Spin-Precessing Compact Binaries: Gravitational Wave Modeling and Information Extraction

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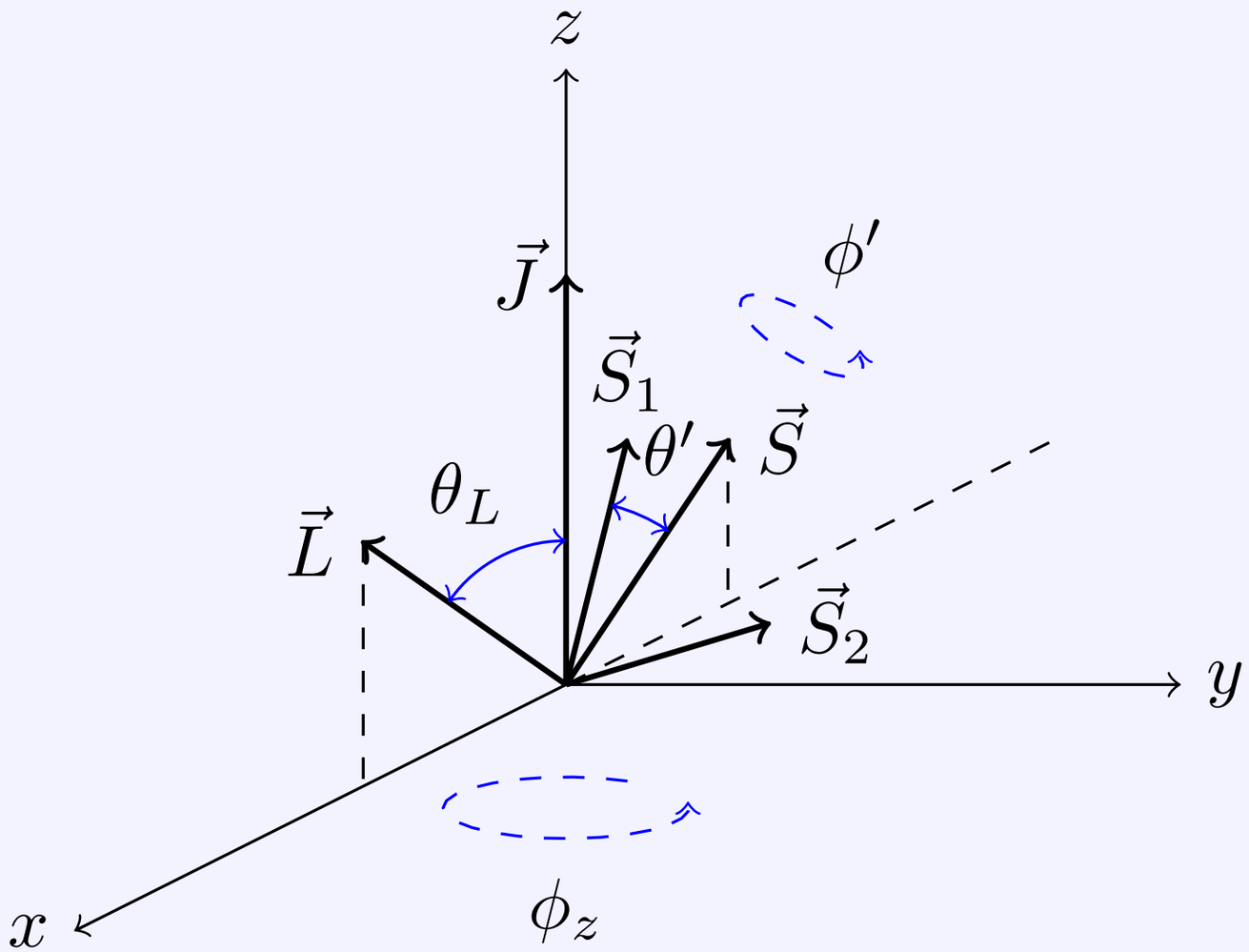
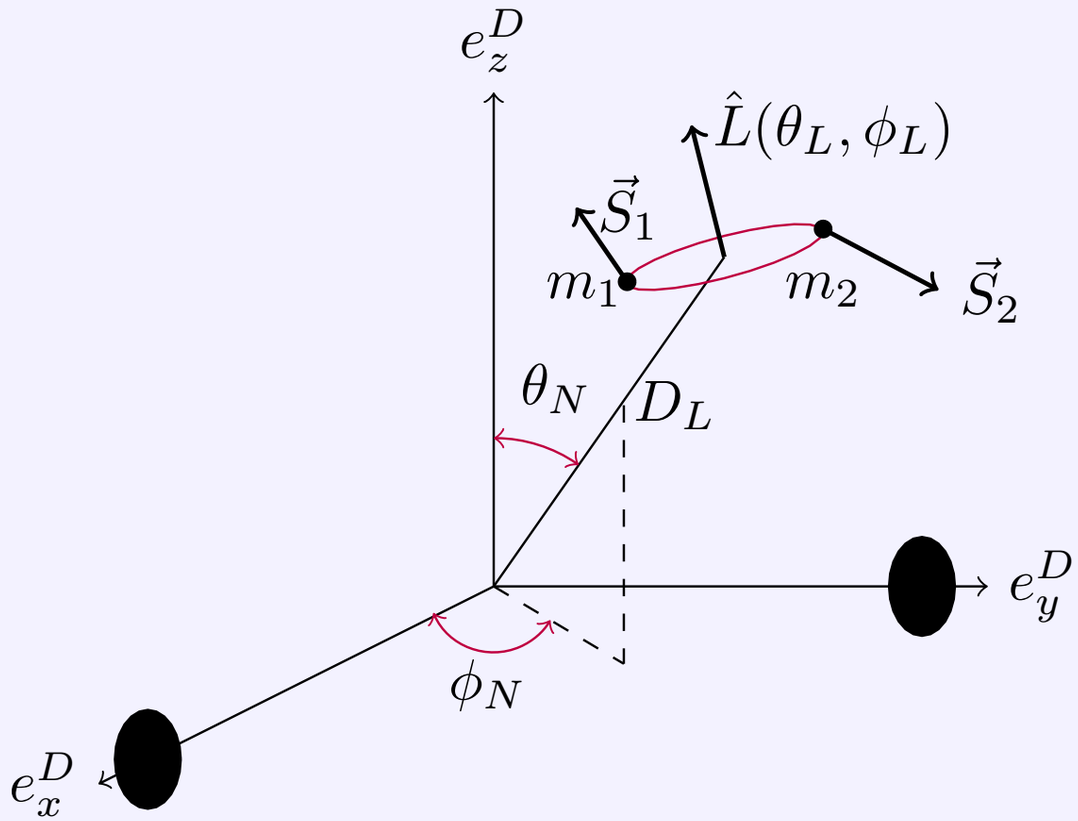
Ole Miss, StronG BaD Workshop, 28/2/2017

Analytic Precessing Waveform



Kesden+ (arxiv:1411.0674)

Modeling Precessing Systems



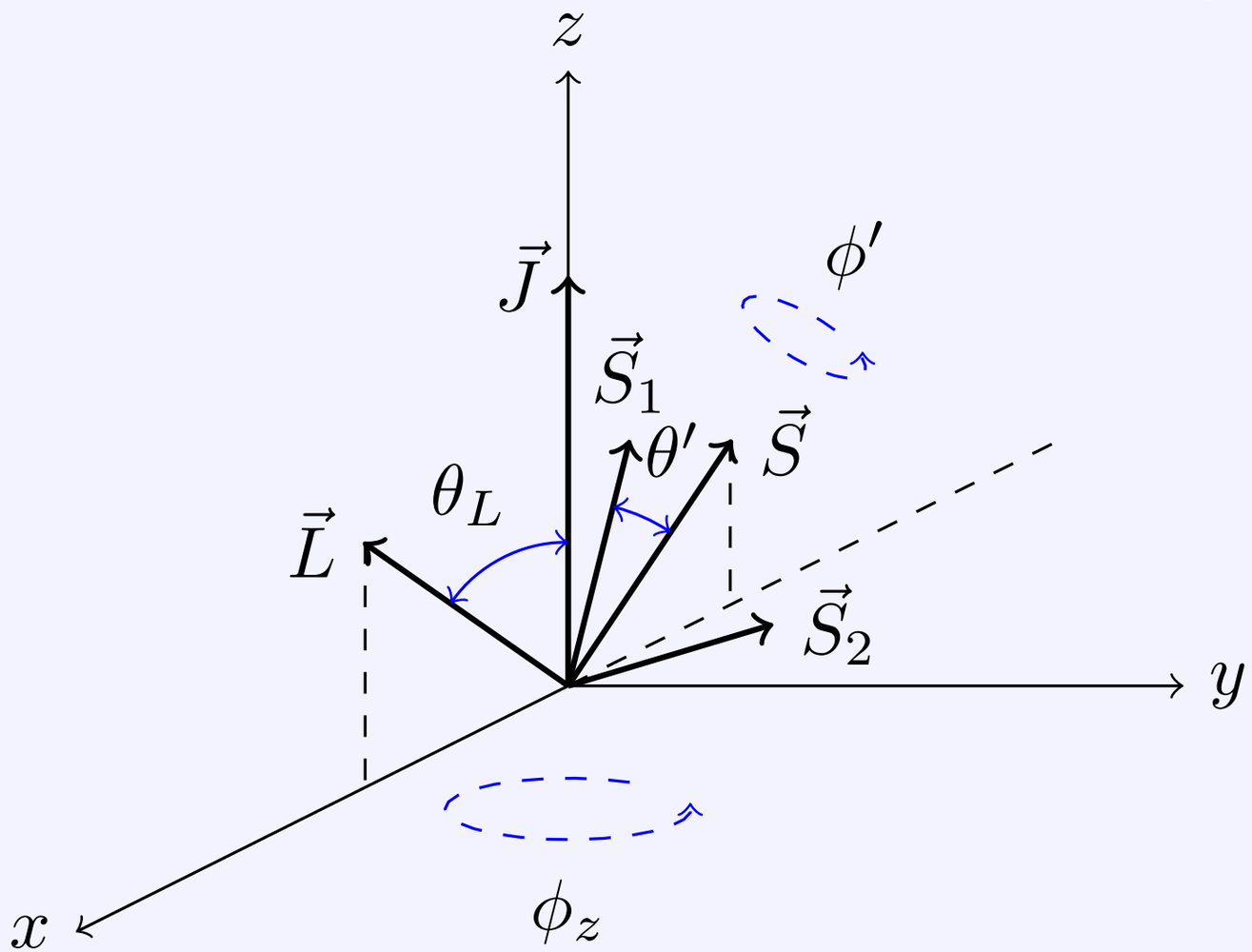
Radiation reaction is a slowly evolving perturbation of precession

$$\dot{\hat{L}} \sim \omega^2(\vec{S}_1 \times \vec{L}) + \omega^2(\vec{S}_2 \cdot \hat{L})(\vec{S}_1 \times \hat{L}) + 1 \leftrightarrow 2$$

$$\dot{\vec{S}}_1 \sim \omega^2(\vec{L} \times \vec{S}_1) + \omega^2(\vec{S}_2 \cdot \hat{L})(\hat{L} \times \vec{S}_1) + \omega^2(\vec{S}_2 \times \vec{S}_1)$$

$$\dot{\omega} \sim \omega^{11/3}(1 + \omega^{2/3} + ...)$$

Ignoring Radiation Reaction



$$\vec{c} = (\vec{J}, L, S_1, S_2, S_{eff}) = \text{const}$$

$$\Downarrow$$

$$\vec{L}(S(t); \vec{c})$$

$$\Downarrow$$

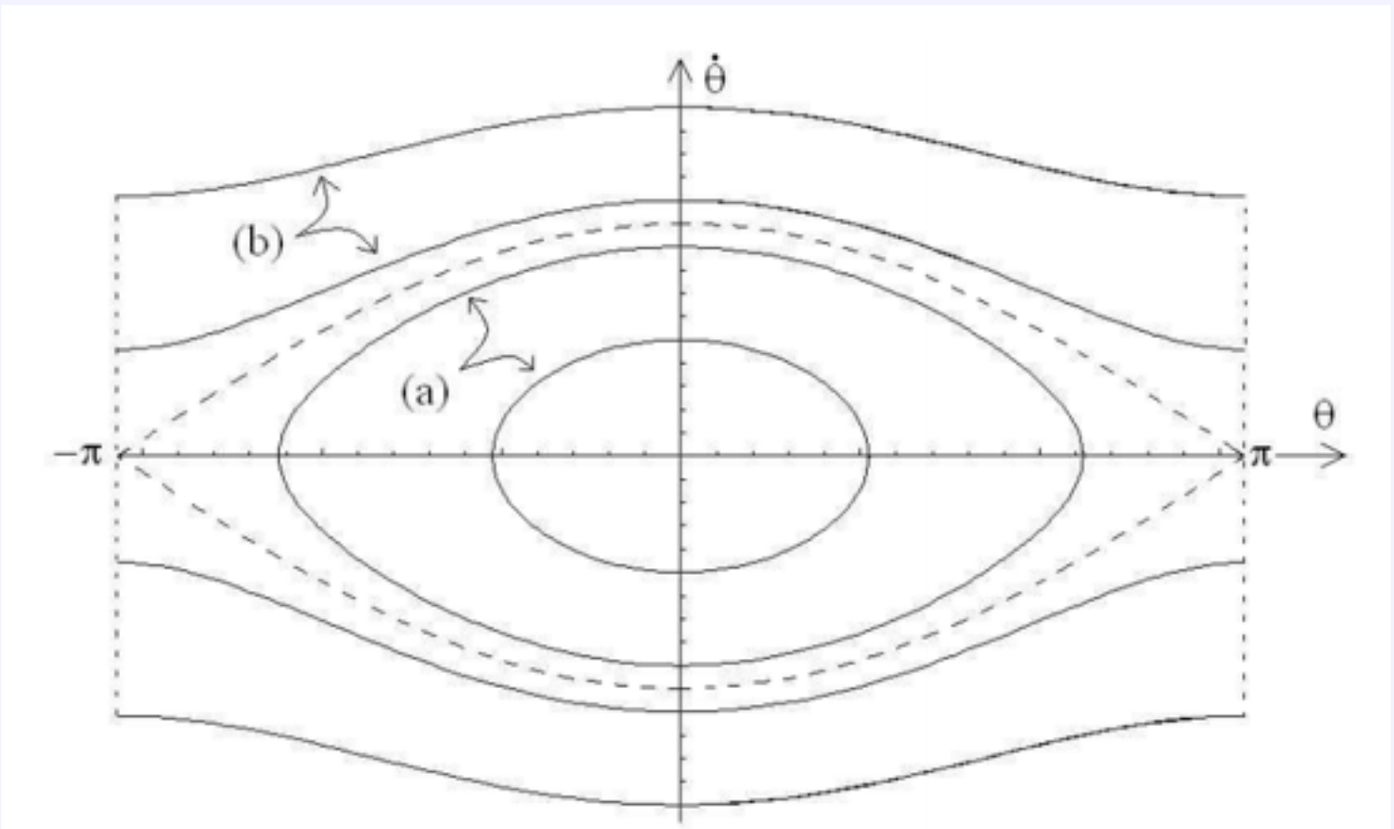
$$R\left[\hat{z}, \phi_z(t)\right] \vec{L}(S(t), \vec{c})$$

Kesden+ (arxiv:1411.0674)

Planar Pendulum

$$\left(\frac{dS^2}{dt}\right)^2 \sim (S^2 - S_+^2)(S^2 - S_-^2)(S^2 - S_3^2)$$

$$S^2(t) = S_+^2 + (S_-^2 - S_+^2)\,\mathrm{sn}^2(\psi(t), m)$$



Brizard (arxiv:0711.4064)

Stop Ignoring Radiation Reaction

$$\dot{\hat{L}} \sim \omega^2(\vec{S}_1 \times \vec{L}) + \omega^2(\vec{S}_2 \cdot \hat{L})(\vec{S}_1 \times \hat{L}) + 1 \leftrightarrow 2$$

$$\dot{\vec{S}}_1 \sim \omega^2(\vec{L} \times \vec{S}_1) + \omega^2(\vec{S}_2 \cdot \hat{L})(\hat{L} \times \vec{S}_1) + \omega^2(\vec{S}_2 \times \vec{S}_1)$$

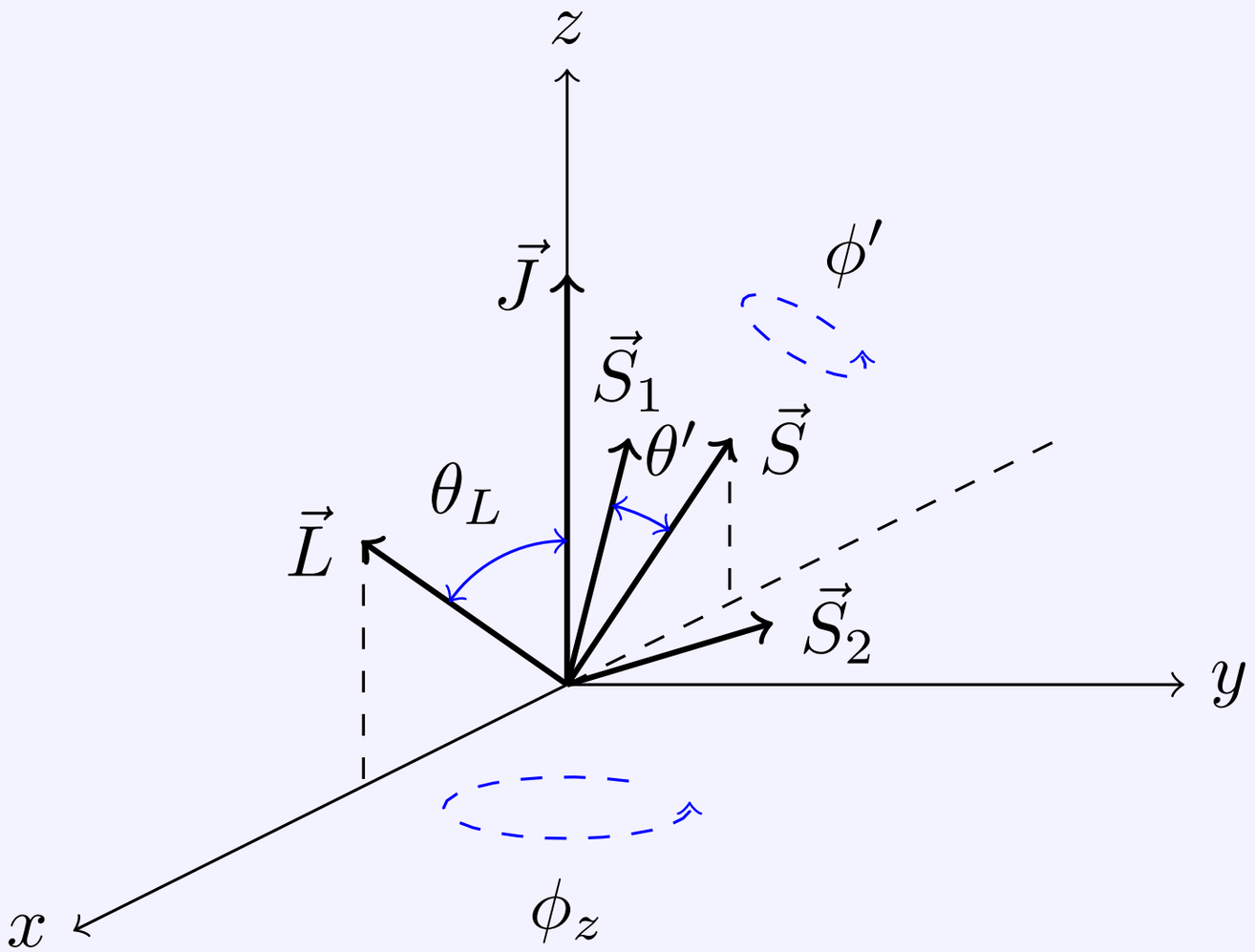
$$\dot{\omega} \sim \omega^{11/3}(1 + \omega^{2/3} + ...)$$

$$\frac{t_{pr}}{t_{rr}} \sim (v/c)^3$$

Work perturbatively

Coordinate System

Does our coordinate system even make sense now?



