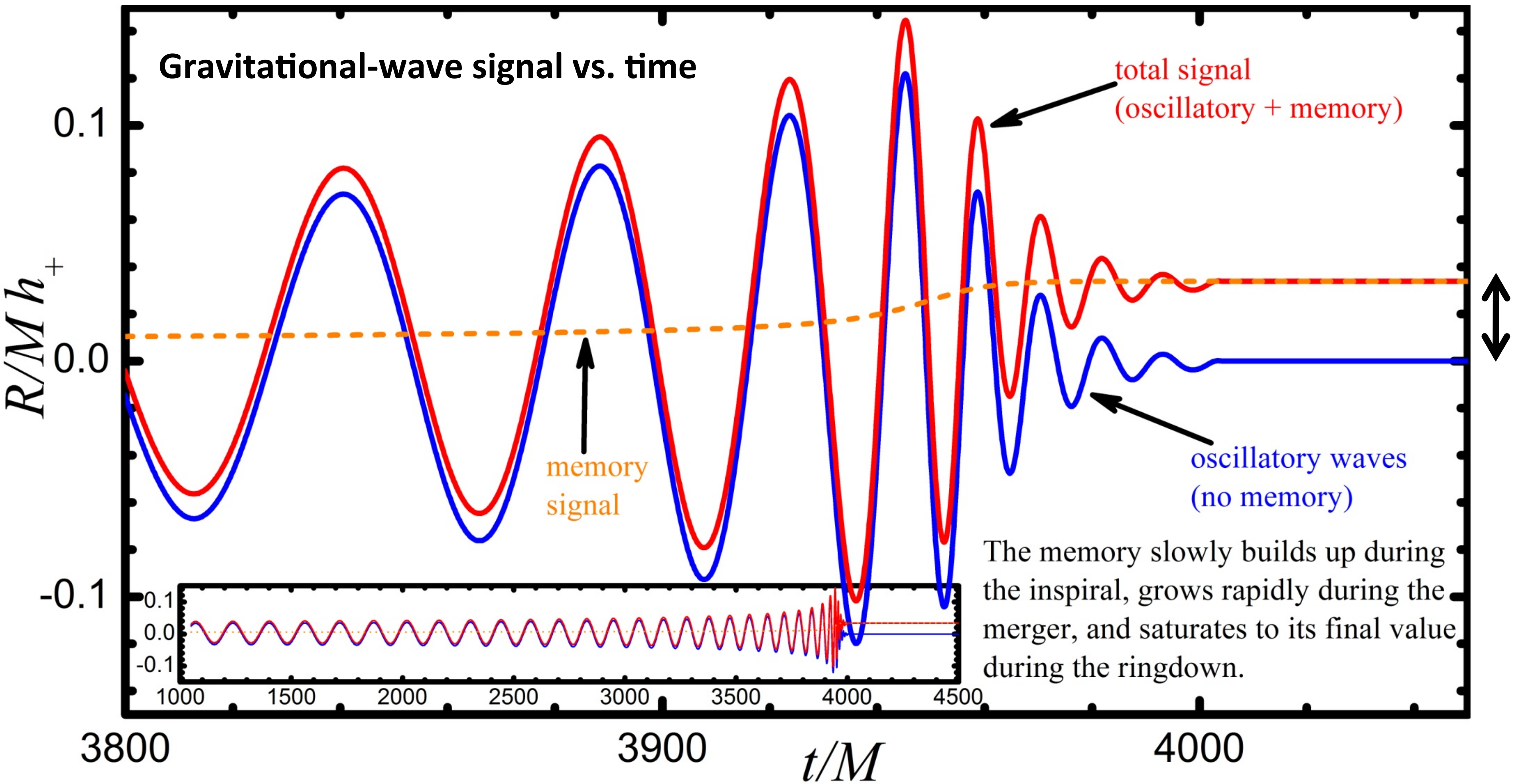
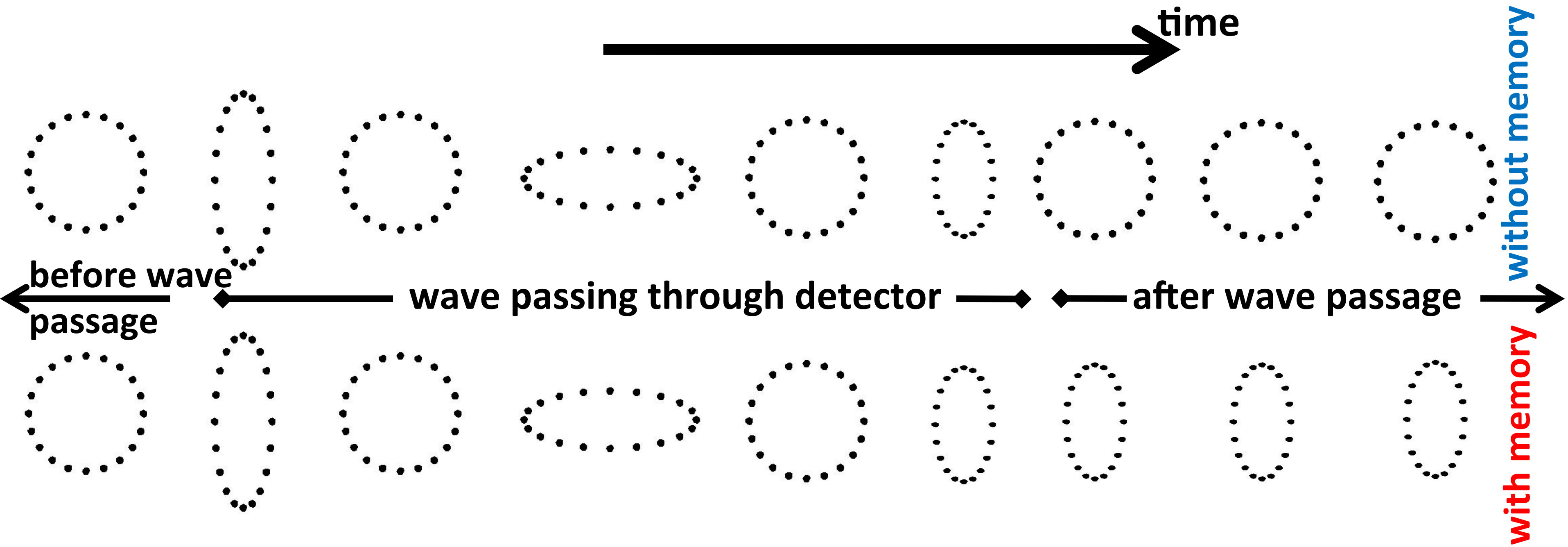
What is the gravitational-wave memory?

example: nonlinear memory from binary black-hole mergers



The wave no longer returns to the zero-point of its oscillation.
This growing-offset is called the **memory**.

Why is this called "memory"?

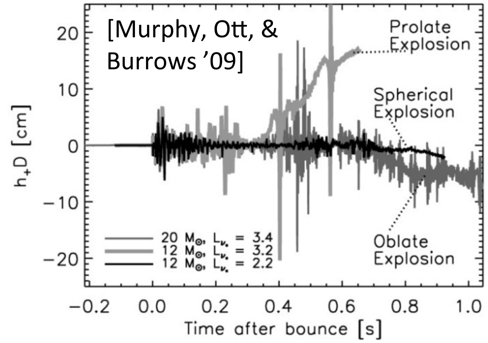


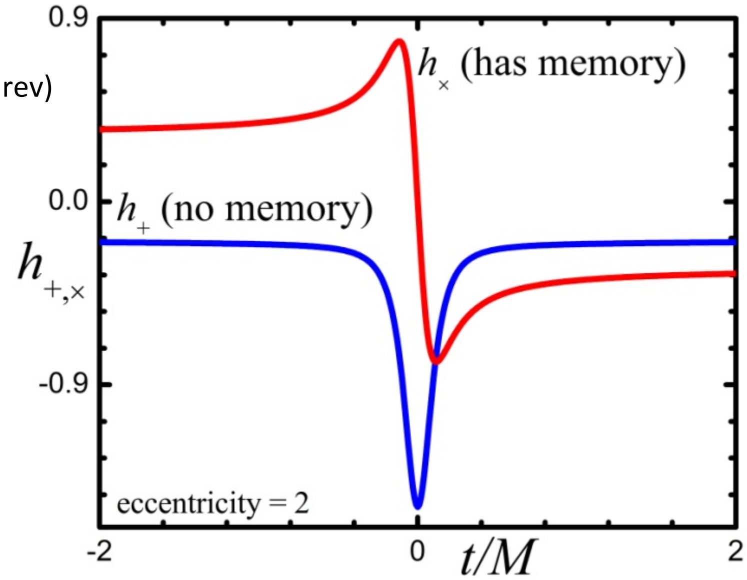
(gravitational-waves propagating into the screen)

The linear and nonlinear memory:

Linear memory: (Braginskii, Grishchuck, Thorne, Zeldovich, Polnarev)

- Arises from the non-oscillatory motion of a source, especially due to unbound masses.
- Ex: hyperbolic orbits, mass/neutrino ejection in supernovas/GRBs





Nonlinear memory: (Christodoulou, Blanchet, Damour)

- Arises from the GWs produced by GWs:

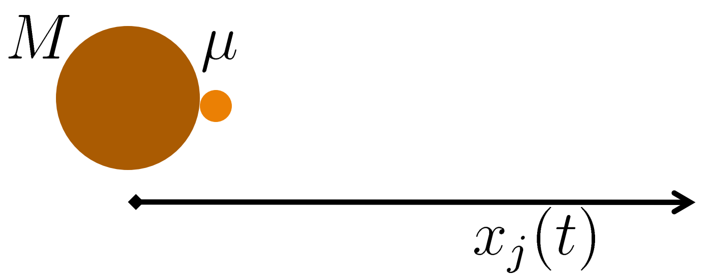
$$T^{jk}_{\rm GW} = \frac{1}{R^2}\frac{dE_{\rm GW}}{dt d\Omega} n_j n_k$$

$$\Box \bar{h}^{jk} = -16\pi(-g)(T^{jk} + T^{jk}_{\rm GW}[\bar{h},\bar{h}]) + O(\bar{h}^2)$$

- "Unbound particles" are the individual "radiated gravitons". [Thorne '92]
- Produced by all sources of GWs.
- Allows us to probe one of the most nonlinear features of GR.

