Astrophysical inference from gravitational wave detections

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Many thanks to...

R. O'Shaughnessy, L. London, J. Healy, and D. Shoemaker.
Precession during merger: Strong polarization changes are observationally accessible features of strong-field gravity during binary black hole merger.

A. Lundgren and **R. O'Shaughnessy**.
A single-spin precessing gravitational wave in closed form.

L. Pekowsky, **R. O'Shaughnessy**, J. Healy, and D. Shoemaker.
Comparing gravitational waves from nonprecessing and precessing black hole binaries

K. Belczynski, M. Dominik, T. Bulik, **R. O'Shaughnessy**, C. L. Fryer, and D. E. Holz.
The effect of metallicity on the detection prospects for gravitational waves.

R. O'Shaughnessy, V. Kalogera, and K. Belczynski.
Binary Compact Object Coalescence Rates: The Role of Elliptical Galaxies.

R. O'Shaughnessy and C. Kim.
Pulsar Binary Birthrates with Spin-opening Angle Correlations.

R. O'Shaughnessy, R. Kopparapu, and K. Belczynski.
Impact of star formation inhomogeneities on merger rates and interpretation of LIGO results.

K. Belczynski, **R. O'Shaughnessy**, V. Kalogera, F. Rasio, R. E. Taam, and T. Bulik.
The Lowest-Mass Stellar Black Holes: Catastrophic Death of Neutron Stars in Gamma-Ray Bursts.

R. O'Shaughnessy, C. Kim, V. Kalogera, and K. Belczynski.
Constraining population synthesis models via observations of compact-object binaries and supernovae.

R. M. O'Leary, **R. O'Shaughnessy**, and F. A. Rasio.
Dynamical Interactions and the Black Hole Merger Rate of the Universe.

Reconstructing the sky location of gravitational-wave detected compact binary systems: methodology for testing and comparison

T. Sidery,[1,*] B. Aylott,[1] N. Christensen,[2] B. Farr,[3,1] W. Farr,[3,1] F. Feroz,[4] J. Gair,[5] K. Grover,[1] P. Graff,[6] C. Hanna,[7] V. Kalogera,[3] I. Mandel,[1] R. O'Shaughnessy,[8] M. Pitkin,[9] L. Price,[10] V. Raymond,[10] C. Röver,[11] L. Singer,[10] M. van der Sluys,[12] R.J.E. Smith,[1] A. Vecchio,[1] J. Veitch,[13] and S. Vitale[14]

Parameter estimation for compact binary coalescence signals with the first generation gravitational-wave detector network

J. Aasi[1], J. Abadie[1], B. P. Abbott[1], R. Abbott[1], T. D. Abbott[2], M. Abernathy[3], T. Accadia[4], F. Acernese[5a,5c], C. Adams[6], T. Adams[7], P. Addesso[58], R. Adhikari[1], C. Affeldt[9,10], M. Agathos[11a], K. Agatsuma[12], P. Ajith[1],

Predictions for the Rates of Compact Binary Coalescences Observable by Ground-based Gravitational-wave Detectors

J. Abadie[29], B. P. Abbott[29], R. Abbott[29], M, Abernathy[66], T. Accadia[27], F. Acernese[19ac], C. Adams[31], R. Adhikari[29], P. Ajith[29], B. Allen[2,78], G. Allen[52], E. Amador Ceron[78], R. S. Amin[34], S. B. Anderson[29],

Parameter Estimation of Gravitational Waves from Precessing BH-NS Inspirals with higher harmonics

Richard O'Shaughnessy[3,*] Ben Farr[4], Evan Ochsner[3], Hee-Suk Cho[5], Chunglee Kim[1,2,†] Vivien Raymond[4], and Chang-Hwan Lee[5]

R. O'Shaughnessy, B. Farr, E. Ochsner, C. Cho, H.S. Kim, and C.H. Lee.
Parameter Estimation of Gravitational Waves from Nonprecessing BH-NS Inspirals

Fraternal twins: Distinguishing two post-Newtonian resonances using gravitational waves

Davide Gerosa,[1,*] Richard O'Shaughnessy,[2,†] Michael Kesden,[3,‡] Emanuele Berti,[4,§] and Ulrich Sperhake[1,4,5,6,¶]

D. Gerosa, M. Kesden, E. Berti, **R. O'Shaughnessy**, and U. Sperhake.
Resonant-plane locking and spin alignment in stellar-mass black-hole binaries:

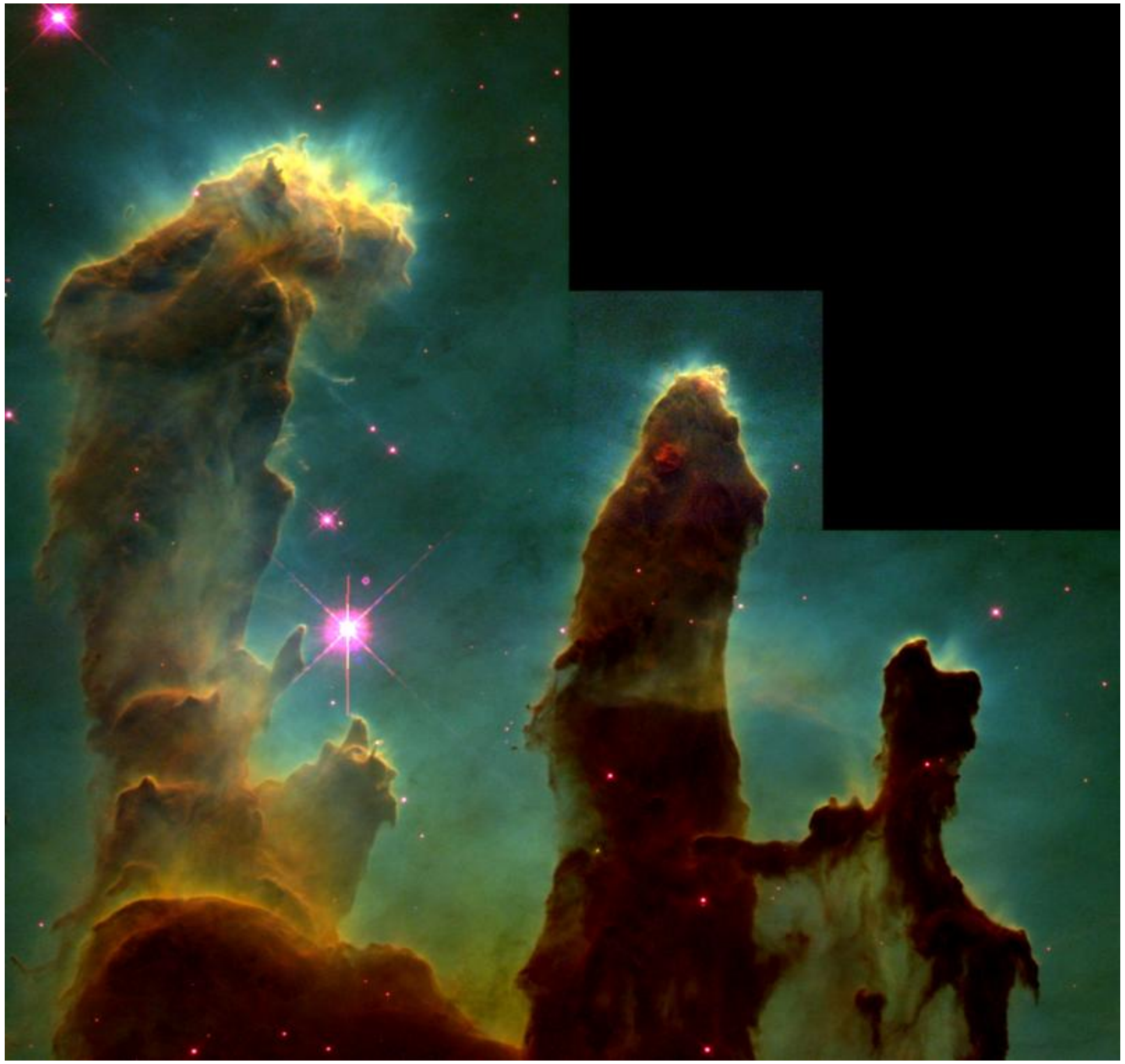
BLINDLY DETECTING ORBITAL MODULATIONS OF JETS FROM MERGING SUPERMASSIVE BLACK HOLES

R. O'Shaughnessy and D. L. Kaplan

Outline

- Why use gravitational waves for astronomy?
    - Rich data, addressing previously inaccessible phenomena
- Gravitational wave astronomy 101
    - Precise measurements, modulo systematics and degeneracies
    - Robust constraints on astrophysics
- Gravitational waves astronomy with precessing binaries
    - Richer signals, tracing different phenomena
- The future
    - Look to the sky, to new scales and new phenomena

Why study gravitational waves?



EM waves

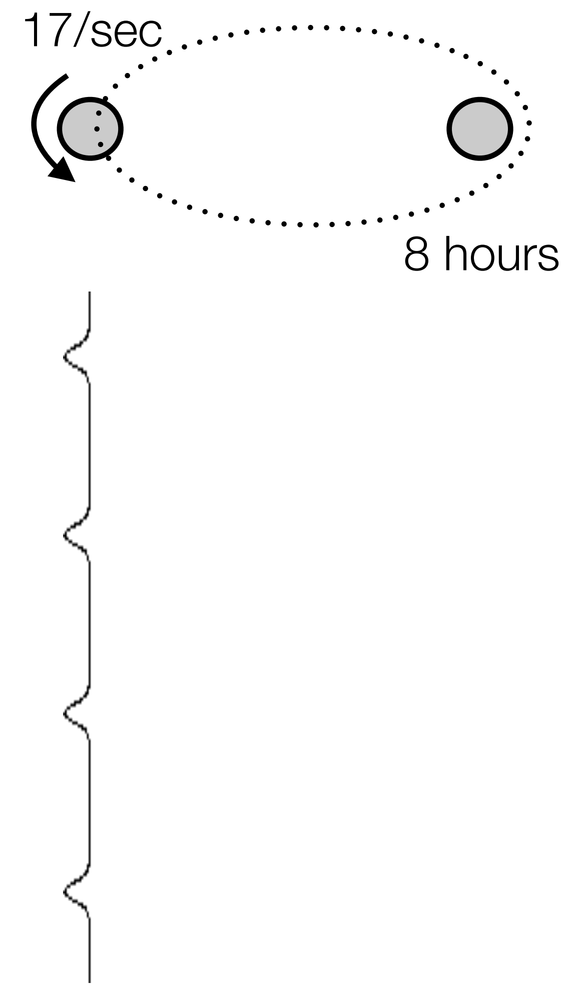
Source

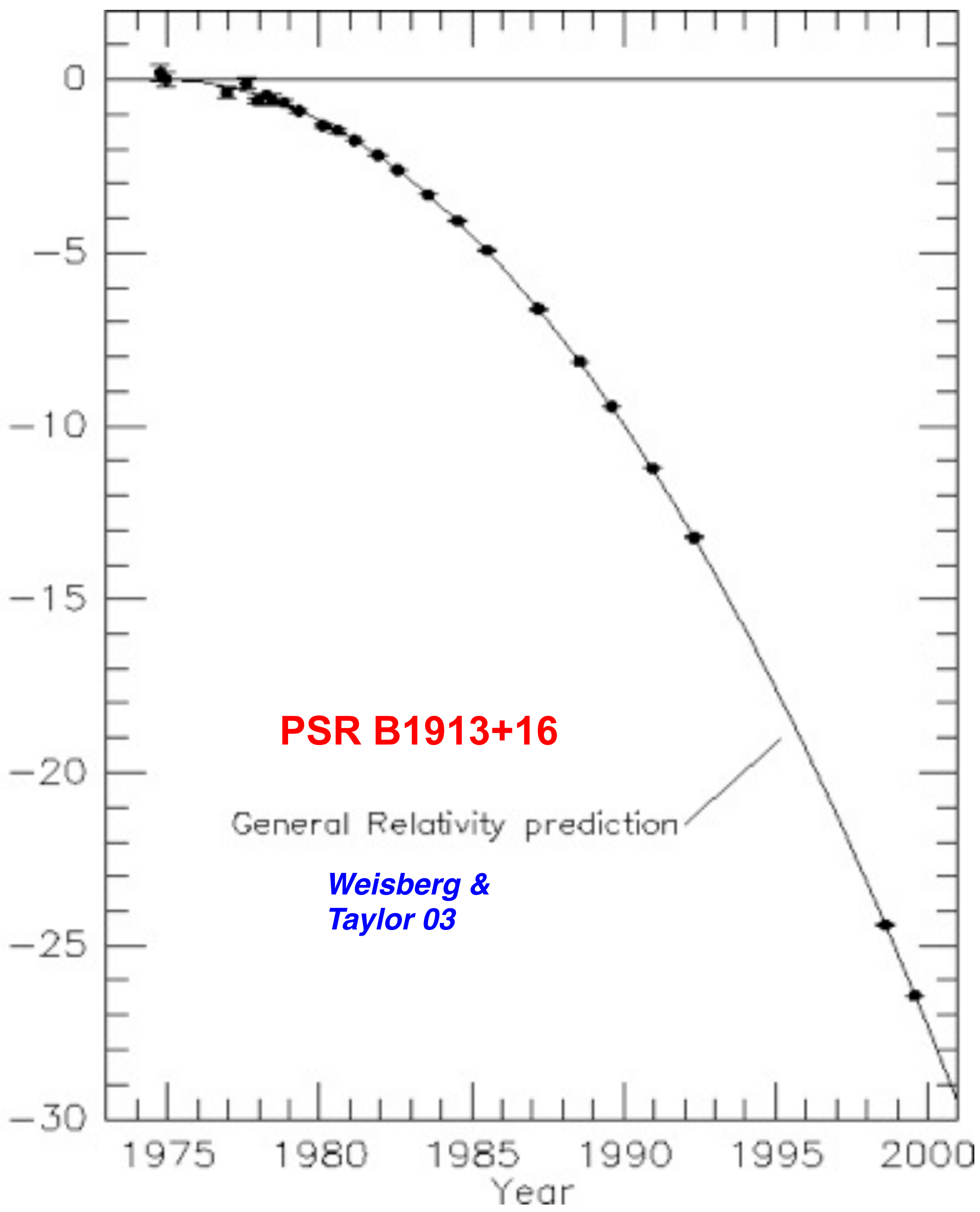
- Any accelerating charge
- Screening limits signal
- Strong coupling
  - Easy to make & detect
  - Easy to **obscure**