$$\langle \dot{\hat{J}} \rangle_{pr} = 0$$

$$\Downarrow$$

$$R\left[\hat{z}, \phi_z(t)\right] \vec{L}(S(t), \vec{c})$$

Precession Phase

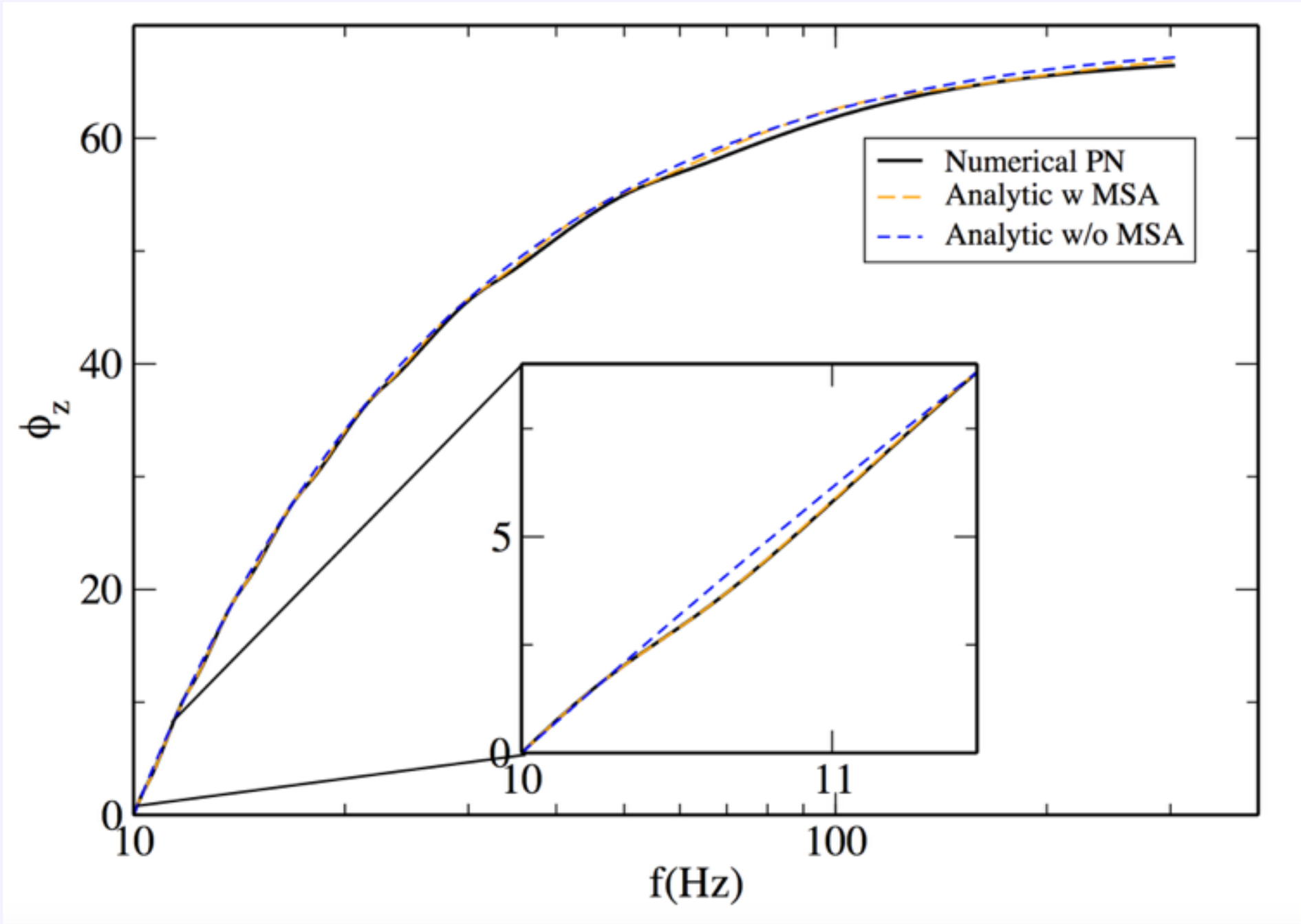
$$\phi_z(t) = \int \Omega_z(S(t), \vec{c}(t))dt$$

t_{pr} t_{rr}

$$\underbrace{\int \langle \Omega_z(t_{rr}) \rangle_{pr} \, dt_{rr}}_{\mathcal{O}(t_{rr}/t_{pr})} + \underbrace{\int \Omega_z(S(t_{pr}), \vec{c}(t_{rr}))dt_{pr} - \int \langle \Omega_z(t_{rr}) \rangle_{pr} dt_{pr}}_{\mathcal{O}(1)}$$

A. Pound

Precession Phase



$$(m_1, m_2) = (10, 5)M_\odot \quad (\chi_1, \chi_2) = (0.7, 0.6) \quad (\theta_1, \theta_2) = (60^\circ, 45^\circ)$$

Building the Waveform

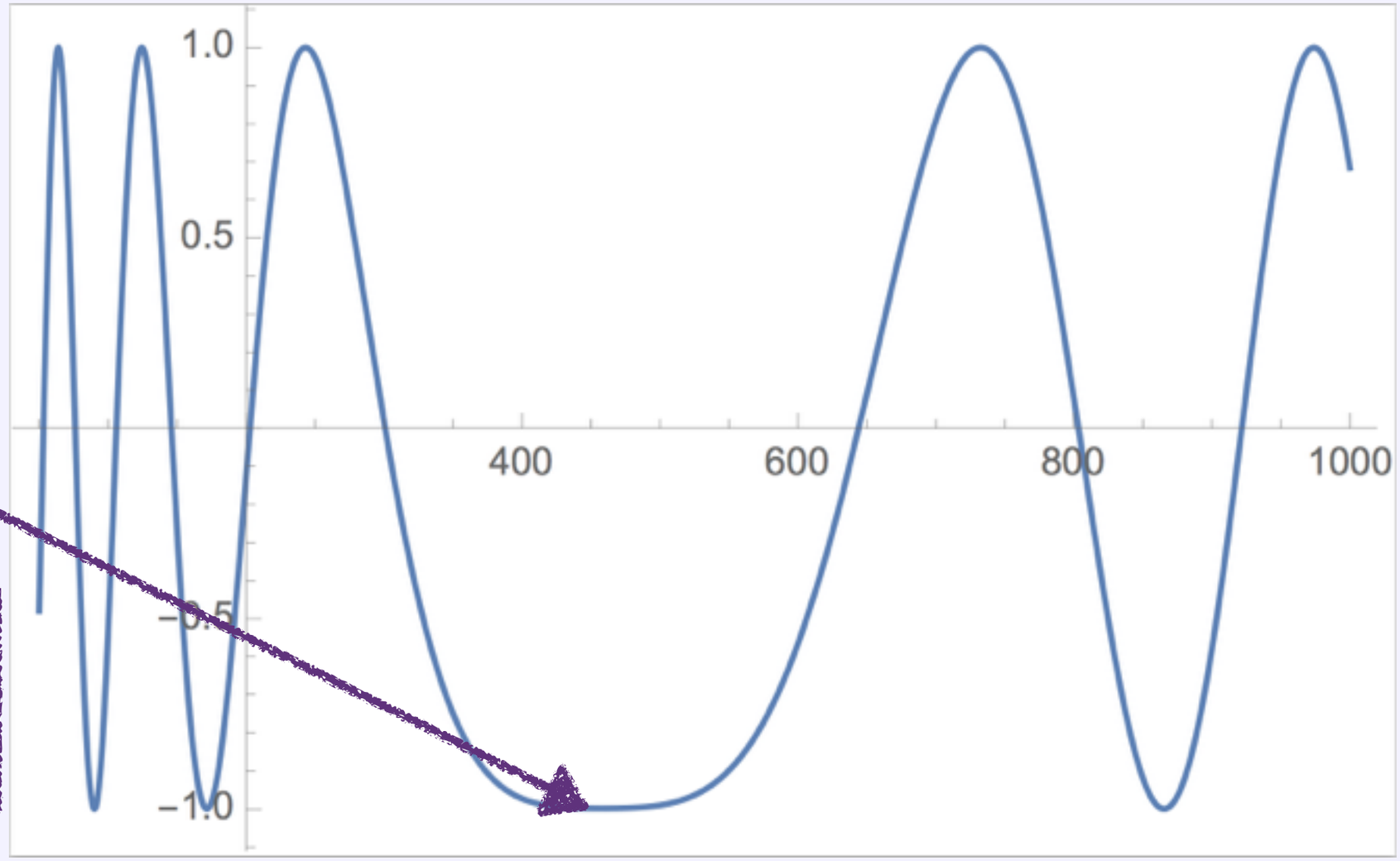
$$\tilde{h}(f) = \int h(t) e^{2\pi i f t} dt$$

$$\sim \int \underbrace{\omega^{2/3}} \overbrace{(1+\cos^2\iota) e^{2\pi i f t - 2i\Phi^{orb}}}^{\text{rapidly-varying phase}} dt$$

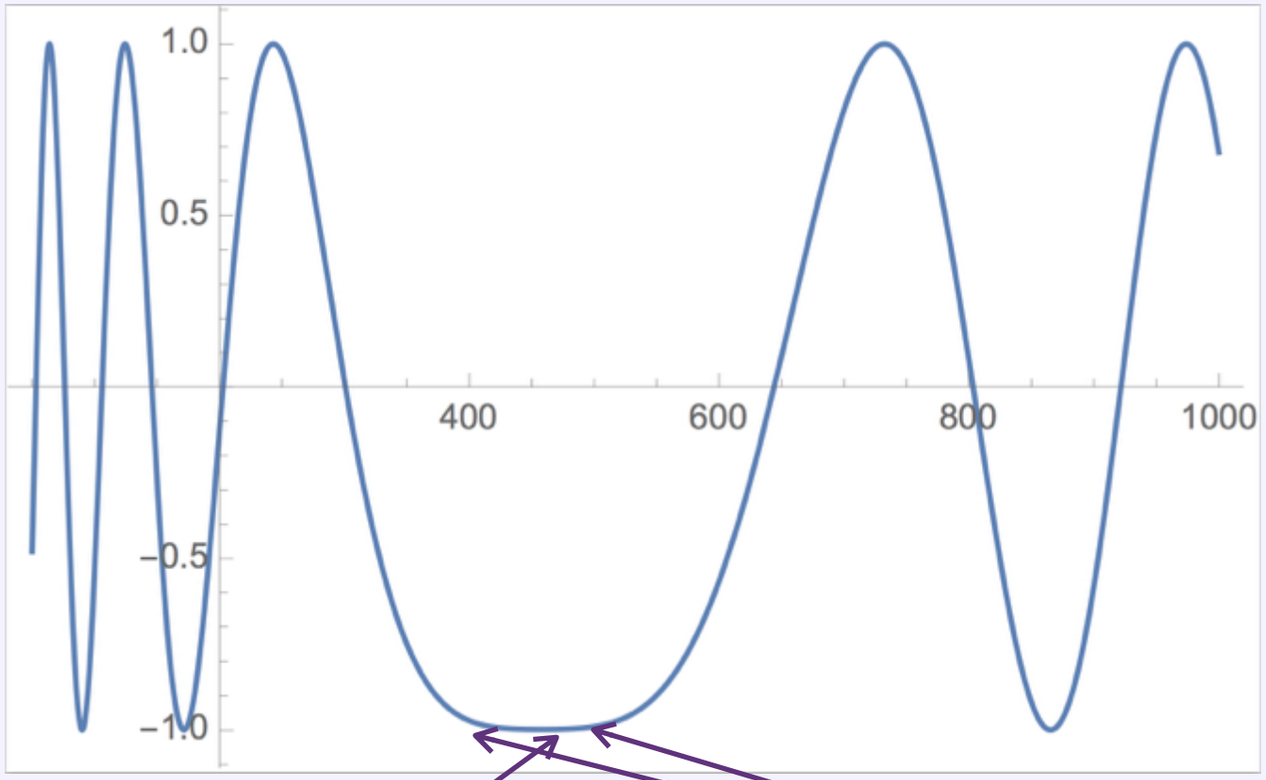
slowly-varying amplitude

Approximate the integral by the contribution of the stationary point only

Stationary Phase Approximation



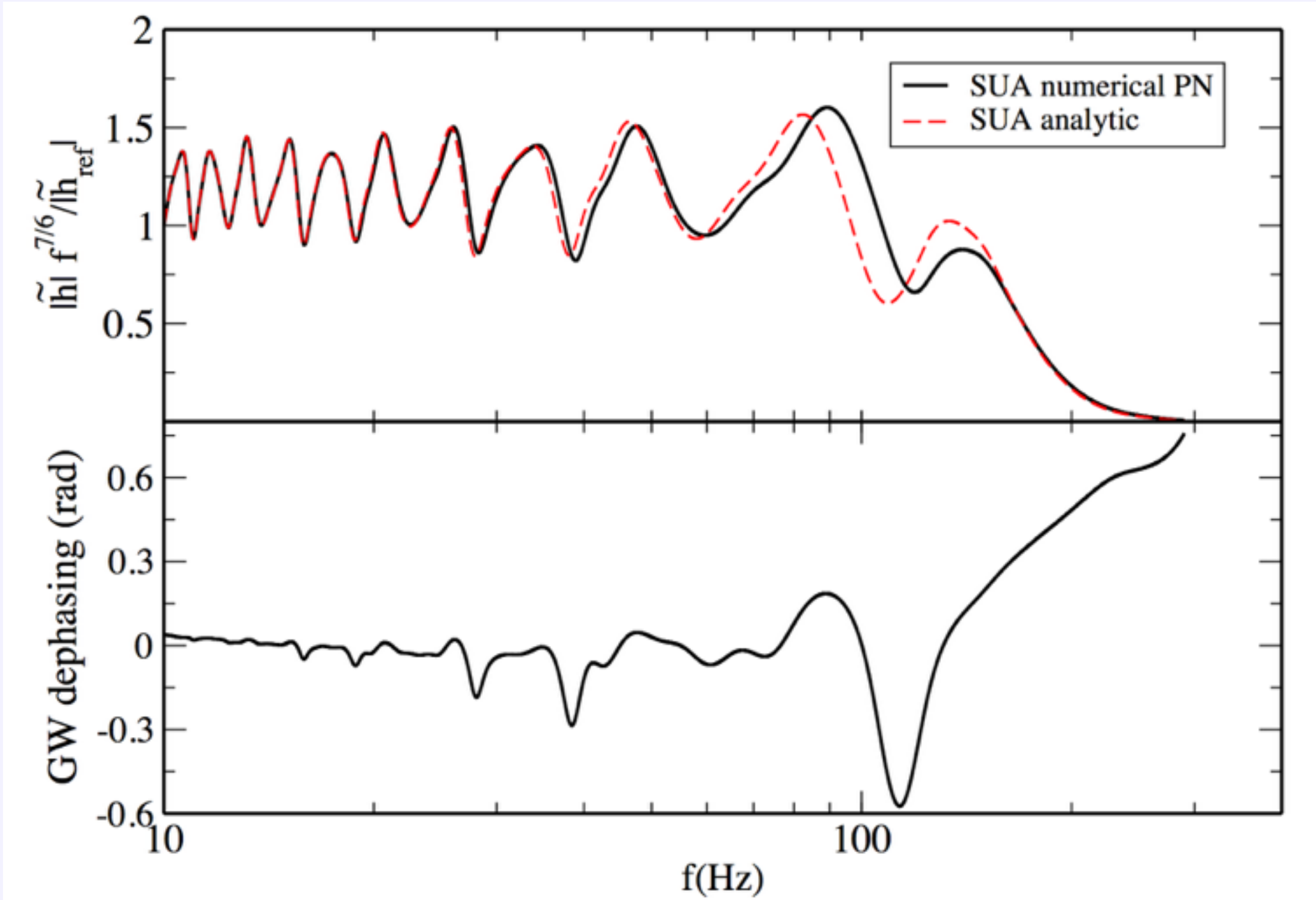
Shifted Uniform Asymptotics



$$\tilde{h}(f) \sim \underbrace{T e^{2\pi i f t_{SP} - 2i\Phi^{orb}}}_{\text{Non precessing SPA}} \underbrace{\sum_{k=-k_0}^{k=k_0} \mathcal{H}(t_{SP} + kT)}_{\text{Precessional modulations}}$$

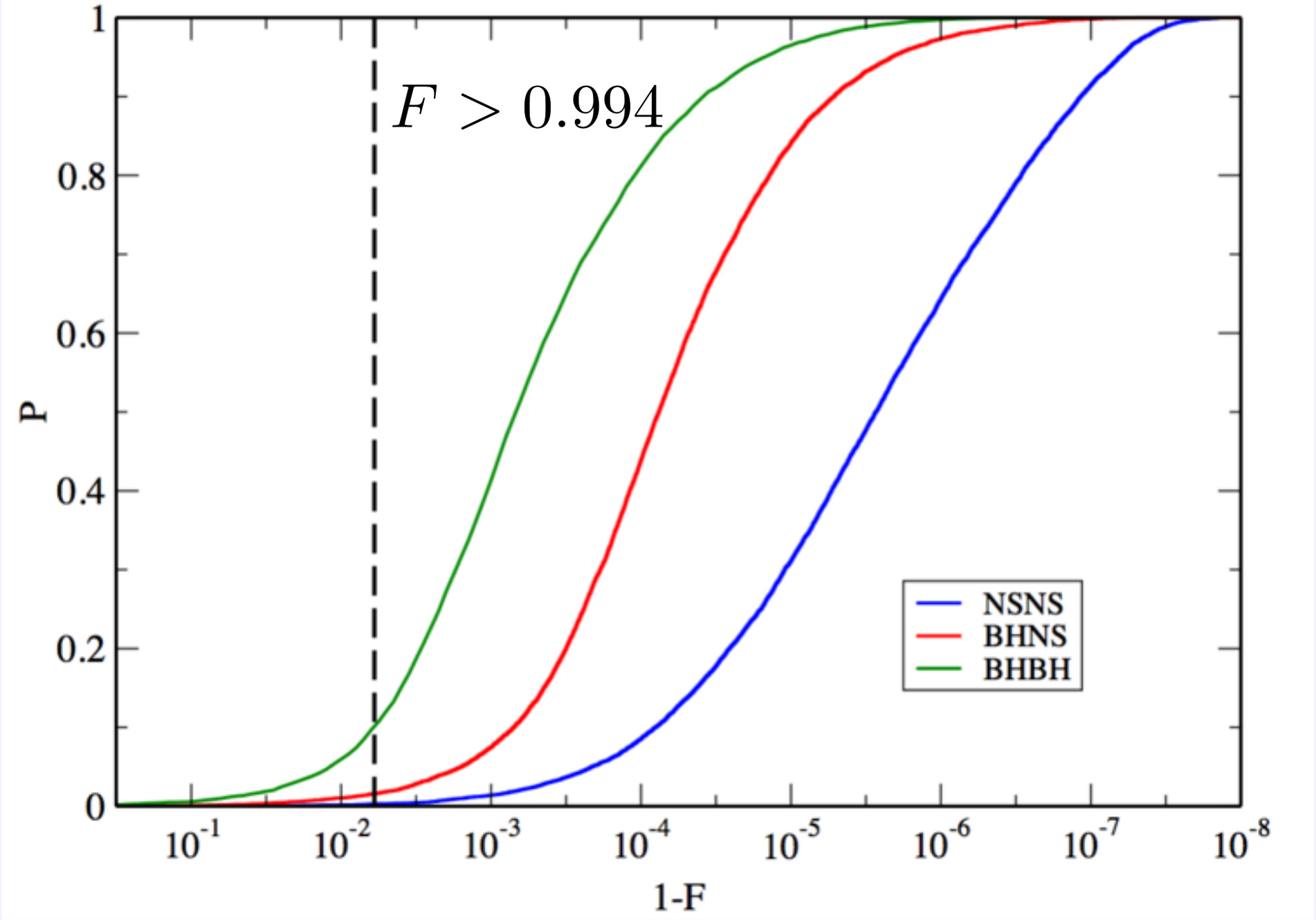
Klein+ (arXiv:1408.5158)

