45-Degree Update of The Wic Fitter Algorithm

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Abstract

This note describes changes in the calculation of the inverse weight matrix due to multiple scattering for the 45-Degree Chamber implementation of the WIC track fitter.

Introduction

The WIC fitter assigns a chi-squared to a hypothetical track trajectory through the Warm Iron Calorimeter as a function of a track parameters {P} and measured hit positions y^*_i:

$$\chi^2 = \sum_{i,j=1}^{N} \left(y_i^* - y_i(\{P\})\right) W_{ij} \left(y_j^* - y_j(\{P\})\right)$$

y_i^* is the i[th] of the N measured coordinates that constitute the track,

$y_i(\{P\})$ is the fitted function of the parameters corresponding to y_i^*,

The parameters set {P} consists of positions and slopes of the starting trajectory projected along two orthogonal directions on a specified reference plane, and the magnitude of the initial momentum,

W is the weight matrix. It has non-diagonal terms arising from the persistence of deflections due to multiple scattering. The correlation between the measurement residues at the ith and jth detector plane due to multiple scattering deviations is represented in the off diagonal elements of the weight matrix:

$$W_{ij}^{-1} = <(y_i^* - <y_i^*>)\,(y_j^* - <y_j^*>)> - <y_i^* - <y_i^*>><y_j^* - <y_j^*>>$$

$$= \delta_{ij}\sigma_i^2 + (14\mathrm{MeV}^2)\hat{m}_i \bullet \hat{m}_j \times$$

$$\int_0^{\min(s_i, s_j)} ds'\left(s_i - s'\right)\left(s_j - s'\right) / \left[\hat{t}(s_i)\cdot\hat{n}_i \hat{t}\left(s_j\right)\cdot\hat{n}_j X_{rad}(s')p(s')^2\right],$$

where the integral is taken over all the material in front of both detector planes i and j,

σ_i is the rms position resolution error assigned to the i[th] and measurement,

s_i, s_j are the path lengths to the points where the track crossed the i^{th} and j^{th} detector planes,

$\hat{m}_i, \hat{m}_j$ are unit vectors along the i^{th} and j^{th} measurement axes,

$\hat{n}_i, \hat{n}_j$ are unit vectors normal to me i^{th} and j^{th} detector planes,

$p(s')$ is the momentum times c, and

$X_{rad}(s')$ is the radiation length at path length s' along the trajectory.

The expression has an obliquity factor

$$\hat{m}_i \bullet \hat{m}_j \,/\, \left(\hat{t} \bullet \hat{n}_i\right)\left(\hat{t} \bullet \hat{n}_j\right)$$

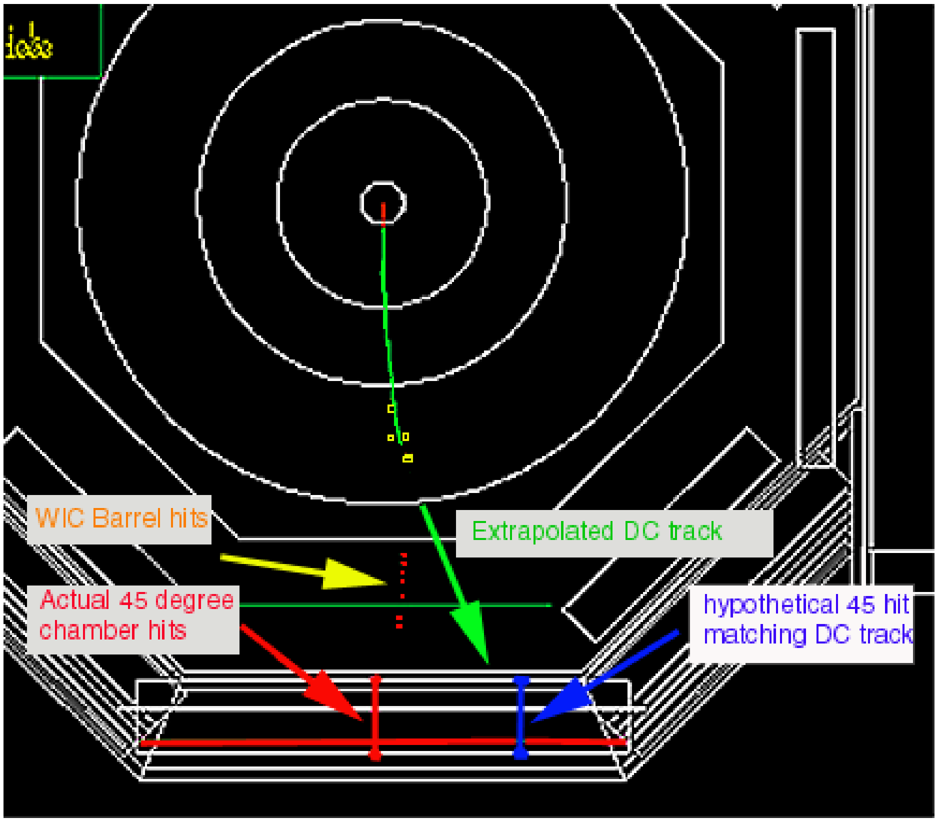
which is appropriate to a fixed target geometry with all the detector planes aligned along a common normal direction and with the track trajectory nearly normal to the detector planes in all cases.