Gravitational waves and compact binaries exist

Hulse and Taylor

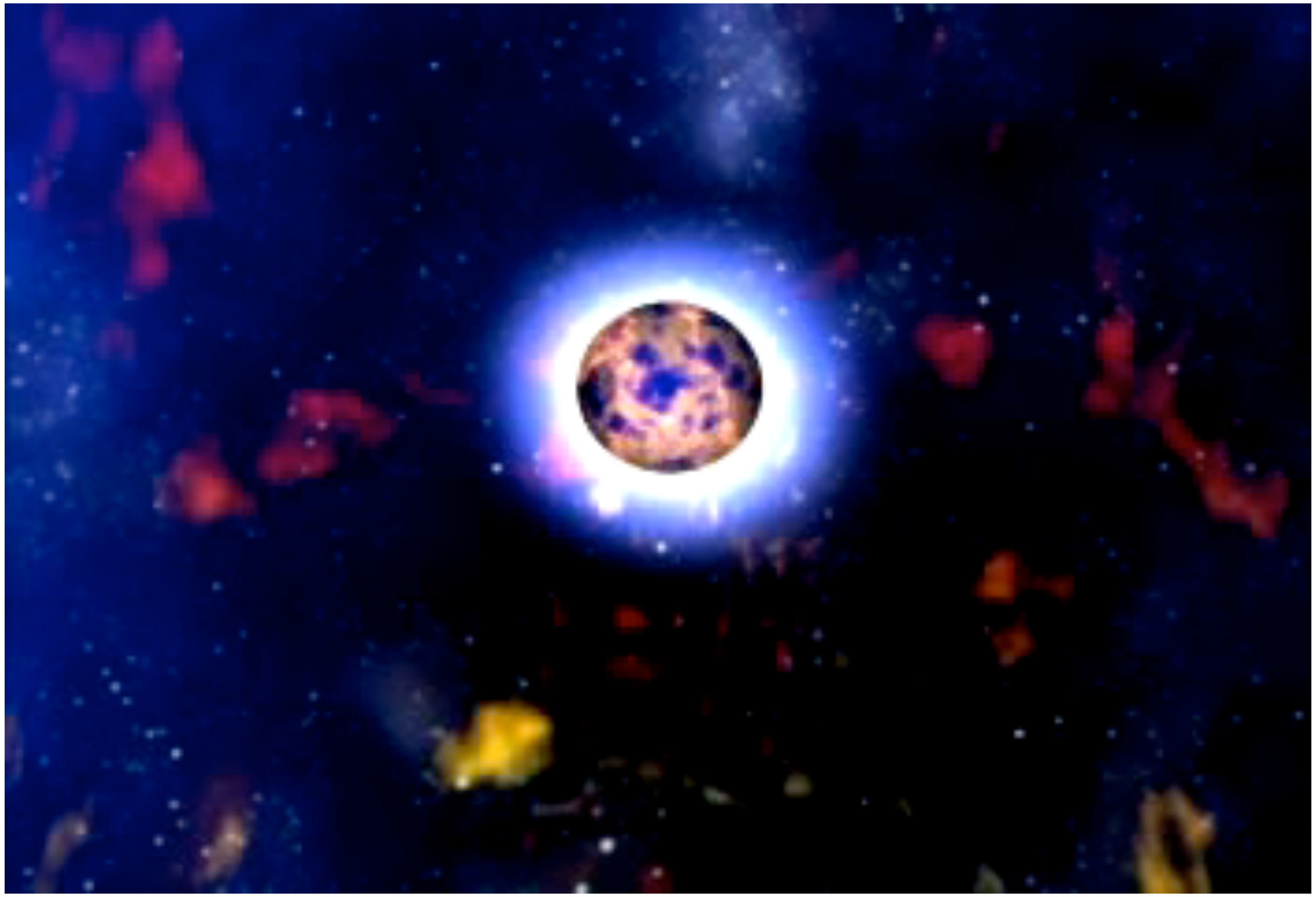
- Observed neutron star binary
- Measured orbit
- Energy **losses** in orbit consistent with GW emission





Some neutron star binaries will merge

- Example orbit: $a \simeq 2.7 R_\odot \ll O(10^3 R_\odot) \simeq R_{\text{giant}}$
  $\tau_{gw} \simeq 0.3\text{Gyr}$



- Are these known explosions seen throughout the universe?

Many binary pulsars

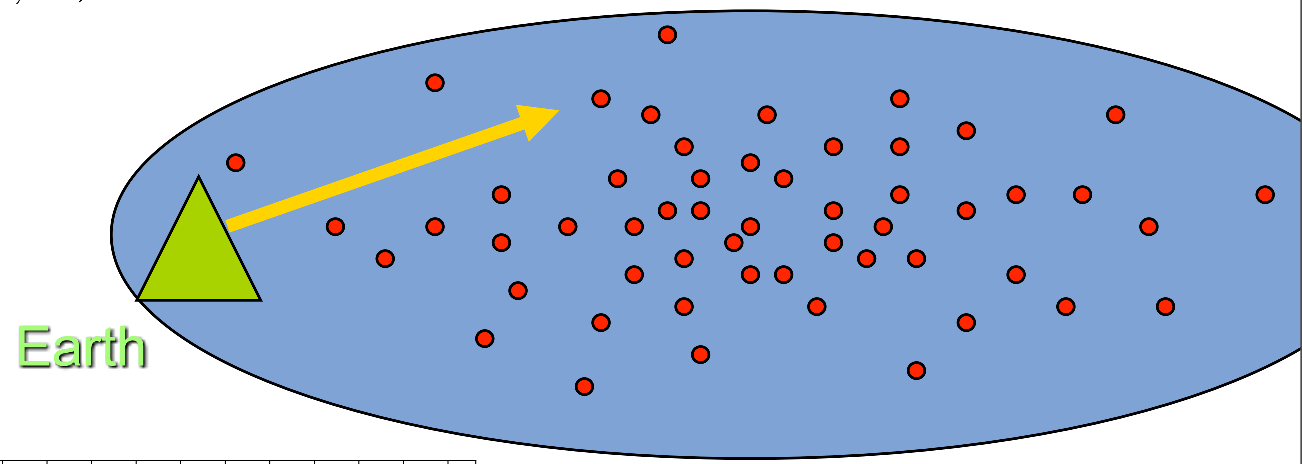
Estimate probability of finding binary pulsars

Infer formation rate

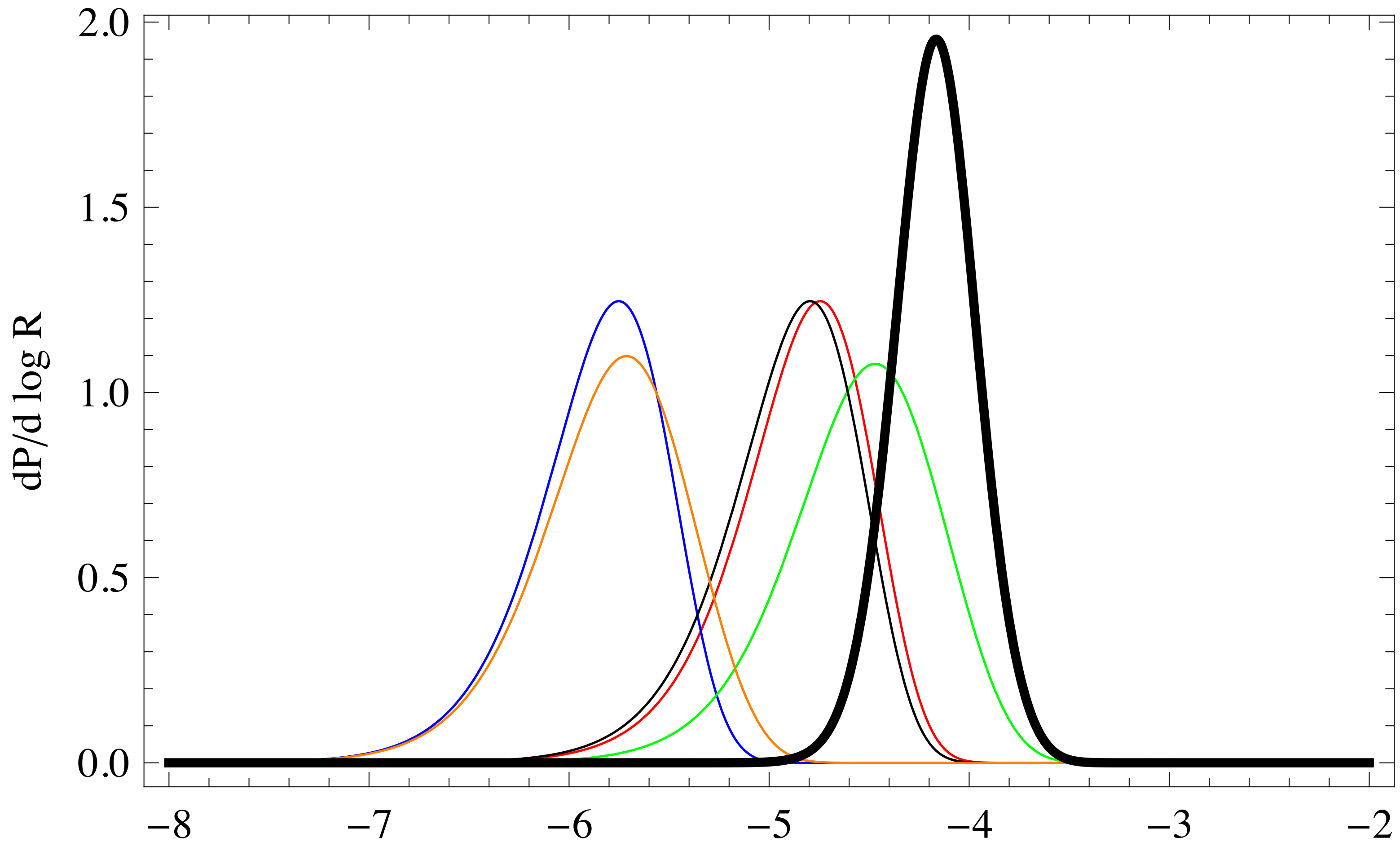
$$\mathcal{P}(\mathcal{R}) = (\tau_{\rm eff}/N_{\rm psr} f_{\rm b,eff})\mathcal{R} e^{-(\tau_{\rm eff}/N_{\rm psr} f_{\rm b,eff})\mathcal{R}}$$

$$\mathcal{R} \simeq f_b N_{seen}/\tau$$

$$f_b = \frac{4\pi}{\Omega_{\rm visible}}$$



100/Myr



log (rate (year))

ROS et al 2010 (ApJ 715 230)

Countdown to detection

Epoch	Run Duration	BNS range (Mpc) LIGO	BNS range (Mpc) Virgo	Number of Detections
2015	3 months	60 ± 20	—	0.0004 - 3
2016–17	6 months	100 ± 20	40 ± 20	0.006 - 20
2017–18	6 months	140 ± 30	70 ± 15	0.02 - 70
2019+	(per year)	200	100 ± 15	0.2 - 200
2022+ (India)	(per year)	200	130	0.4 - 400

using "low" rate from 1003.2480, worst noise curve

using "high" rate, best noise curve

GW detections by 2017-2018?

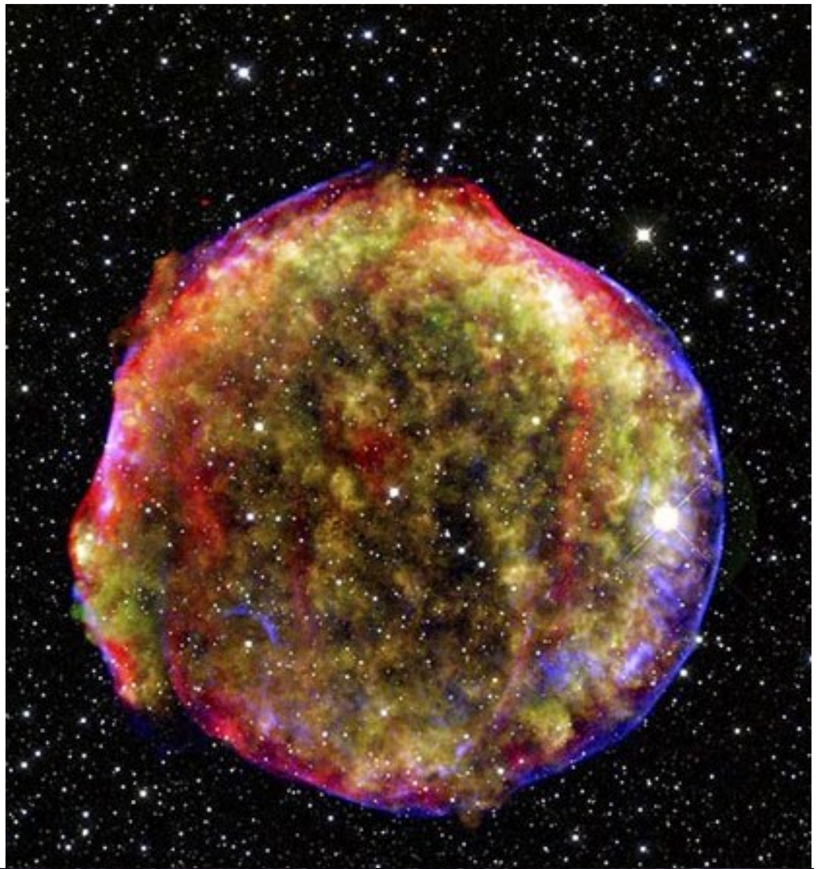
LVC: Living Reviews 2013

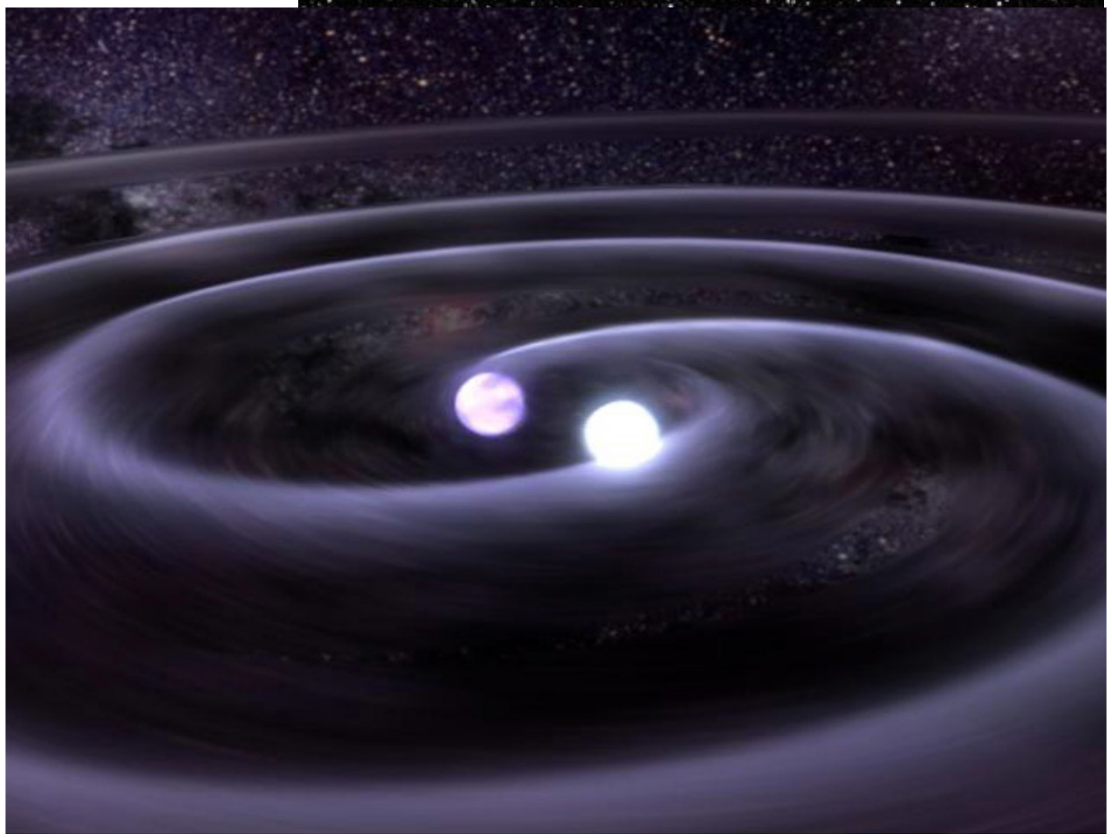
Binary star evolution

Key fact: stars evolve and **expand**

Typical life of interacting binary system

- Star 1 ages, expands. Potential mass transfer 1->2
- Star 1 explodes
- Star 2 ages, expands. Potential mass transfer 2->1
- Star 2 explodes
- Compact binary left behind
  - **May** merge via GW....