Waveform



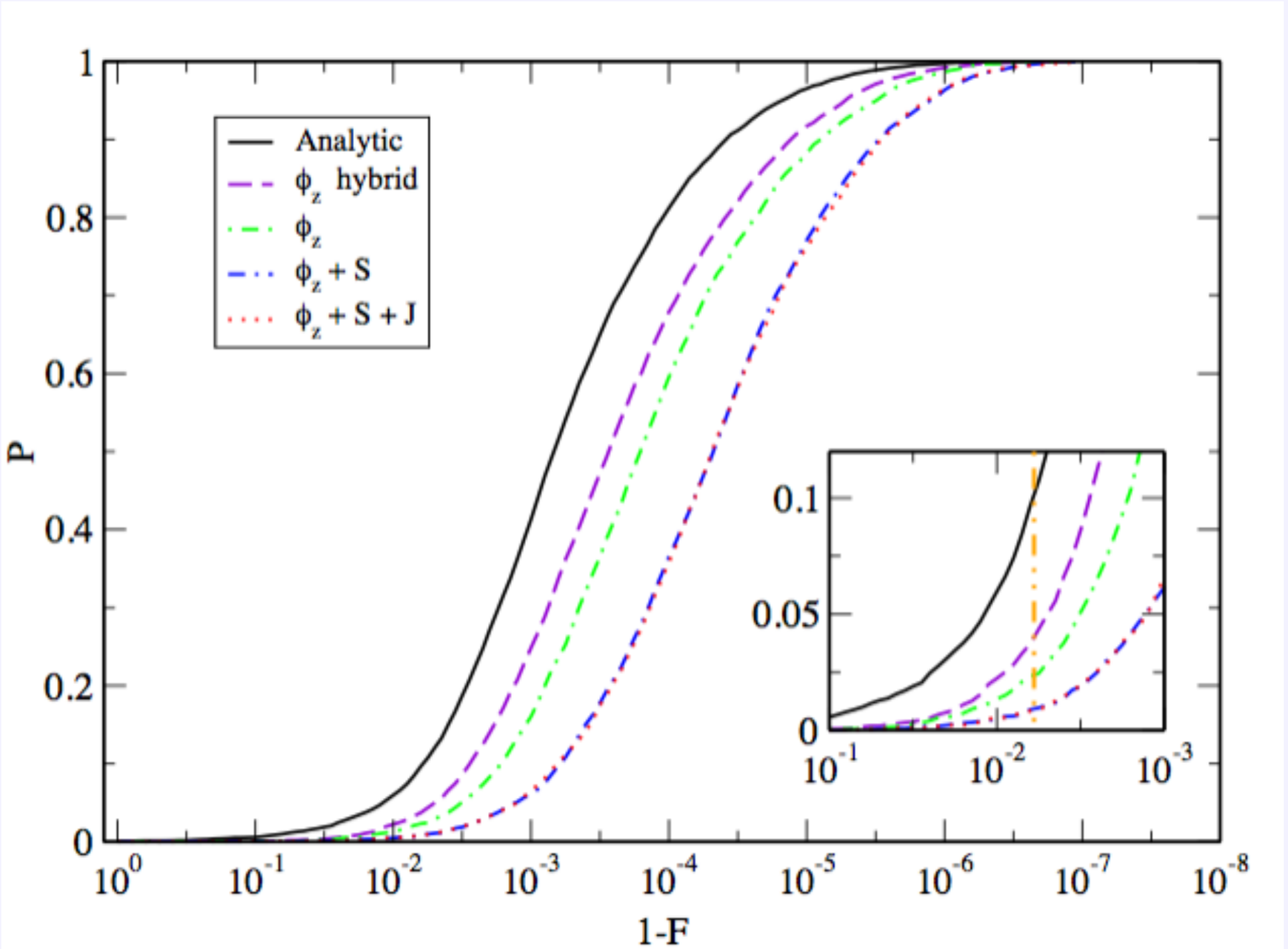
$$(m_1, m_2) = (10, 5)M_\odot \quad (\chi_1, \chi_2) = (0.7, 0.6) \quad (\theta_1, \theta_2) = (60^\circ, 45^\circ)$$

Faithfulness $F \sim \langle h_1 | h_2 \rangle$



NSNS ~0.3% BHNS ~1.6% BHBH ~10%

Analytic Control



Analytic 10%

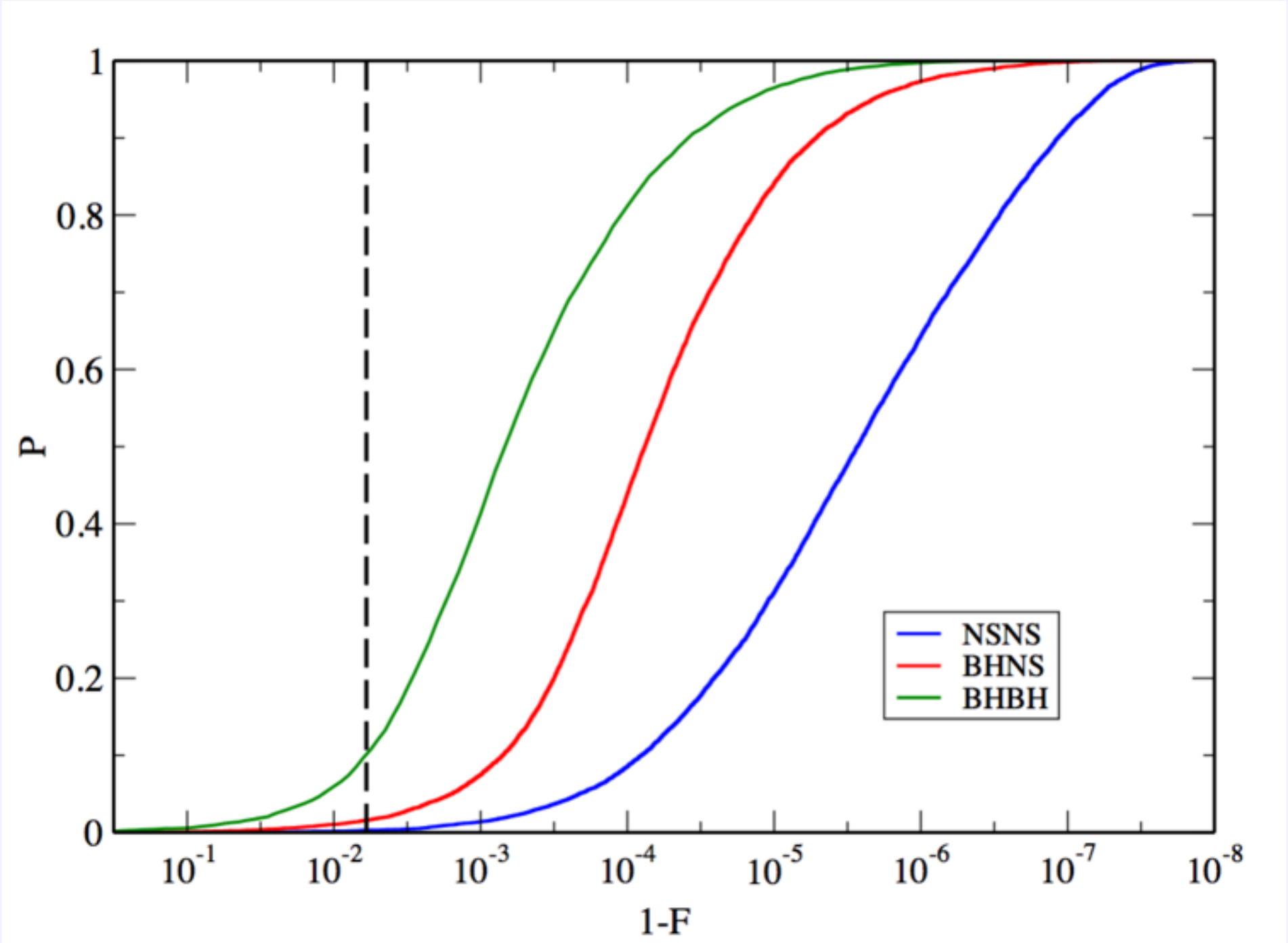
Precession phase integral 4%

Precession phase full 2%

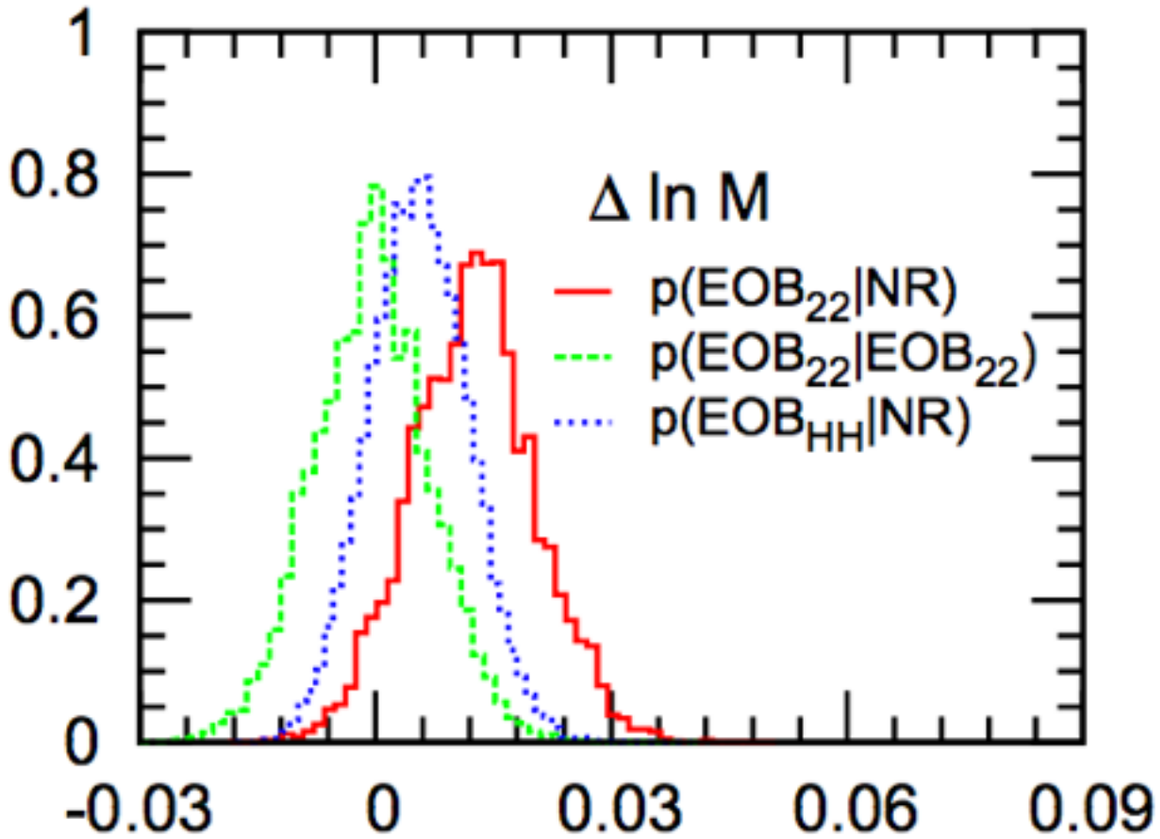
S Magnitude 1%

J Magnitude 1%

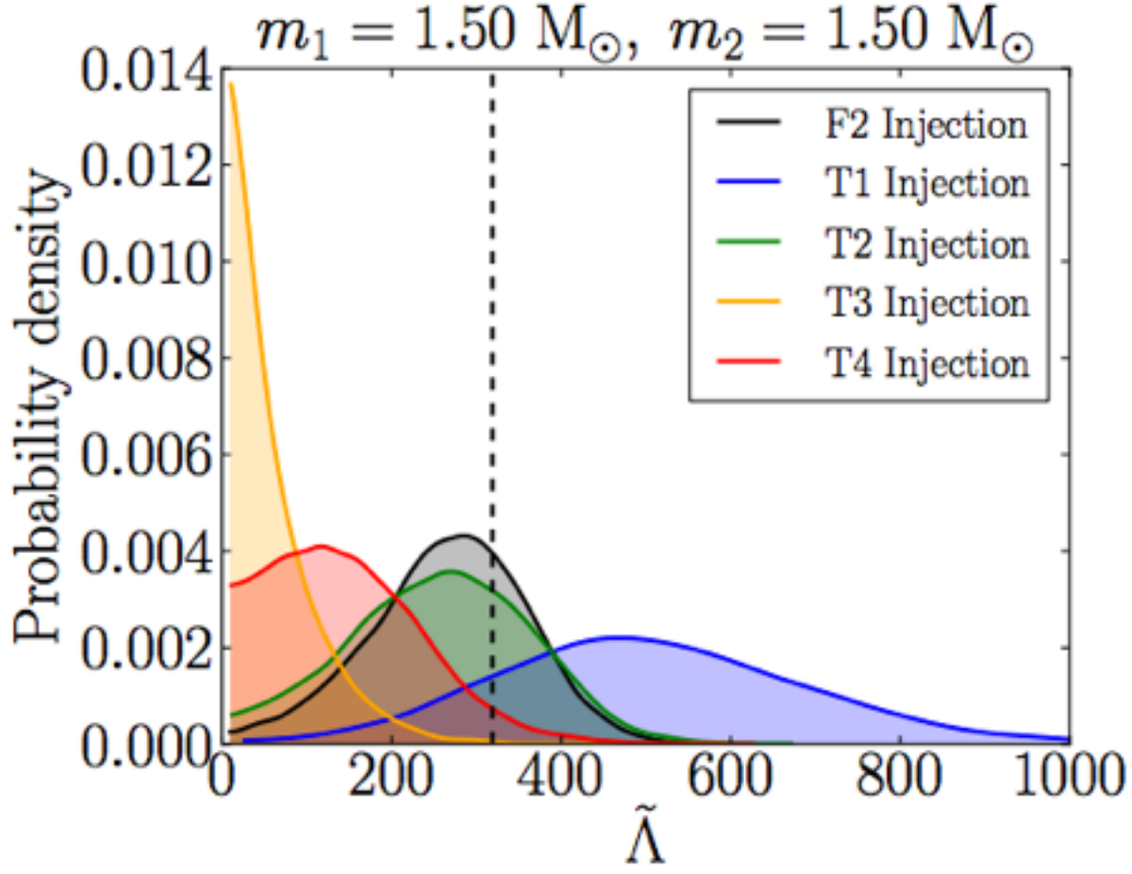
Assessing Systematics



How good is a waveform?

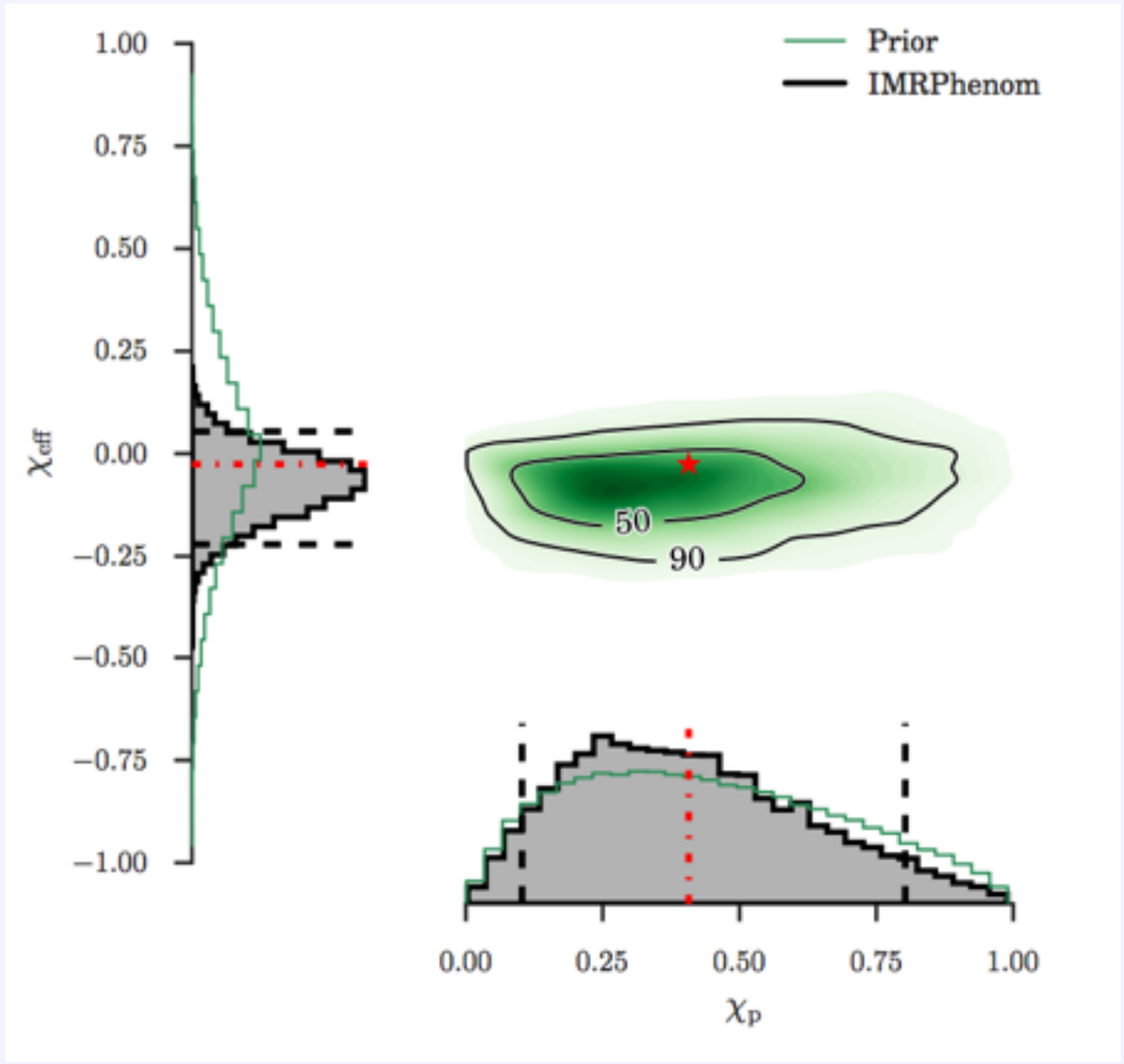


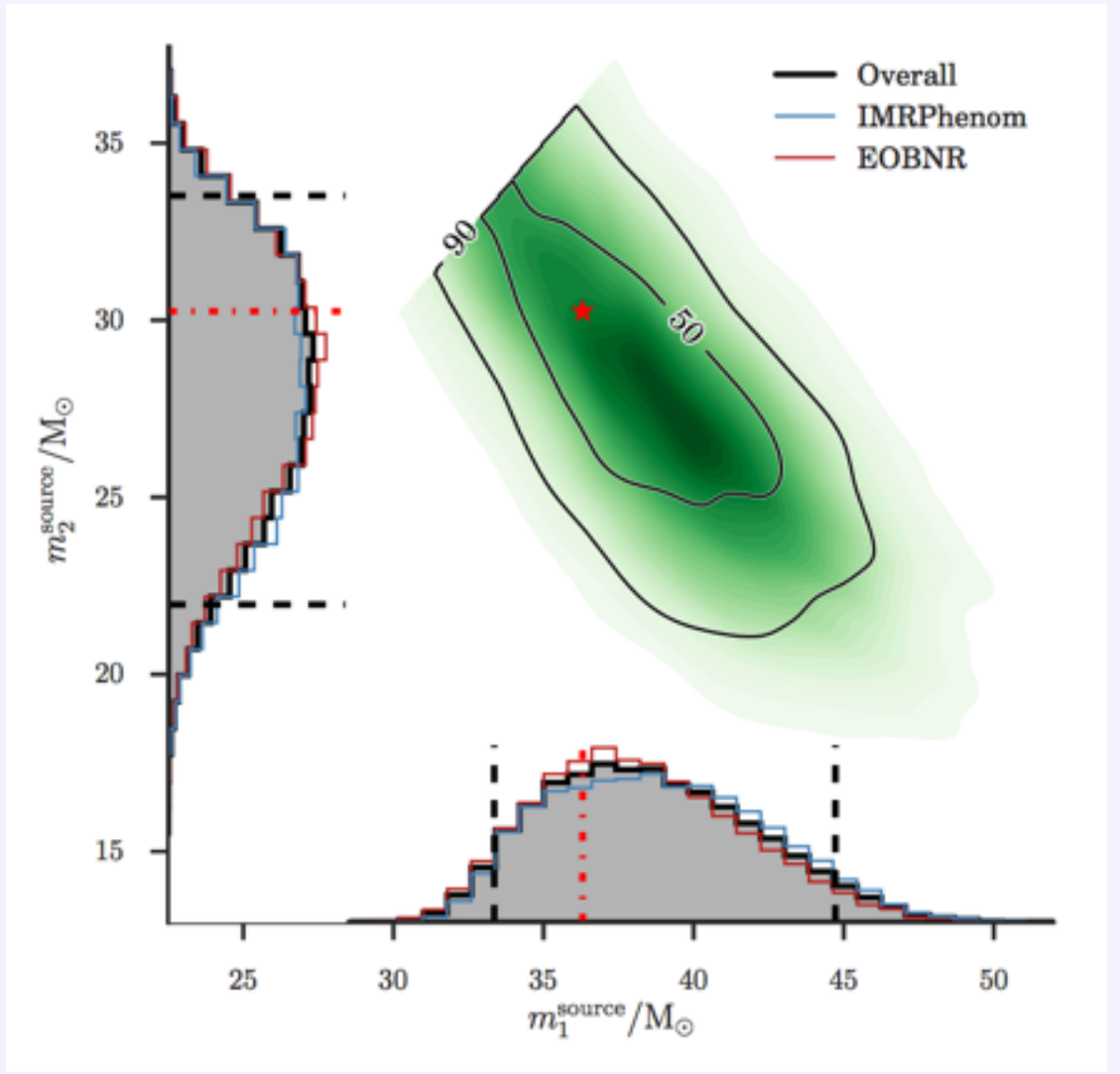
Littenberg+ (arXiv:1210.0893)



Wade+ (arXiv:1402.5156)

How good is a waveform?