Understanding the memory:
the linear memory effect

[Zel'Dovich & Polnarev '74; Braginsky & Grishchuk '85; Braginsky & Thorne '87]



$$\dot{x}_j(t) \to v^j_\infty$$

$$h^{\rm TT}_{jk} \approx \frac{2}{R}\ddot{\mathcal{I}}^{\rm TT}_{jk} \qquad \mathcal{I}^{\rm TT}_{jk} = \mu[x_j x_k]^{\rm TT}$$

$$\ddot{x}_j = -\frac{M}{r^3}x_j$$

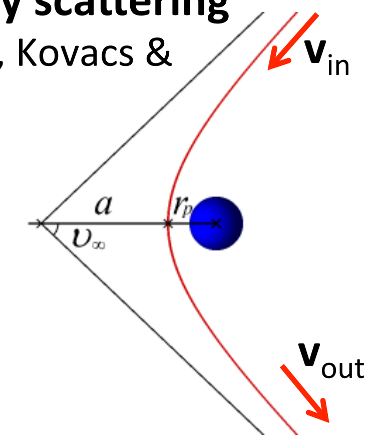
$$\begin{aligned}\ddot{\mathcal{I}}^{\rm TT}_{jk} &= \mu[x_j\ddot{x}_k + \ddot{x}_j x_k + 2\dot{x}_j\dot{x}_k]^{\rm TT} \\ &= 2\mu\left[\dot{x}_j\dot{x}_k - \frac{M}{r^3}x_j x_k\right]^{\rm TT}\end{aligned} \longrightarrow \quad \Delta h^{\rm TT}_{jk} = \frac{4\mu}{R}\Delta[v^j v^k]^{\rm TT}$$

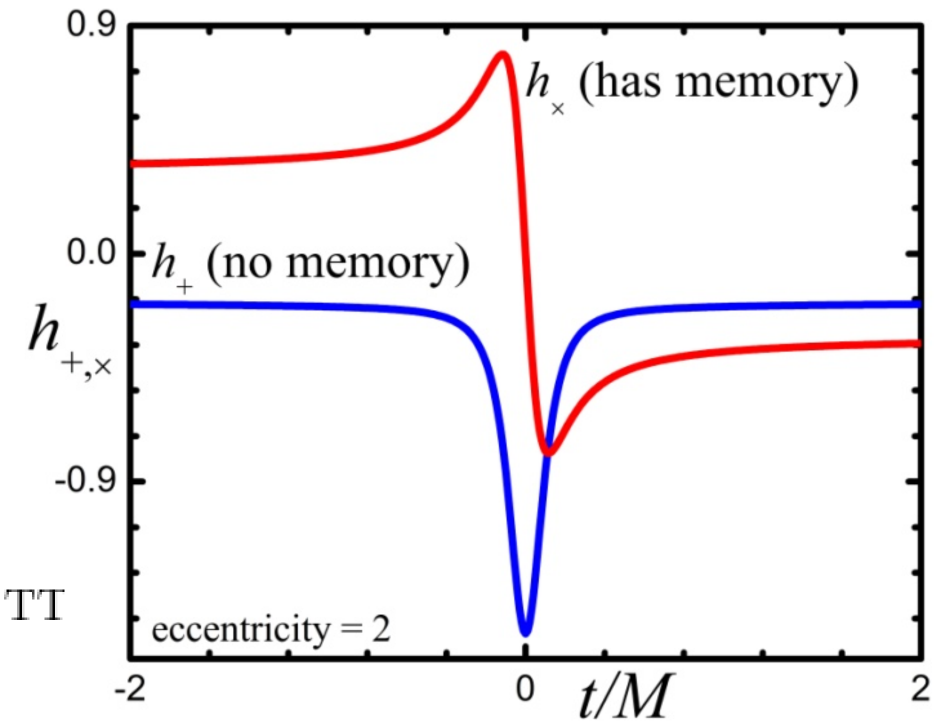
Understanding the memory:
the linear memory effect

[Zel'Dovich & Polnarev '74; Braginsky & Grishchuk '85; Braginsky & Thorne '87]

Hyperbolic orbit/two-body scattering

[Turner '77, Turner & Will '78, Kovacs & Thorne '78]





$$h_{jk}^{\mathrm{TT}} \approx \frac{2}{R}\ddot{\mathcal{I}}_{jk}^{\mathrm{TT}} \qquad \mathcal{I}_{jk}^{\mathrm{TT}} = \mu[x_j x_k]^{\mathrm{TT}}$$

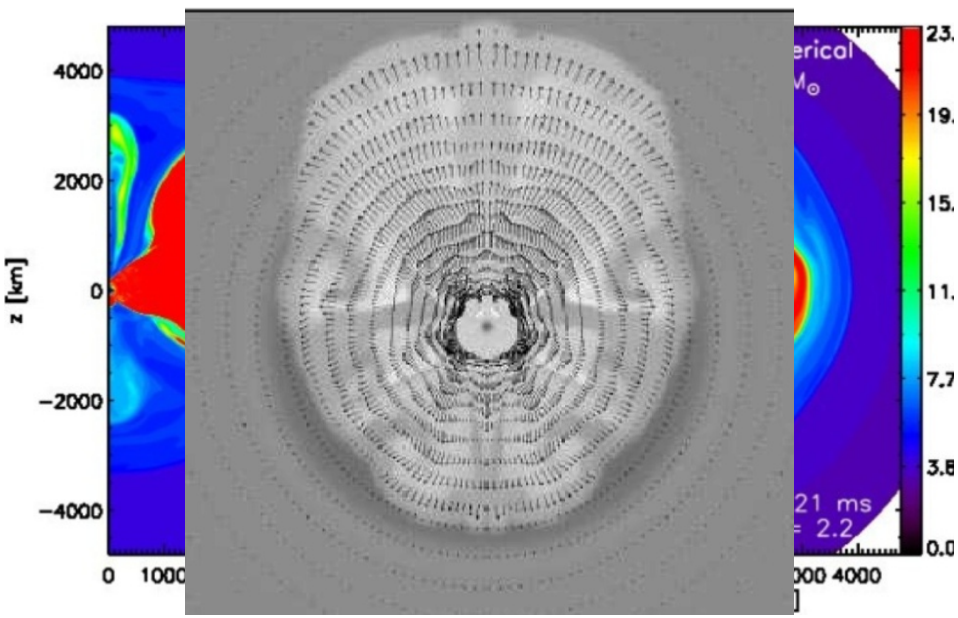
$$\ddot{x}_j = -\frac{M}{r^3}x_j$$

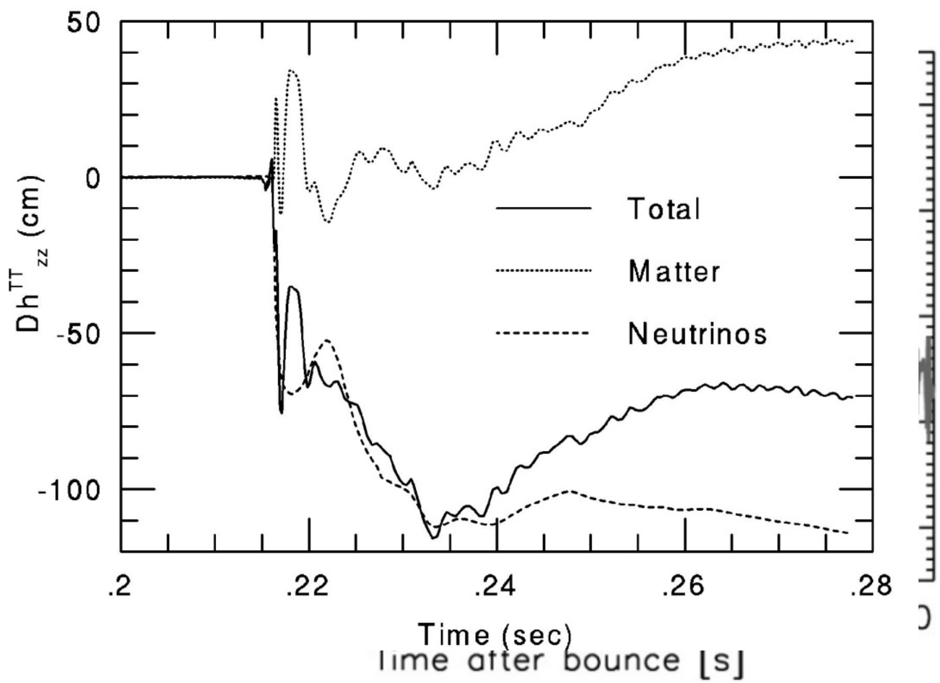
$$\ddot{\mathcal{I}}_{jk}^{\mathrm{TT}} = \mu[x_j\ddot{x}_k + \ddot{x}_j x_k + 2\dot{x}_j\dot{x}_k]^{\mathrm{TT}}$$

$$= 2\mu\left[\dot{x}_j\dot{x}_k - \frac{M}{r^3}x_j x_k\right]^{\mathrm{TT}} \longrightarrow \qquad \Delta h_{jk}^{\mathrm{TT}} = \frac{4\mu}{R}\Delta[v^j v^k]^{\mathrm{TT}}$$

Understanding the memory:
the linear memory effect

Supernova simulations and GRBs also show a linear memory due to the asymmetric ejection of masses or neutrinos [Epstein '78, Burrows & Hayes '96, Murphy, Ott, & Burrows '09, Segalis & Ori '01, Sago et al '04]:





Understanding the memory:
the linear memory effect

General formula for the memory jump in a system w/ N components [Braginsky & Thorne '87, Thorne '92]

$$\Box \bar{h}_{ij} = -16\pi \sum_{A=1}^{N} T_{ij}^{\mathrm{pp},A}$$

$$\Delta h_{ij} = \lim_{t\to+\infty} h_{ij}(t) - \lim_{t\to-\infty} h_{ij}(t)$$

$$\Delta h_{ij}^{\mathrm{TT}} = \Delta \sum_{A=1}^{N} \frac{4M_A}{R\sqrt{1-v_A^2}} \left[\frac{v_A^j v_A^k}{1-\boldsymbol{N}\cdot\boldsymbol{v}_A} \right]^{\mathrm{TT}}$$

Understanding the memory:
the nonlinear memory

[Christodoulou '91; Blanchet & Damour '92]

Mathematically, the nonlinear memory arises from the contribution of the **gravitational-wave stress-energy** to Einstein's equations:

harmonic gauge EFE...

$$\Box \bar{h}^{\alpha\beta} = -16\pi(-g)(T^{\alpha\beta} + t^{\alpha\beta}_{\rm LL}) - \bar{h}^{\alpha\mu}{}_{,\nu}\bar{h}^{\beta\nu}{}_{,\mu} + \bar{h}^{\mu\nu}\bar{h}^{\alpha\beta}{}_{,\mu\nu}$$

...has a nonlinear source from the GW stress-energy tensor...

$$T^{\rm gw}_{jk} = \frac{1}{R^2}\frac{dE^{\rm gw}}{dt d\Omega} n_j n_k$$

solve EFE

$$\bar{h}_{jk}(t,\boldsymbol{x}) = 4\int \frac{(-g)[T_{jk}(t',\boldsymbol{x}') + t^{\rm LL}_{jk}(t',\boldsymbol{x}') + \ldots]}{|\boldsymbol{x}-\boldsymbol{x}'|}\delta(t'-t-|\boldsymbol{x}-\boldsymbol{x}'|)\, d^4x'$$

[Wiseman & Will '91]

$$\delta h^{\rm TT}_{jk} = \frac{4}{R}\int_{-\infty}^{T_R} dt' \left[\int \frac{dE^{\rm gw}}{dt' d\Omega'} \frac{n'_j n'_k}{(1-\boldsymbol{n}'\cdot\boldsymbol{N})} d\Omega'\right]^{\rm TT}$$

Understanding the memory:
the nonlinear memory

[Christodoulou ‘91; Blanchet & Damour ‘92]

Mathematically, the nonlinear memory arises from the contribution of the **gravitational-wave stress-energy** to Einstein’s equations:

Nonlinear memory can be related to the “linear” memory if we interpret the component masses as the individual radiated gravitons (Thorne’92):

$$\Delta h_{ij}^{\mathrm{TT}} = \Delta \sum_{A=1}^{N} \frac{4M_A}{R\sqrt{1-v_A^2}} \left[\frac{v_A^j v_A^k}{1-\boldsymbol{v}_A \cdot \boldsymbol{N}}\right]^{\mathrm{TT}}$$

$$\Delta h_{ij}^{\mathrm{TT}} = \Delta \sum_{A=1}^{N} \frac{4E_A}{R} \left[\frac{n_A^j n_A^k}{1-\boldsymbol{n}_A \cdot \boldsymbol{N}}\right]^{\mathrm{TT}}$$

$$v_A^j \to c n_A^j$$

$$\frac{M_A c^2}{\sqrt{1-v_A^2}} \to E_A$$

$$\delta h_{jk}^{\mathrm{TT}} = \frac{4}{R} \int_{-\infty}^{T_R} dt' \left[\int \frac{dE^{\mathrm{gw}}}{dt' d\Omega'} \frac{n'_j n'_k}{(1-\boldsymbol{n}' \cdot \boldsymbol{N})} d\Omega'\right]^{\mathrm{TT}}$$

Understanding the memory: the nonlinear memory

[Christodoulou '91; Blanchet & Damour '92]

Can also think of it as a nonlinear correction to the multipoles:

$$T^{\rm gw}_{\alpha\beta} \propto \frac{dE^{\rm gw}}{dt d\Omega} \sim O(h^2)$$

$$\ddot{\mathcal{I}}_{jk} \to \ddot{\mathcal{I}}_{jk} + U^{\rm gw}_{jk}$$

$$h^{\rm TT}_{jk} \approx \frac{2}{R}\ddot{\mathcal{I}}^{\rm TT}_{jk}$$

- Memory piece scales like the radiated energy.

$$\Delta h^{(\rm mem)} \sim \frac{\Delta E^{\rm gw}}{R}$$

- So the nonlinear memory is present in *all* GW sources.
- The effect is hereditary (depends on entire past evolution).