New Obliquity Factor

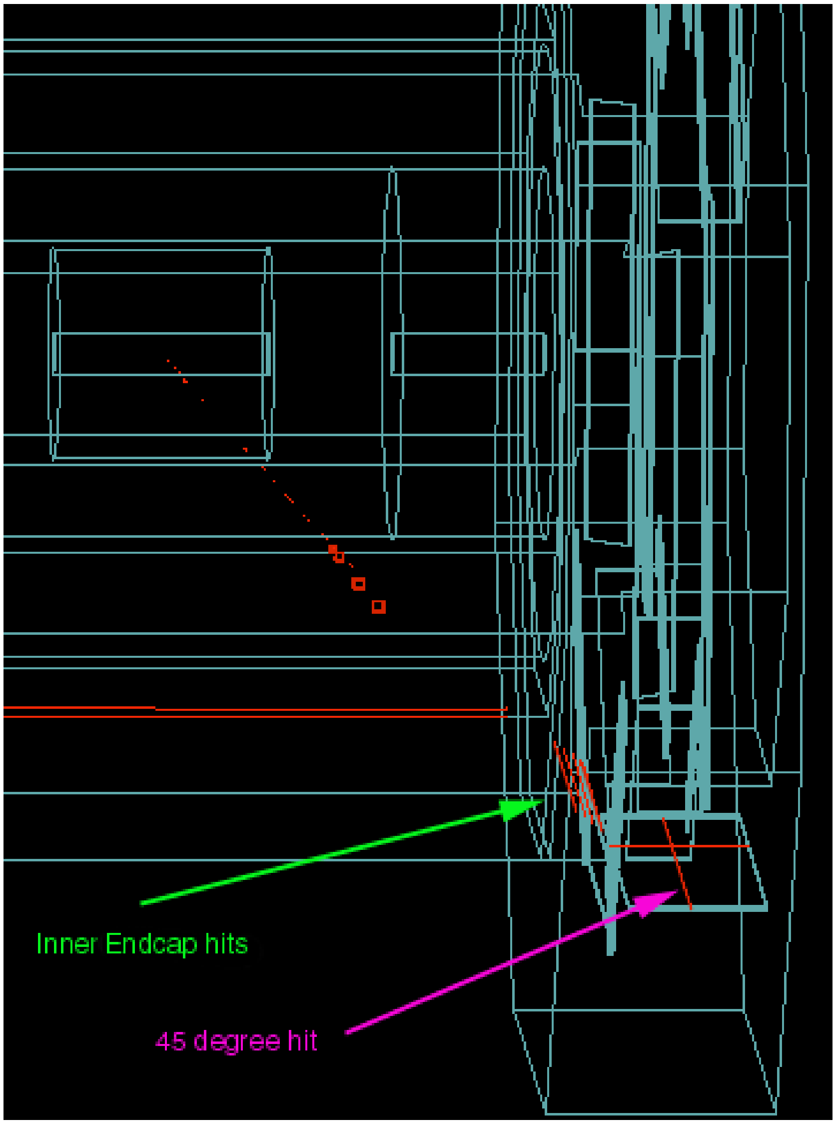
In the region of the barrel and end cap joint the assumption of small incident angle is not applicable. Furthermore the barrel, end cap, and 45-degree chambers do not have a common normal direction. Some pairs of hits are not appropriately correlated as a consequence.

