International Journal of Modern Physics A
Vol. 20, No. 16 (30 June 2005) 3851–3856

World Scientific
www.worldscientific.com

6D IONIZATION MUON COOLING WITH TABLETOP RINGS

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Progress on six dimensional ionization muon cooling with relatively small rings of magnets is described. Lattices being explored include scaling sector cyclotrons with edge focusing and strong focusing, fixed field alternating gradient (FFAG) rings. Ionization cooling is provided by high pressure hydrogen gas which removes both transverse and longitudinal momentum. Lost longitudinal momentum is replaced using radio frequency (RF) cavities, giving a net transverse emittance reduction. The longer path length in the hydrogen of higher momentum muons decreases longitudinal emittance at the expense of transverse emittance. Thus emittance exchange allows these rings to cool in all six dimensions and not just transversely. Alternatively, if the RF is located after the ring, it may be possible to cool the muons by stopping them as they spiral adiabatically into a central swarm. As $p \to 0$, $\Delta p \to 0$. The resulting cooled muons can lead to an intense muon beam which could be a source for neutrino factories or muon colliders.

Keywords: beam cooling; cyclotron; muon; black hole.

1. Introduction

Muons at rest have a 2.2 μs lifetime; cooling an ensemble of muons must be completed faster than that. Ionization cooling can help.[1] Random muon motion is removed by passage through a low Z material, such as hydrogen, and coherent motion is added with RF acceleration. Designs to cool muons in six dimensions using

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linear helical channels[2] at 100 MeV kinetic energies and using frictional cooling[3] at 1 keV kinetic energies are under investigation. A number of muon cooling rings have been simulated at various levels.[4] In a ring structure, the same magnets and RF cavities may be reused each time a muon orbits. Transverse cooling can naturally be exchanged for longitudinal cooling by allowing higher momentum muons to pass through more material. Thus rings can cool in all six dimensions. The rings reported here are the smallest to date, and are basically radial sector cyclotrons.

Small emittance bunches of cold muons are useful for a neutrino factory[5] and are required for a muon collider.[6] At a neutrino factory, accelerated muons are stored in a racetrack to produce neutrino beams ($\mu^- \to e^- \overline{\nu}_e \, \nu_\mu$ and $\mu^+ \to e^+ \nu_e \, \overline{\nu}_\mu$). Neutrino oscillations have been observed[7] and need more study. Further exploration at a neutrino factory could reveal CP violation in the lepton sector,[8] and will be particularly useful if the ν_e to ν_τ coupling, θ_{13}, is small.[9] A muon collider can do s-channel scans to split the H^0 and A^0 Higgs doublet.[10] Above the ILC's 800 GeV there are a large array of supersymmetric particles that might be produced[11] and, if large extra dimensions exist, so could mini black holes.[12] Note that the energy resolution of a 4 TeV muon collider is not smeared by beamstrahlung.

2. Hydrogen Gas Filled Sector Cyclotrons with Internal RF

A low field ring is being designed using ICOOL[13] for optimization and SYNCH[14] for lattices. It could be built to demonstrate six dimensional muon cooling. Relatively constant β functions allow a continuous placement of the energy absorber. Hydrogen gas cools best and also prevents breakdown in the 201 MHz RF cavities.[2] Skew quadrupole magnets mix vertical and horizontal betatron oscillations. The parameters for the ring appear in Table 1. Simulated 6D cooling is shown in Fig. 2.

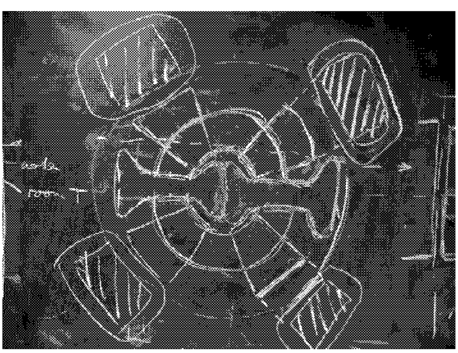

Fig. 1. A way of fitting a pair of large 201 MHz RF cavities into a small sector cyclotron. The cavities fill the center of the cyclotron. The four hashed areas around the periphery are return yokes for the magnets.