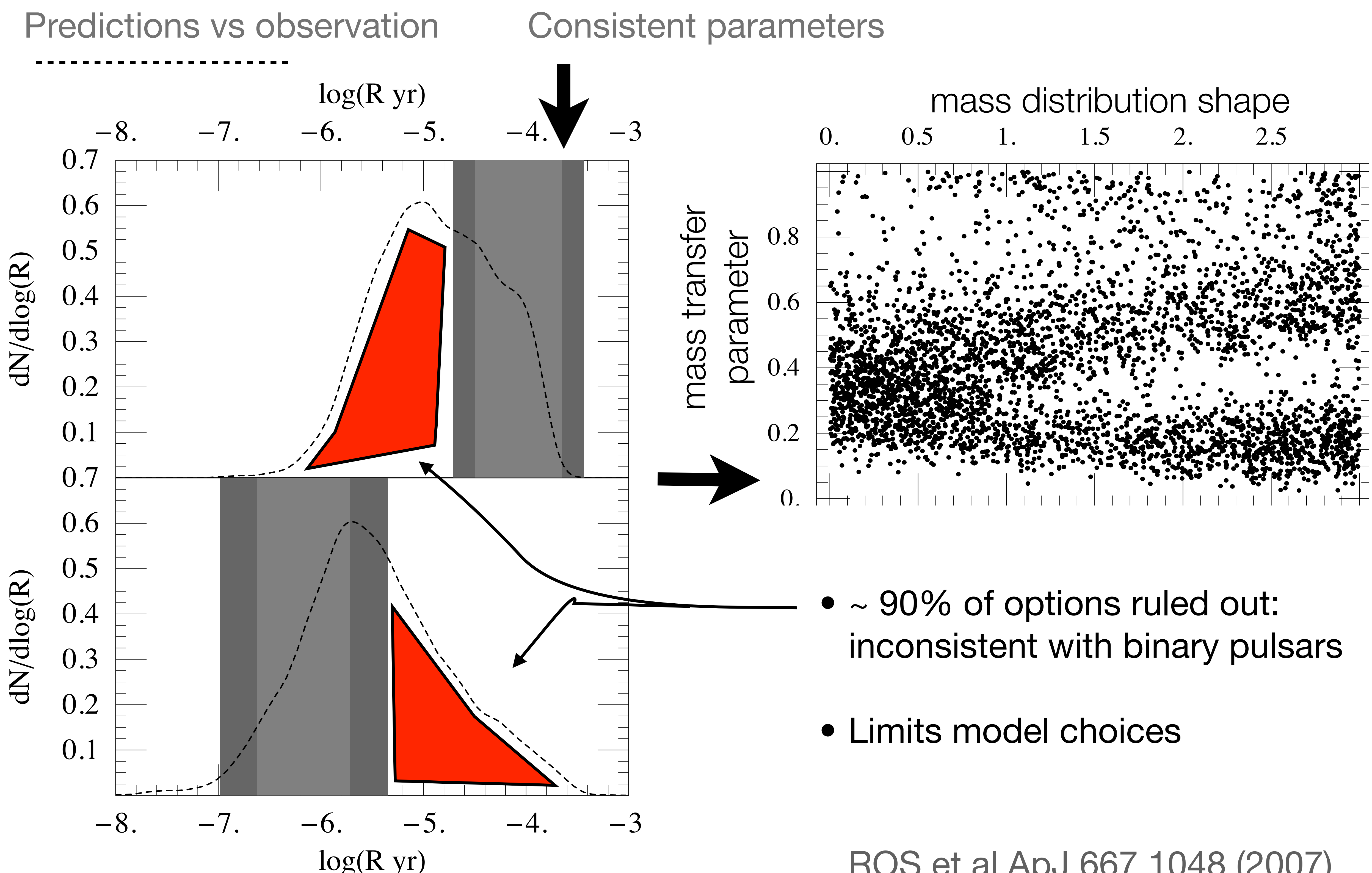




All ingredients are poorly modeled & measured

- Initial conditions
- Massive (rotating) star evolution
- Interactions
- Explosions

Learning from nature: Binary pulsars in the galaxy



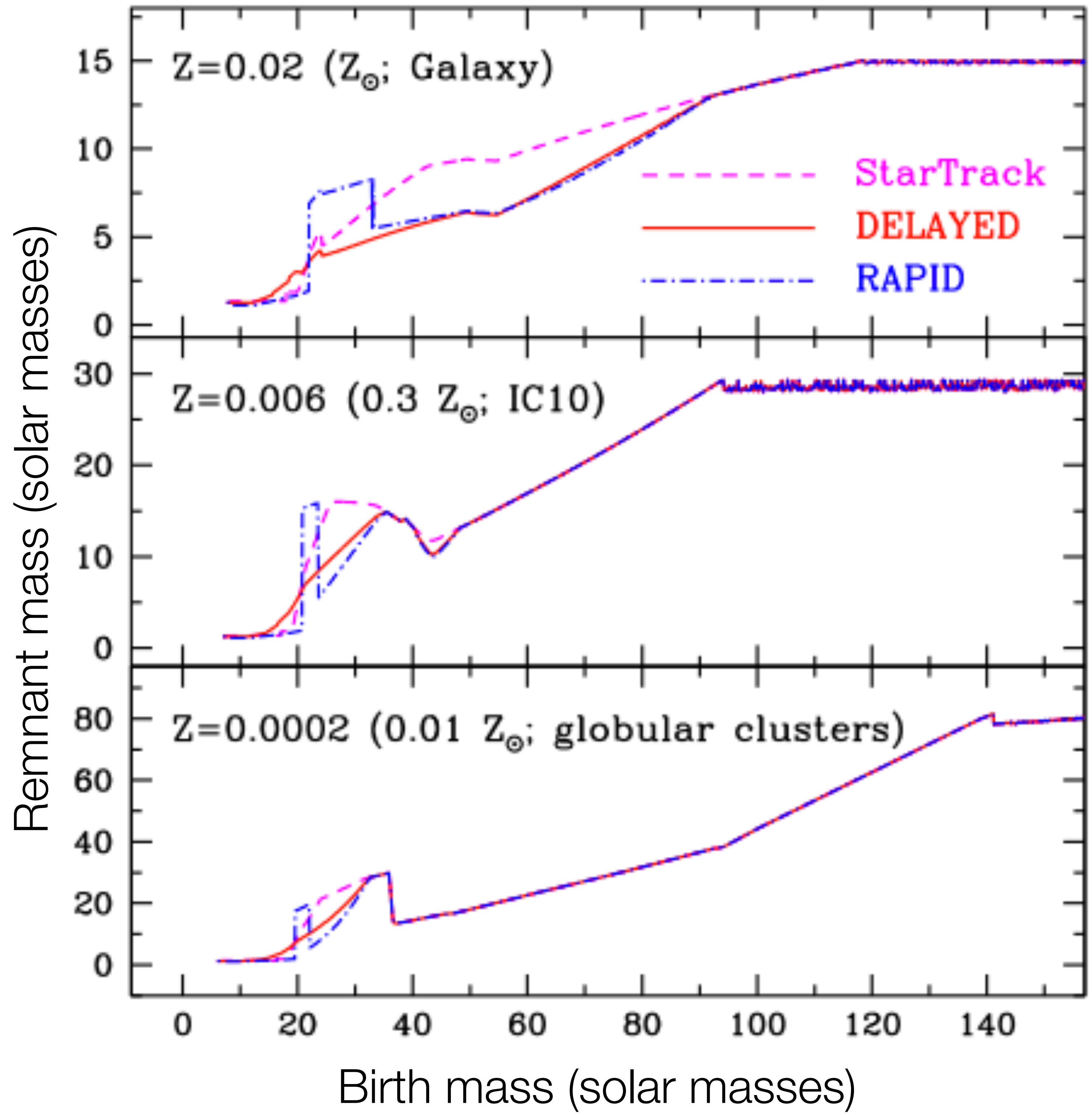
- ~ 90% of options ruled out: inconsistent with binary pulsars
- Limits model choices

ROS et al ApJ 667 1048 (2007)

Probe uncertain physics with masses

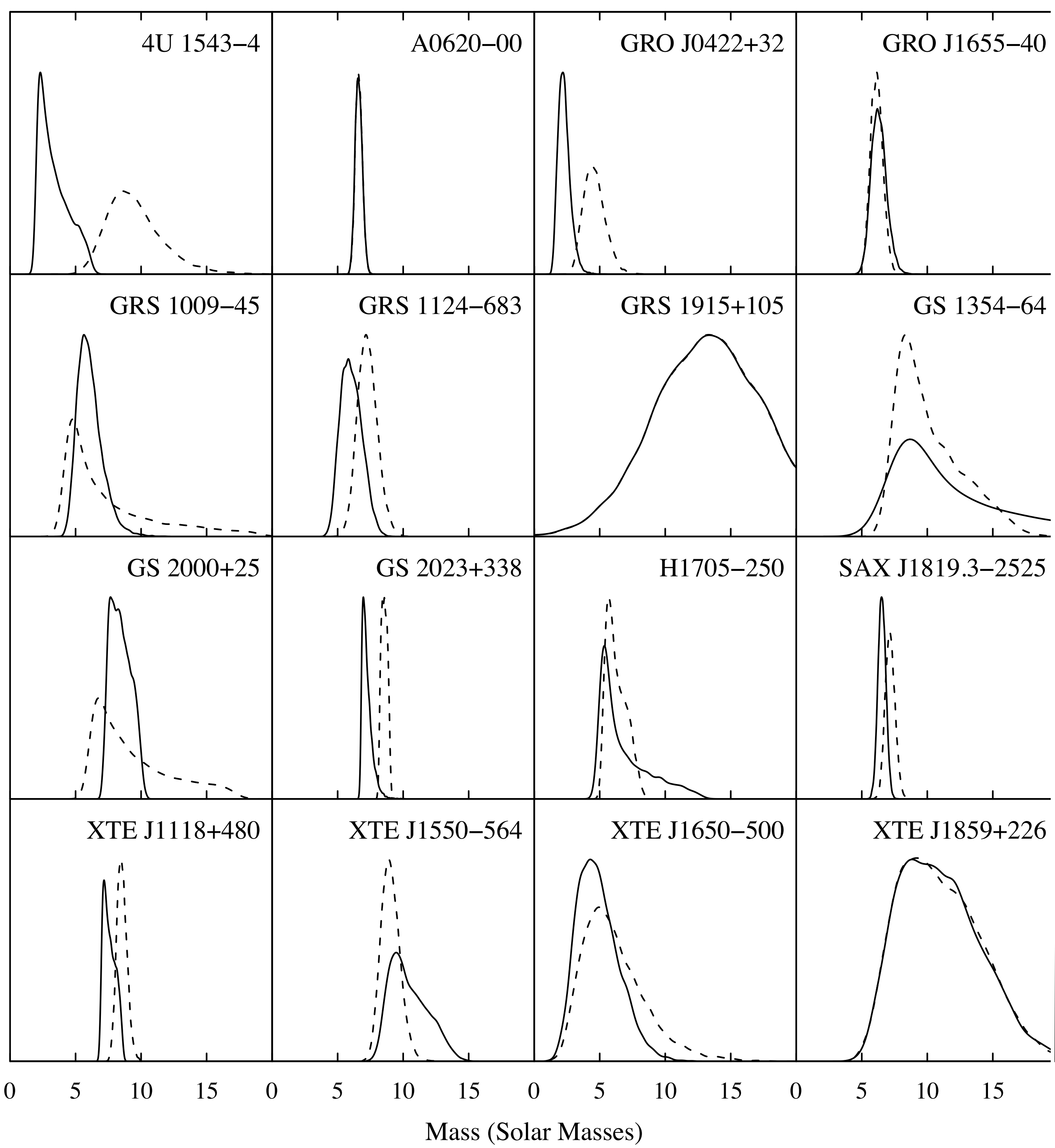
- Mass loss depends on composition (Z)