Understanding the memory:
the nonlinear memory: inspiralling binaries

Although it arises from a 2.5PN correction to the multipole moments, for inspiralling binaries the nonlinear affects the waveform at ***leading*** (Newtonian) order:

$$h_+ = -2\frac{\mu}{R}v_{\rm orb}^2\left[(1+\cos^2\Theta)\cos[2\varphi(t)-2\Phi] + \frac{1}{96}\sin^2\Theta(17+\cos^2\Theta) + O(v_{\rm orb}^{1/2})\right]$$

[Wiseman & Will '91]

Why?

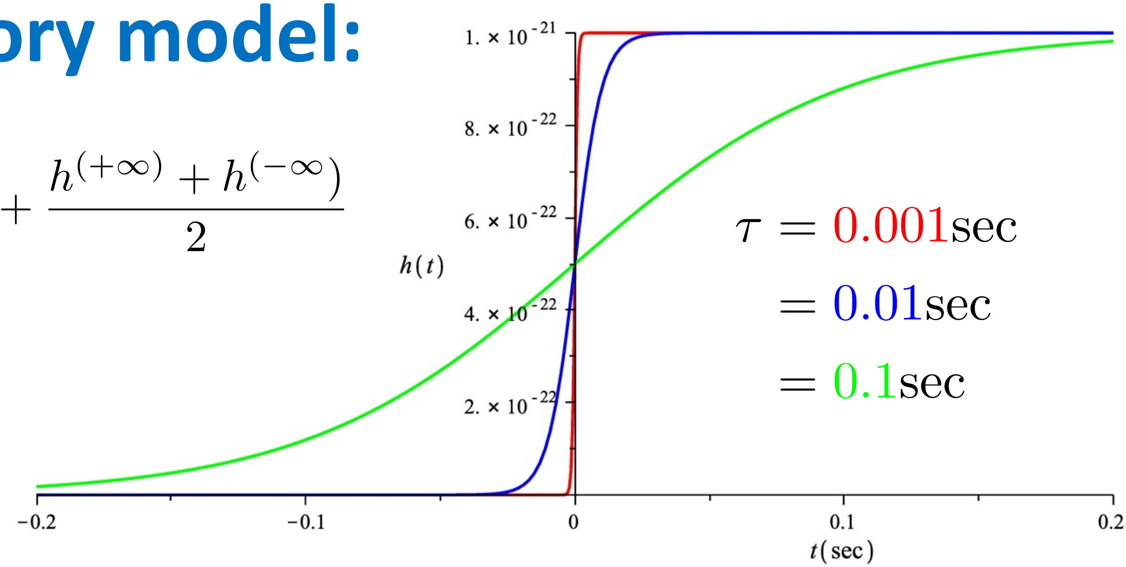
$$\Delta h^{jk}_{\rm mem} \sim \frac{\Delta E_{\rm GW}}{R} \qquad \Delta E_{\rm GW} \sim \Delta E_{\rm binding} \sim \frac{\mu M}{r} \sim \mu v_{\rm orb}^2$$

$$h^{ij}_{\rm oscil.} \propto \frac{1}{R}\ddot{\mathcal{I}}_{ij} \sim \frac{\mu}{R}v_{\rm orb}^2$$

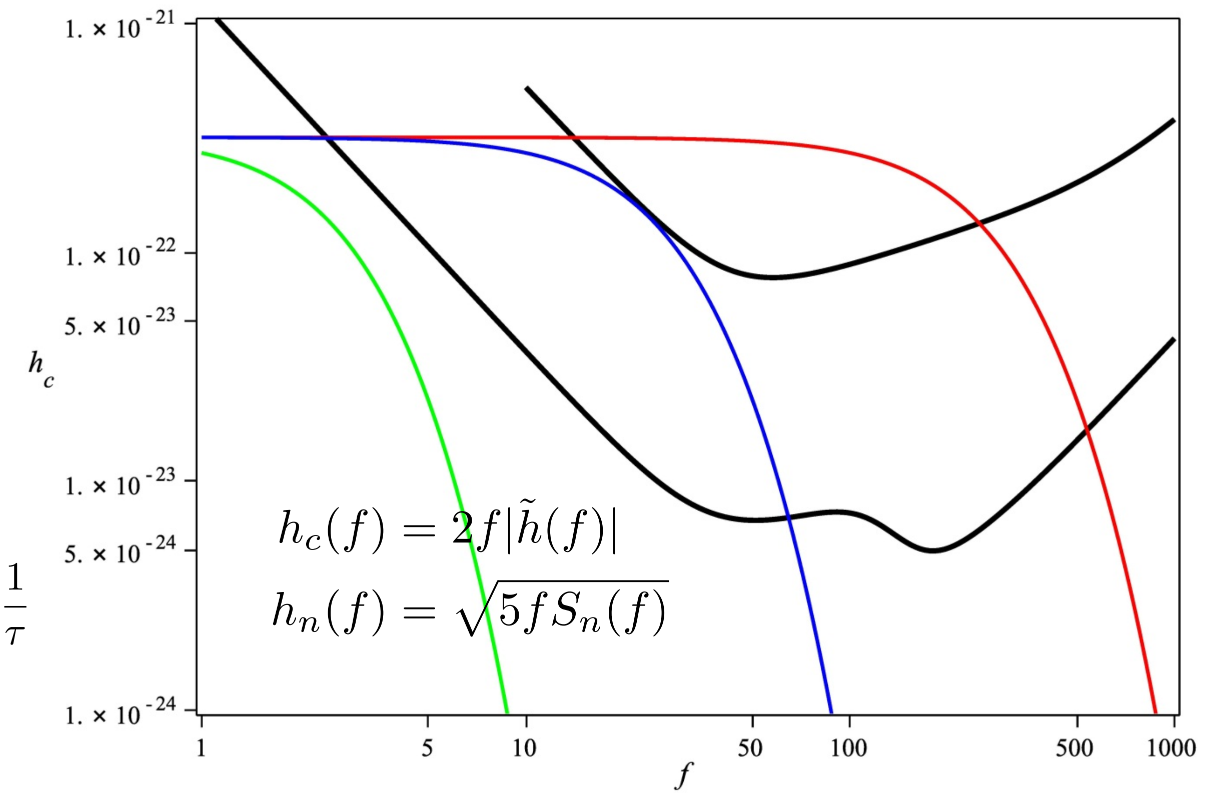
Simple analytic memory model:

$$h^{(\mathrm{mem})}(t) = \frac{h^{(+\infty)} - h^{(-\infty)}}{2} \tanh\left(\frac{t}{\tau}\right) + \frac{h^{(+\infty)} + h^{(-\infty)}}{2}$$

$$\tilde{h}^{(\mathrm{mem})}(f) = \frac{\Delta h^{(\mathrm{mem})}}{2} i\pi\tau \,\mathrm{csch}(\pi^2 \tau f)$$
$$\approx i\frac{\Delta h^{(\mathrm{mem})}}{2\pi f}\left[1 - \frac{\pi^2}{6}(\tau f)^2\right]$$



$$\Delta h^{(\mathrm{mem})} \equiv h^{(+\infty)} - h^{(-\infty)}$$



Step function approximation:

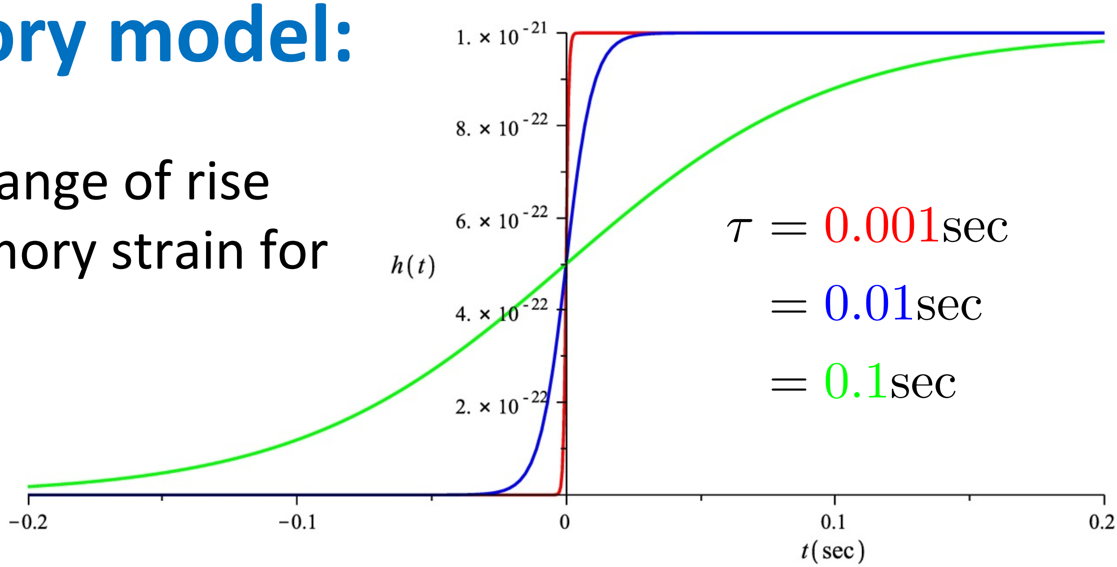
$$h^{(\mathrm{mem})}(t) = \Delta h^{(\mathrm{mem})}\Theta(t)$$

$$\tilde{h}^{(\mathrm{mem})} = \frac{i\Delta h^{(\mathrm{mem})}}{2\pi f}\ ,\ 0 < f < f_c,\ f_c \sim \frac{1}{\tau}$$

[MF, PRD ’11]

Simple analytic memory model:

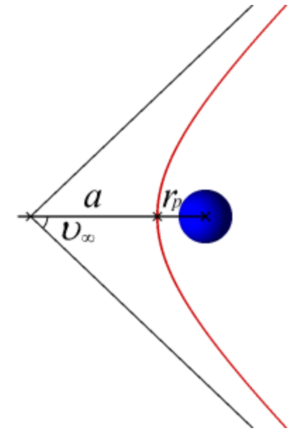
Using step function model and range of rise times, critical characteristic memory strain for <SNR>=1 is:



$$h_c^{(\mathrm{mem})} \approx \begin{cases} 6\times 10^{-23}\text{ - }10^{-22}, & \text{aLIGO} \\ 5\times 10^{-24}\text{ - }10^{-23}, & \text{ET} \\ 8\times 10^{-22}\text{ - }2\times 10^{-21}, & \text{LISA} \end{cases}$$