Figure 1 is event display of a 3.5 GeV Monte Carlo muon. The particle trajectory through the drift chamber is shown in green. The yellow boxes are minimum ionizing depositions in the Liquid Argon Calorimeter, the yellow arrow indicates some wire hits in the WIC for wires parallel to the beam axis (the red dots) and the red lines indicate wire hits in the 45-degree chambers.

If the track through the drift chamber is extrapolated through the WIC, it follows more or less the green arrow. Due to multiple scattering, however, the actual hit is much further to the left as indicated by the red arrow. A hypothetical 45-degree hit, indicated by the blue arrow, would align better with the track trajectory. The fitter however, should prefer the actual hit because it is much better correlated with the hits in the barrel WIC as indicated by the yellow arrow.



The weight matrix as constituted doesn't correlate certain of 45-degree chamber hits with the relevant endcap and barrel hits. Figure 2. below represents another 3.5 GeV muon in the WIC. The measurement axis for the inner endcap hits is vertical and the 45-degree chamber hit has a horizontal measurement axis; there is no correlation in the weight matrix because the obliquity factor is zero for two detector planes with orthogonal measurement axes. The 45-degree hit lines up well with the endcap hits and poorly with the extrapolated chamber track. Without the correlations being included the 45-degree hits tend to pull the WIC track off of the correct trajectory in the direction of the multiple scatter.



We calculated a new obliquity factor with fewer simplifying assumptions:

$$\left[(\hat{\alpha}_1 \bullet \hat{m}_1)(\hat{\alpha}_2 \bullet \hat{m}_2)\cos\Delta\phi / \left((\hat{t} \bullet \hat{n}_1)(\hat{t} \bullet \hat{n}_2)\right)\right] +$$

$$\left[(\beta_1 \bullet \hat{m}_1)(\hat{\beta}_2 \bullet \hat{m}_2)\cos\Delta\phi\right] +$$

$$\left[(\hat{\alpha}_1 \bullet \hat{m}_1)(\hat{\beta}_2 \bullet \hat{m}_2)\sin\Delta\phi / (\hat{t} \bullet \hat{n}_1)\right] -$$

$$\left[(\hat{\alpha}_2 \bullet \hat{m}_2)(\hat{\beta}_1 \bullet \hat{m}_1)\sin\Delta\phi / (\hat{t} \bullet \hat{n}_2)\right]$$

with the following definitions:

$$\hat{\alpha}_1 = \frac{(\hat{n}_1 \bullet \hat{t})\hat{n}_1 - \hat{t}}{|\hat{t} \times \hat{n}_1|}$$

$$\hat{\alpha}_2 = \frac{(\hat{n}_2 \bullet \hat{t})\hat{n}_2 - \hat{t}}{|\hat{t} \times \hat{n}_2|}$$

$$\hat{\beta}_1 = \frac{(\hat{t} \times \hat{n}_1)}{|\hat{t} \times \hat{n}_1|}$$

$$\hat{\beta}_2 = \frac{(\hat{t} \times \hat{n}_2)}{|\hat{t} \times \hat{n}_2|}$$

$$\cos\Delta\phi = \frac{\hat{n}_1 \bullet \hat{n}_2 - (\hat{t} \bullet \hat{n}_1)(\hat{t} \bullet \hat{n}_2)}{|\hat{t} \times \hat{n}_1||\hat{t} \times \hat{n}_2|}$$

$$\sin\Delta\phi = \frac{\hat{t} \bullet (\hat{n}_2 \times \hat{n}_1)}{|\hat{t} \times \hat{n}_1||\hat{t} \times \hat{n}_2|}$$

The calculation is in the next section.