- Different SN mechanisms change mass promptly accreted onto core



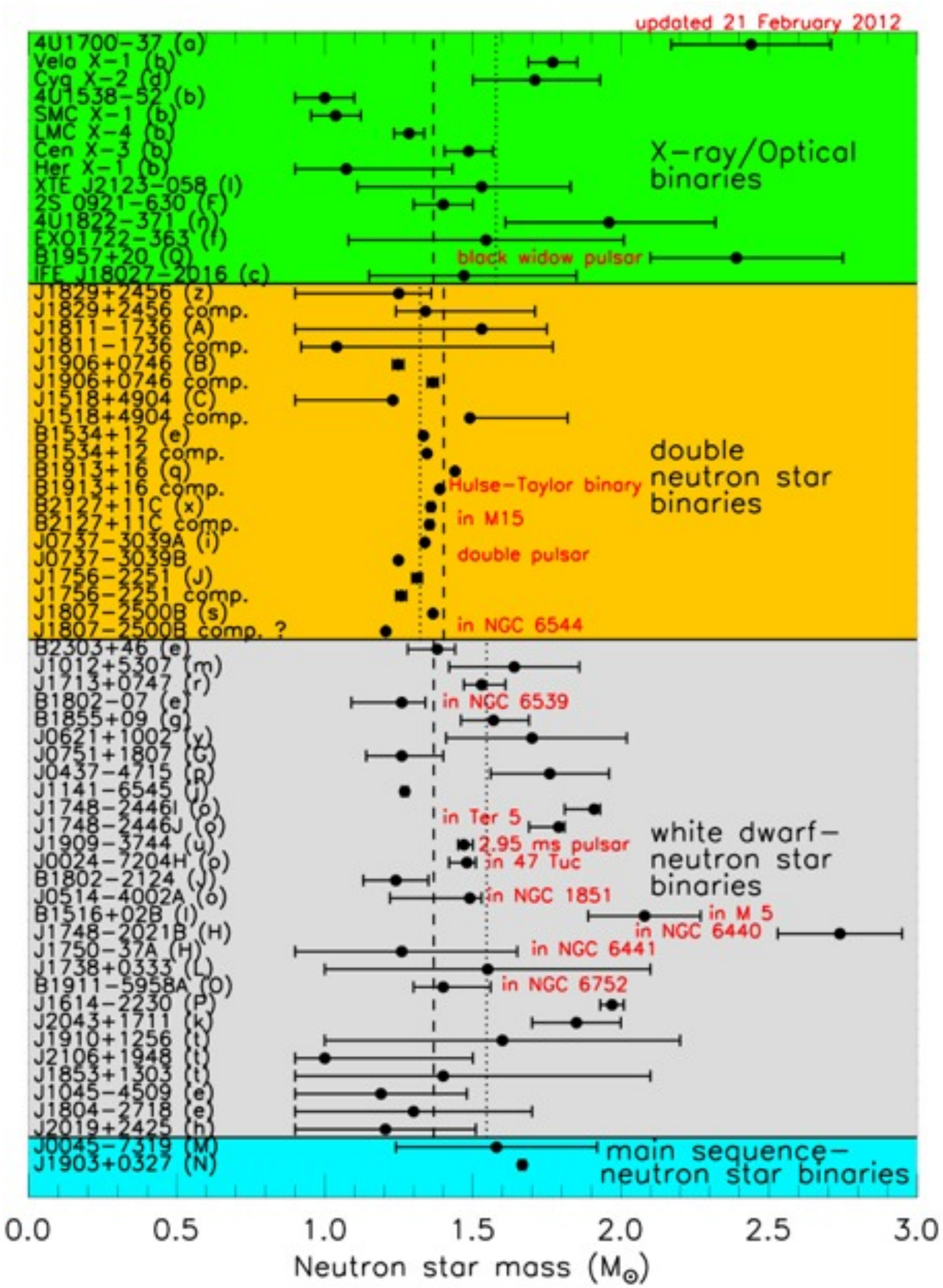
Fryer et al 2012; see also Ott et al;Ugliano et al

Masses of black holes



Kreidberg et al ApJ 757 36 (2012)

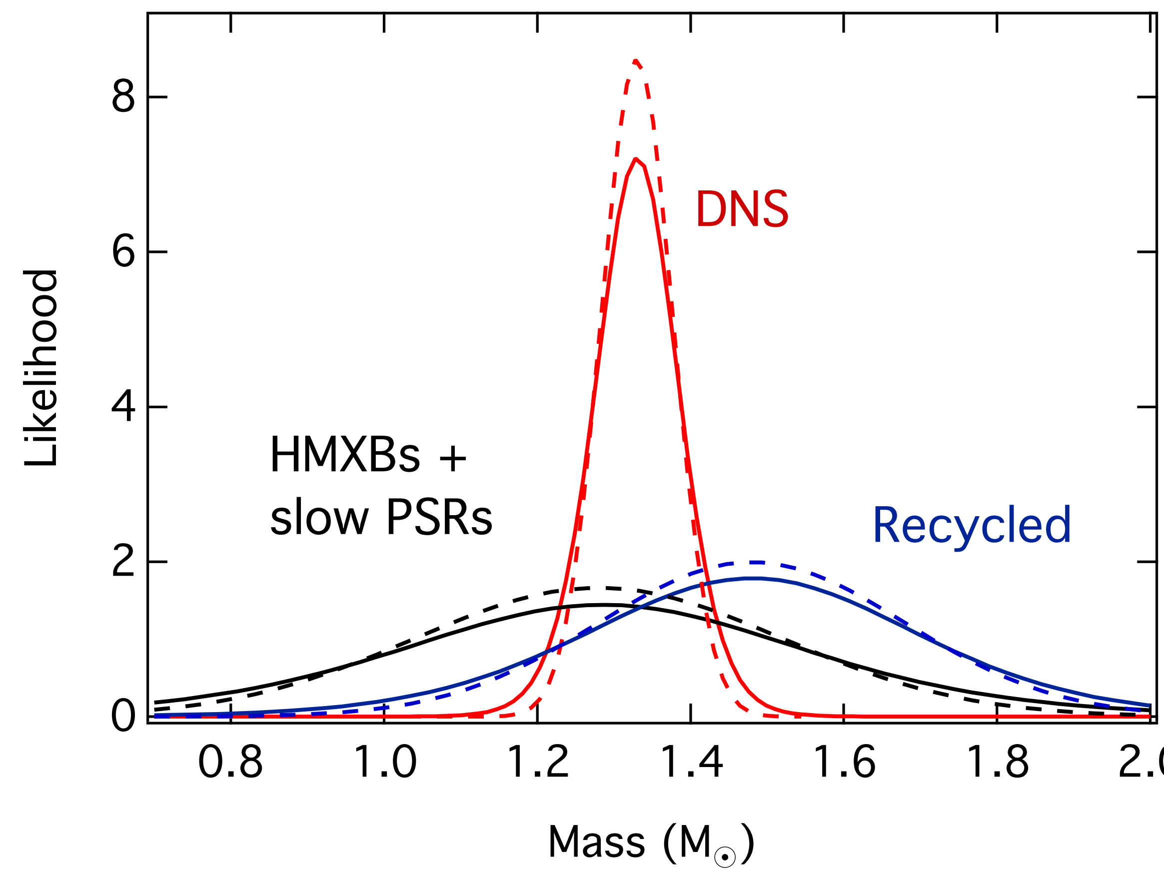
Masses of neutron stars



J. Lattimer

http://www.stellarcollapse.org/nsmasses

- <u>One</u> interpretation of data



Ozel et al ApJ 757 55 (2012)

What are general relativity and gravitational waves?

General relativity

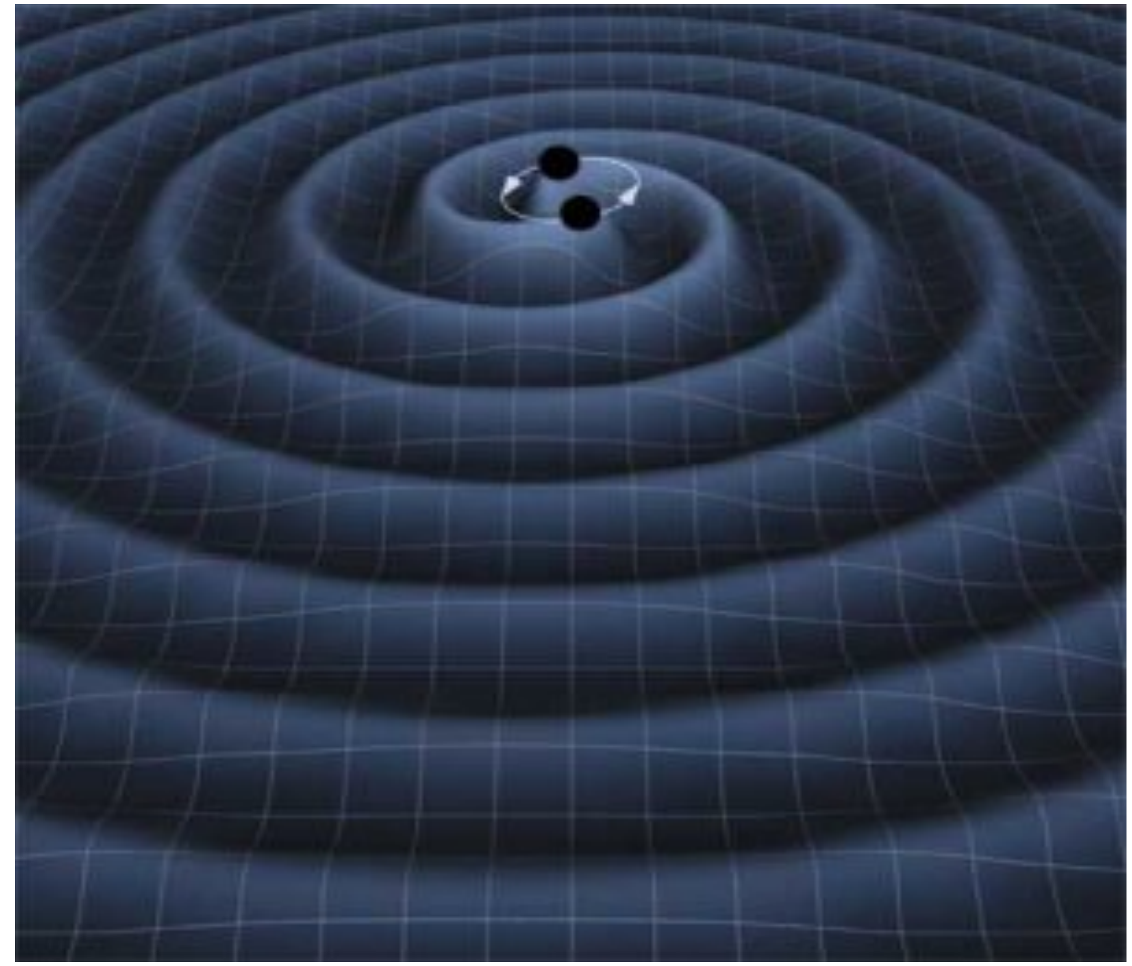
- "Evolution" of distance (metric **g**)

$$R_{ab} - \frac{g_{ab}}{2} R = \cancel{8\pi T_{ab}}$$

- Linearize about "normal" space

$$g = \eta + \epsilon h$$

$$\nabla^2 h_{ab} \simeq 0$$



Gravitational waves [this talk]

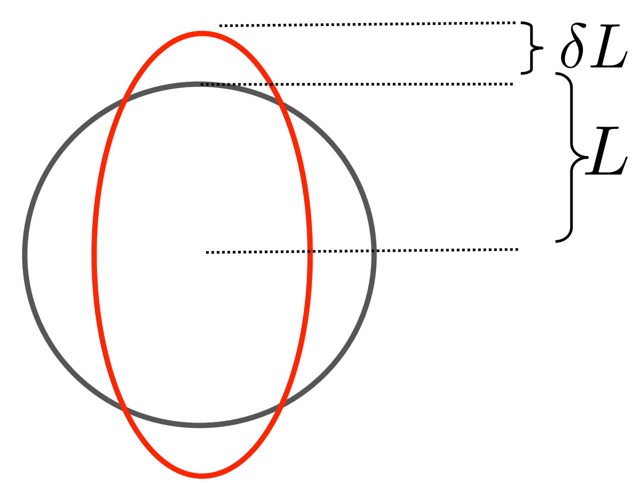
- Identify propagating functions, e.g.

$$h(\hat{z}) = \frac{1}{2}[h_{xx} - h_{yy} - 2ih_{xy}]$$

  - h is complex: 2 "polarizations"

$${}^{(-2)}\nabla^2 h \simeq 0$$

- Physics: "length changes":
  - and "acceleration"