[MF, PRD ’11]

Memory sources: gravitational scattering



$$h_{c,e_t\gg 1}^{(\mathrm{lin.mem})} \propto \eta \frac{M}{R}\frac{M}{r_p}$$

$$h_{c,e_t=1}^{(\mathrm{nonlin.mem})} \propto \eta^2 \frac{M}{R}\left(\frac{M}{r_p}\right)^{7/2}$$

Note that the nonlinear is suppressed by several orders of magnitude in hyperbolic/ parabolic binaries.

$$h_{c,e_t\gg 1}^{(\mathrm{lin.\ mem})} \approx 10^{-21}\left(\frac{\eta}{0.25}\right)\left(\frac{M/10M_\odot}{R/10\,\mathrm{Mpc}}\right)\left(\frac{20M}{r_p}\right)$$

$$h_{c,e_t=1}^{(\mathrm{nonlin.\ mem})} \approx 2\times 10^{-22}\left(\frac{\eta}{0.25}\right)^2\left(\frac{M/10M_\odot}{R/10\,\mathrm{kpc}}\right)\left(\frac{20M}{r_p}\right)$$

[MF, PRD ’11]

Memory sources: supernovae

[Yakunin et al ’10]

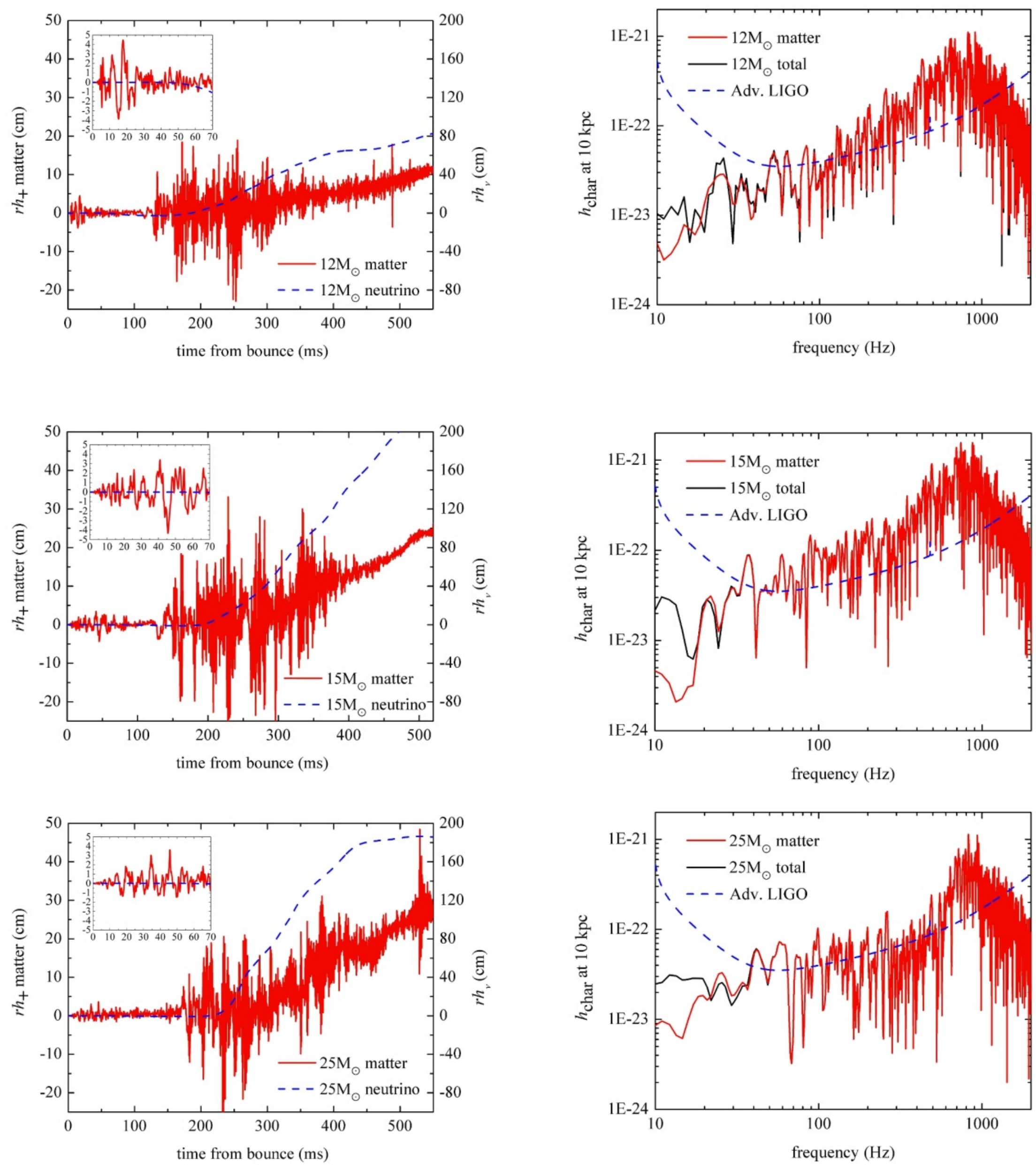
Simulations from multiple groups show a memory effect due to anisotropic matter or neutrino emission:
[Burrows & Hayes ‘94, Murphy, Ott, Burrows ’09, Kotake et al ‘09, Muller & Janka ’97, Yakunin et al ‘10]

$$\Delta h^{(\text{mem})}_{\text{matter}} \sim 10^{-21}\left(\frac{10\,\text{kpc}}{R}\right)$$

$$\Delta h^{(\text{mem})}_{\nu} \sim 7\times 10^{-21}\left(\frac{10\,\text{kpc}}{R}\right)$$

$$\text{but}\quad f_c \lesssim 10\,\text{Hz}$$

Size of memory varies among simulations depending on input physics.
[reviews by Ott’09 & Kotake ‘11]



Nonlinear memory for inspiralling binaries:

Survey of previous and recent work

Part I: Inspiral memory in PN approximation

- needed to fully describe waveform amplitude corrections (including at 0PN order)
- provide input to merger/ringdown calculation

Part II: memory from merger/ringdown

- provides full memory signal; grows rapidly during merger
- semi-analytic descriptions or full NR

Part III: detectability estimate

- apply above models to evaluate detection prospects

Nonlinear memory for inspiralling binaries:

Survey of previous and recent work

✓ 0PN inspiral, circular, nonspinning: Wiseman & Will ’91

✓ 3PN inspiral, circular, nonspinning: MF ’09a

✓ 0PN inspiral, eccentric, nonspinning: MF ’11

✓ merger/ringdown, nonspinning, equal-mass: MF ’09b, ‘10

✓ merger/ringdown, aligned-spins, equal-masses: Pollney & Reisswig ‘11

✓ crude detectability estimates for LISA & LIGO: MF ‘09, ‘11, Pollney & Reisswig

✓ estimates of recoil-induced QNM Doppler shift and memory: MF ’09c

✓ pulsar timing studies/searches: Seto ‘09, van Haasteren & Levin ‘10, Pshrikov et al’10, Cordes & Jenet ‘12, Madison, Cordes, Chatterjee ‘14, Wang et al ’15, Arzoumanian et al ’15

✓Lasky et. al ’16: aLIGO detectability via combinations of multiple events.

[See also mathematical aspects of memory: Bieri, Garfinkle, Tolish, Wald.]

nonlinear memory from circular binaries:
3PN h_{lm} modes and polarization

$$h_{+,\times} = \frac{2\eta M x}{R} H_{+,\times} \; + \; O\left(\frac{1}{R^2}\right), \; \text{where} H_{+,\times} = \sum_{n=0}^{\infty} x^{n/2} H_{+,\times}^{(n/2)}.$$

[MF, Phys. Rev. D '09]

$$H_+^{(0,\mathrm{mem})} = \alpha \frac{1}{96} s_\Theta^2 (17 + c_\Theta^2),$$

$$H_+^{(0.5,\mathrm{mem})} = 0,$$

$$H_+^{(1,\mathrm{mem})} = \alpha s_\Theta^2 \left[-\frac{354\,241}{2\,064\,384} - \frac{62\,059}{1\,032\,192} c_\Theta^2 - \frac{4195}{688\,128} c_\Theta^4 + \left(\frac{15\,607}{73\,728} + \frac{9373}{36\,864} c_\Theta^2 + \frac{1575}{8192} c_\Theta^4 \right) \eta \right],$$

$$H_+^{(1.5,\mathrm{mem})} = 0,$$

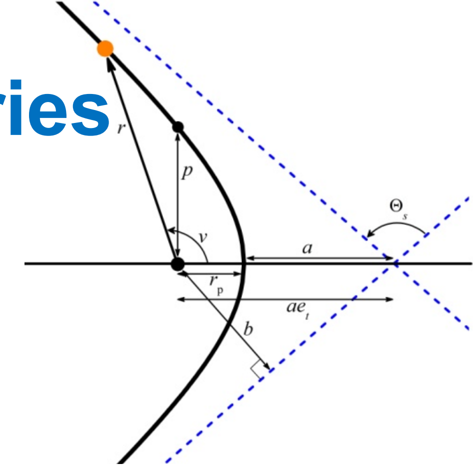
$$H_+^{(2,\mathrm{mem})} = \alpha s_\Theta^2 \left[-\frac{3\,968\,456\,539}{9\,364\,045\,824} - \frac{170\,408\,175}{4\,682\,022\,912} c_\Theta^2 + \frac{122\,150\,837}{3\,121\,348\,608} c_\Theta^4 + \frac{75\,601}{15\,925\,248} c_\Theta^6 + \left(-\frac{7\,169\,749}{18\,579\,456} - \frac{13\,220\,477}{18\,579\,456} c_\Theta^2 + \frac{1\,345\,405}{6\,193\,152} c_\Theta^4 - \frac{25\,115}{884\,736} c_\Theta^6 \right) \eta + \left(\frac{10\,097}{147\,456} + \frac{5179}{36\,864} c_\Theta^2 + \frac{44\,765}{147\,456} c_\Theta^4 + \frac{3395}{73\,728} c_\Theta^6 \right) \eta^2 \right],$$

$$H_+^{(2.5,\mathrm{mem})} = -\alpha \frac{5\pi}{21\,504} (1 - 4\eta) s_\Theta^2 \left(509 + 472 c_\Theta^2 + 39 c_\Theta^4 \right),$$

$$H_+^{(3,\mathrm{mem})} = \alpha s_\Theta^2 \left\{ -\frac{69\,549\,016\,726\,181}{46\,146\,017\,820\,672} + \frac{6\,094\,001\,938\,489}{23\,073\,008\,910\,336} c_\Theta^2 - \frac{1\,416\,964\,616\,993}{15\,382\,005\,940\,224} c_\Theta^4 - \frac{2\,455\,732\,667}{78\,479\,622\,144} c_\Theta^6 - \frac{9\,979\,199}{2\,491\,416\,576} c_\Theta^8 + \left[\frac{1\,355\,497\,856\,557}{149\,824\,733\,184} - \frac{3485\pi^2}{9216} + \left(-\frac{3\,769\,402\,979}{4\,682\,022\,912} - \frac{205\pi^2}{9216} \right) c_\Theta^2 + \frac{31\,566\,573\,919}{49\,941\,577\,728} c_\Theta^4 + \frac{788\,261\,497}{3\,567\,255\,552} c_\Theta^6 + \frac{302\,431}{9\,437\,184} c_\Theta^8 \right] \eta + \left(\frac{5\,319\,395}{28\,311\,552} - \frac{24\,019\,355}{99\,090\,432} c_\Theta^2 - \frac{4\,438\,085}{3\,145\,728} c_\Theta^4 - \frac{3\,393\,935}{7\,077\,888} c_\Theta^6 - \frac{7835}{98\,304} c_\Theta^8 \right) \eta^2 + \left(\frac{1\,433\,545}{63\,700\,992} + \frac{752\,315}{15\,925\,248} c_\Theta^2 + \frac{129\,185}{2\,359\,296} c_\Theta^4 + \frac{389\,095}{1\,179\,648} c_\Theta^6 + \frac{9065}{131\,072} c_\Theta^8 \right) \eta^3 \right\},$$