Weight Matrix Calculation

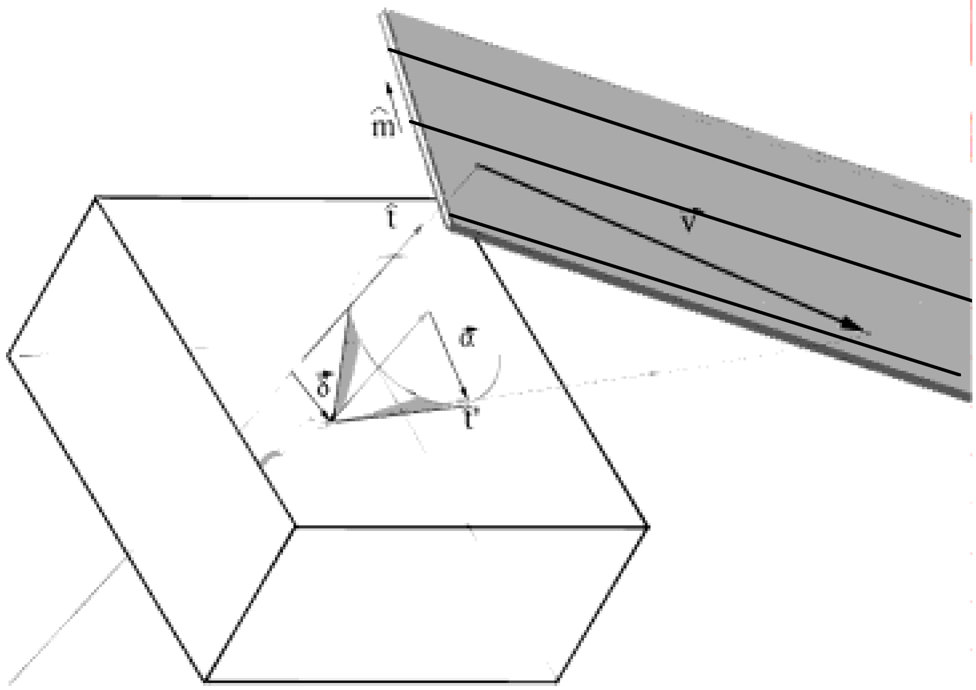
Multiple scattering effects are included in the calculation of the inverse of the weight matrix. This is done by estimating the effect of multiple scattering on residuals in the measurement axis of each detector plane.

A muon moves through a block of matter a distance Δz in the direction of the incident track. Due to multiple scattering, when the particle exits the block, it has been displaced by an impact parameter δ and deflected through an angle α. Its new direction is represented as **t′**.

Fermi calculated the joint probability density for α and δ to be:

$$f(\vec{\alpha},\vec{\delta}) = \frac{12}{\pi\theta_s^4} e^{-\frac{4}{\theta_s^2}\left[\alpha^2 + \frac{3\delta^2}{\Delta z^2} - \frac{3\vec{\alpha}\cdot\vec{\delta}}{\Delta z^3}\right]}$$