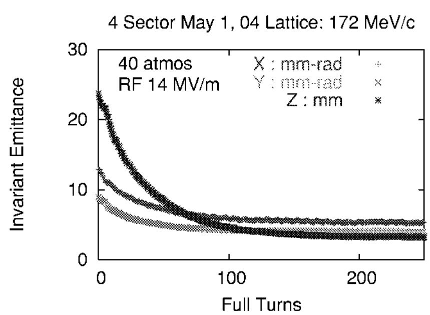

Fig. 2. X, Y, and Z emittance versus orbits for the low field ring with four sectors that is described in Table 1.

Table 1. Scaling sector cyclotron parameters. The cooling merit factors include transmission loss; the X, Y, and Z cooling factors do not. The low field ring is designed to test 6D cooling.

	Low Field Ring	High Field Ring	FFAG Ring
Dipole Field	1.8 T	5.2 T	2.6 T
Magnetic Elements	hard edge	hard edge	hard edge
Sectors	4	6	12
Focusing	edge	edge	alternating gradient
β_x Range	38 → 92 cm	26 → 36 cm	44 → 65 cm
β_y Range	54 → 66 cm	30 → 32 cm	26 → 42 cm
p(central orbit)	172 MeV/c	250 MeV/c	250 MeV/c
Hydrogen Pressure	40 Atm. @ 300^0 K	100 Atm. @ 300^0 K	100 Atm. @ 300^0 K
Peak RF Gradient	14 MV/m	45 MV/m	8 MV/m
Total RF Length	1.6 m	0.8 m	3.6 m
Ring Circumference	3.81 m	1.95 m	6.0 m
X Aperture	±20 cm	±25 cm	±25 cm
Y Aperture	±15 cm	±15 cm	±15 cm
p_z Acceptance	±10 MeV/c	±10 MeV/c	±10 MeV/c
Orbits	100	250	40
X, Y, Z Cooling Factors	2.4, 2.3, 7.4	2, 13, 20	16, 15, 2
Muon Decay Included	no	yes	yes
Cooling Merit Factor	20	400	120

The equation describing transverse cooling (with energies in GeV) is:

$$\frac{d\epsilon_n}{ds} = -\frac{1}{\beta^2}\frac{dE_\mu}{ds}\frac{\epsilon_n}{E_\mu} + \frac{1}{\beta^3}\frac{\beta_\perp (0.014)^2}{2\, E_\mu m_\mu\, L_R}, \tag{1}$$

where β = v/c, ϵ_n is the normalized emittance, $\beta_\perp$ is the betatron function (focal length) at the absorber, dE_μ/ds is the energy loss, and L_R is the radiation length of the material. The first term in this equation is the cooling term, and the second is the heating term due to multiple scattering. This heating term is minimized if $\beta_\perp$ is small (strong-focusing) and L_R is large (a low-Z absorber). The equilibrium emittance is achieved when the heating and cooling terms balance.

A higher field ring with smaller values of $\beta_\perp$ can give a higher cooling merit factor. Simulations show that high RF gradients are required as noted in Table 1.

A scaling fixed field, alternating gradient (FFAG) ring[15] can allow the use of lower RF gradients and magnetic fields. For the FFAG ring in Table 1, a focusing parameter, $n = -(r/B)(dB/dr) = -0.6$, was used. Each sector has a focusing-defocusing-focusing (FDF) geometry with three magnets. There are no open slots between sectors, so the RF cavities would have to placed within the magnets.