THIS COMPLETES THE WAVEFORM AMPLITUDE TO 3PN ORDER.

nonlinear memory from eccentric binaries



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$$\Delta h^{(\mathrm{mem})} = \int_{-\infty}^{T_R} \dot{h}^{(\mathrm{mem})}\, dt \quad \left(\frac{M}{R}\right)\left(\frac{M}{p}\right.$$

$$T_{\mathrm{orb}} \sim M\left(\frac{p}{M}\right)^{3/2}$$

$$T_{\mathrm{rr}} \sim \frac{M}{\eta}\left(\frac{p}{M}\right)^{5}$$

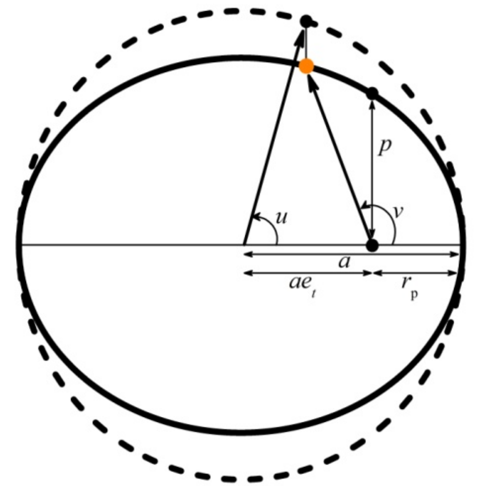
Pa ry

$$\Delta h^{(\mathrm{mem}),\mathrm{hyper.}} \sim \dot{h}^{(\mathrm{mem})} T_{\mathrm{orb}}$$

Ell y

$$\Delta h^{(\mathrm{mem}),\mathrm{ellip.}} \sim \dot{h}^{(\mathrm{mem})} T_{\mathrm{rr}}$$

$$\frac{T_{\mathrm{orb}}}{T_{\mathrm{rr}}} \sim \eta\left(\frac{M}{p}\right)^{5/2}$$



Nonlinear memory case:

Hyperbolic/parabolic orbits: $\Delta h^{(\mathrm{mem})} \propto \eta^2 \left(\frac{M}{R}\right) \boxed{\left(\frac{M}{p}\right)^{7/2}} F(e)$

2.5PN

Elliptical orbits:

$$h_{20}^{(\mathrm{mem})} = -\frac{2}{7}\sqrt{\frac{10\pi}{3}}\frac{\eta M^2}{R p_0} e_0^{12/19}(304+121e_0^2)^{870/2299}\int_{e_0}^{e(t)} de \frac{1}{e^{31/19}}\frac{(192+580e^2+73e^4)}{(304+121e^2)^{3169/2299}}$$

$$\Delta h^{(\mathrm{mem})} \propto \eta \left(\frac{M}{R}\right)\boxed{\left(\frac{M}{p_0}\right)}\left[1-\left(\frac{e_0}{e(t)}\right)^{12/19}\right]$$

0PN

[MF PRD'11]

Spin-orbit corrections to nonlinear memory (inspiral):

Aligned-spin case: [w/ Xinyi Guo]

$$h_+^{(\mathrm{mem})} = -\frac{2\eta M}{R} v^2 \sin^2\Theta \left[H_+^{0\mathrm{PN,nonspin}} + v^2 H_+^{1\mathrm{PN,nonspin}} + v^3 H_+^{1.5\mathrm{PN,spin}} + \cdots v^6 H_+^{3\mathrm{PN,nonspin}} \right]$$

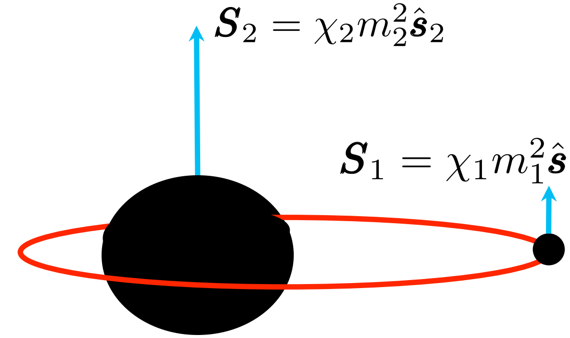
$$H_+^{0\mathrm{PN,nonspin}} = \frac{17+\cos^2\Theta}{96}$$

$$H_+^{1\mathrm{PN,nonspin}} = F(\cos^2\Theta, \eta)$$

$$H_+^{1.5\mathrm{PN,spin}} = \frac{1}{768} \sum_{i=1,2} \chi_i \kappa_i \left[369 \frac{m_i^2}{M^2} + 351\eta + \cos^2\Theta \left(23 \frac{m_i^2}{M^2} + 57\eta \right) \right]$$

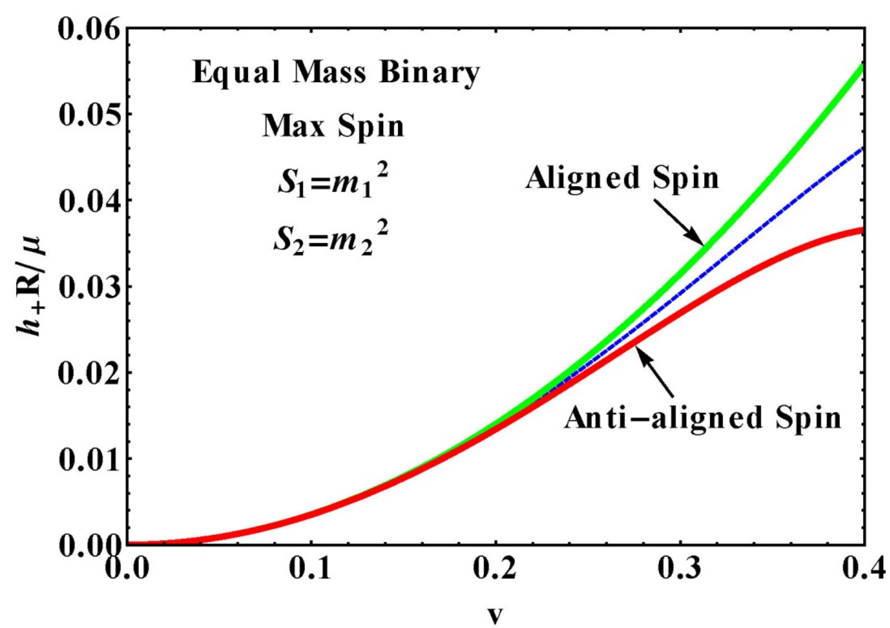
$\boldsymbol{S}_2 = \chi_2 m_2^2 \hat{\boldsymbol{s}}_2$

$\boldsymbol{S}_1 = \chi_1 m_1^2 \hat{\boldsymbol{s}}_1$



$$\kappa_i = \hat{\boldsymbol{L}}_N \cdot \hat{\boldsymbol{s}}_i$$

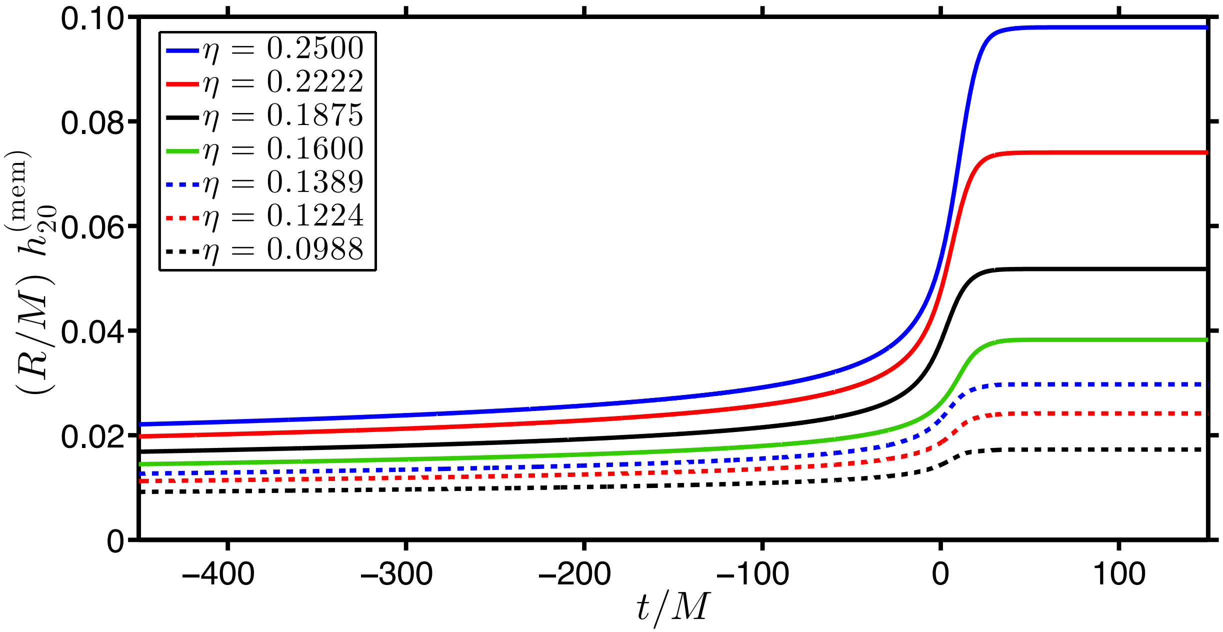
- Spin correction maximized for maximally spinning, aligned binaries.
- Spin terms produce ~20% maximum correction at Schwarzchild ISCO.
- Small-inclination angle case also computed analytically. (Depends on perpendicular spin components.)
- Generic precessing case computed numerically.