$$\theta_s \equiv \frac{21MeV}{\beta p}\frac{1}{\sqrt{X_{RAD}}}$$

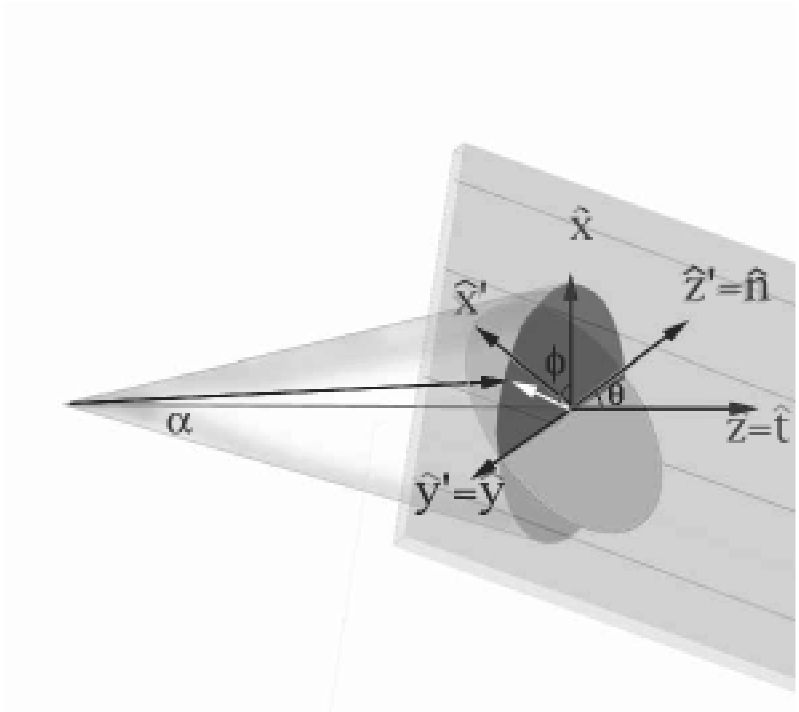


α is perpendicular to **t** and has magnitude α. For small enough α This vector is $\hat{t}' - \hat{t}$. The vector **δ** is perpendicular to **t** and has the magnitude delta. And **V** is the vector connecting the points where the undeviated track would intersect the plane to the place where the multiple scattered track actually intersects it.

The weight matrix will be:

$$W_{ij}^{-1} = \sum_{\Delta Z} \left\langle \left(\vec{V}_i \cdot \hat{m}_i\right)\left(\vec{V}_j \cdot \hat{m}_j\right) \right\rangle_{\phi_\alpha, \phi_\delta, \alpha, \delta} + \delta_{ij}\sigma_i^2$$

The summation is the part due to multiple scattering and the delta term is due to measurement uncertainties, including the strip width and uncorrelated alignment errors. To find the average in the first term we set up the following coordinate systems:



$$\hat{z} = \hat{t}$$

$$\hat{x} = \frac{\hat{t} \times \hat{n}}{\left|\hat{t} \times \hat{n}\right|} \times \hat{t}$$

$$\hat{z}' = \hat{n}$$

$$\hat{x}' = \frac{\hat{t} \times \hat{n}}{\left|\hat{t} \times \hat{n}\right|} \times \hat{n}$$

$$\hat{y} = \hat{y}' = \frac{\hat{t} \times \hat{n}}{\left|\hat{t} \times \hat{n}\right|}$$

$\hat{n}$ is the unit vector normal to the chamber plane and $\hat{t}$ is the unit vector in the track trajectory direction. ϕ is the angle between the $\hat{x}$ axis and the vector α .

The coordinate axes are related by:

$$x = x' \cos\theta + z' \sin\theta$$

$$y = y'$$

$$z = z' \cos\theta - x' \sin\theta$$

Points on the cone obey:

$$x = (s+z)\tan\alpha\cos\phi$$
$$y = (s+z)\tan\alpha\sin\phi$$

At the detector plane, $z' = 0$, or z=-xtanα

Points on the intersection of that plane and the cone obey:

$$x_p = x'\cos\theta = (z_p + s)\tan\alpha\cos\phi$$
$$y_p = y' = (z_p + s)\tan\alpha\sin\phi$$
$$z_p = -x'\sin\theta = -x_p\tan\theta$$

Substituting and rearranging:

$$x_p = \frac{s\tan\alpha\cos\phi}{1+\tan\theta\tan\alpha\cos\phi}$$

One can now obtain a new expression for z on the plane:

$$z_p = \frac{-s\tan\alpha\cos\phi\tan\theta}{1+\tan\theta\tan\alpha\cos\phi}$$

and for y on the plane

$$y_p = \frac{-z_0\tan\alpha\sin\phi}{1+\tan\theta\tan\alpha\cos\phi}$$

For small angles of deflection the second term in the denominator of each expression is small.

$$x_p \approx s\alpha\cos\phi$$
$$y_p \approx s\alpha\sin\phi$$
$$z_p \approx -s\alpha\cos\phi\tan\theta$$

In the primes coordinate system the vector V_α (the α contribution to V) is (x’,y’,0) . Then :

$$\vec{V}_\alpha \bullet \hat{m} = m_{x'}V_{\alpha x'} + m_{y'}V_{\alpha y'} + 0$$

so with $V_{\alpha x'} = \dfrac{x_p}{\cos\theta}$

$V_{\alpha y'} = y' = y$

$$V_{\alpha x'} = \frac{s\alpha\cos\phi}{\cos\theta}$$
$$V_{\alpha y'} = s\alpha\sin\phi$$