3. Sector Anti–Cyclotrons with External RF

An anti-cyclotron has been used to slow LEAR anti-protons at CERN.[16,17] An annular quasipotential well, $U(r, z)$, is formed which ferries anti-protons towards the center of an azimuthally symmetric cyclotron. The radius of the annulus decreases with the decreasing angular momentum of the $\overline{p}$.

$$U(r,z) = V(r,z) - (1/(2\eta r^2))\,(L_g/M + \eta\, r A_\theta)^2, \tag{2}$$

where $\eta = e/M$ and $L_g = L_z - e\, r\, A_\theta$ is a generalized angular momentum. The radial well deepens with decreasing radius and the vertical well grows shallower (see Fig. 2 of Ref. 16). Particles must adiabatically spiral to the center. If dE/dx is too large, particles will not stay in the magnetic wells. The final $\overline{p}$ swarm has a radius of 1.5 cm, a height of 4 cm, and a kinetic energy of 2 keV. A long bunch train is coalesced into a single swarm, which is roughly the same diameter as the incoming beam. The spiral time is 20 μs with 0.3 mbar hydrogen and about 1 μs with 10 mbar hydrogen. Given the dependence of the cyclotron frequency on mass, $f = \omega/2\pi = qB/2\pi m$, the spiral time for a muon is nine time less than for a $\overline{p}$. The gas pressure in the center must be low, both to allow a particle to spiral all the way in before stopping, and to allow reasonable kicker voltages for extraction. An 80 ns electric kicker pulse rising to 500 V/cm in 20 ns is employed. The $\overline{p}$'s move 32 cm in 500 ns. Given that $F = ma$, muons will go farther.

The anti-cyclotron has now been moved from LEAR at CERN to PSI where it is being used to slow negative muons to kinetic energies of a few keV.[18] Three centimeter diameter beams with 30 000 μ^-/s below 50 keV and 0.8 cm diameter beams with 1000 μ^-/s in the 3 to 6 keV kinetic energy range are output for use. A static electric field continuously ejects the muons. The energy absorber and the electrode consist of a single 30 μg/cm^2 formvar foil with a 3 nm nickel film produced by 30 minutes of sputtering.

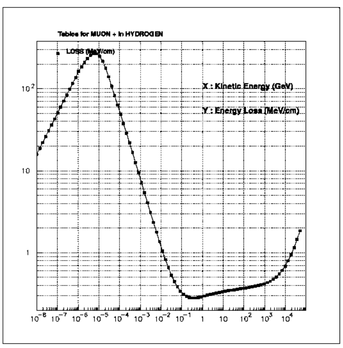

Fig. 3. Plot of μ^+ energy loss (MeV/cm) in liquid hydrogen versus kinetic energy (GeV) generated using GEANT3. The default value of the GEANT3 variable "CUTMUO" was decreased from 10 MeV to 10 eV to permit the propagation of slow muons. The energy turnover at 8 keV corresponds to a momentum of 1.3 MeV/c. $p = \sqrt{2mE} = \sqrt{2 \times 105.7 \times 0.008}$. Liquid helium, aluminum, copper, and iron show similar results.

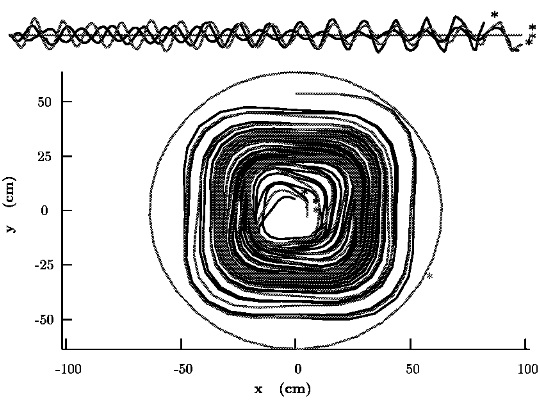

Fig. 4. ICOOL[13] simulation of single turn, energy loss injection. Three muons with identical momenta are injected into a low field cyclotron with four sectors like the one in Table 1, but with soft edged magnetic fields. The inward spirals differ because of multiple scattering and straggling. The energy loss is caused by radial LiH wedges surrounded by hydrogen gas. The amount of matter encountered in a given orbit decreases adiabatically with radius. The upper trace shows that vertical motion is completely contained within ± 5 cm along the 70 m spiral. The fractional energy loss required in the first turn for injection increases with the width of the muon beam and decreases as the cyclotron's magnetic field is lowered.