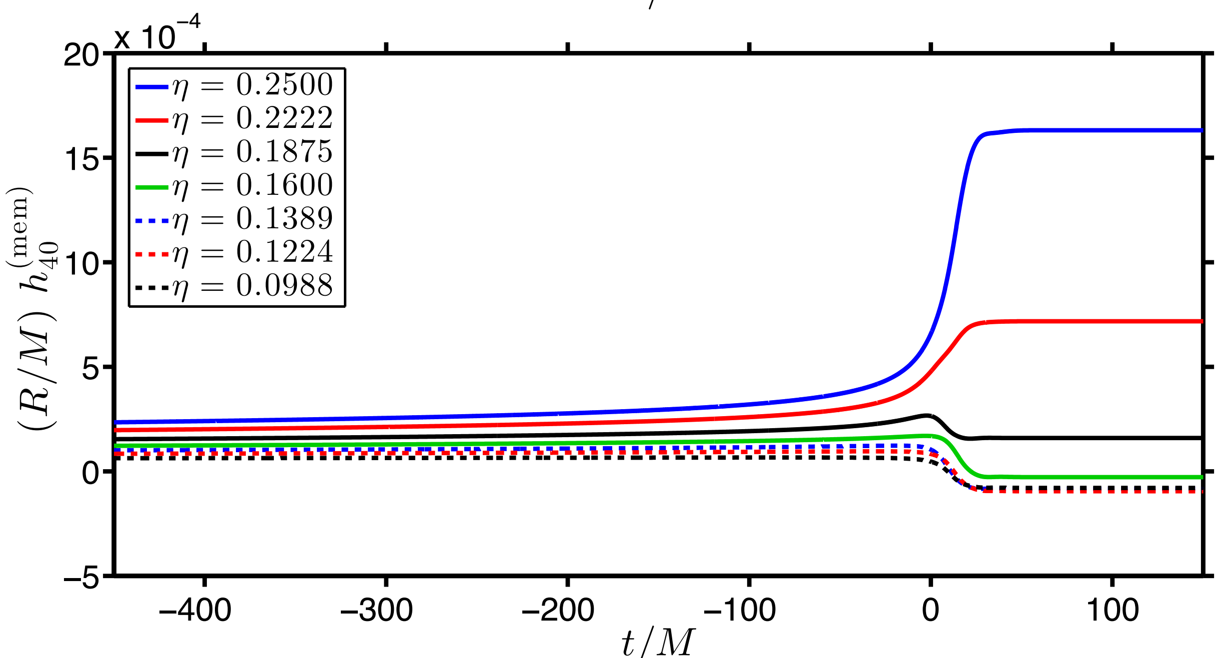


Merger/ringdown memory (nonspinning):

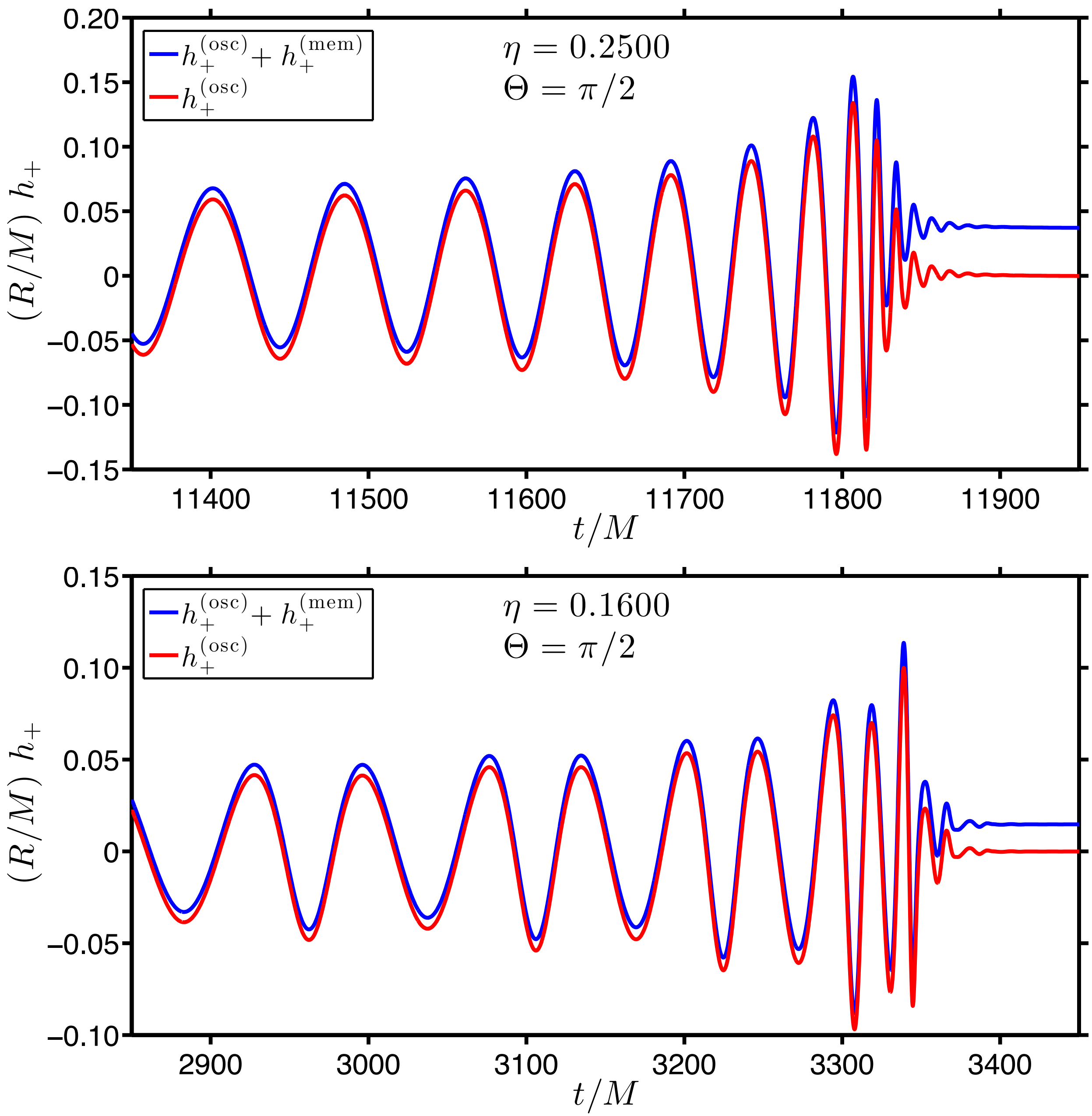
[w/ Goran Dojcinoski]

- Express $m=0$ memory modes in terms of oscillatory modes.
- Use h_{lm} from SXS catalog.
- Match to inspiral memory.



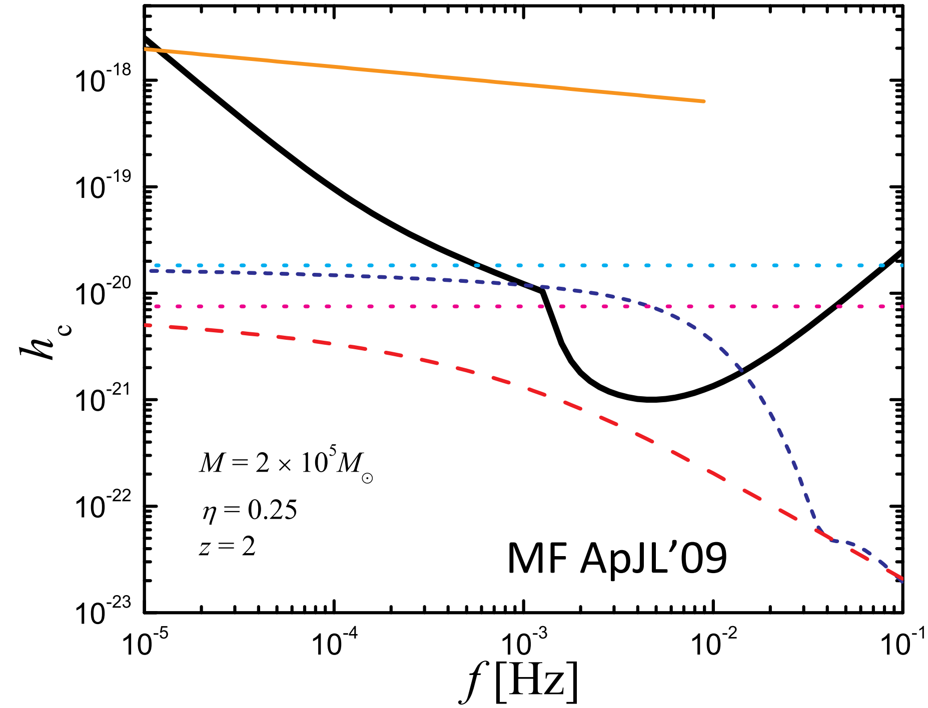


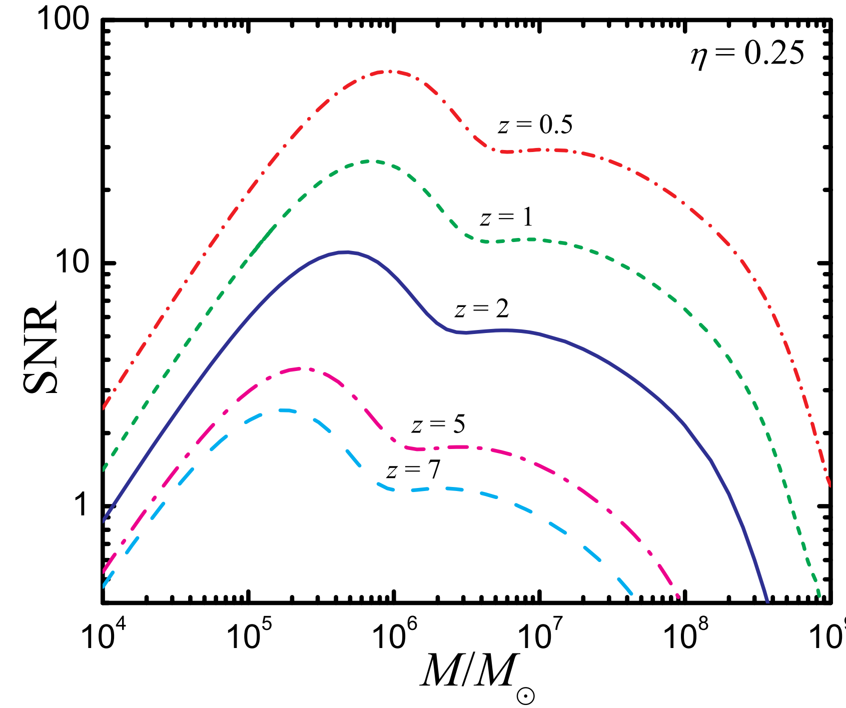
Merger/ringdown memory (nonspinning):



Detectability of memory:

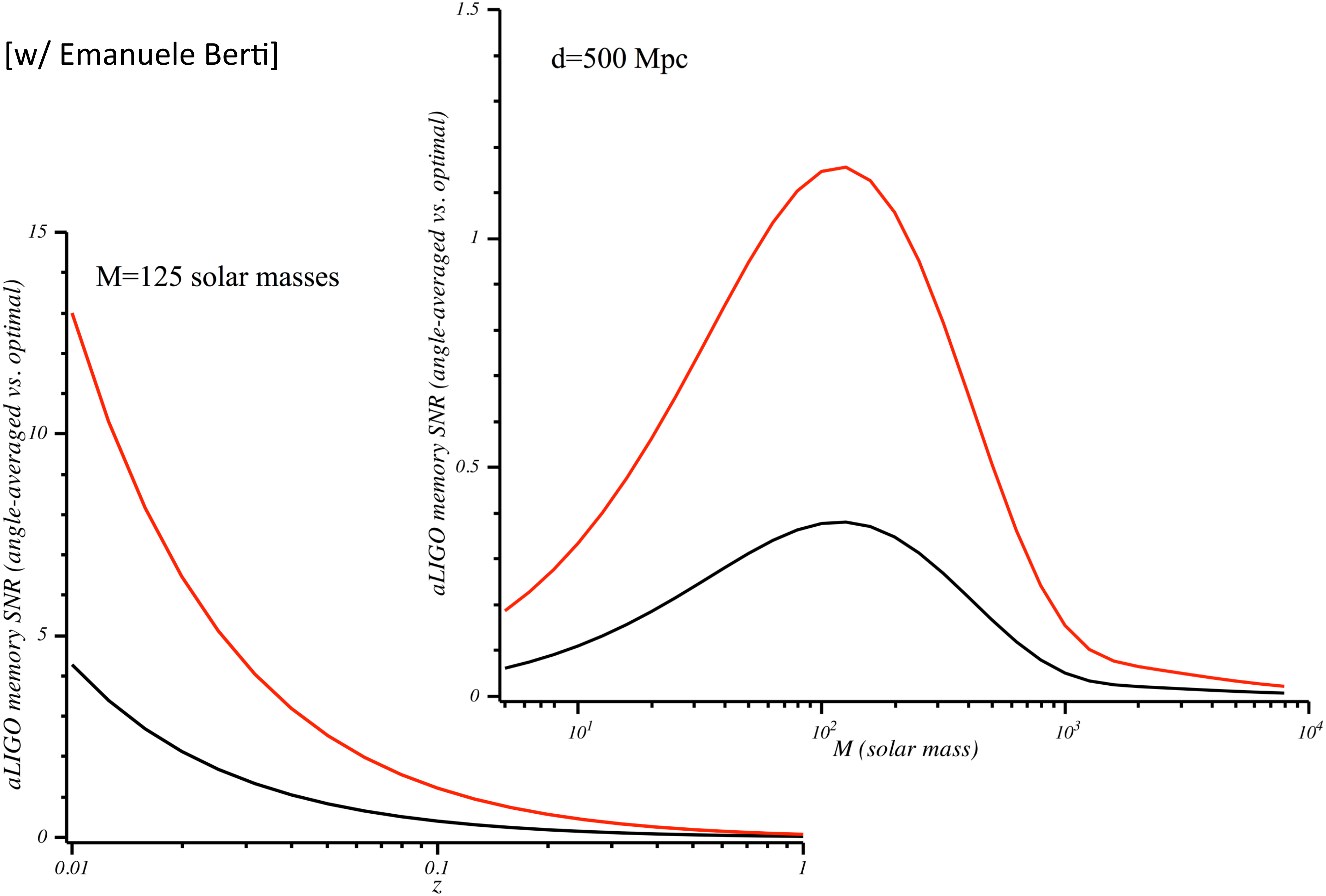
- Use analytic model from MF ApJL '09 to compute SNR for equal-mass case (extension to other mass ratios via new waveforms in progress).
- MF ApJL '09 focused on detectability by LISA. (SMBH memory easily seen to z=2.)
- Also estimated aLIGO SNR of 8 for 100 $M_\odot$ binary at 20 Mpc.
- Next: extend the analysis to future ground-based detectors...



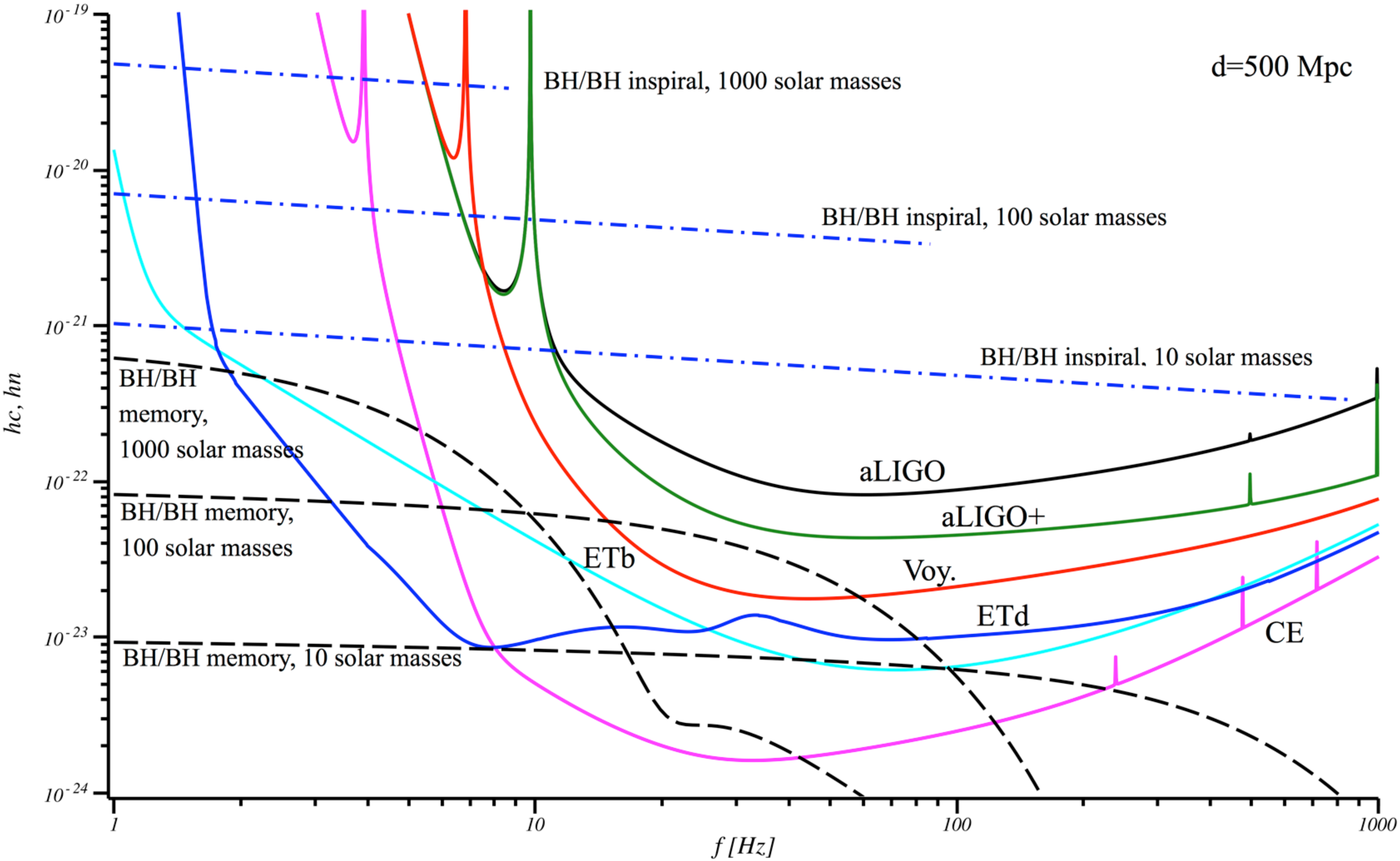


Detectability: aLIGO (preliminary)

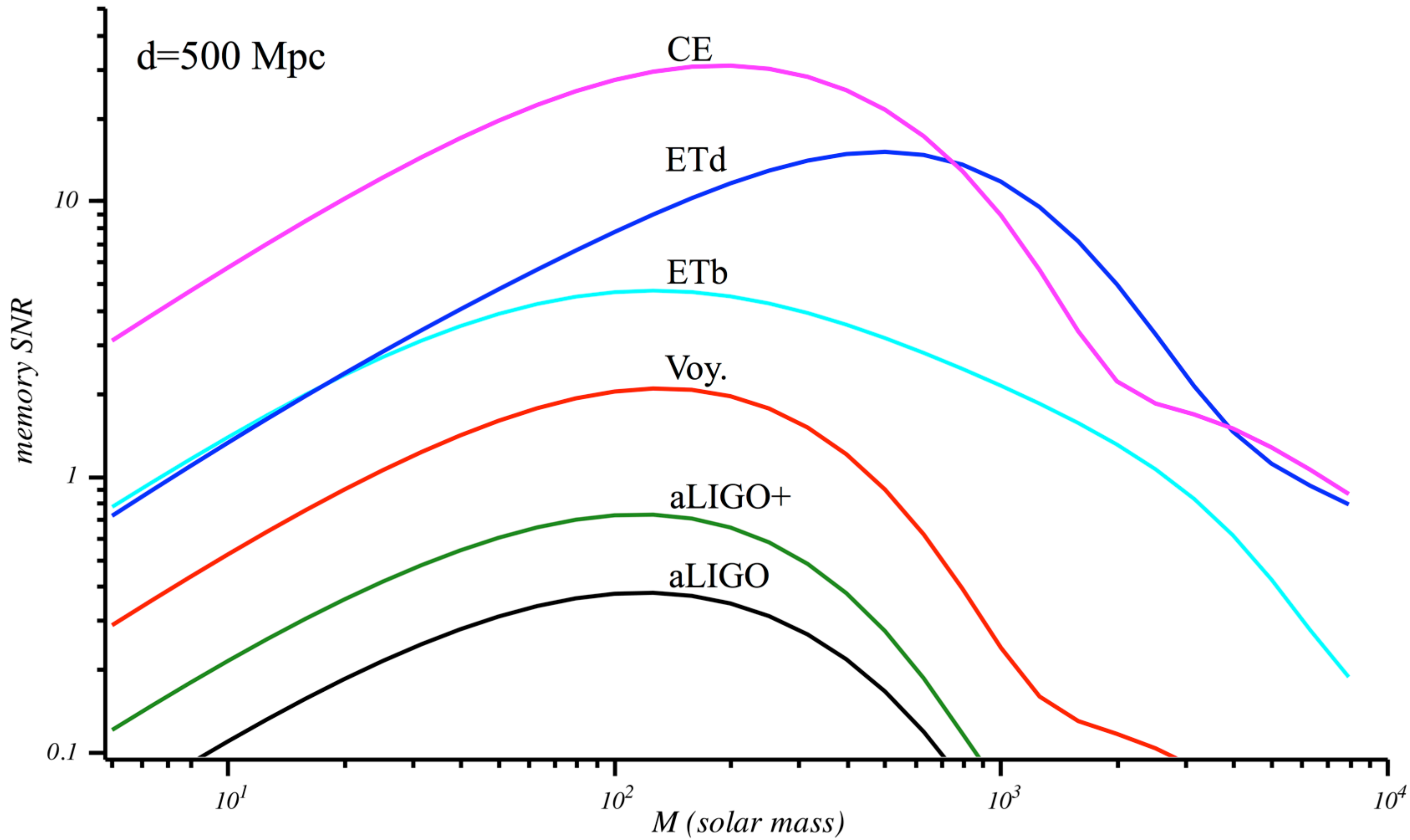
[w/ Emanuele Berti]



Detectability: future ground-based (preliminary)



Detectability: future ground-based (preliminary)



- Good prospects for most sensitive 3rd generation detectors.
- For masses ∼5 to 4000 solar masses, memory SNR ∼O(1%) of inspiral SNR.

Detectability: stacking multiple events

Lasky, et al., PRL '16

- Build evidence for nonlinear memory via stacking of multiple events.
- Need to also measure higher-order modes to break degeneracy w/ polarization angle.