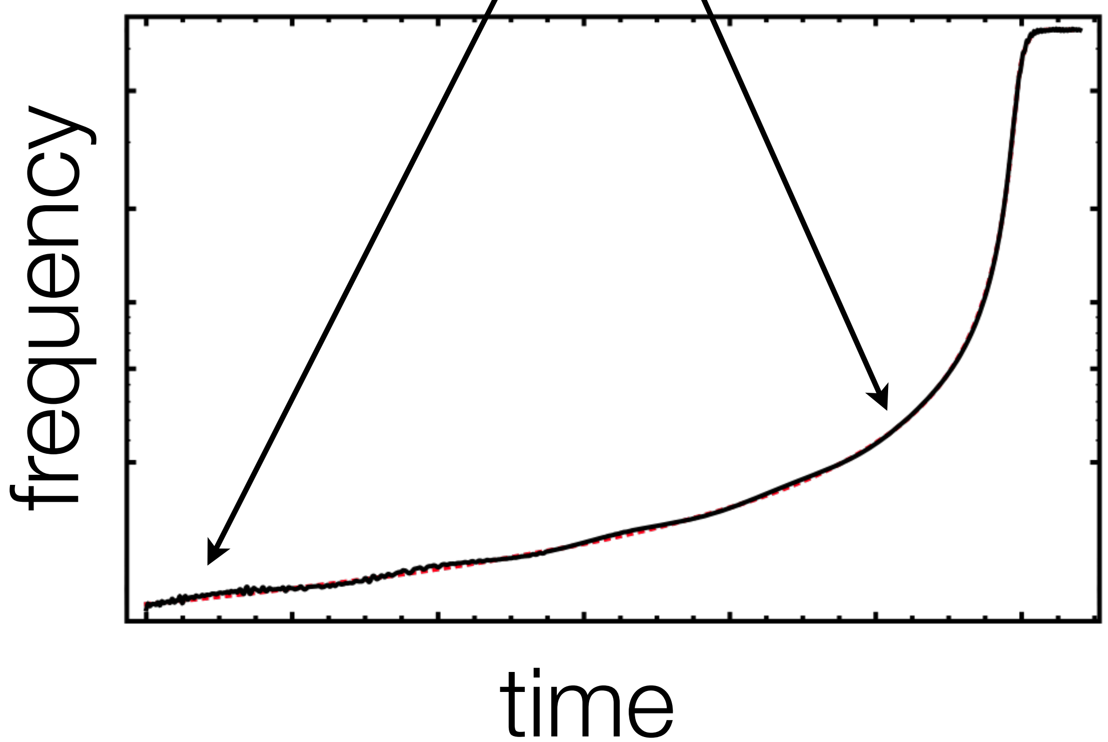


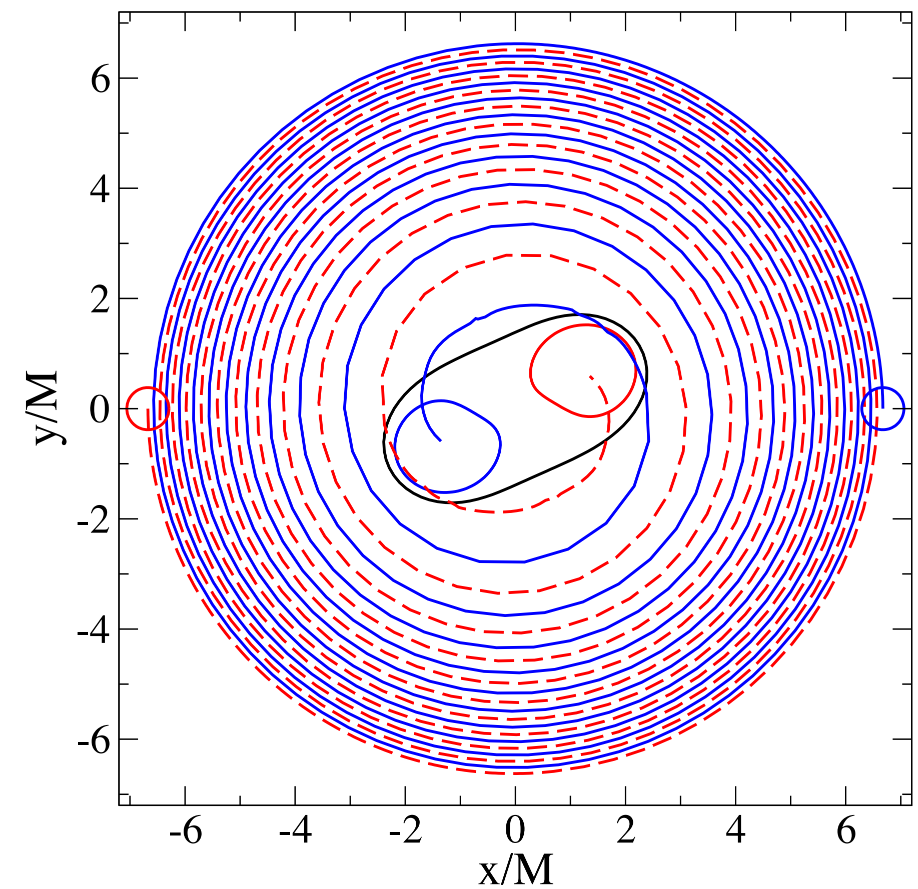
$$h = \frac{\delta L}{L}$$

$$\psi_4 = \partial_t^2 (h_+ - i h_\times)$$

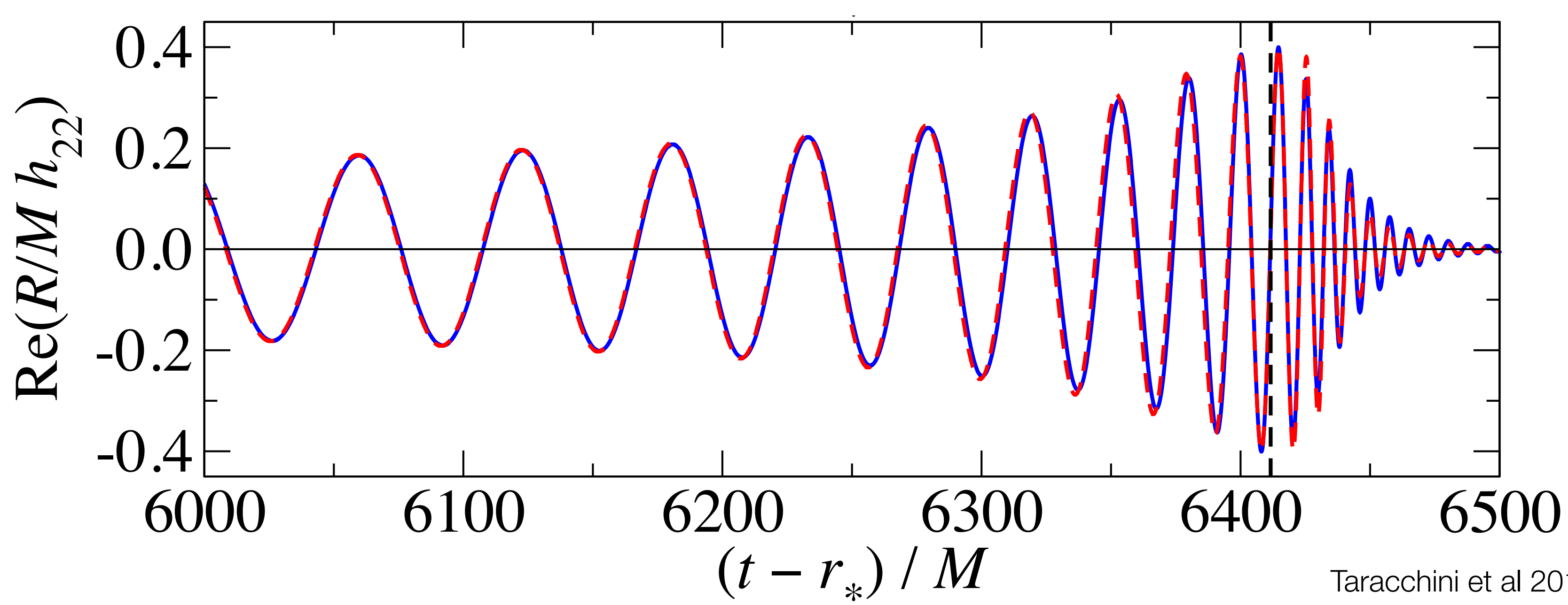
Binary inspiral and merger

- Shrinking binary spirals in, “chirping”





Chu et al 2009 (0909.1313)



Taracchini et al 2013 (1311.2544)

Inferring source parameters

- Evidence for signal

$$Z(d|H_1) \quad \equiv \quad \frac{p(\{d\}|H_1)}{p(\{d\}|H_0)} = \int d\lambda p(\vec{\lambda}|H_1) \underbrace{\frac{p(\{d\}|\vec{\lambda}, H_1)}{p(\{d\}|H_0)}}_{\text{posterior distribution}}$$

H_1 : with signal
H_0 : no signal

- Inputs:
  - Prior knowledge $p(\lambda|H_1)$ about distribution of λ
  - Signal model $h(\lambda)$
  - Noise model $p(\{d\}|H_0)$ $\quad p(\{d\}|\vec{\lambda}, H_1) = p(\{d - h(\vec{\lambda})\}|H_0)$
  - Algorithm for integral/exploration in many dimensions

Measuring gravitational waves

Detector

- Nearly gaussian, stationary

$$\langle n^*(f)n(f)\rangle = \frac{1}{2}S_h(|f|)\delta(f-f')$$

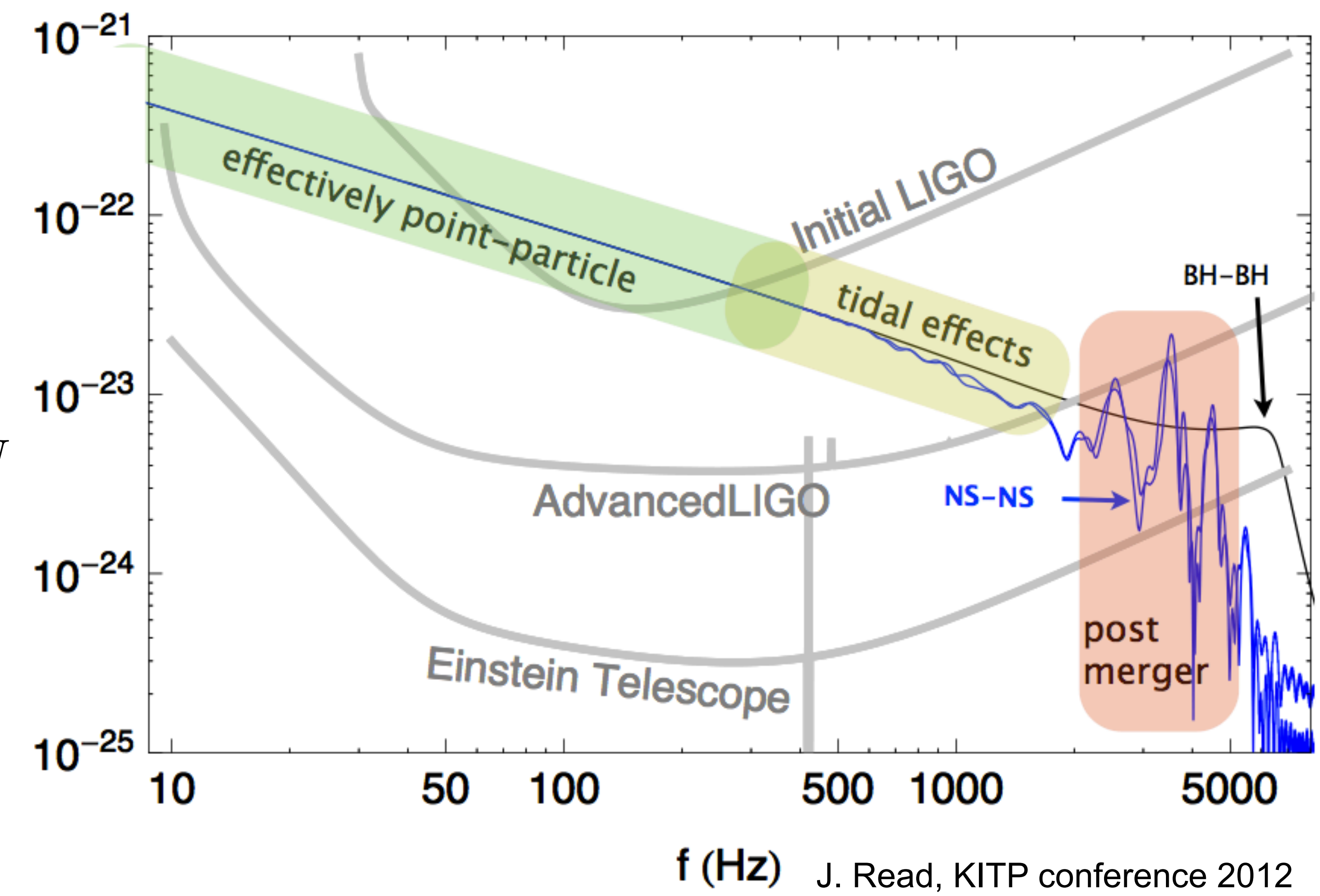
$$p(\{d\}|H_0) \propto \exp -\frac{\langle d|d\rangle}{2} dd_1 dd_2 \ldots dd_N$$

$$\langle a|b\rangle \equiv 2\int_{-\infty}^{\infty} df \frac{a^*(f)b(f)}{S_h(|f|)}$$

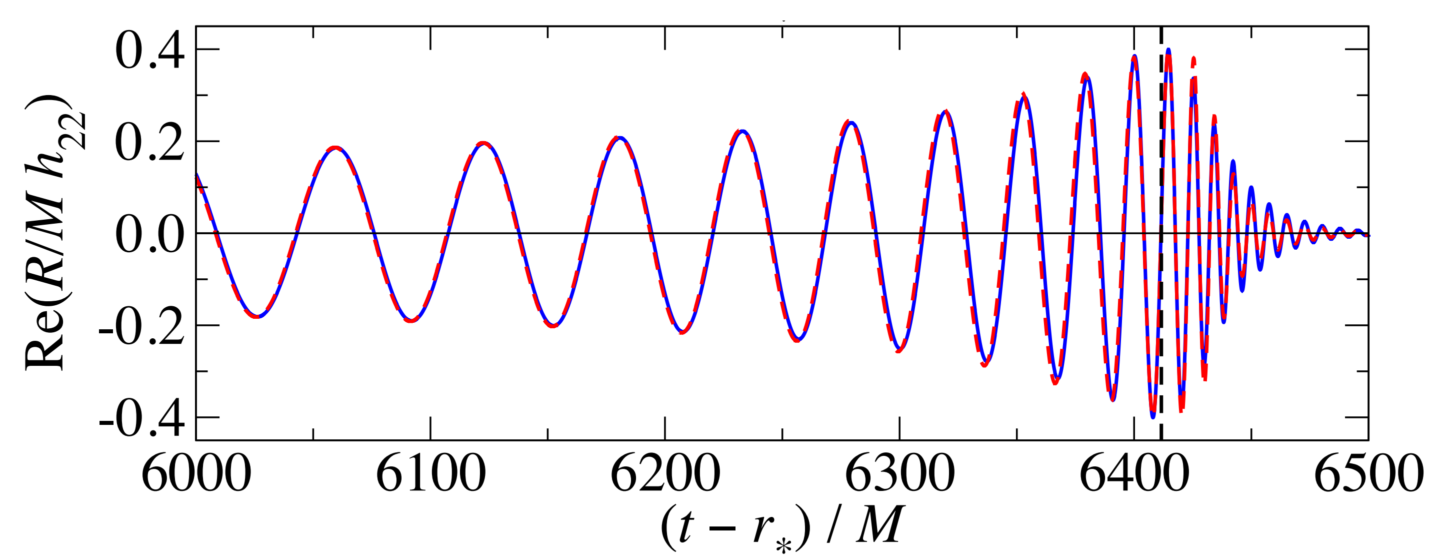
- Band limited

Signal

- More cycles at low frequency
- “Typical” merger physics not in band
  - “Input” binary dominates
- Orbital phase: degenerate evolution



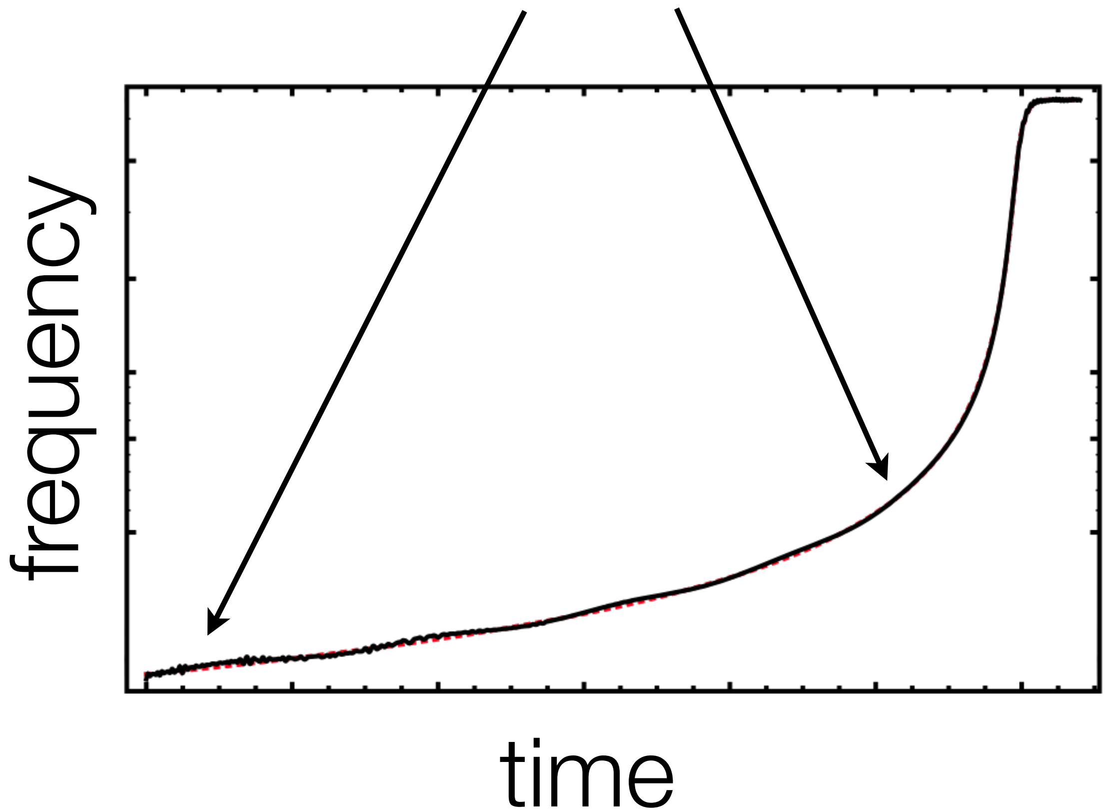
J. Read, KITP conference 2012



Taracchini et al 2013 (1311.2544)