Then

$$(\vec{m}\cdot\vec{V}_\alpha) = m_{x'}V_{\alpha x'} + m_{y'}V_{\alpha y'}$$
$$= \left(\frac{(\hat{t}\times\hat{n})\times\hat{n}}{|\hat{t}\times\hat{n}|}\right)\bullet\hat{m}\frac{s\alpha\sin\phi}{\cos\vartheta} +$$
$$\frac{(\hat{t}\times\hat{n})\bullet\hat{m}}{|\hat{t}\times\hat{n}|}s\alpha\sin\phi$$

The expression for $\vec{m}\cdot\vec{V}_\delta$can be seen from the expression for $m\cdot\vec{V}_\alpha$ if we recognize that taking the small angle α and large value for s in a limit so that as α→ 0, sα→δ.

$$(\vec{m}\cdot\vec{V}) = \left(\frac{\hat{t}\times\hat{n}}{|\hat{t}\times\hat{n}|}\times\hat{n}\right)\cdot\hat{m}\frac{s\alpha\cos\phi_\alpha + \delta\cos\phi_\delta}{\cos\theta} + \frac{(\hat{t}\times\hat{n})\cdot\hat{m}}{|\hat{t}\times\hat{n}|}(s\alpha\sin\phi_\alpha + \delta\sin\phi_\delta)$$

In the inverse weight matrix:

$$W_{ij}^{-1} = \sum_{\Delta Z}\left\langle(\vec{V}_i\cdot\hat{m}_i)(\vec{V}_j\cdot\hat{m}_j)\right\rangle_{\phi_\alpha,\phi_\delta,\alpha,\delta} + \delta_{ij}\sigma_i^2$$

The two detector planes i and j will have different normal directions. Therefore they will have different orientations for their axes. Their unprimed axes - $\hat{x}_i\hat{y}_i$ and $\hat{x}_j\,\hat{y}_j$ lie in a plane however. We define the angle between the two axes $\hat{y}_i$ and $\hat{y}_j$ is Δ.

We define the angle by:

$$\hat{y}_1\cdot\hat{y}_2 = \frac{\hat{t}\times\hat{n}_1}{|\hat{t}\times\hat{n}_1|}\cdot\frac{\hat{t}\times\hat{n}_2}{|\hat{t}\times\hat{n}_2|} = \cos\Delta\phi$$

along with

$$\sin\theta = \hat{x}_1 \cdot \hat{y}_2 = \frac{\left(\hat{t} \times \hat{n}_1\right) \times \hat{t}}{\left|\hat{t} \times \hat{n}_1\right|} \cdot \frac{\hat{t} \times \hat{n}_2}{\left|\hat{t} \times \hat{n}_2\right|}$$

so that we can consistently use:
$\phi_1 + \Delta\phi = \phi_2$.
Using:

$$\left(\hat{n}_1 - \left(\hat{n}_1 \cdot \hat{t}\right)\hat{t}\right) \cdot \left(\hat{t} \times \hat{n}_2\right) = \hat{n}_{1\cdot}\left(\hat{t} \times \hat{n}_2\right)$$

We can reduce the expression for sinΔϕ to:

$$\sin\Delta\phi = x_1 \cdot y_2 = \frac{t \cdot (\hat{n}_2 \times \hat{n}_1)}{\sin\theta_1 \sin\theta_2}$$

and using:

$$\left(\hat{t} \times \hat{n}_1\right) \cdot \left(\hat{t} \times \hat{n}_2\right) = (\hat{t} \cdot \hat{t})(\hat{n}_1 \cdot \hat{n}_2) - (\hat{t} \cdot \hat{n}_1)(\hat{t} \cdot \hat{n}_2)$$

$$\cos\Delta\phi = \frac{(\hat{n}_1 \cdot \hat{n}_2) - (\hat{t} \cdot \hat{n}_1)(\hat{t} \cdot \hat{n}_2)}{\sin\theta_1 \sin\theta_2}$$

Now we have:

$$\phi_{\alpha_j} = \phi_{\alpha_i} + \Delta\phi$$
$$\phi_{\delta_j} = \phi_{\delta_i} + \Delta\phi$$