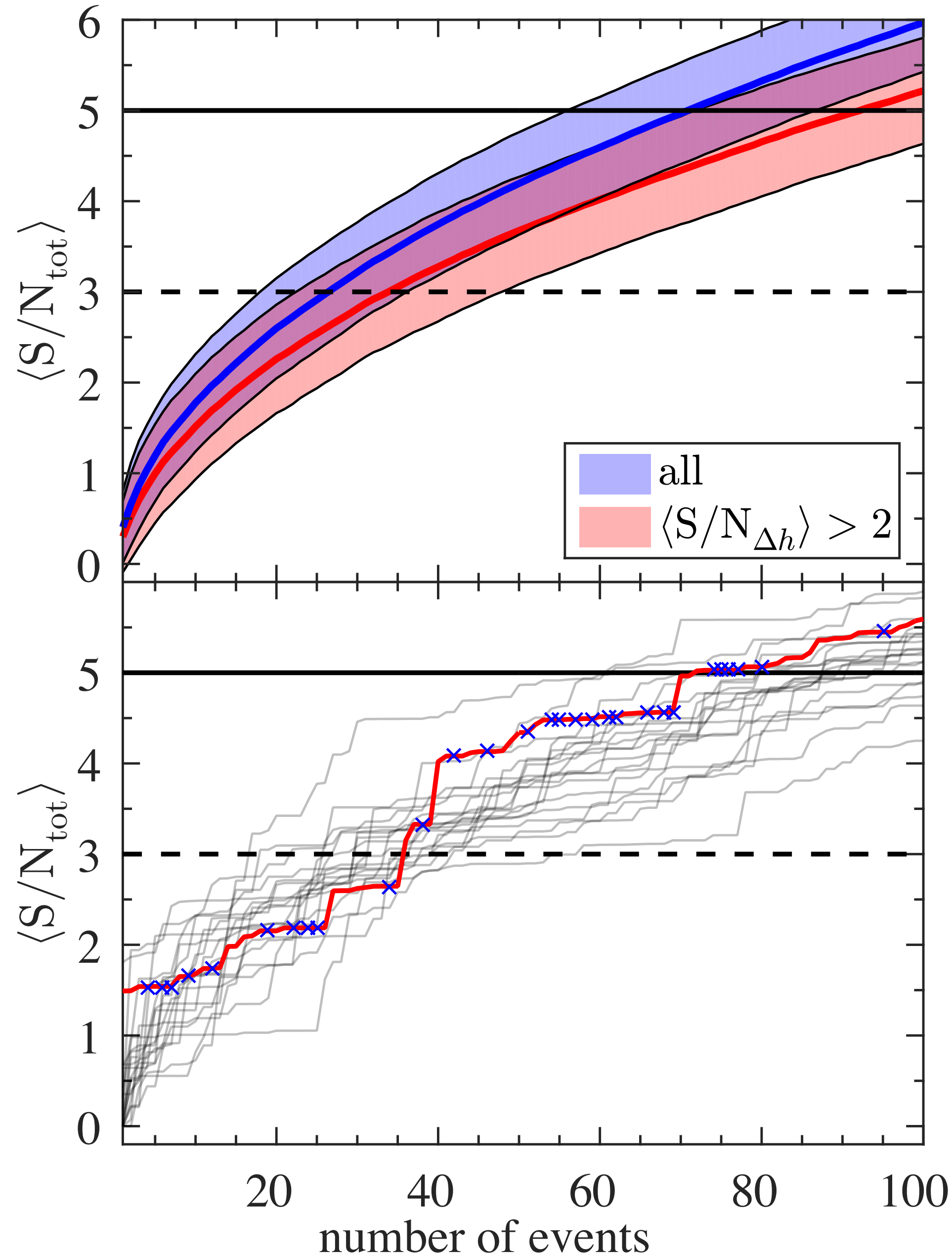
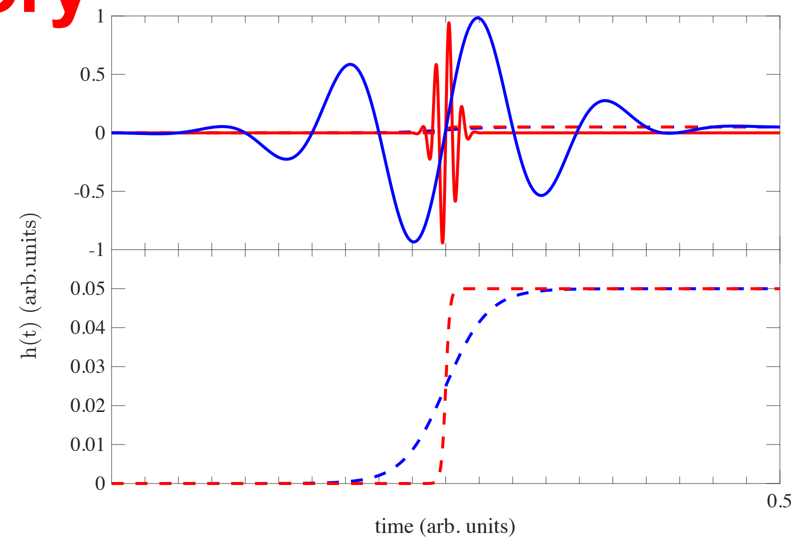


FIG. 3: Evolution of the cumulative signal-to-noise $\langle S/N_{tot} \rangle$ as a function of the number of binary black hole mergers. All binaries have the same distance and mass as the maximum likelihood parameters of GW150914, but have random distributions of inclination, polarisation and sky position. In the top panel, the solid curves represent the expectation value and the shaded region is the one-sigma uncertainties. The blue curve sums the memory signal-to-noise contribution from *all* binaries, and the red curve assigns memory $\langle S/N \rangle = 0$ for those binaries where the polarisation angle, and hence the sign of the memory cannot be determined. The bottom panel shows 20 individual realisations of the red curve in the top panel. One particular realisation is highlighted in red; the binaries assigned $\langle S/N \rangle = 0$ are shown with blue crosses. In both panels, the horizontal dashed and solid lines show $\langle S/N_{tot} \rangle = 3$ and 5 respectively.

Detectability: "orphan" memory

McNeill, Thrane, Lasky '17

- High frequency detectors could detect bursts from exotic sources (DM collapse in stars, ...)
- Memory component of high-frequency burst could be in the LIGO band.



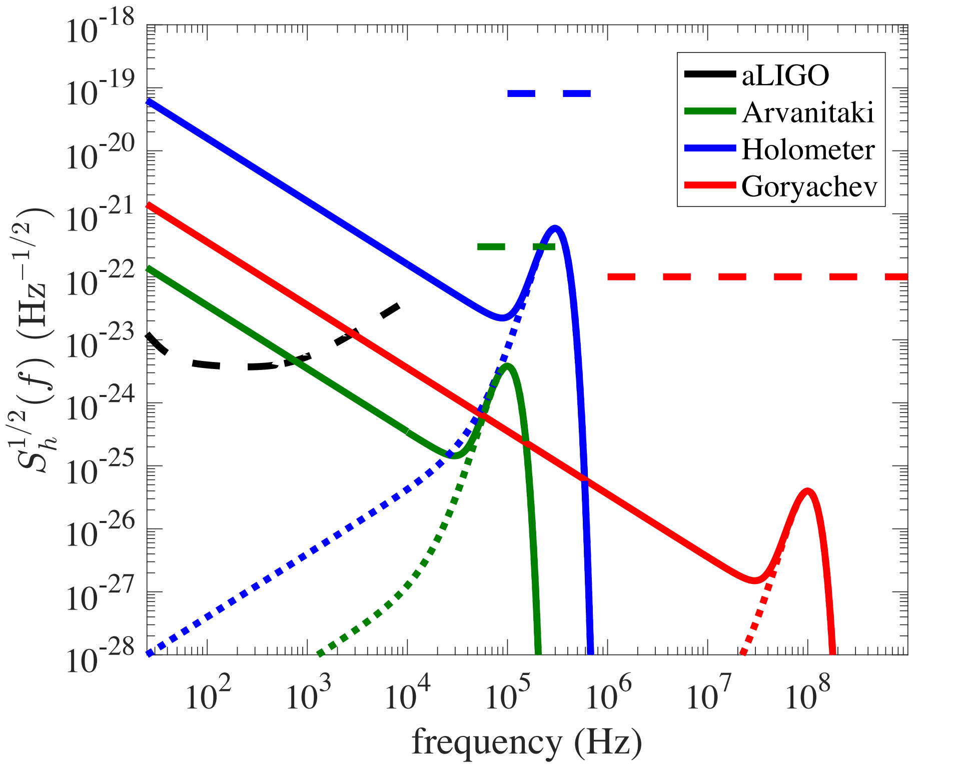


FIG. 3: Strain amplitude spectral density. The dashed curves represent the noise in three different detectors: Advanced LIGO (black) and three dedicated high-frequency detectors (colored). For each dedicated detector, we plot the amplitude spectral density for a sine-Gaussian burst in the middle of the observing band (colored dotted peaks). The peak height is tuned so that the oscillatory burst can be observed with a signal-to-noise ratio $\langle S/N \rangle = 5$. The solid colored lines shows the amplitude spectral density when we include the memory calculated with our fiducial value of κ. The memory bursts produce large signals in Advanced LIGO, with $\langle S/N \rangle$ ranging from 300 to 10^5.

Spin memory:

- Motivated by papers: Strominger, Zhiboedov, Pasterski.
- Related works by Flanagan & Nichols, Madler & Winicour
- Recent paper by Nichols '17 explains "spin memory" in PN context:

It is the *"nonlinear, nonhereditary memory"* discussed in MF PRD '09 & originally found in 2.5PN order amplitude correction of Arun et al '04.

Current multipole moments can also source linear and nonlinear memory effects—these are (I think) what the more recent literature refers to as "spin memory":

$$\mathcal{U}_L = \mathcal{I}_L^{(l)} + U_L^{(\mathrm{tail})} + U_L^{(\mathrm{nonlin.\ mem})} + \cdots$$

$$\mathcal{V}_L = \mathcal{J}_L^{(l)} + V_L^{(\mathrm{tail})} + V_L^{(\mathrm{nonlin.\ spin\, mem})} + \cdots$$

Leads to polarization correction [Arun et. al.'04]

$$\begin{aligned}\mathcal{V}_{ijk}(T_R) = \mathcal{S}^{(3)}_{ijk}(T_R) + \frac{G}{c^3}\Big\{ 2\mathcal{M}\int_{-\infty}^{T_R}\Big[\ln\Big(\frac{T_R-\tau}{2\tau_0}\Big)+\frac{5}{3}\Big] \\ \times \mathcal{S}^{(5)}_{ijk}(\tau)d\tau + \frac{1}{10}\epsilon_{ab<i}\mathcal{M}^{(5)}_{j\underline{a}}\mathcal{M}_{k>b} \\ - \frac{1}{2}\epsilon_{ab<i}\mathcal{M}^{(4)}_{j\underline{a}}\mathcal{M}^{(1)}_{k>b} - 2\mathcal{S}_{<i}\mathcal{M}^{(4)}_{jk>}\Big\} + O(5),\end{aligned} \qquad (5.2)$$

where

$$\mathcal{S}_i = \mathcal{J}_i = \eta M \epsilon_{iab} x_a v_b + O(c^{-2}), \qquad (5.3)$$

$$h_\times^{(\mathrm{nonlin.\ spinmem})} = -\frac{12}{5}\eta^2\frac{M}{R}x^{7/2}\sin^2\Theta\cos\Theta$$

Detection is difficult (need multiple event and ET) since this is a 2.5PN effect instead of a 0PN effect (like the hereditary nonlinear memory). [See Nichols '17]

[MF PRD '09]

Summary:

- Linear and nonlinear memories are interesting non-oscillatory components to the gravitational-wave signal.

- Linear memory has the potential to tell us about non-periodic sources (binary scattering, supernovae, GRB jets, ...)

- Nonlinear memory let's us probe nonlinear wave generation in GR ("waves that produce waves").

- Detection of linear memory relies on "getting lucky" with a nearby source.

- Nonlinear memory from BBH mergers is clearly detectable by 3rd generation detectors or LISA;
potentially within reach of LIGO with ~100 detections.

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