One thing we measure reliably: “Chirp” mass

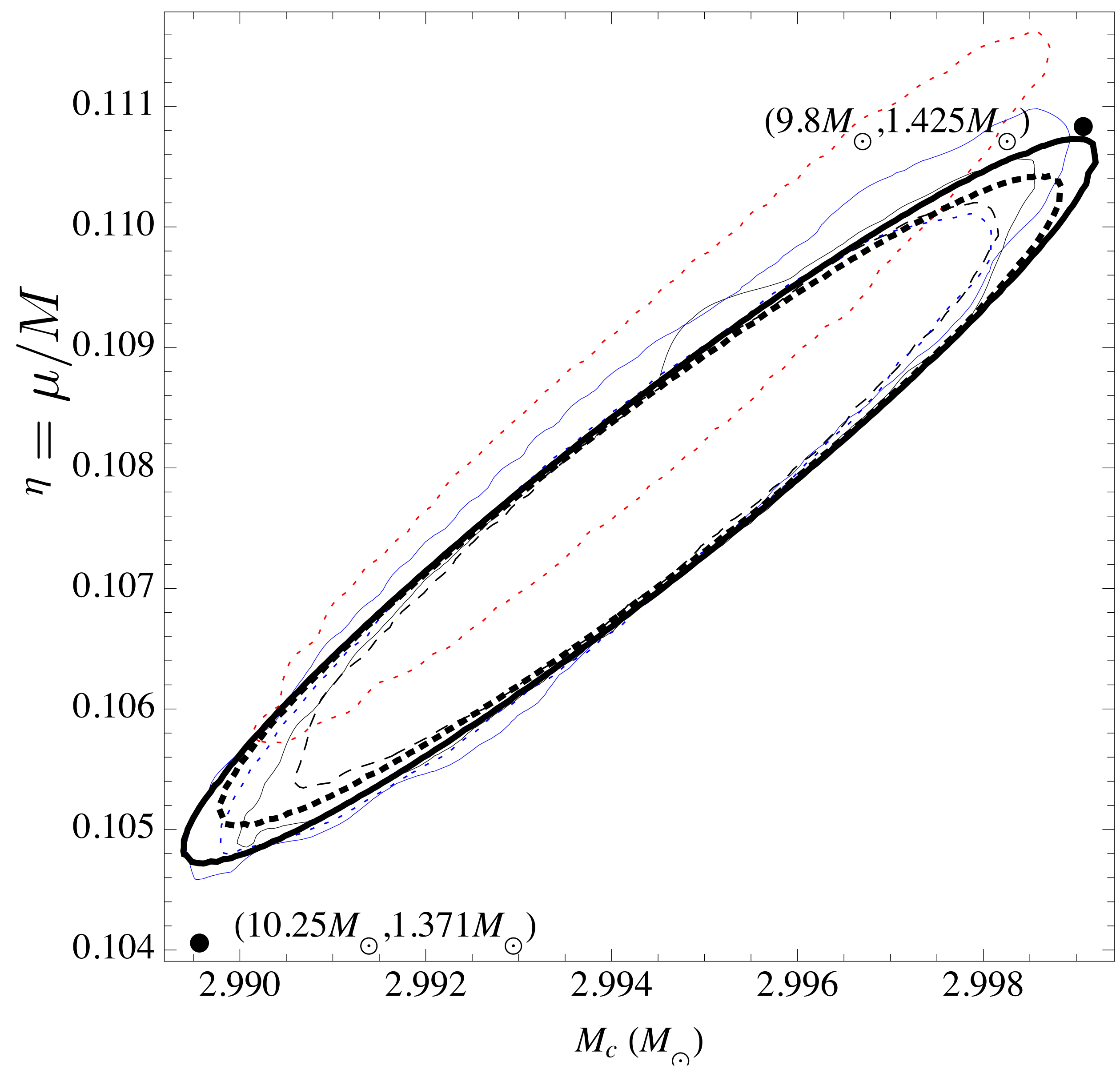
- Shrinking binary “chirps”



- Chirp rate (df/dt) set by “chirp mass”
  - “Exactly” measurable
- Fisher matrix

$$\Gamma_{ab} = 2\int_{-\infty}^{\infty} \frac{\partial_a h^* \partial_b h}{S_h} df$$

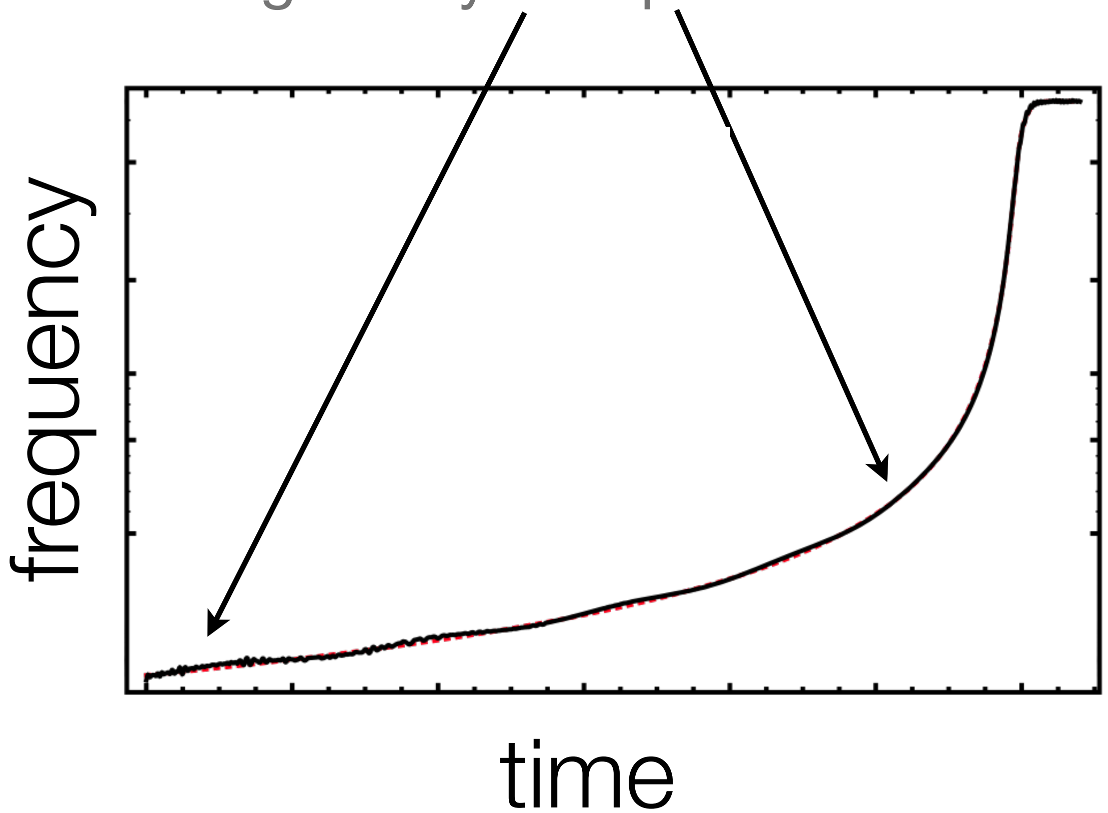
BH-NS, no spin

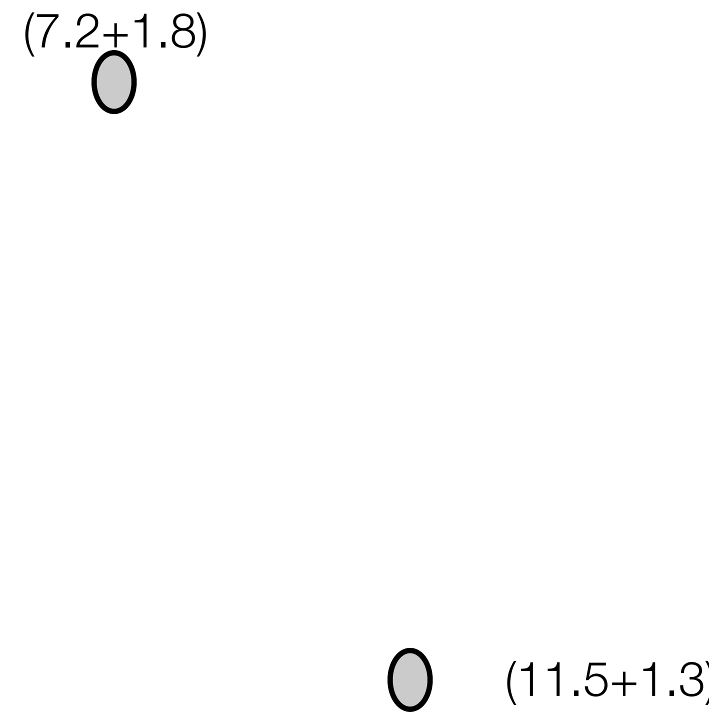


$$\mathcal{M}_c = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

What can we learn from the “chirp”?

- Shrinking binary “chirps”





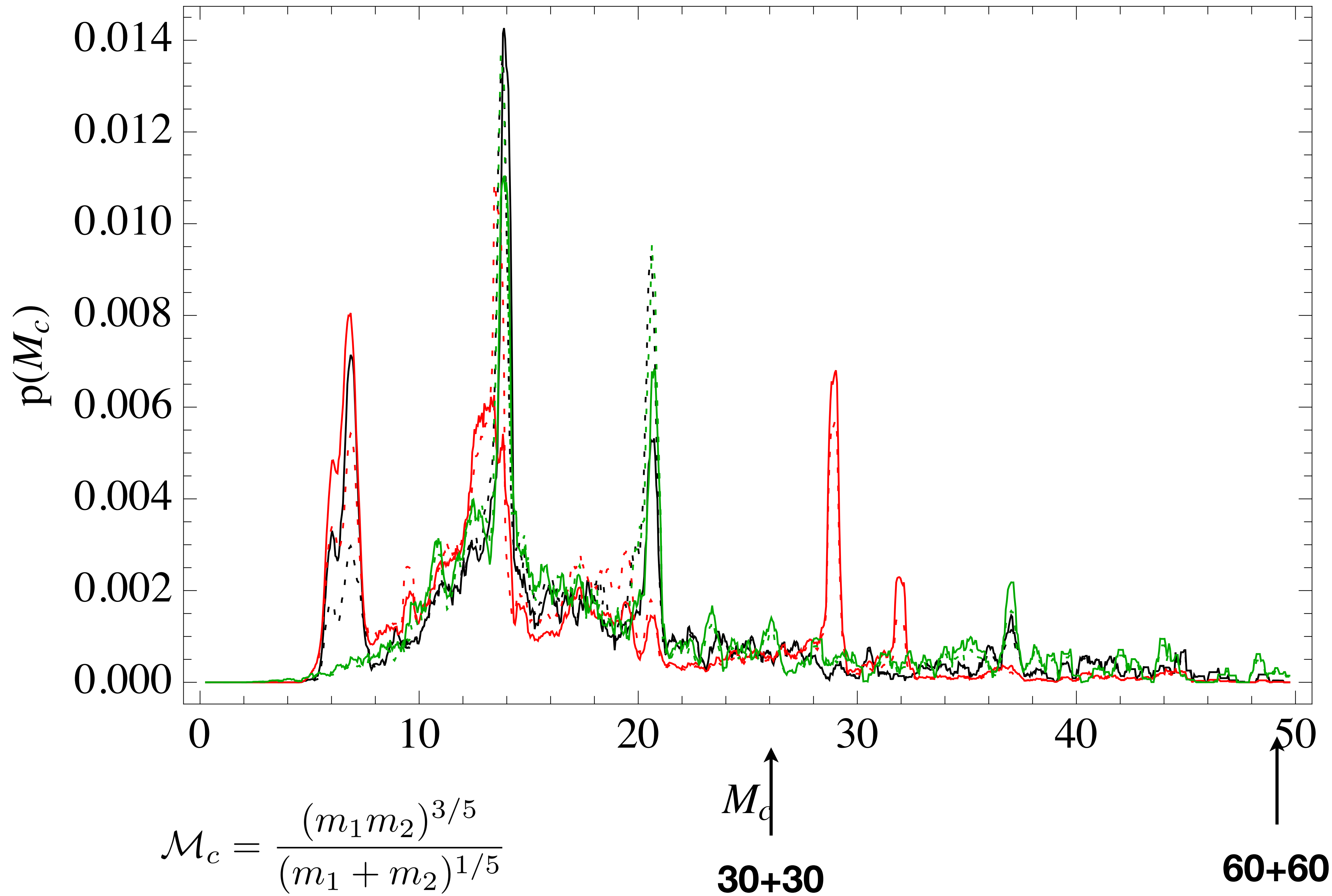
- Measure masses, spins, tides, ...
  - Adding parameters (spin) degrades measurement accuracy
- Fisher matrix

$$\Gamma_{ab} = 2\int_{-\infty}^{\infty} \frac{\partial_a h^* \partial_b h}{S_h} df$$

ROS et al 2013

Distinguishing between formation models?