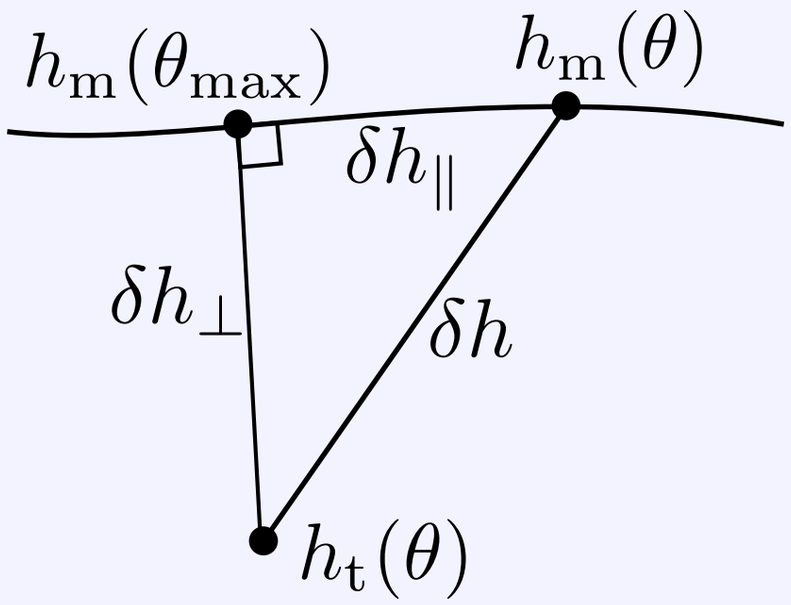
LIGO, VIRGO (arXiv:1611.07531)

Sufficient but not necessary

$$\langle \delta h | \delta h \rangle < 1 \;\Rightarrow\; 1 - F < \frac{1}{2\mathrm{SNR}^2}$$

Lindblom+ (arXiv:0809.3844)

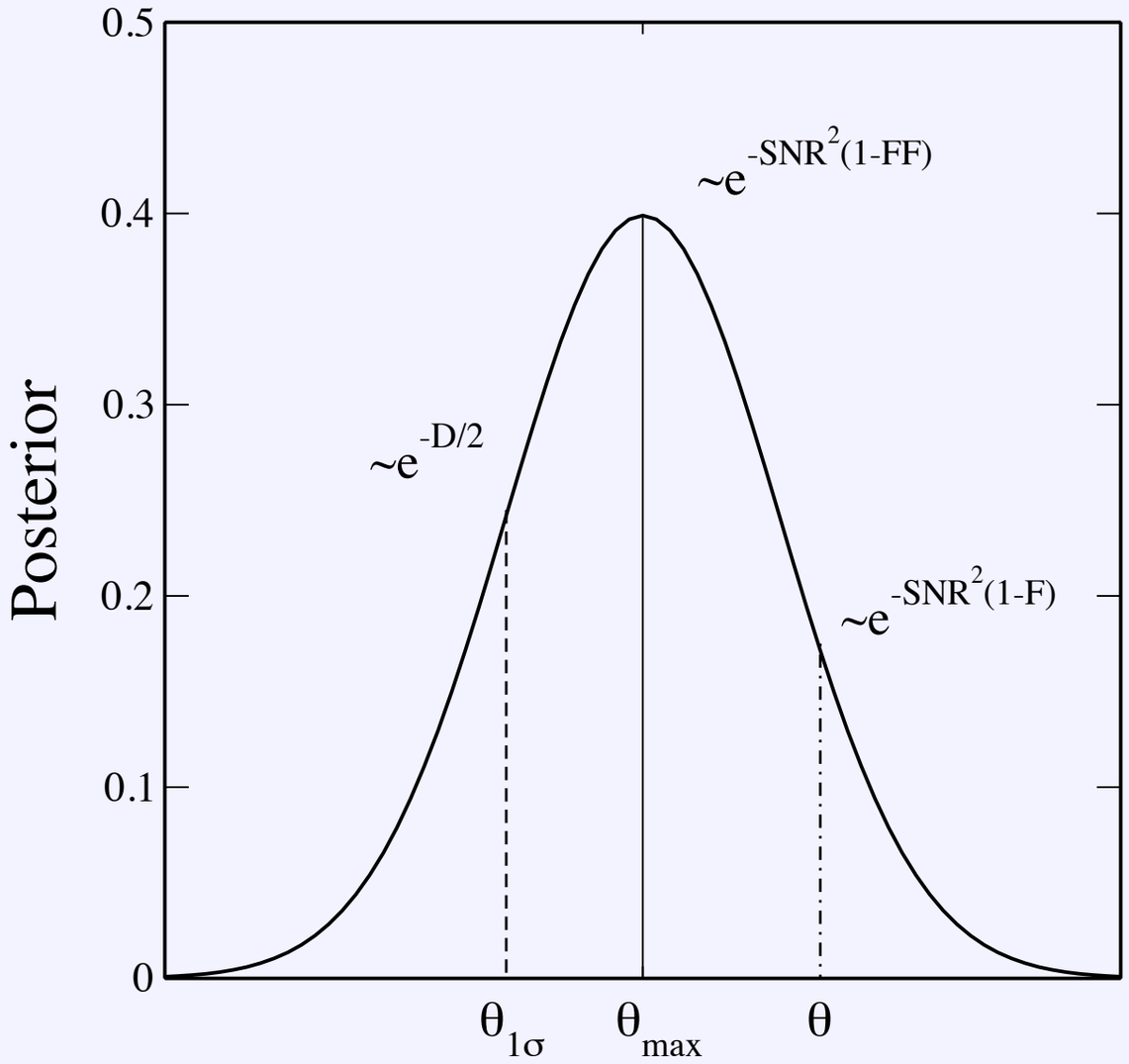
Flanagan, Hughes (arXiv:gr-qc/9710129)



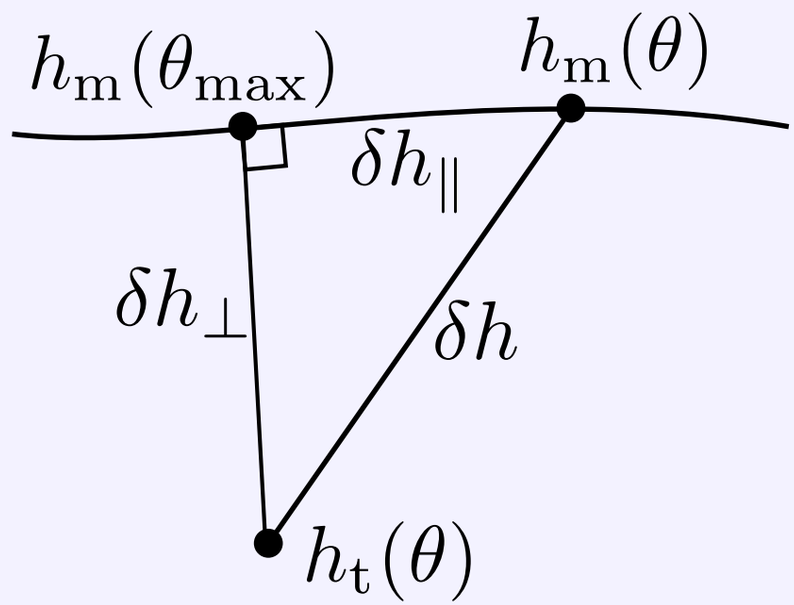
1. Ignores dimensionality
2. Too strong

Baird+ (arXiv:1211.0546)

Sufficient and necessary



$$FF - F < \frac{D}{2\mathrm{SNR}^2}$$



What's on LIGO's horizon?

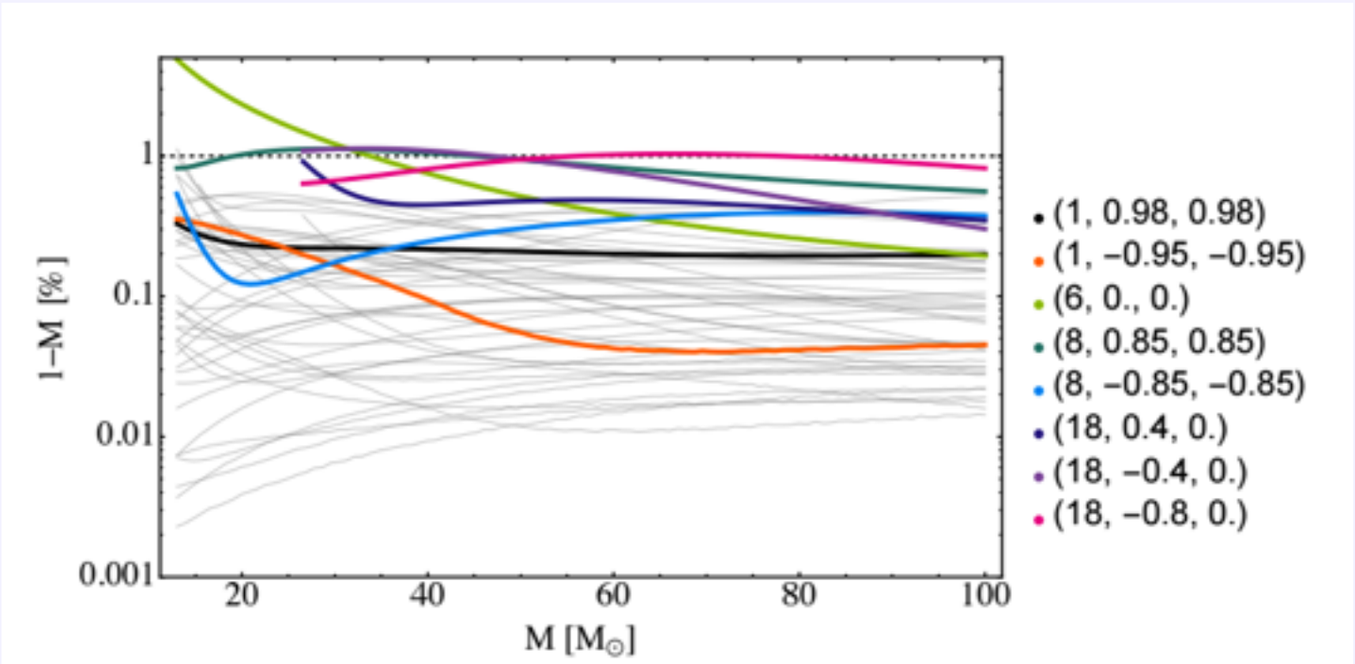
$$\mathrm{SNR} = 25, \quad FF - F > 1 - 0.994$$

$$\mathrm{SNR} = 100, \quad FF - F > 1 - 0.9996$$

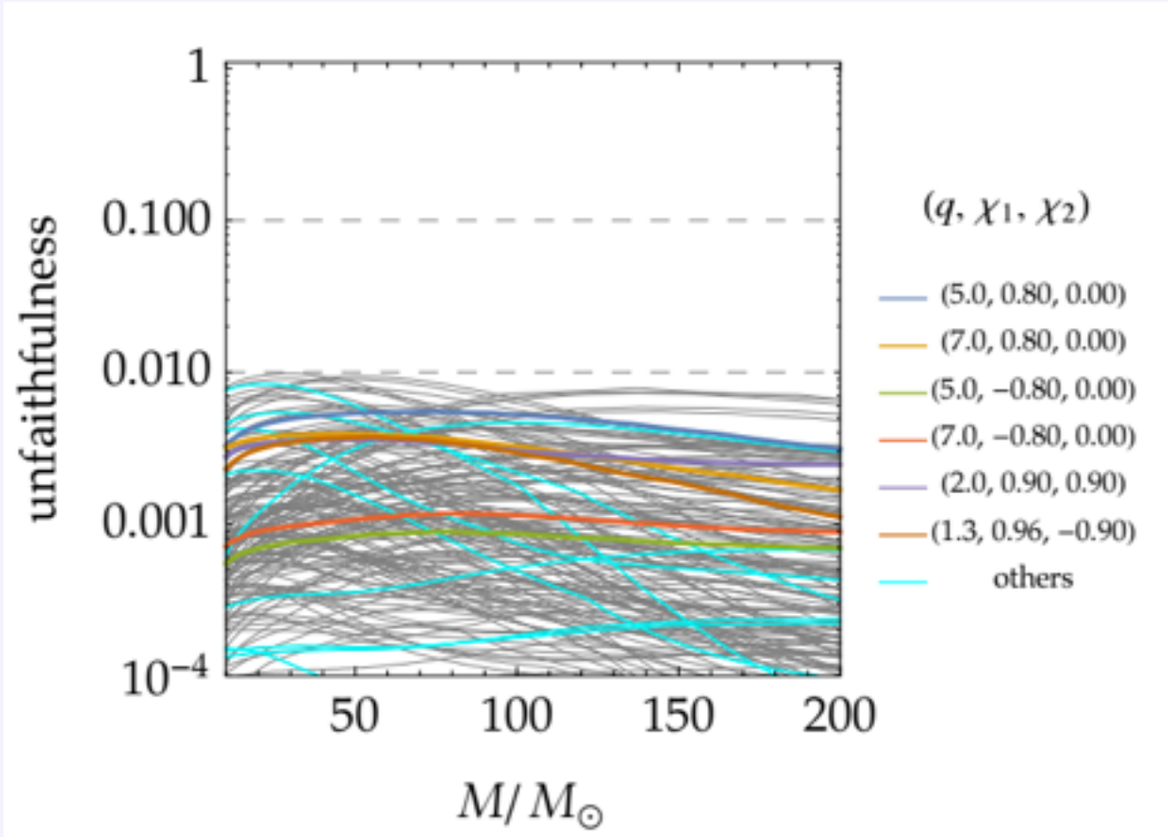
$$\mathrm{SNR} = 500, \quad FF - F > 1 - 0.999984$$

Analytical Models

$$\mathrm{SNR} = 100, \quad FF - F > 1 - 0.9996$$

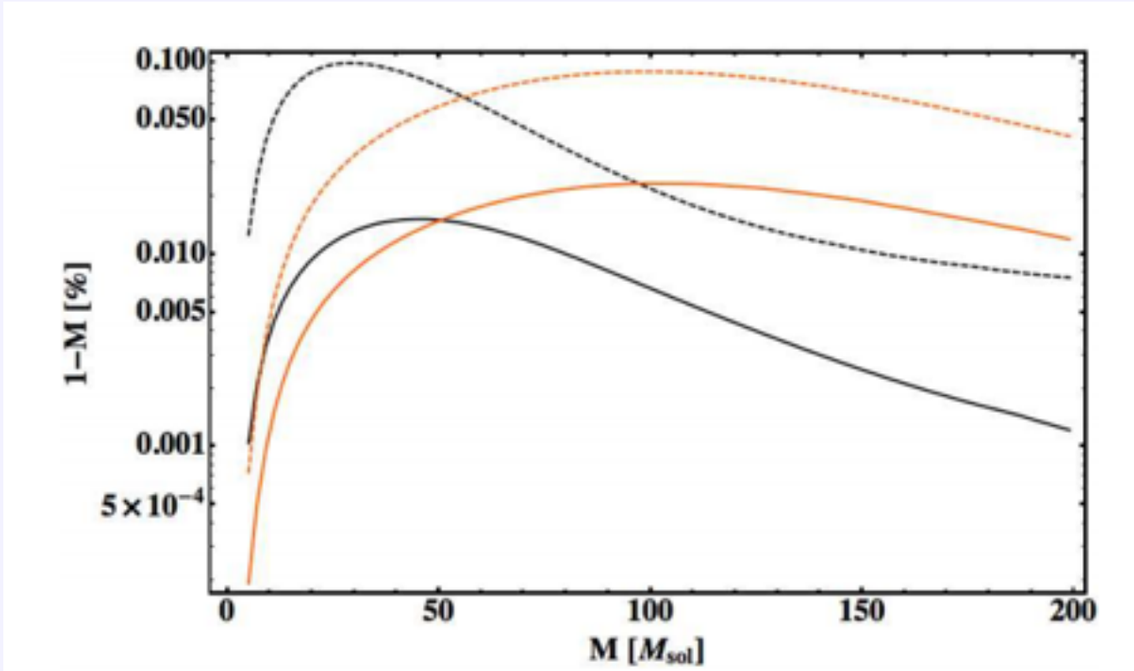


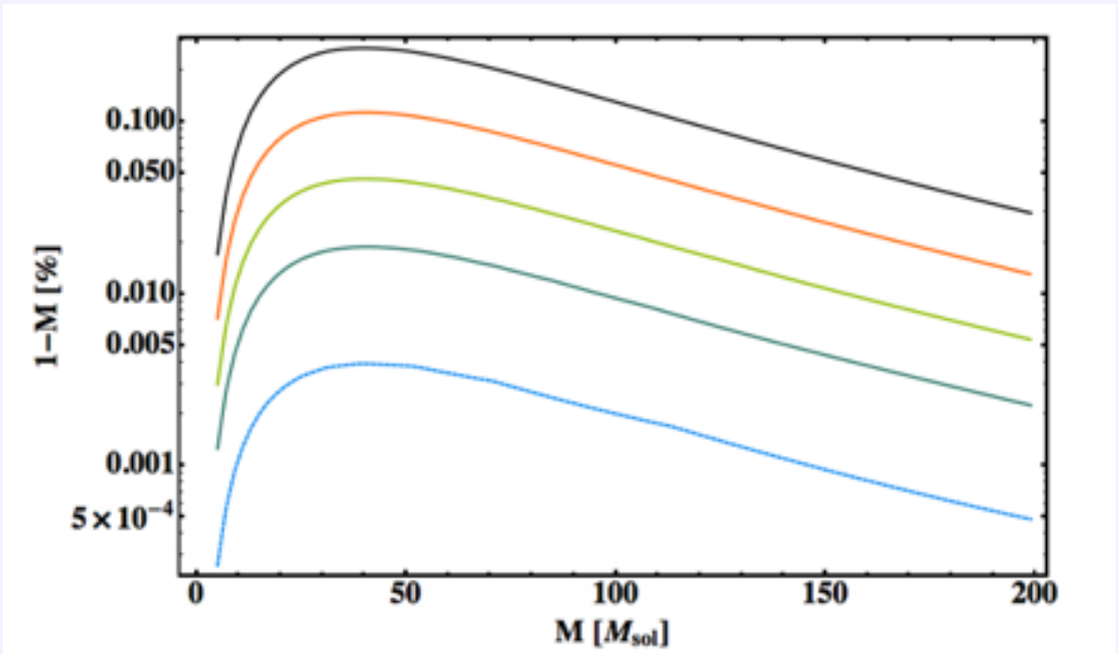
Khan+ (arXiv:1508.07253)