Table 2. Slow muon sticking solutions. Charged foils might repel muons and prevent them from stopping in the foils. In muon catalyzed fusion, a μ^- is typically freed 150 times before it sticks to an ^4He nucleus.

Positively charged foils for μ^+.
Helium to slow μ^+e^- formation.
Laser disassociate μ^+e^-.
Negatively charged foils for μ^-.
DT or D_2 gas. Fusion frees μ^-.

Table 3. Emittance reduction goals for two anti-cyclotrons used in series. Between the anti-cyclotrons, Busch's theorem gives a Δp_ϕ kick, muons are accelerated, and Δp_x is traded for Δx and Δp_y for Δy. Emittance goes as $(\Delta p_x \, \Delta x)\,(\Delta p_y \, \Delta y)\,(\Delta p_z \, \Delta z)$. A muon collider needs a factor of 10^6 in cooling.

	First Anti-Cyclotron	Second Anti-Cyclotron
Δp_x	$50 \rightarrow 1$ MeV/c	$15 \rightarrow 1$ MeV/c
Δp_y	$50 \rightarrow 1$ MeV/c	$15 \rightarrow 1$ MeV/c
Δp_z	$50 \rightarrow 1$ MeV/c	$1 \rightarrow 1$ MeV/c
Δx	$15 \rightarrow 10$ cm	$5 \rightarrow 3$ cm
Δy	$15 \rightarrow 10$ cm	$5 \rightarrow 3$ cm
Δz	$1000 \rightarrow 10$ cm	$10 \rightarrow 3$ cm

A scaling sector anti-cyclotron could allow greater acceptance than the azimuthally symmetric anti-cyclotron now running at PSI. For a given $\int \mathbf{B} \cdot d\ell$, the ratio of the fields in the hills and valleys can be adjusted to maximize acceptance. Only radial and not spiral sectors have been explored so far. The sector anti-cyclotron may be able to function as a damped harmonic oscillator to lower the amplitude of horizontal and vertical betatron motion as a bunch train of muons spirals into a single central swarm. With 10^{12} muons in a swarm, space charge is a concern. A conductor might be used for neutralization. Movement of 10^{12} electrons in 100 ns only requires 1.6 amps of current. Muons must spiral in fast enough to minimize decay loss, but must not stop before reaching the central swarm. So, the density of the absorber must decrease smoothly with radius. Radial LiH wedges immersed in a gas or high to low pressure gases separated by beam pipes might meet this criteria. The sector cyclotron geometry must transform into an azimuthally symmetric magnetic bottle as the muons approach the central swarm. Otherwise, as shown by GEANT3 simulations, muons will escape though the valleys. In the transition region the field might roughly resemble a hexapole or octupole field as used in an Electron Cyclotron Resonance Ion Source (ECRIS).[19] Busch's theorem (eqn. 3)[20] has the effect of increasing the emittance as muons leave a magnetic field. A half Tesla field and a 10 cm radius give an 8 MeV/c azimuthal kick. One might be able to use radial iron fins in the exit port to alleviate this effect or reverse and increase the magnitude of the magnetic field to capture the unwanted angular momentum in an absorber just after extraction. Using low fields with cylindrical swarms that have small diameters works for sure. An RF quadrupole is perhaps a natural choice for acceleration that would immediately follow the extraction electric kicker.

$$\dot{\phi} = [e/(2\pi \gamma \, m \, r^2(s))][\Phi(s) - \Phi_k], \quad L_z = xp_y - yp_x = r^2 \gamma \, m \, \dot{\phi} = -e \, B \, r^2/2 \quad (3)$$

In summary, progress on large acceptance tabletop muon rings is underway, including energy loss injection (see Fig. 4), 6D cooling, and electric kicker extraction.
Many thanks to Juan Gallardo and Franz Kottmann for useful suggestions.

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