Dominik et al (in prep)



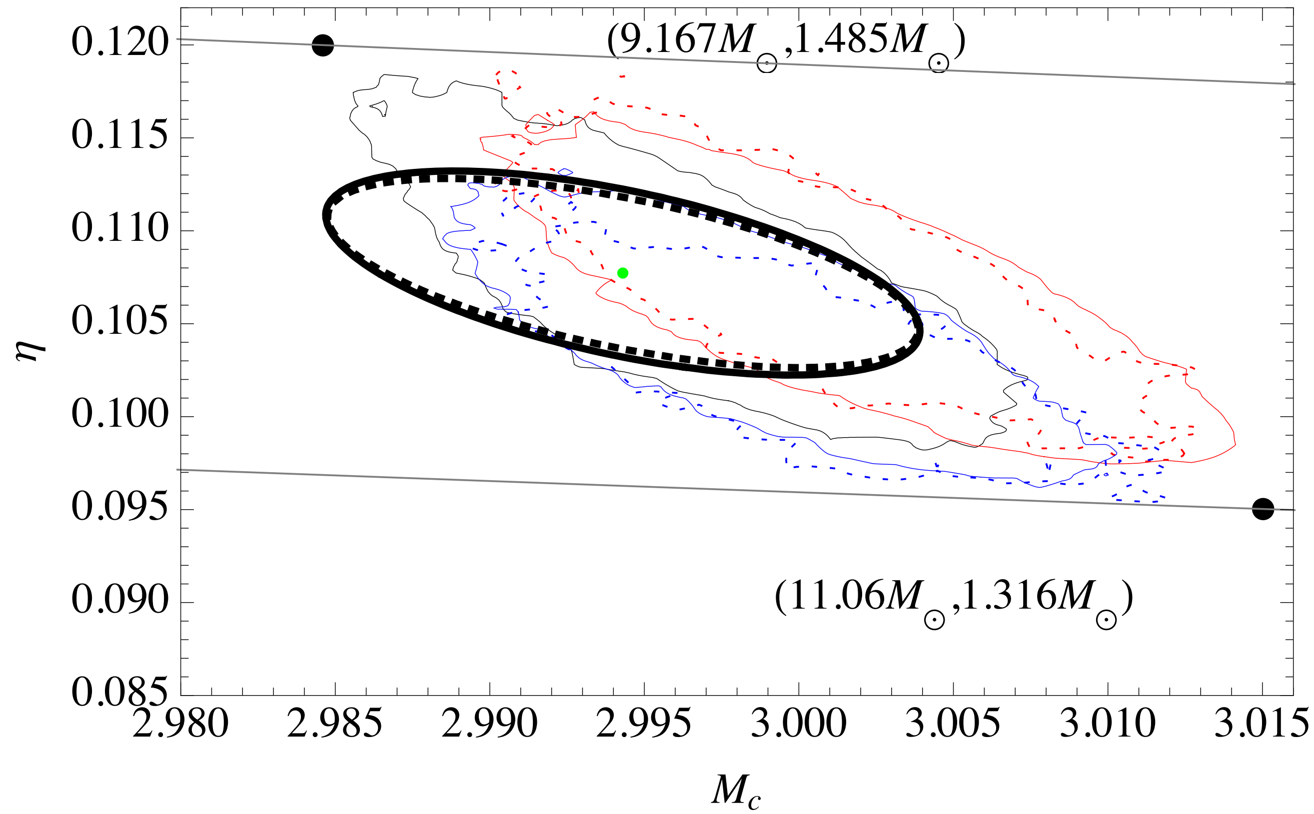
$$\mathcal{M}_c = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

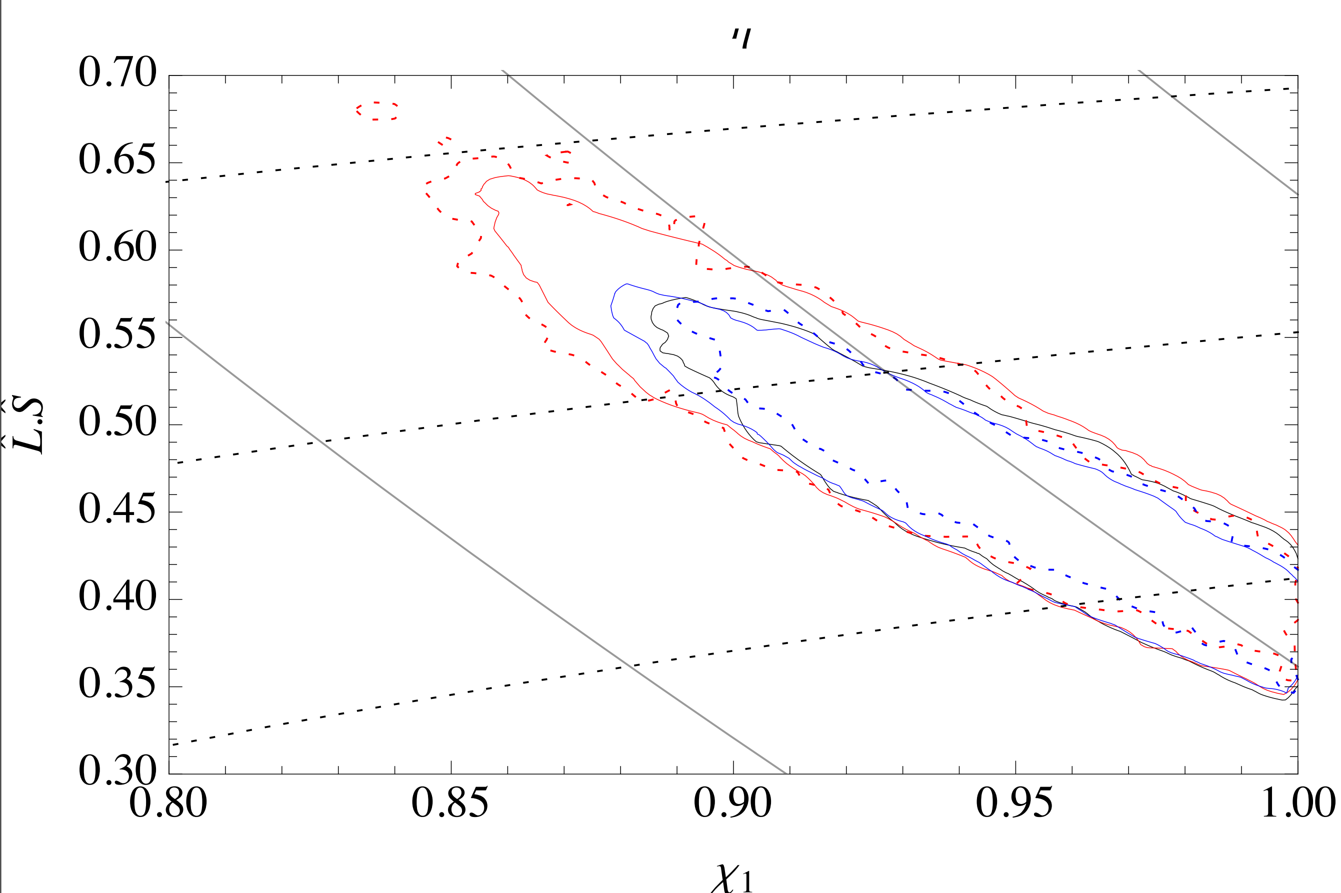
Reminder

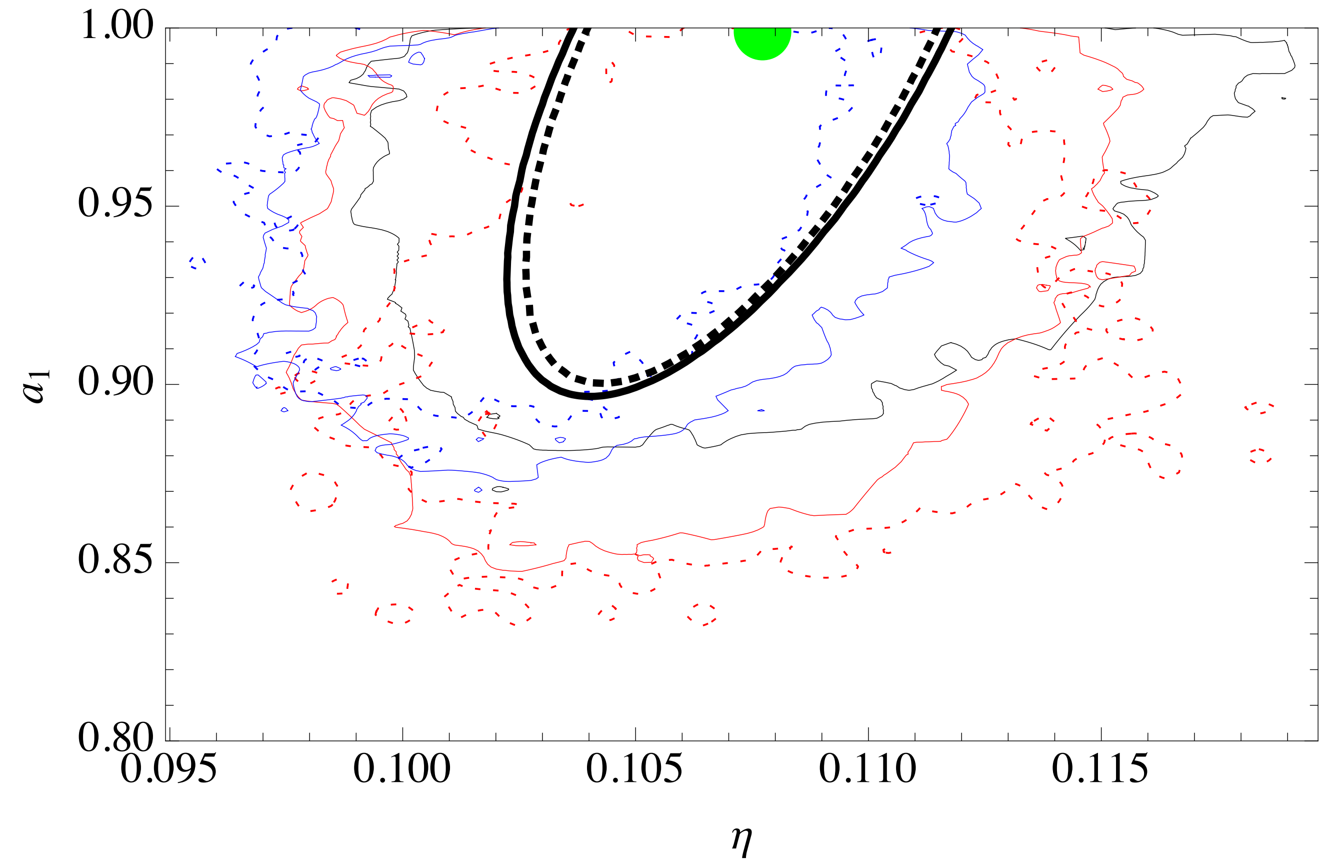
- Why use gravitational waves for astronomy?
    - Rich data, addressing previously inaccessible phenomena
- Gravitational wave astronomy 101
    - Precise measurements, modulo systematics and degeneracies
    - Robust constraints on astrophysics
- Gravitational waves astronomy with precessing binaries

More accurate mass, spin measurements

- Companion mass similar to NS?
- BH spin
- Spin-orbit misalignment



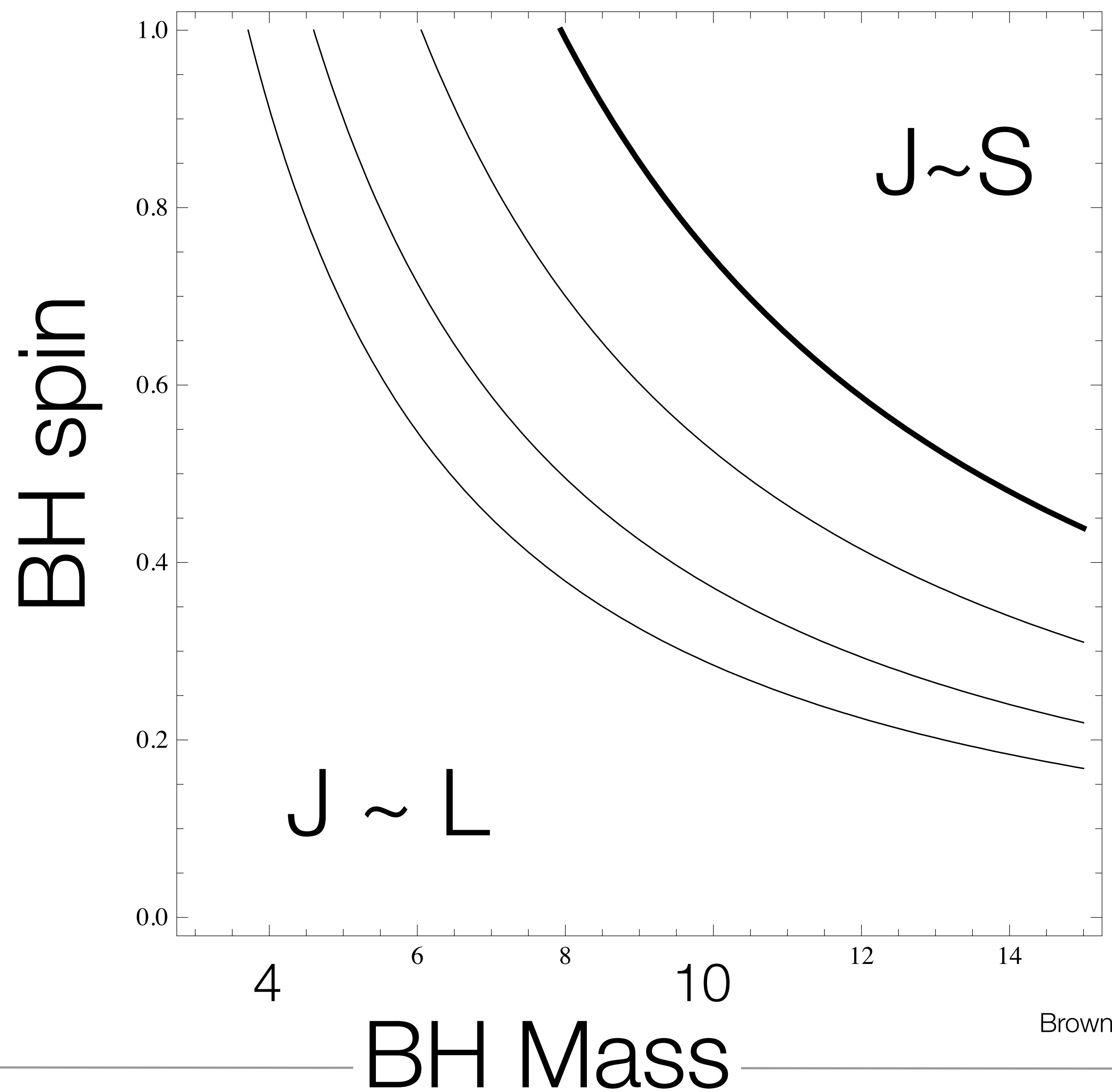




How do binaries evolve when radiating?