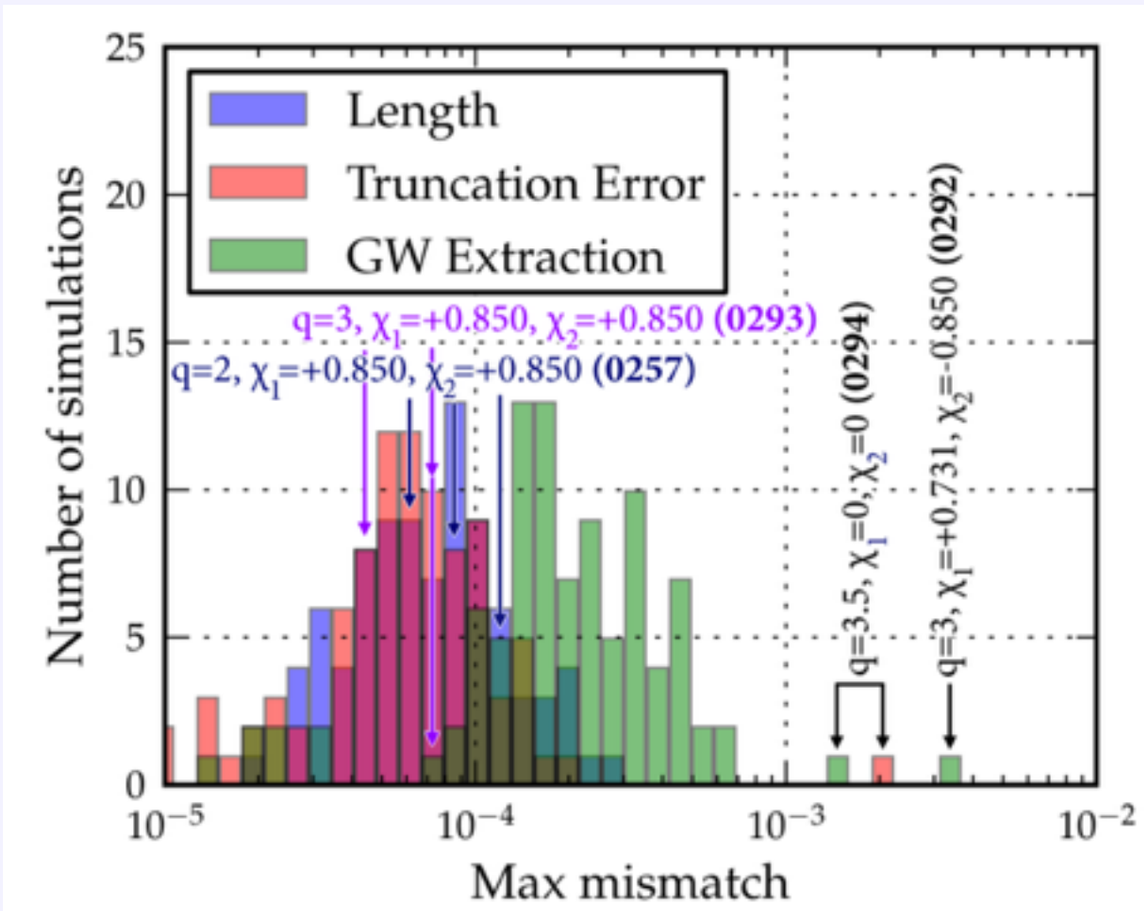
Bohe+ (arXiv:1611.03703)

Numerical Relativity: Convergence



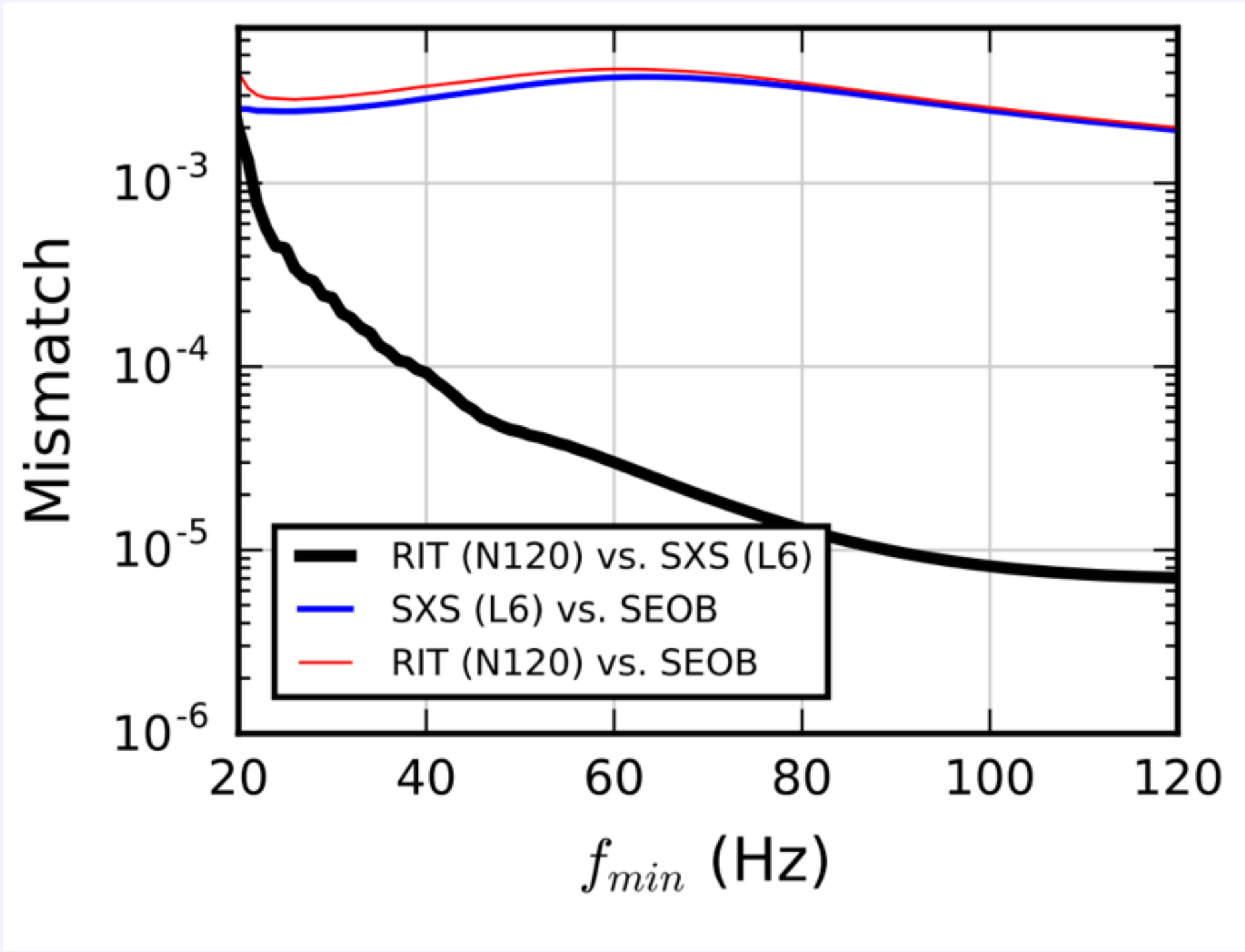


Khan+ (arXiv:1508.07253)



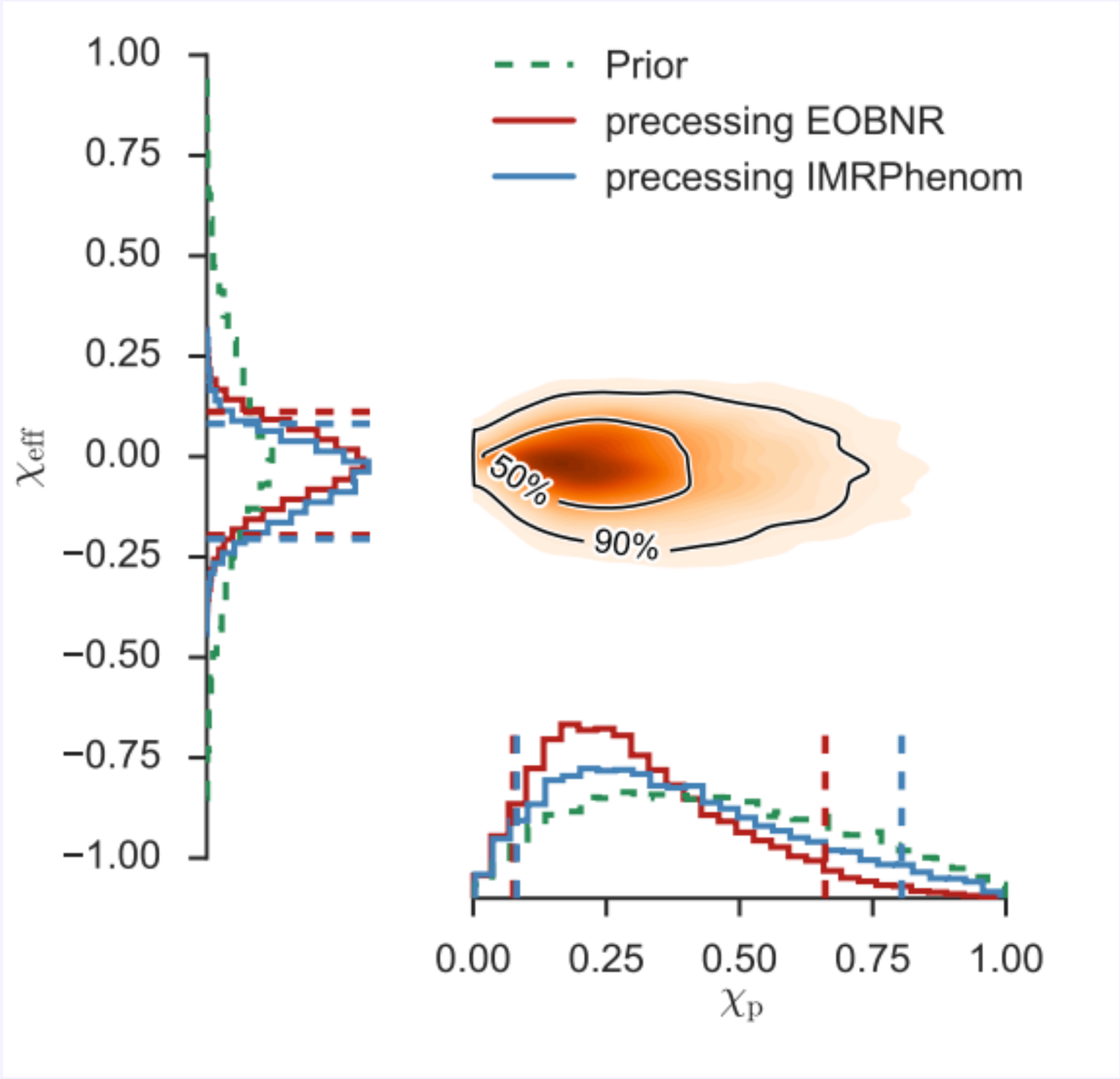
Tsu+ (arXiv:1512.06800)

Numerical Relativity: Comparisons



Lovelace+ (arXiv:1607.05377)

What we can measure

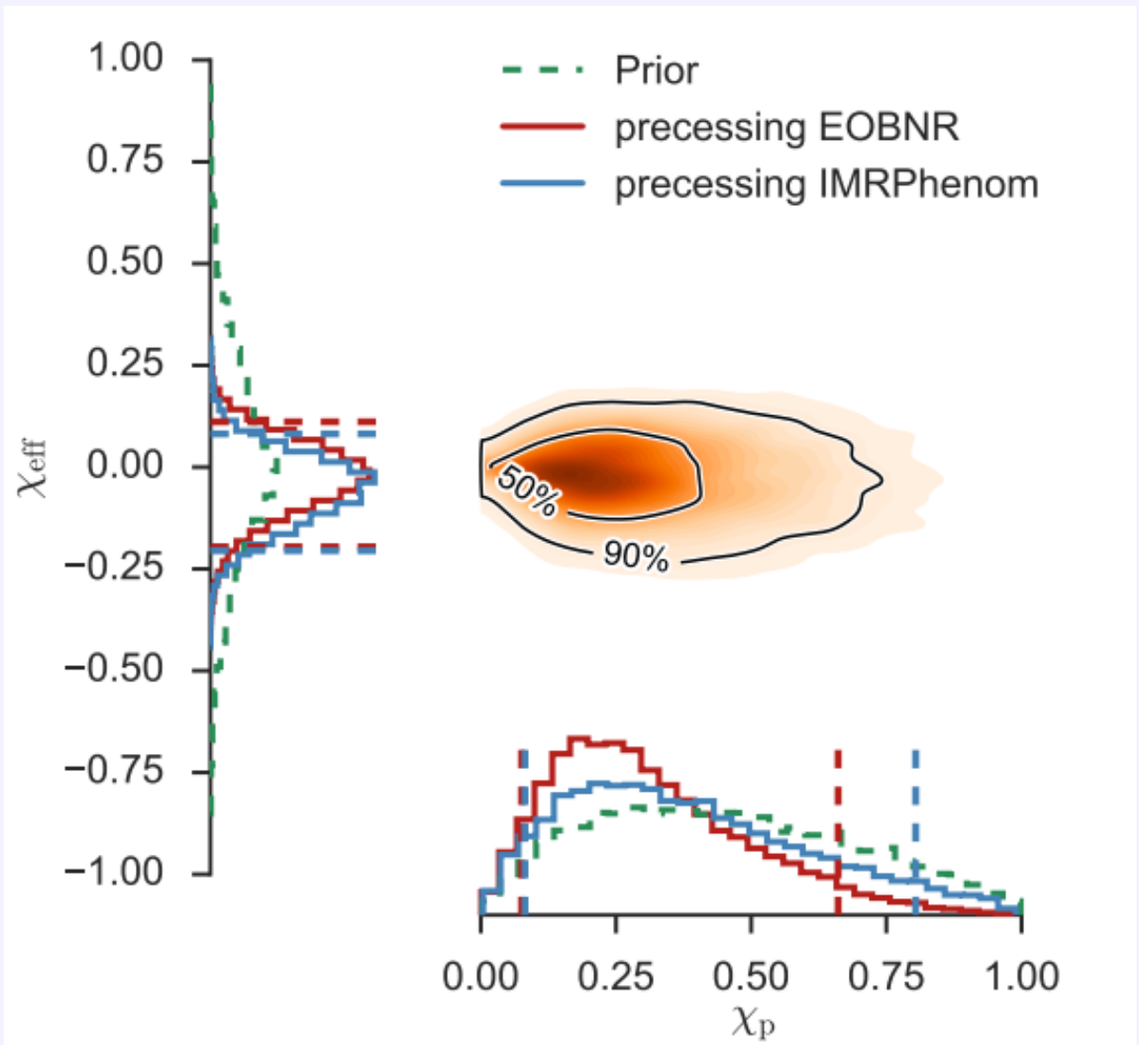


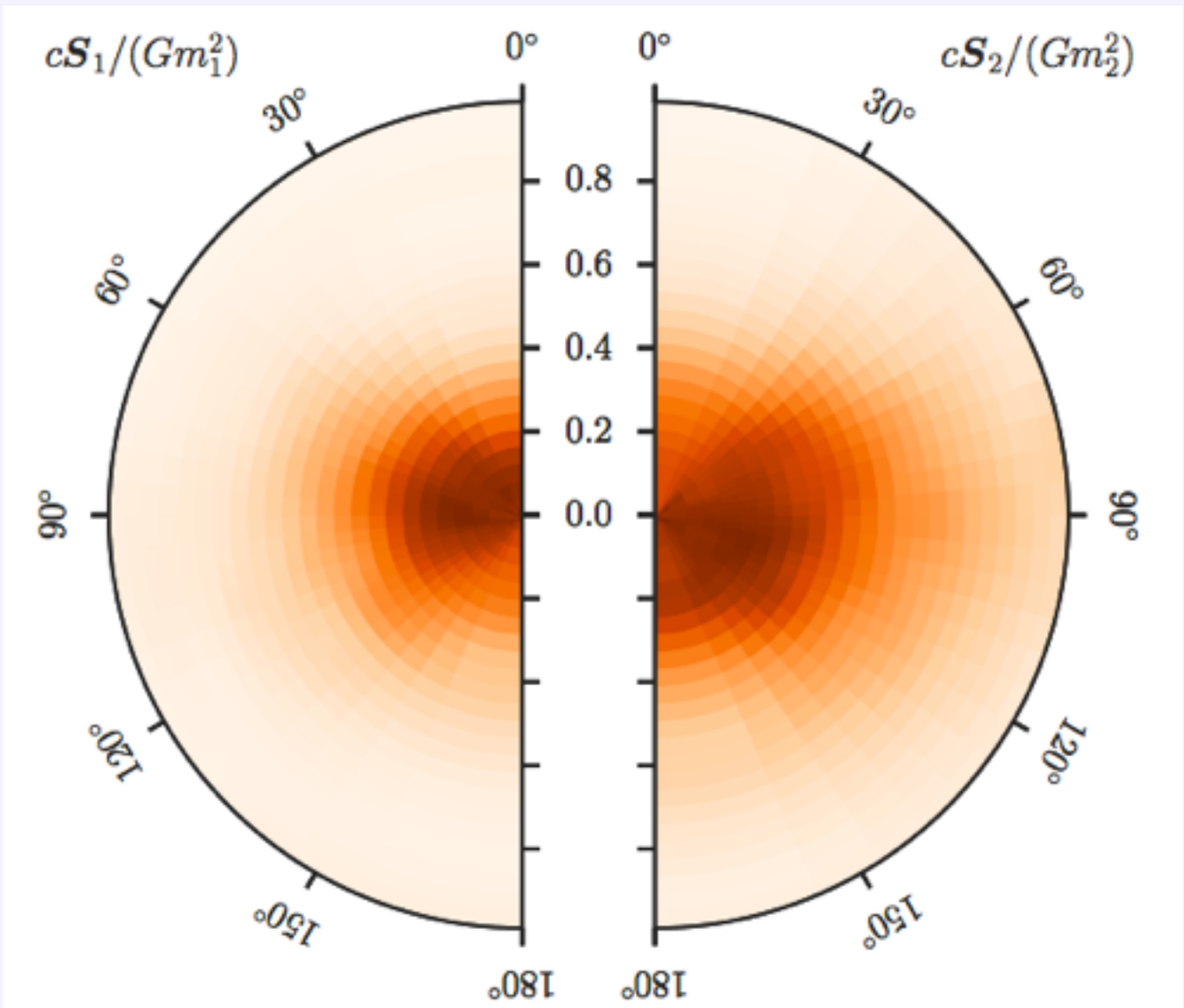
$$\chi_{eff} = \frac{m_1 \vec{\chi}_1 \cdot \hat{L} + m_2 \vec{\chi}_2 \cdot \hat{L}}{m_1 + m_2}$$

$$\chi_p = \frac{\max(B_1 S_{1\perp}, B_2 S_{2\perp})}{B_1 m_1^2}$$

LIGO, VIRGO (arXiv:1606.01210)