We define:

$$F_1(\hat{t}, \hat{n}, \hat{m}) = \left(\frac{(\hat{t} \times \hat{n}) \times \hat{n}}{\left|\hat{t} \times \hat{n}\right|\left|\hat{t} \cdot \hat{n}\right|}\right) \cdot \hat{m}$$

and

$$F_2(\hat{t}, \hat{n}, \hat{m}) = \left(\frac{(\hat{t} \times \hat{n})}{\left|\hat{t} \times \hat{n}\right|}\right) \cdot \hat{m}$$

The we have:

$$W_{ij}^{-1} = \delta_{ij}\sigma_i^2 +$$
$$\sum_{\Delta Z}\Big\langle\Big(F_1(\hat{t}_1,\hat{n}_1,\hat{m}_1)\big(S_1\alpha\cos\phi_\alpha + \delta\cos\phi_\delta\big) + F_2(\hat{t}_1,\hat{n}_1,\hat{m}_1)\big(S_1\alpha\cos\phi_\alpha + \delta\cos\phi_\delta\big)\Big)$$
$$*\Big(F_1(\hat{t}_2,\hat{n}_2,\hat{m}_2)\big(S_2\alpha\cos(\phi_\alpha + \Delta\phi) + \delta\cos(\phi_\delta + \Delta\phi)\big) + F_2(\hat{t}_2,\hat{n}_2,\hat{m}_2)\big(S_2\alpha\cos(\phi_\alpha + \Delta\phi) + \delta\cos(\phi_\delta + \Delta\phi)\big)\Big)\Big\rangle$$

Using the identities:

$$\cos(\phi_\alpha + \Delta\phi) = \cos\phi_\alpha\cos\Delta\phi - \sin\phi_\alpha\sin\Delta\phi$$
$$\sin(\phi_\alpha + \Delta\phi) = \cos\phi_\alpha\sin\Delta\phi + \sin\phi_\alpha\cos\Delta\phi$$

and transforming to cartesian coordinates ($\alpha,\delta,\phi_\alpha,\phi_\delta$ to $\alpha_x,\delta_x,\alpha_y,\delta y$) and integrating:

$$W_{ij}^{-1} = \delta_{ij}\sigma_i^2 + \sum_{\Delta Z}\Lambda\Big(\langle\alpha^2\rangle S_1S_2 + \langle\delta^2\rangle + \langle\alpha\delta\rangle(S_1 + S_2)\Big)$$

Where $\Lambda = F_{11}F_{12}\cos\Delta\phi - F_{12}F_{21}\sin\Delta\phi + F_{11}F_{22}\sin\Delta\phi + F_{21}F_{22}\cos\Delta\phi$

and :

$$\langle\alpha^2\rangle = A^2\Delta z / X$$
$$\langle\alpha\delta\rangle = \tfrac{1}{2}A^2\Delta z^2 / X$$
$$\langle\delta^2\rangle = \tfrac{1}{3}A^2\Delta z^3 / X$$

$$F_1(\hat{t},\hat{n},\hat{m}) = \left(\frac{(\hat{t}\times\hat{n})\times\hat{n}}{|\hat{t}\times\hat{n}||\hat{t}\cdot\hat{n}|}\right)\cdot\hat{m} = \frac{-\hat{t}\cdot\hat{m}}{|\hat{t}\times\hat{n}||\hat{t}\cdot\hat{n}|}$$
$$F_2(\hat{t},\hat{n},\hat{m}) = \left(\frac{(\hat{t}\times\hat{n})}{|\hat{t}\times\hat{n}|}\right)\cdot\hat{m} = \left(\frac{(\hat{n}\times\hat{m})}{|\hat{t}\times\hat{n}|}\right)\cdot\hat{t}$$

These results for the obliquity factor Λ correspond to the value quoted above. The α^2 term gives the dominant contribution to the inverse weight now of the new form:

$$W_{ij}^{-1} = \delta_{ij}\sigma_i^2 + (14\mathrm{MeV}^2)\times\int_0^{\min(s_i,s_j)}\Lambda ds'(s_i - s')(s_j - s') / \left[X_{rad}(s')p(s')^2\right]$$