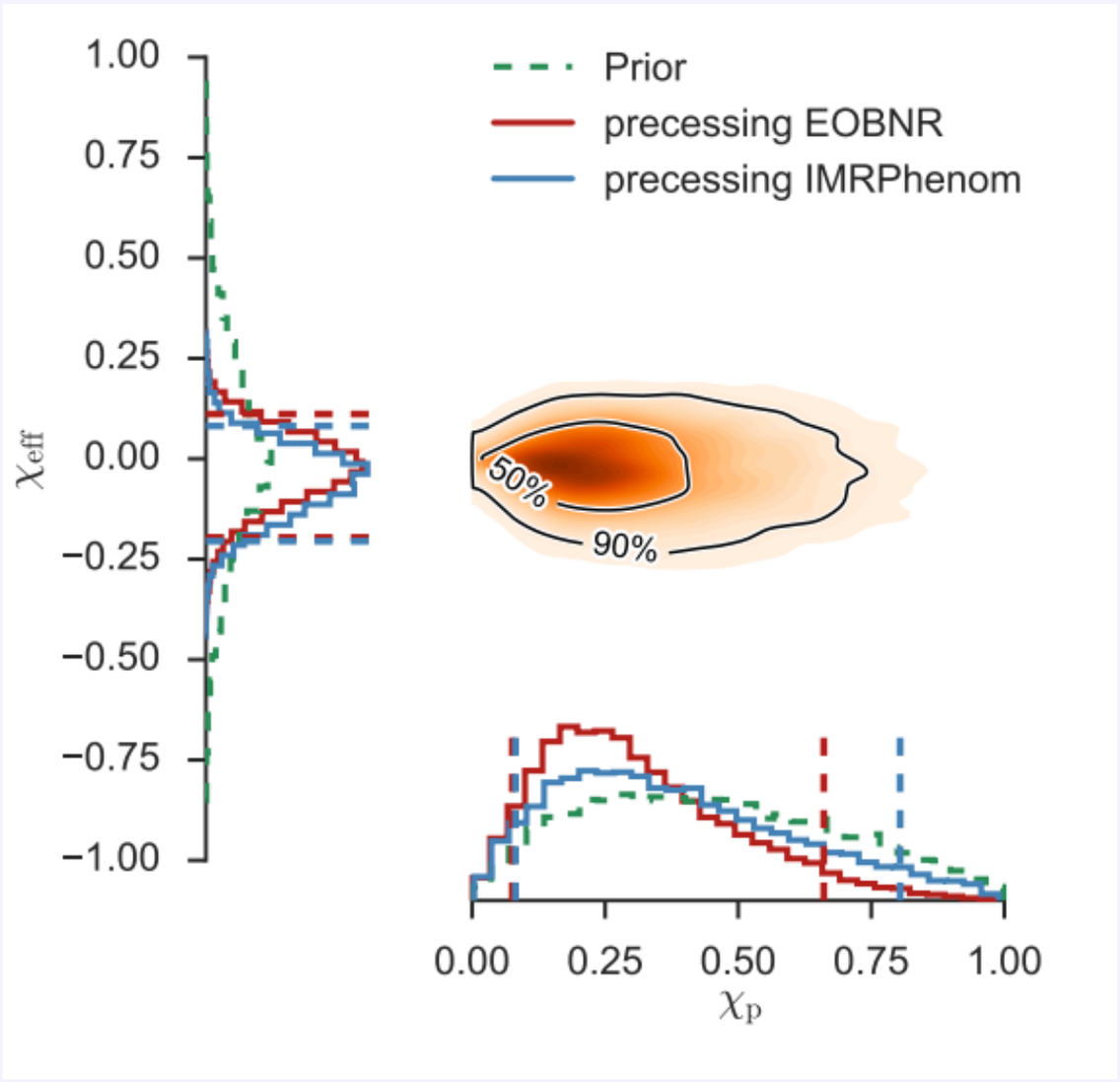
Spin Disk

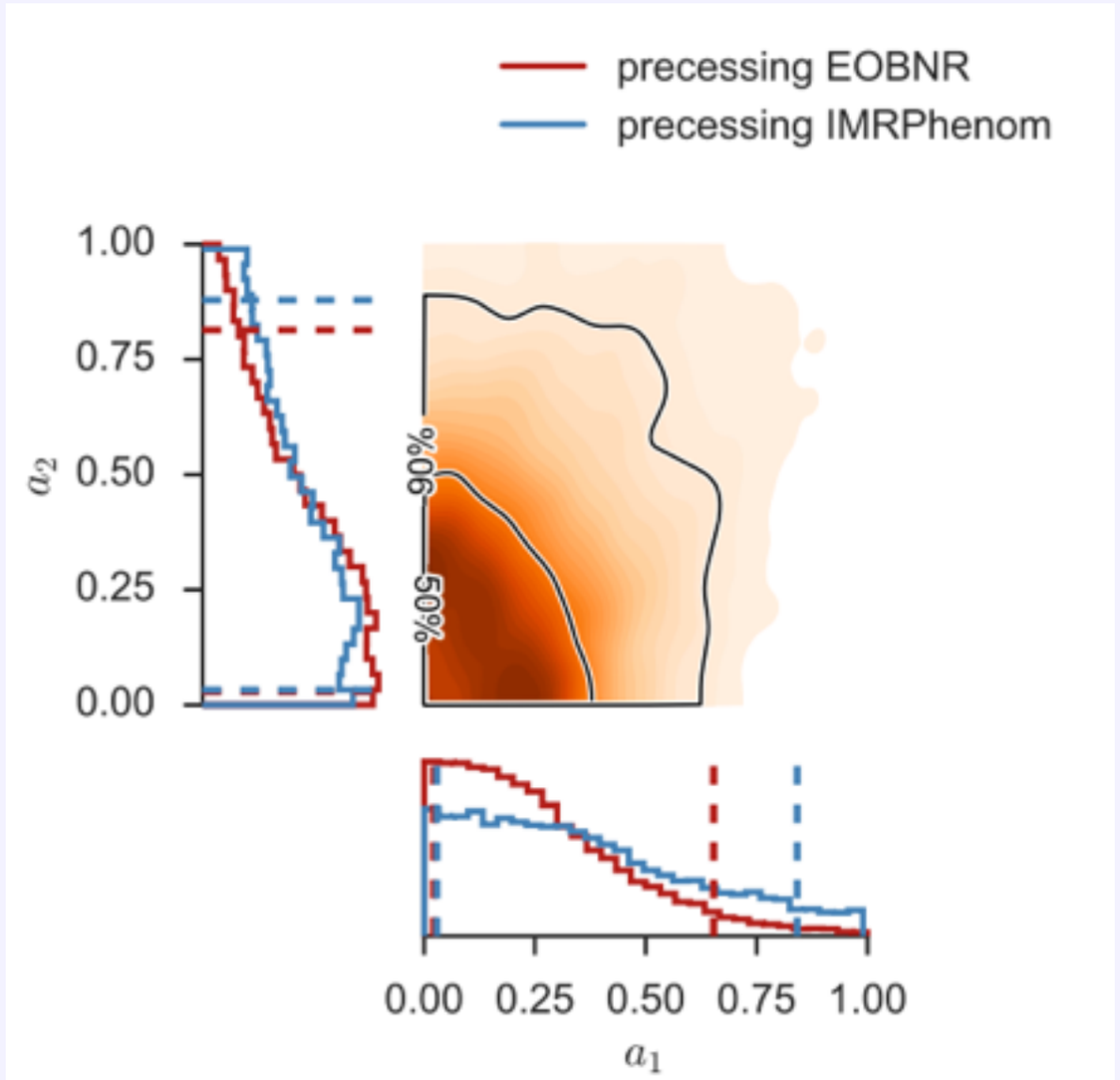




LIGO, VIRGO (arXiv:1606.01210)

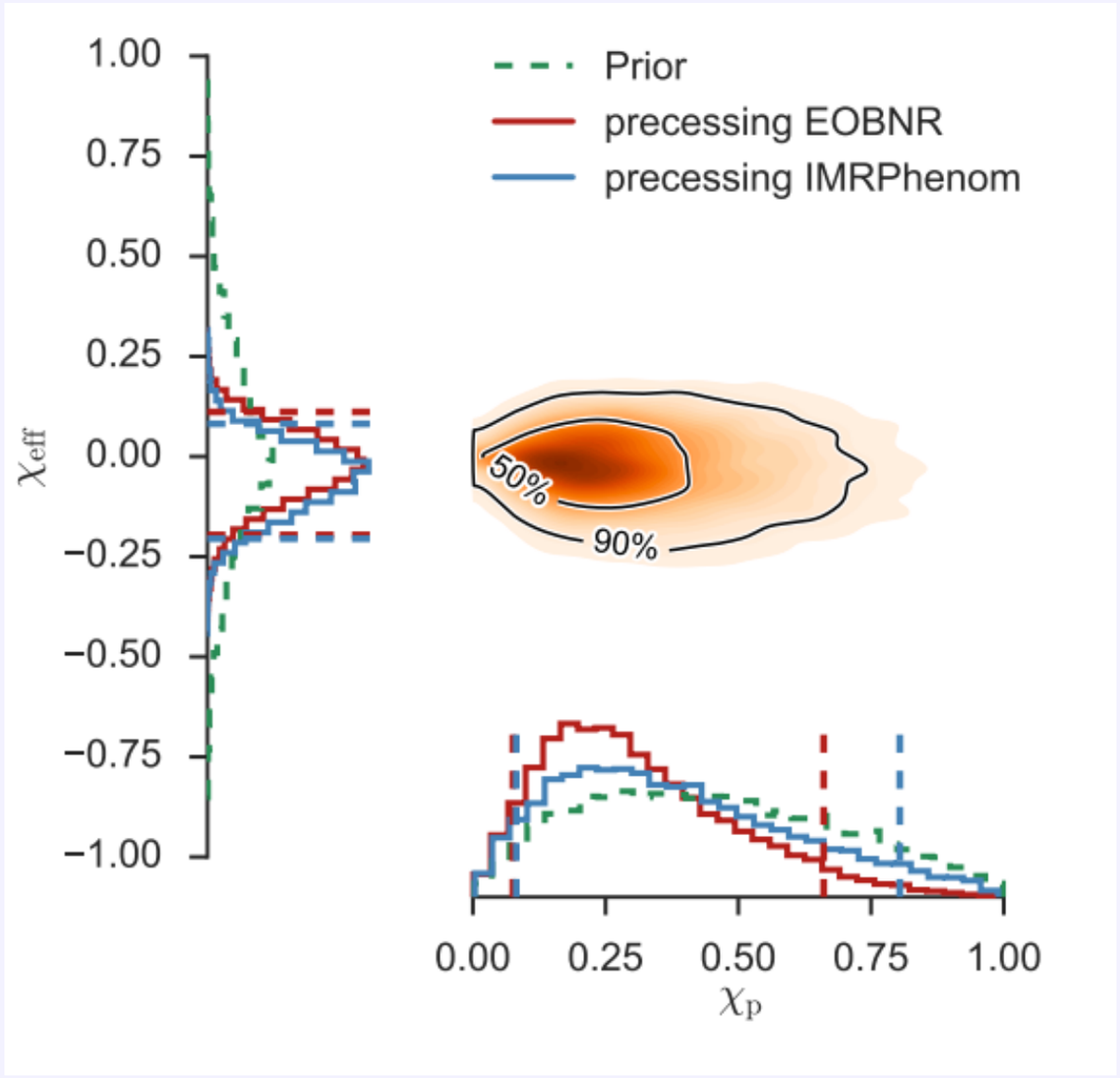
Spin Magnitude

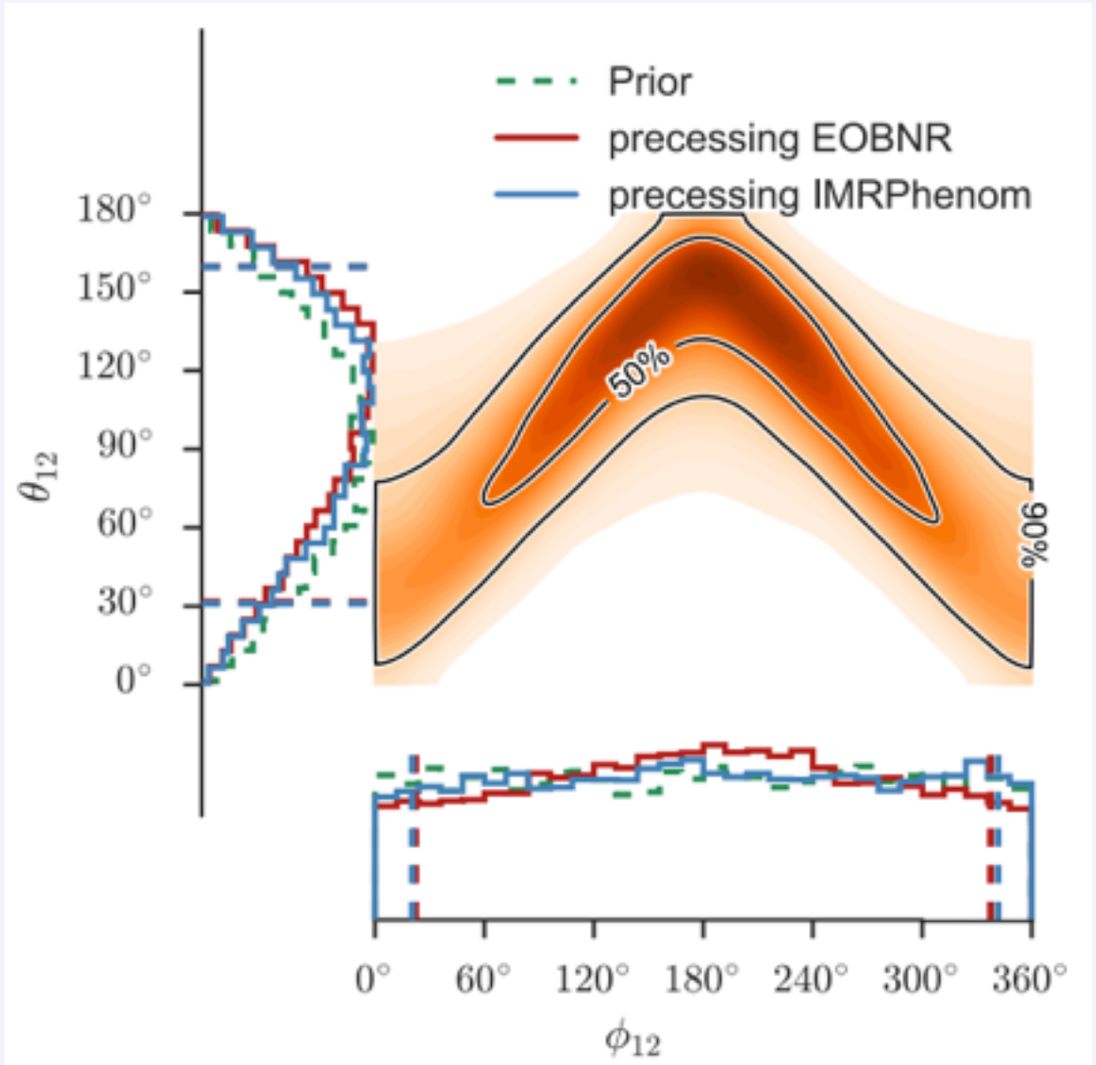




LIGO, VIRGO (arXiv:1606.01210)

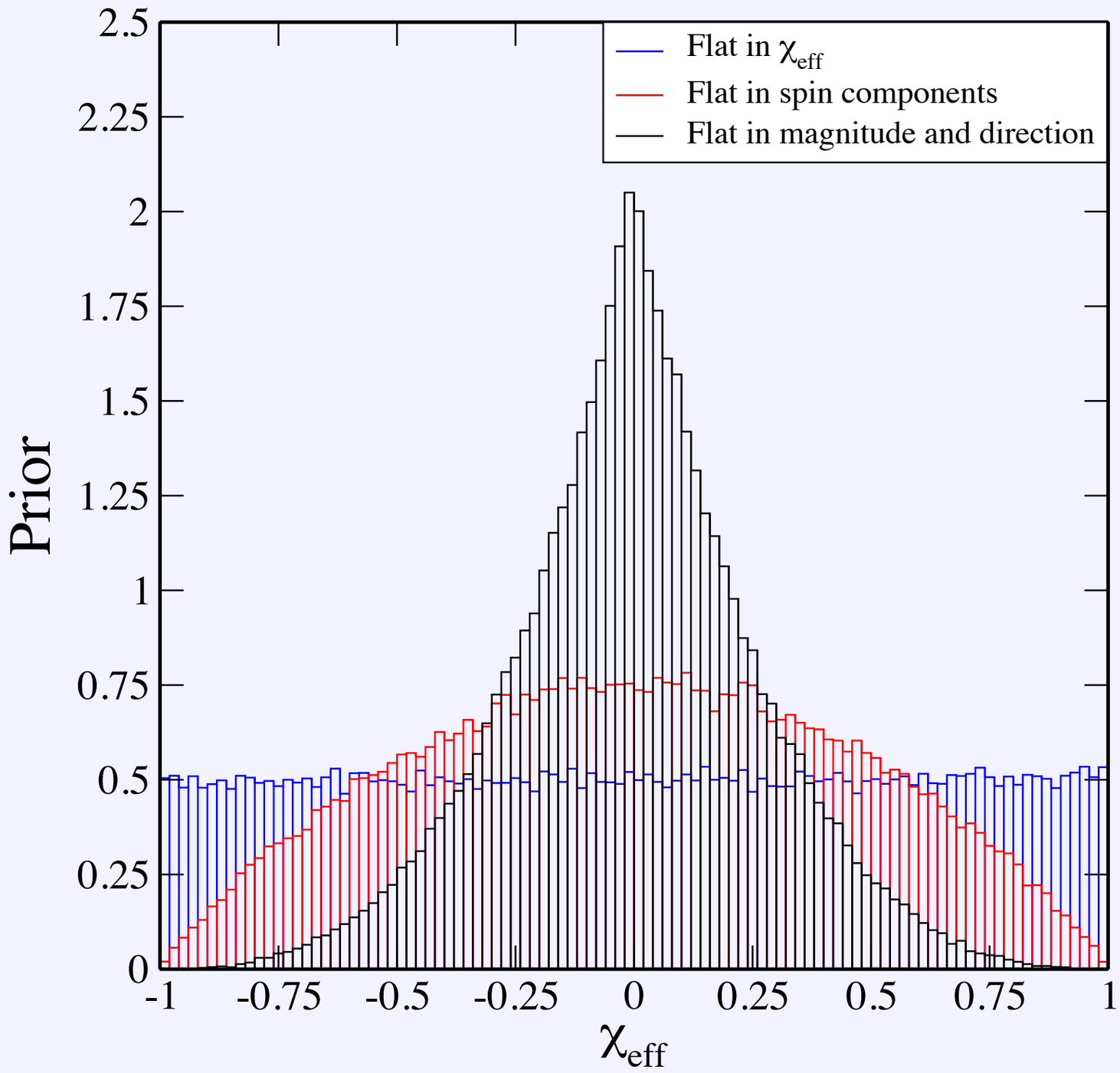
Spin Angles





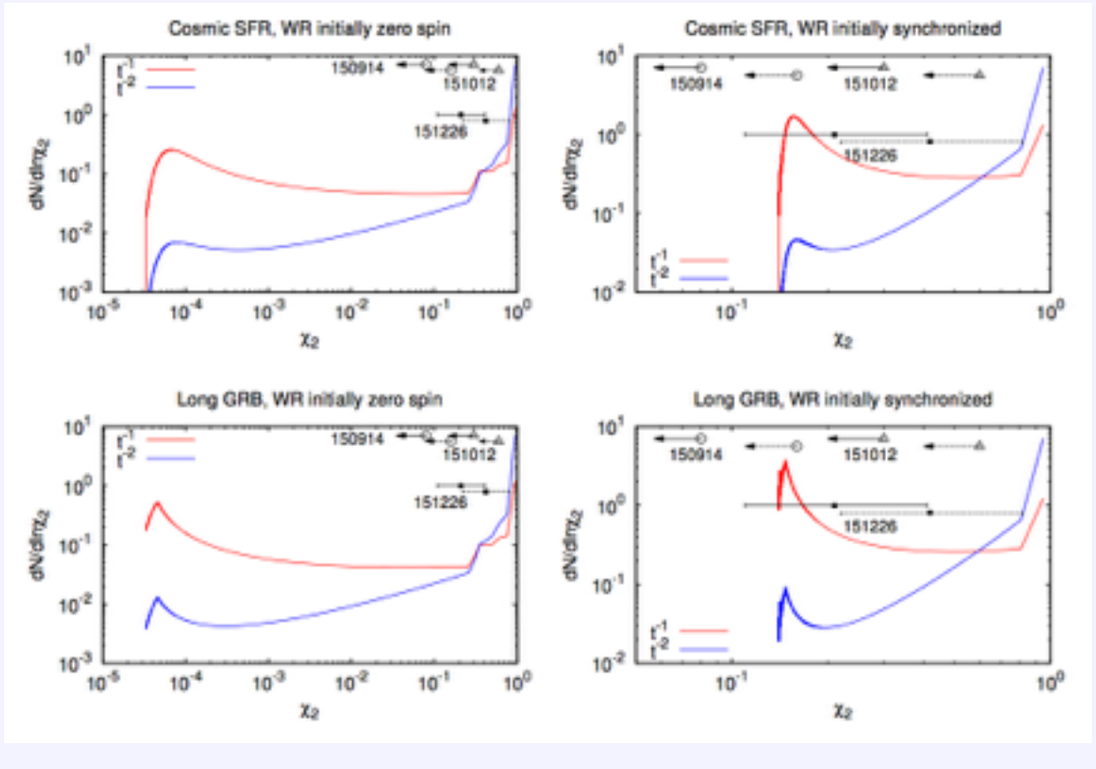
LIGO, VIRGO (arXiv:1606.01210)

Spin Priors

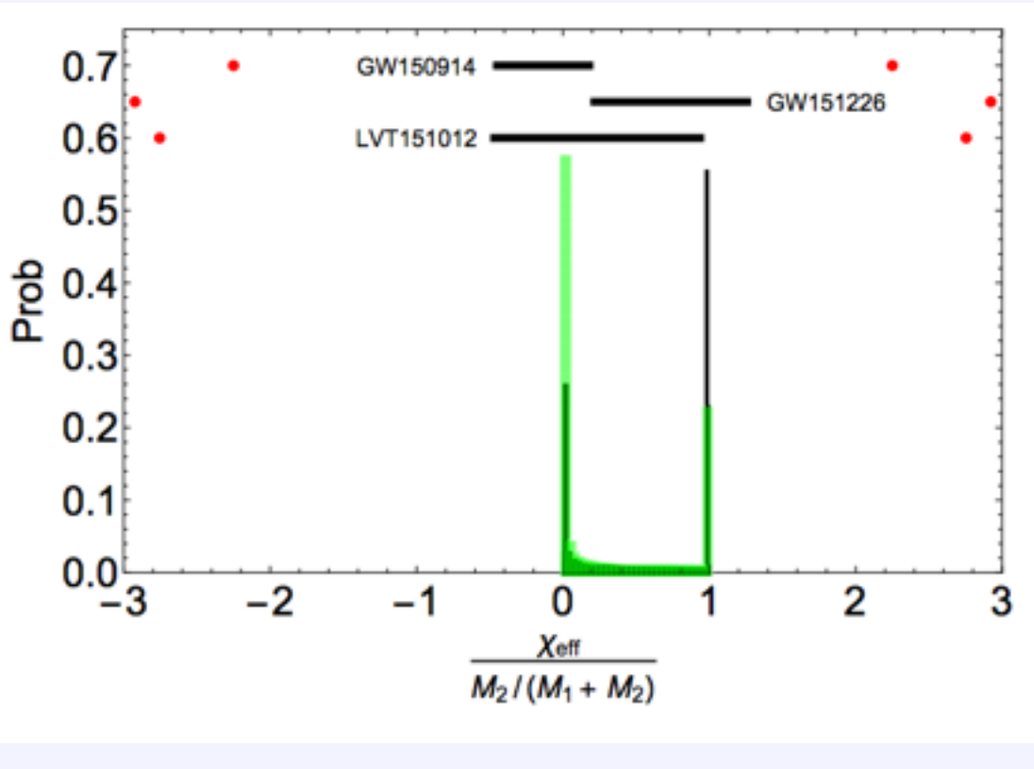


$$d\phi \, \sin\theta d\theta \, d\chi \neq d\phi \, \sin\theta d\theta \, \chi^2 d\chi$$

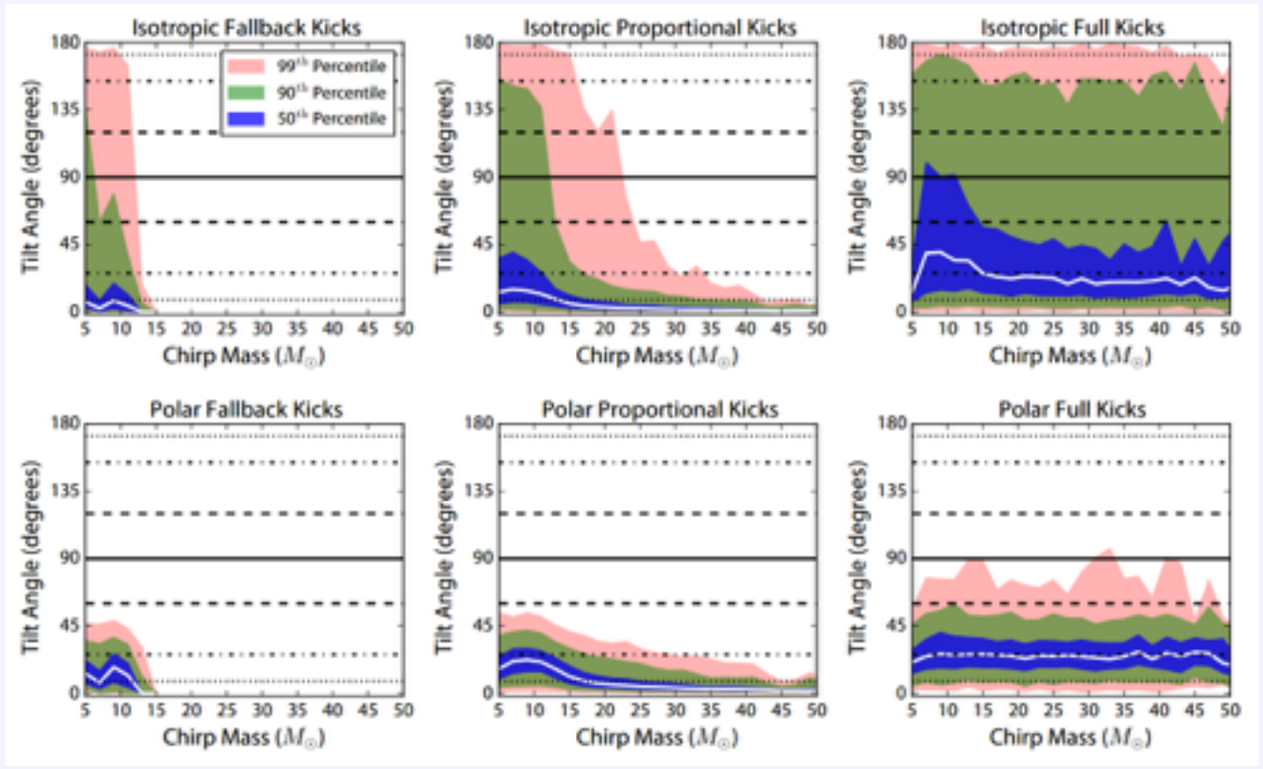
Astrophysical Spin Priors



Hotokezaka+ (arXiv:1702.03952)

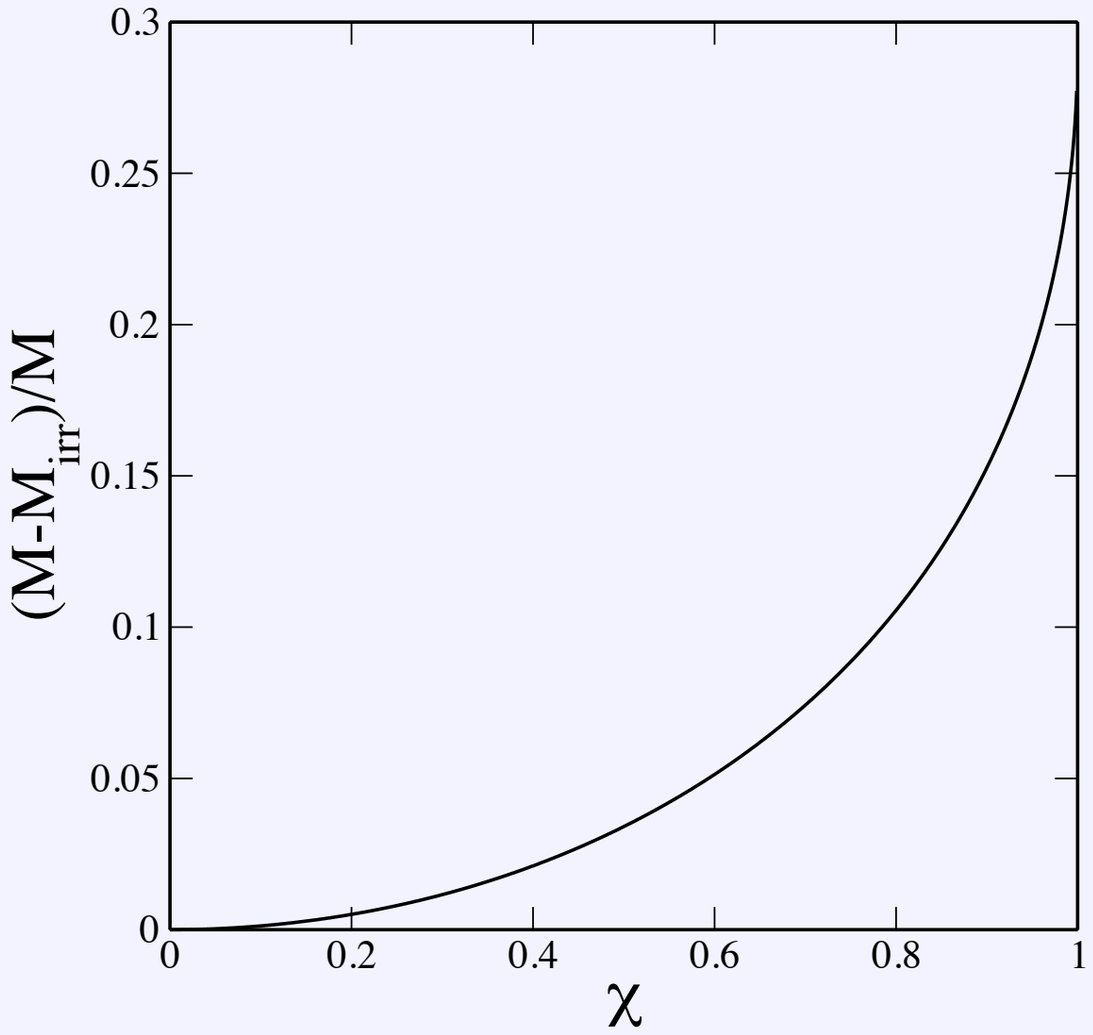


Zaldarriaga+ (arXiv:1702.00885)

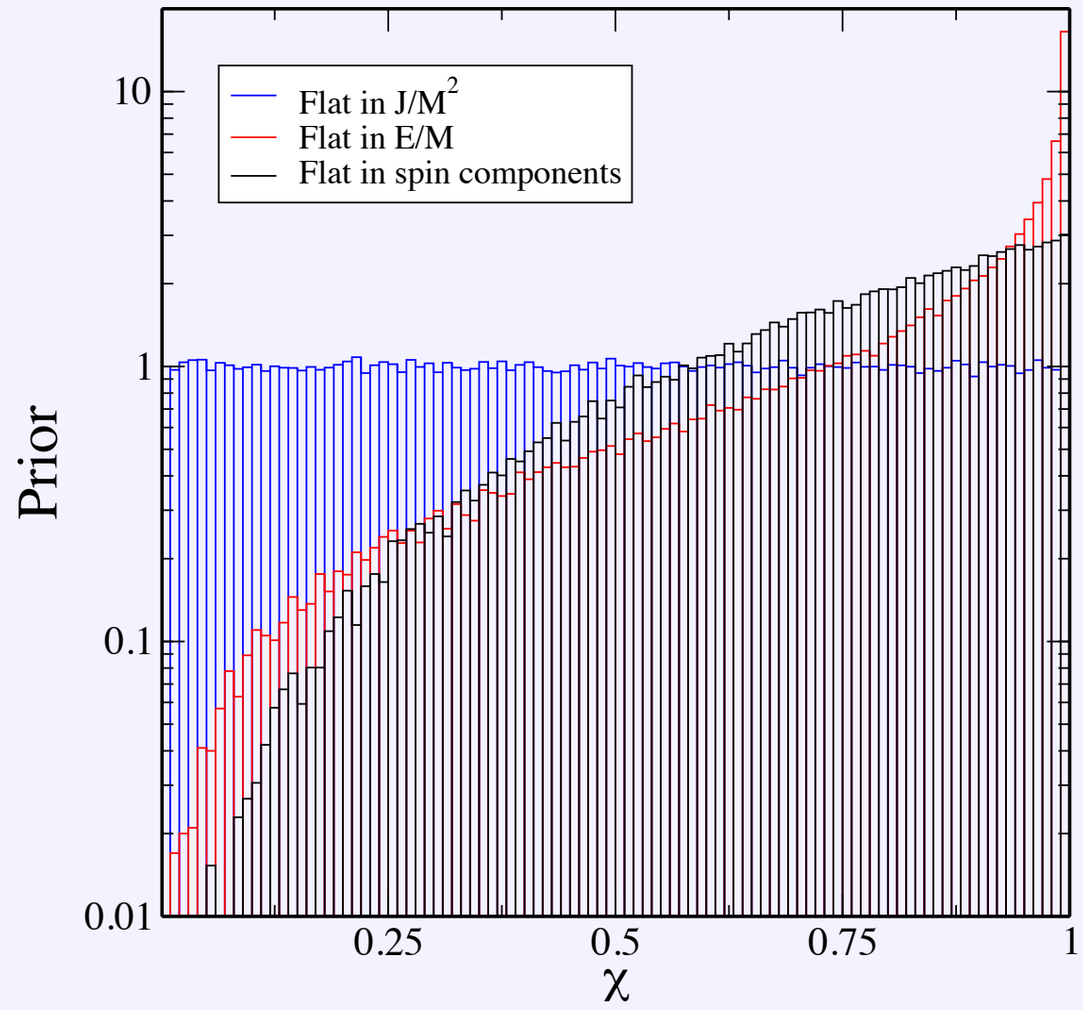


Rodriguez+ (arXiv:1609.05916)

Theoretical Spin Priors

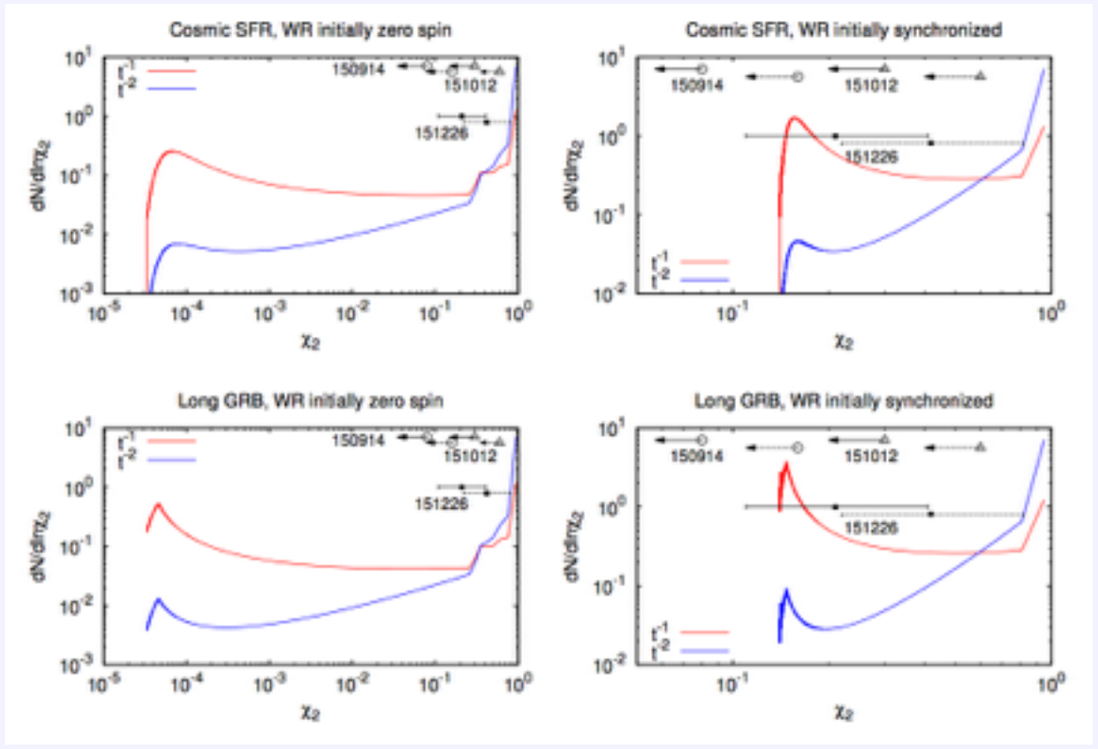


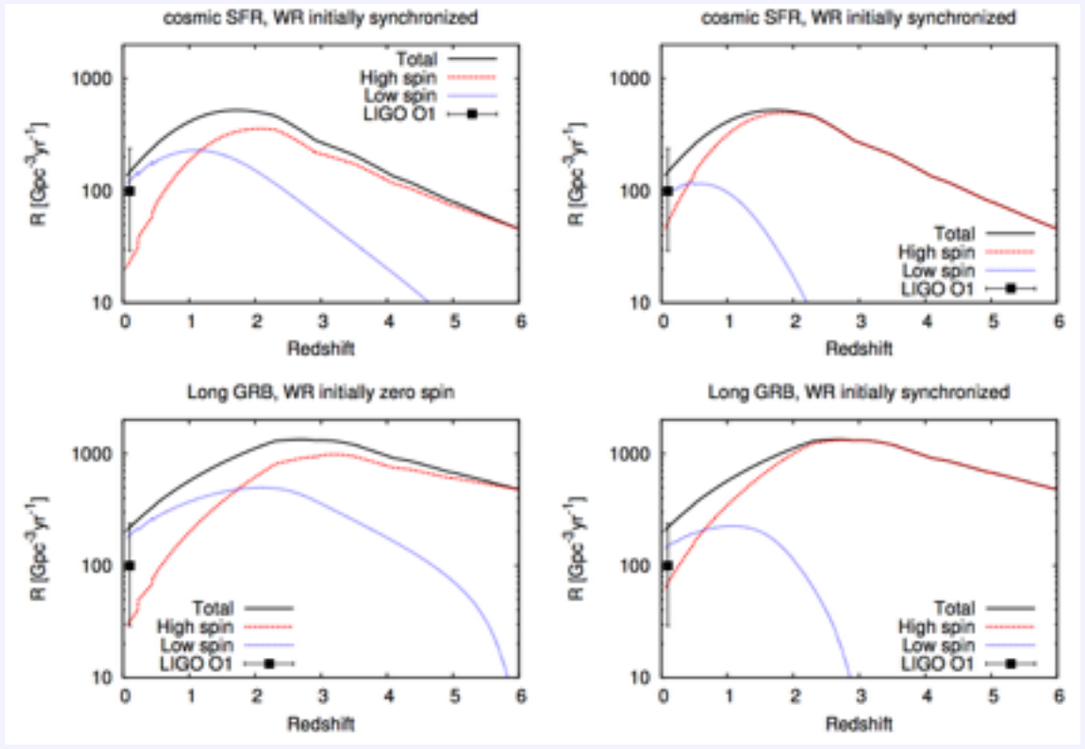
$$M_{\rm irr} = \frac{M}{\sqrt{2}}\sqrt{1+\sqrt{1-\chi^2}}$$



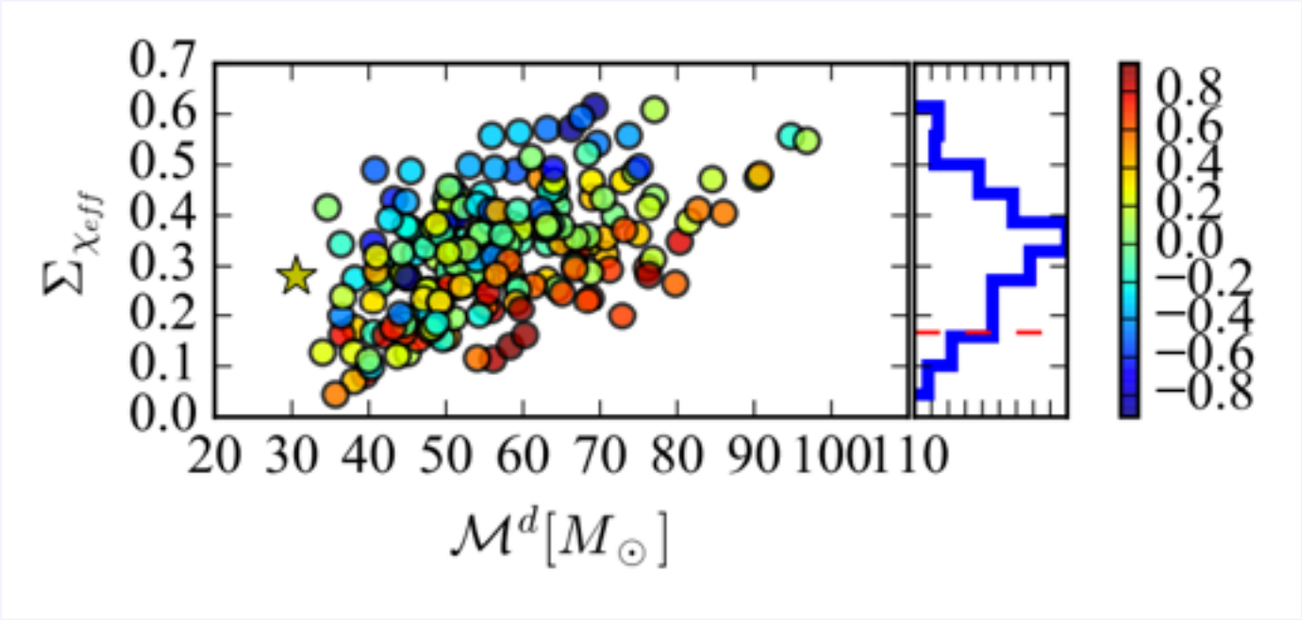
More Questions

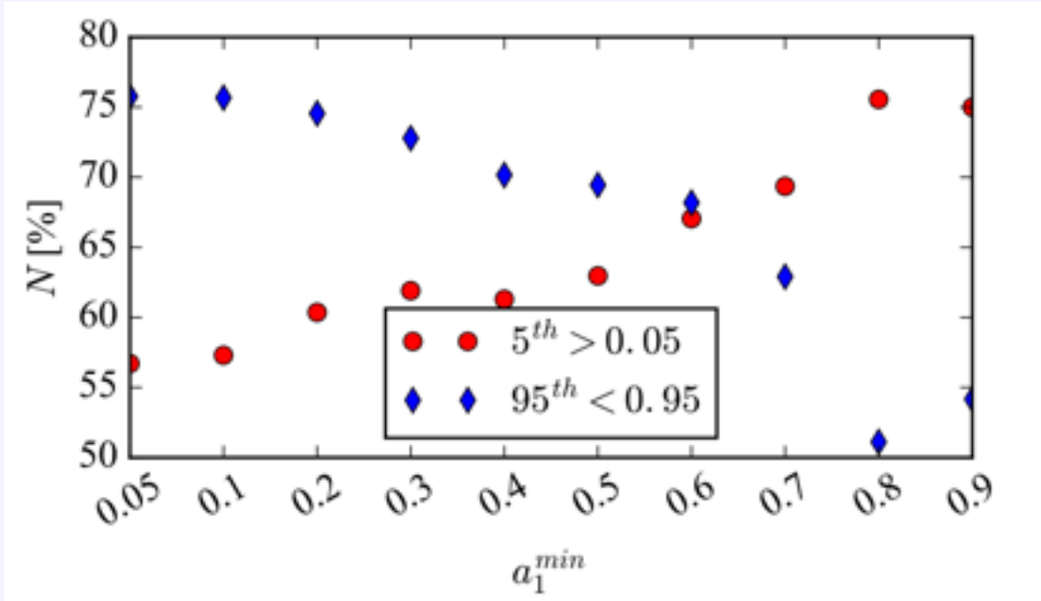
More Questions: Astrophysical Inference





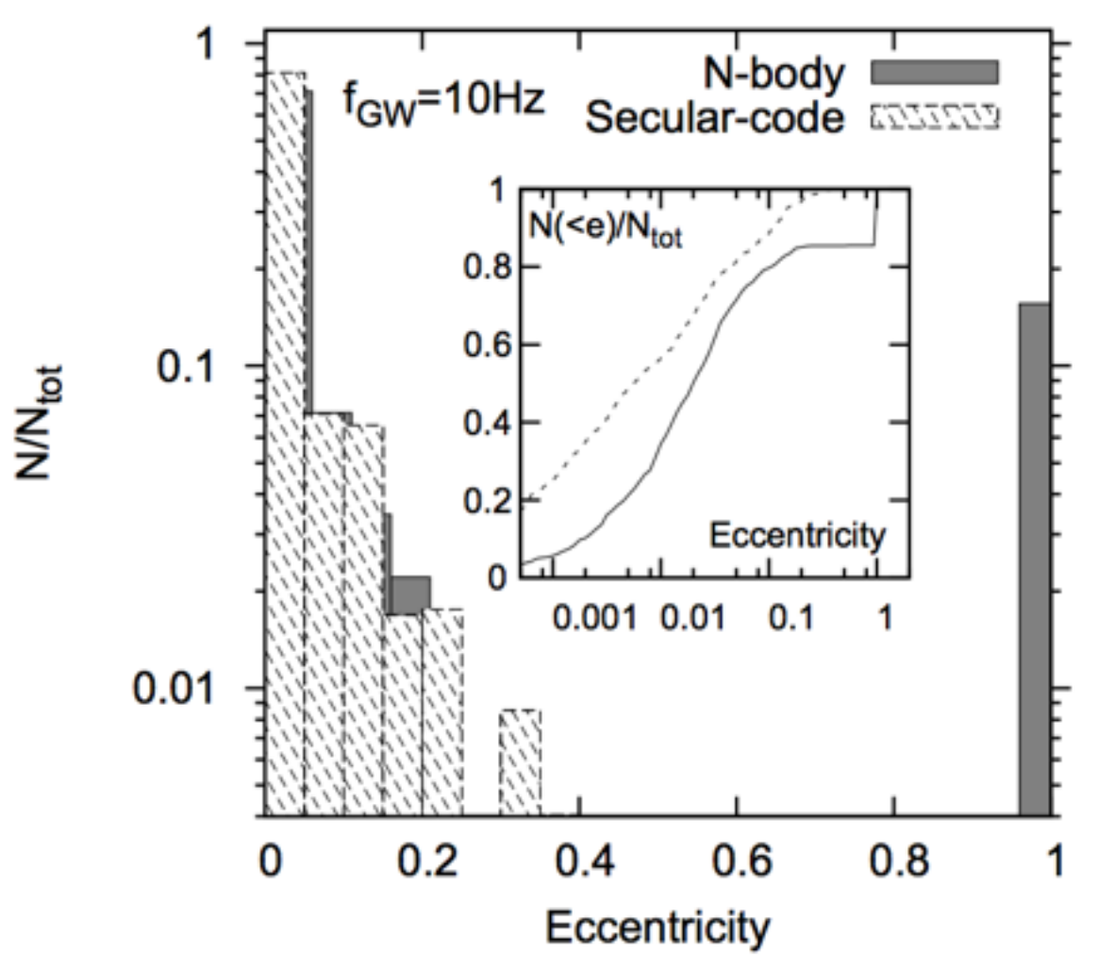
Hotokezaka+ (arXiv:1702.03952)





Vitale+ (arXiv:1611.01122)

More Questions: Eccentricity



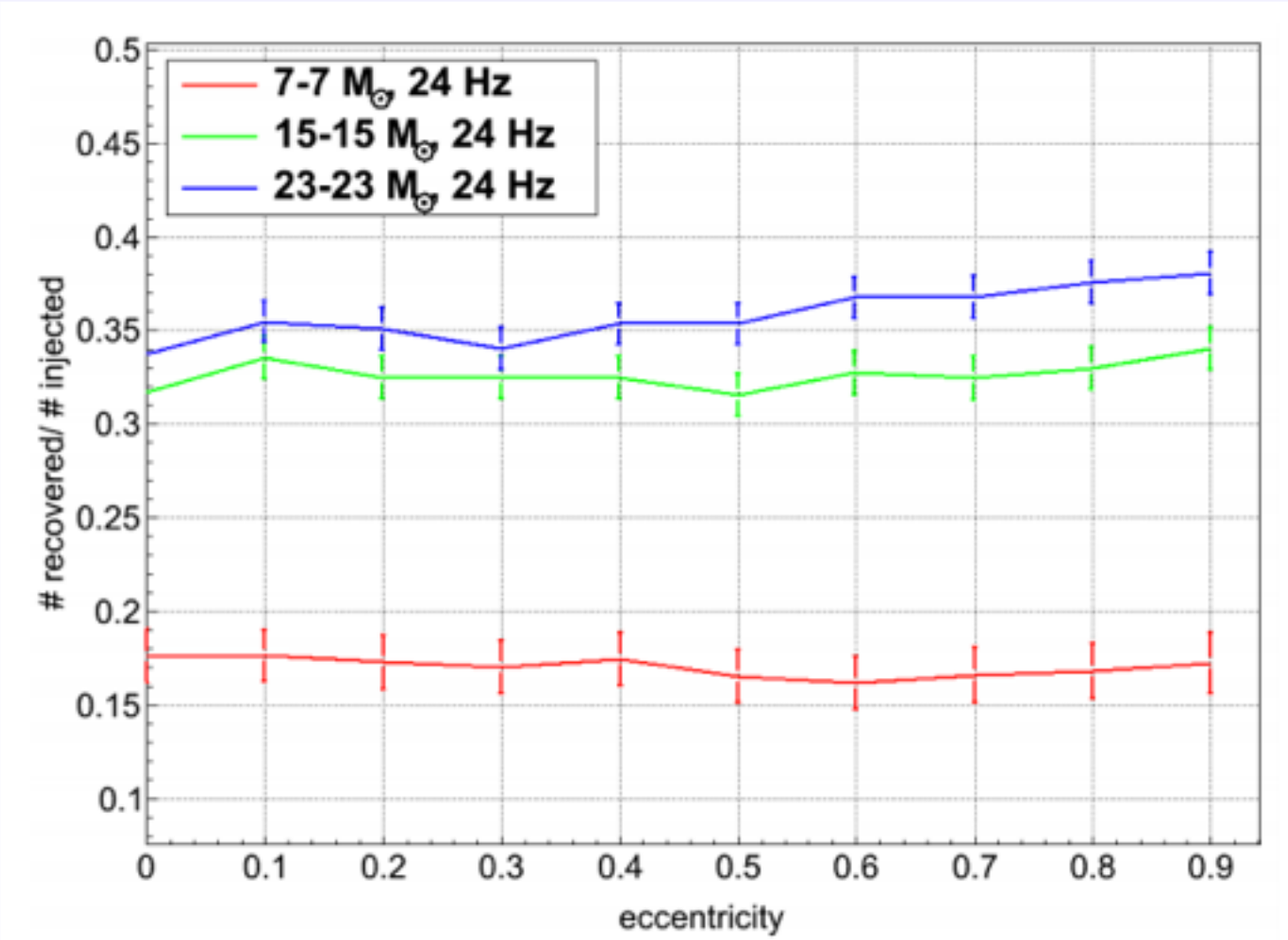
Antonini+ (arXiv:1509.05080)

How important are spins for small eccentricities?

How important are spins for eccentricities near 1?

How about LISA?

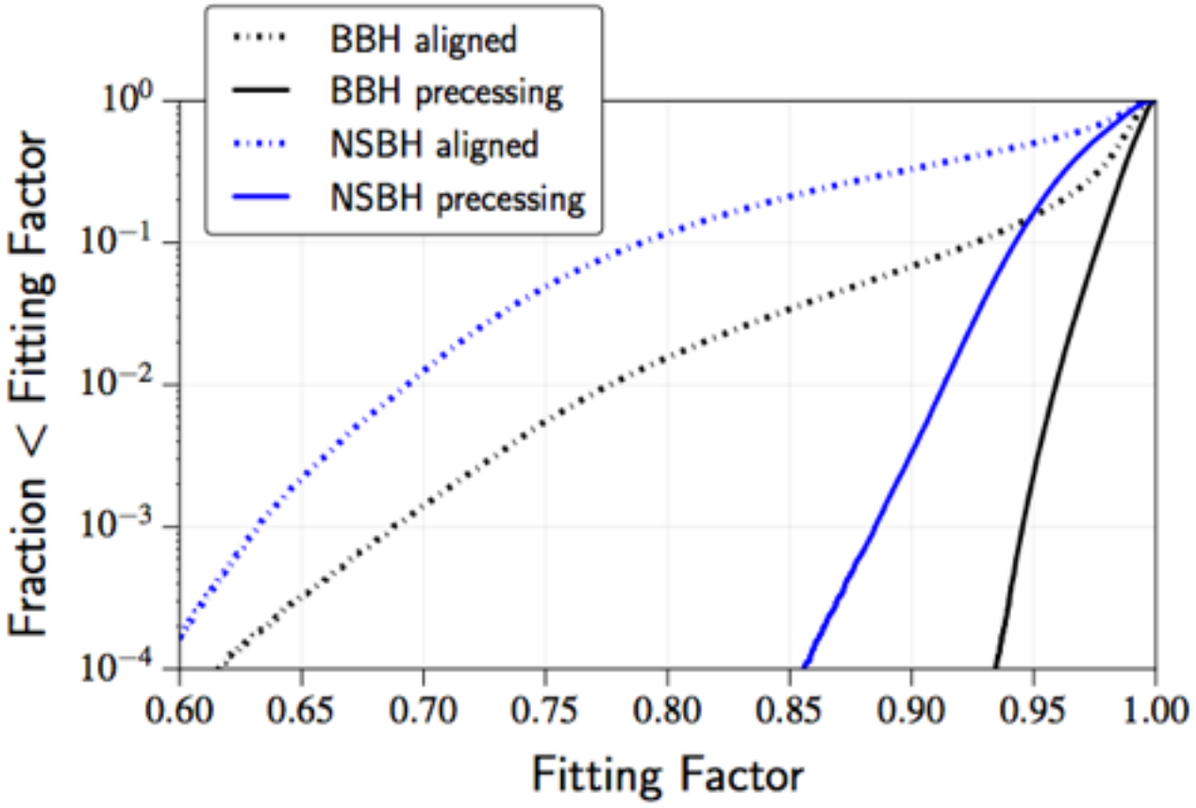
More Questions: Eccentricity



Tiwari+ (arXiv:1511.09240)

Can unmodeled burst searches recover eccentric precessing systems more efficiently?

More Questions: Searches

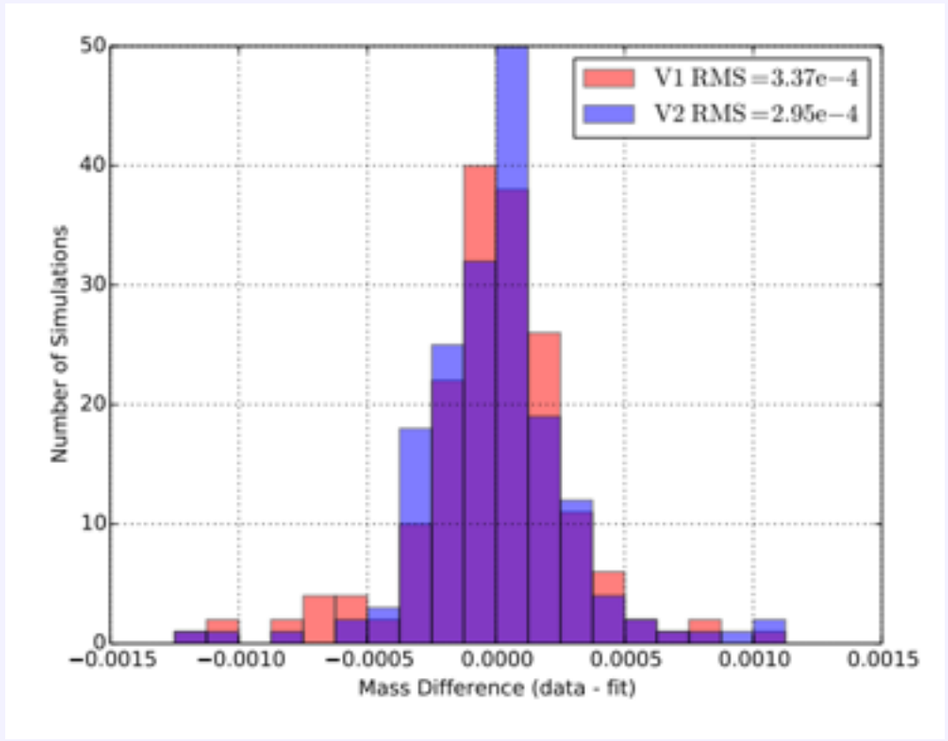


Harry+ (arXiv:1603.02444)
Indik+ (arXiv:1612.05173)

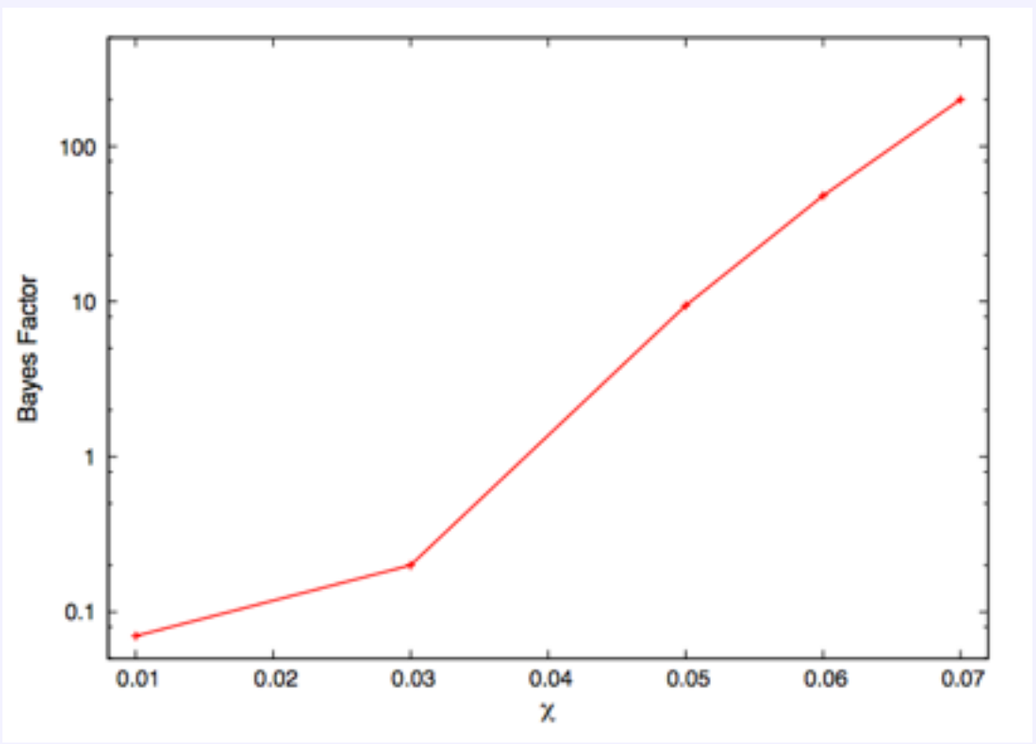
Can we have a fully precessing search?

Do we need it?

More Questions: Testing GR



Healy+ (arXiv:1610.09713)
Keitel+ (arXiv:1612.09566)
Hofmann+ (arXiv:1605.01938)



Sampson+ (arXiv:1311.4898) Agathos+ (arXiv:1311.0420)

Are there configurations that make TGR tests easier?

How do systematics affect TGR?

Thank you!