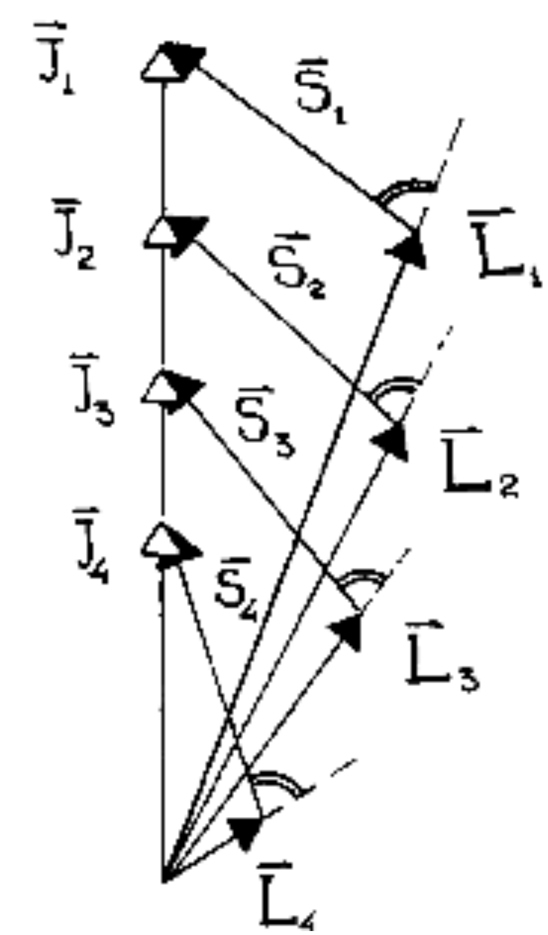
- Orbit shrinks
  - Spin angular momentum more significant
  - L<S easier if unequal mass or high spin

$$J = L + S_1 + S_2$$



Brown et al 2012

Example: BH-NS (40 Hz)



Apostolatos et al 1994

Putting it together

- Kinematics
  - Before merger, orbit shrinks & precesses
- GW signal in all directions have common trends
  - Amplitude
    - Inspiral : up
    - Merger: peak
    - Ringdown: exponential decay
  - Frequency “chirps” to merger

GW amplitude (log scale)

ROS et al 1209.3712

time

Putting it together

- Kinematics
  - Before merger, orbit shrinks & precesses
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  - Amplitude
    - Inspiral : up
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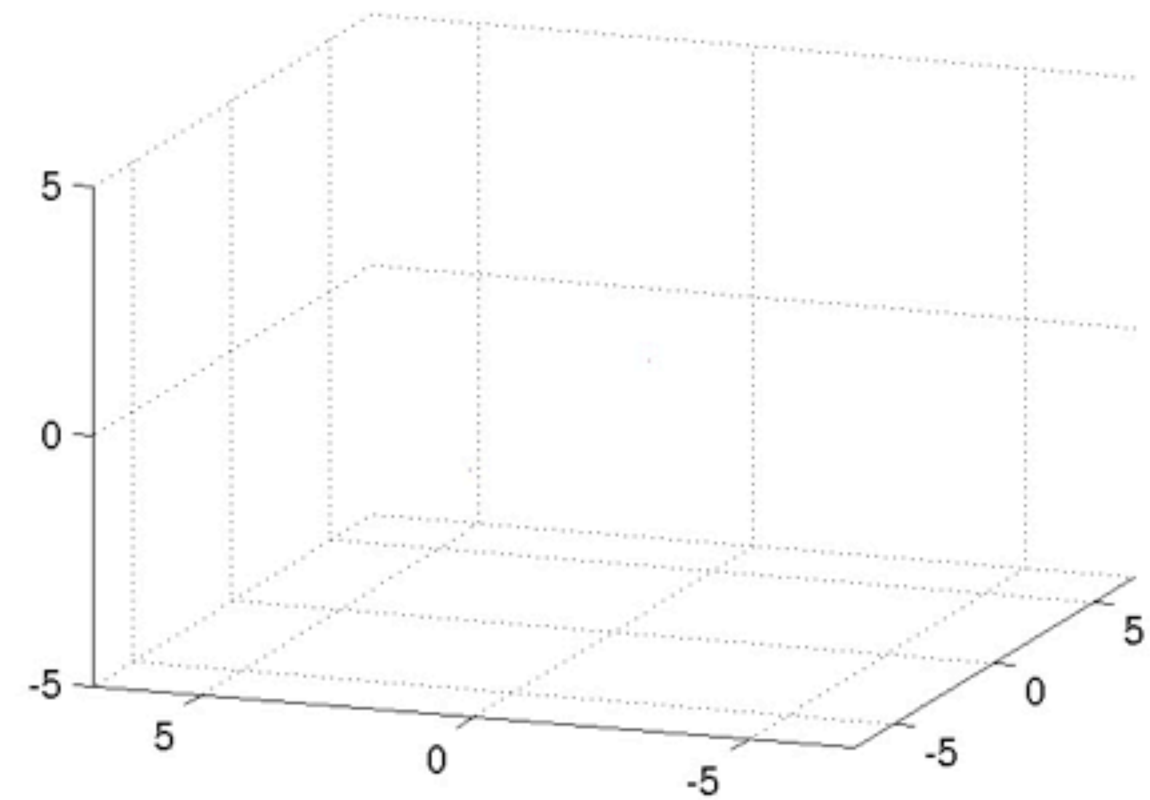
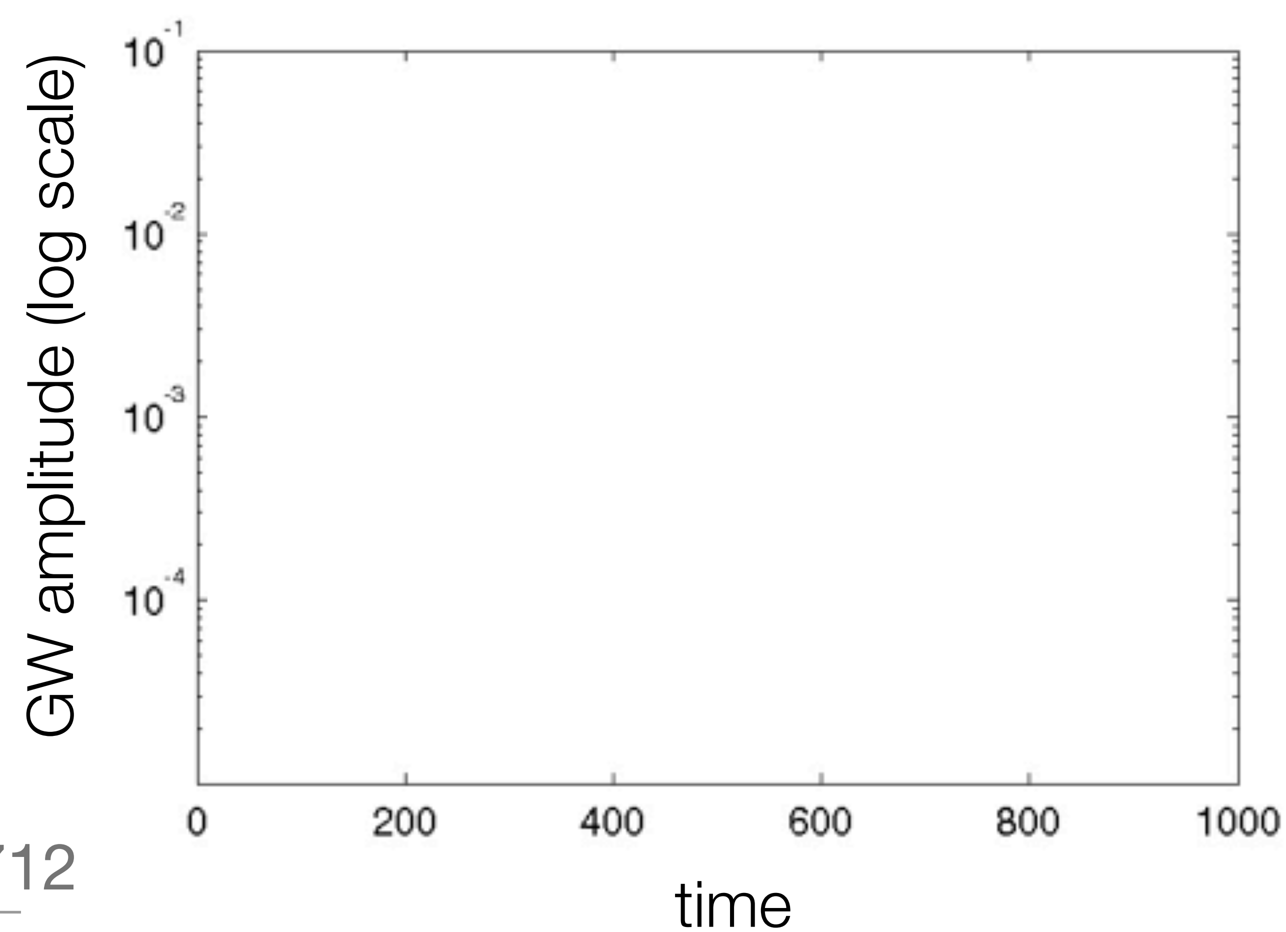
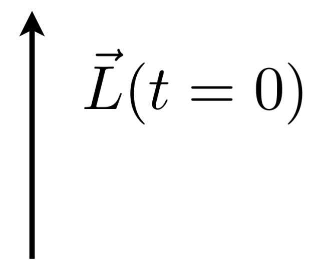




ROS et al 1209.3712

What does a GW from a binary look like?

- Each direction **distinctively modulated**



ROS et al 1209.3712

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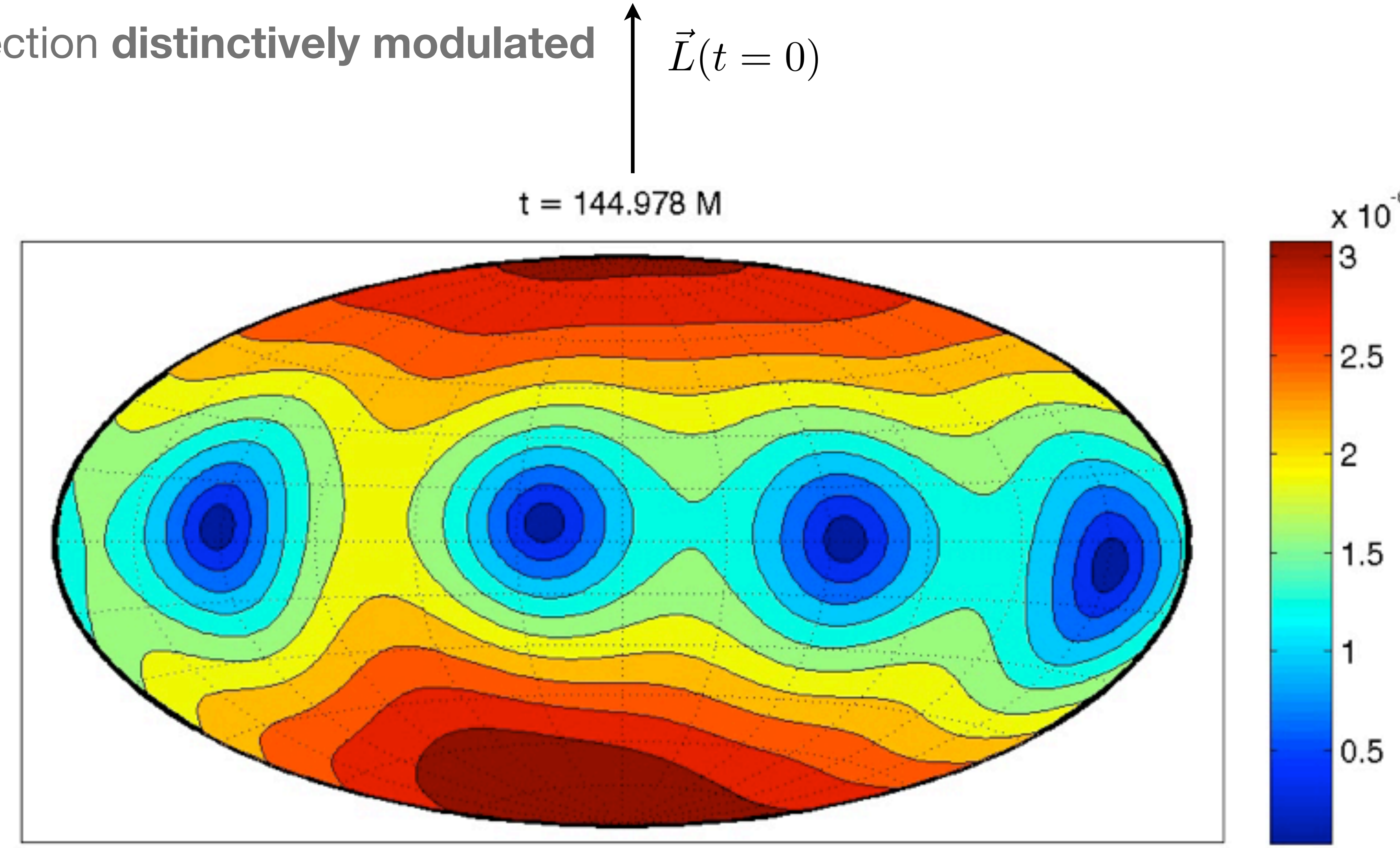
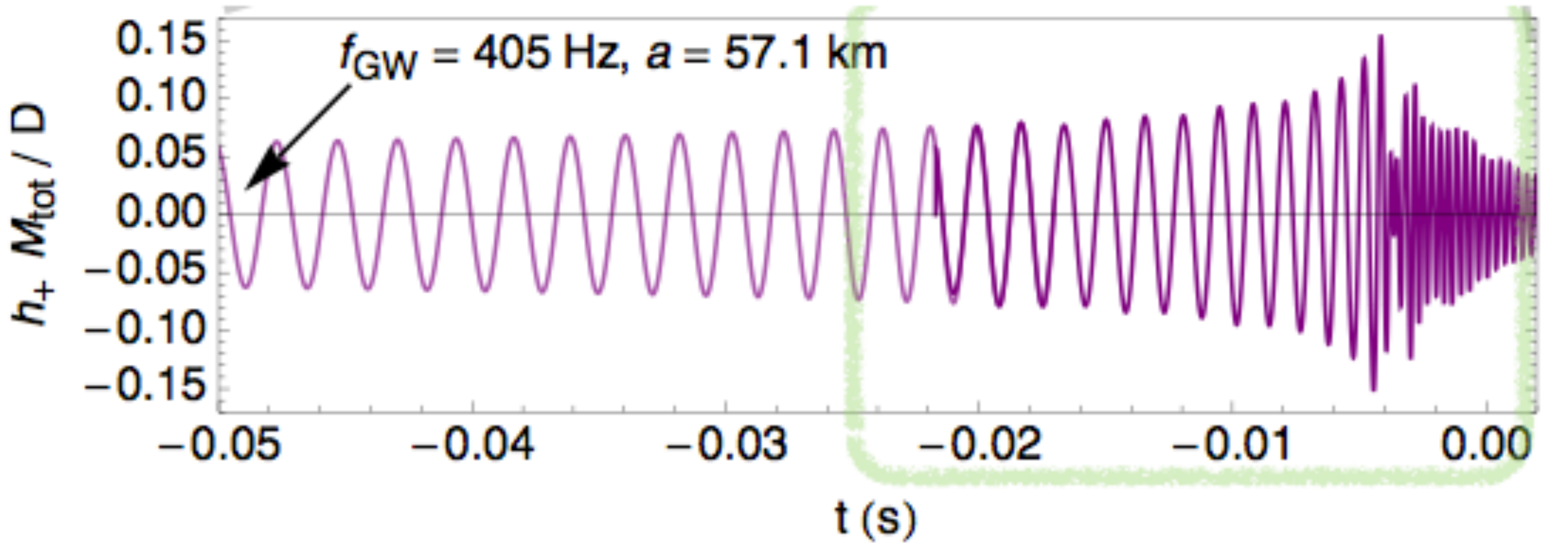


ROS et al 1209.3712

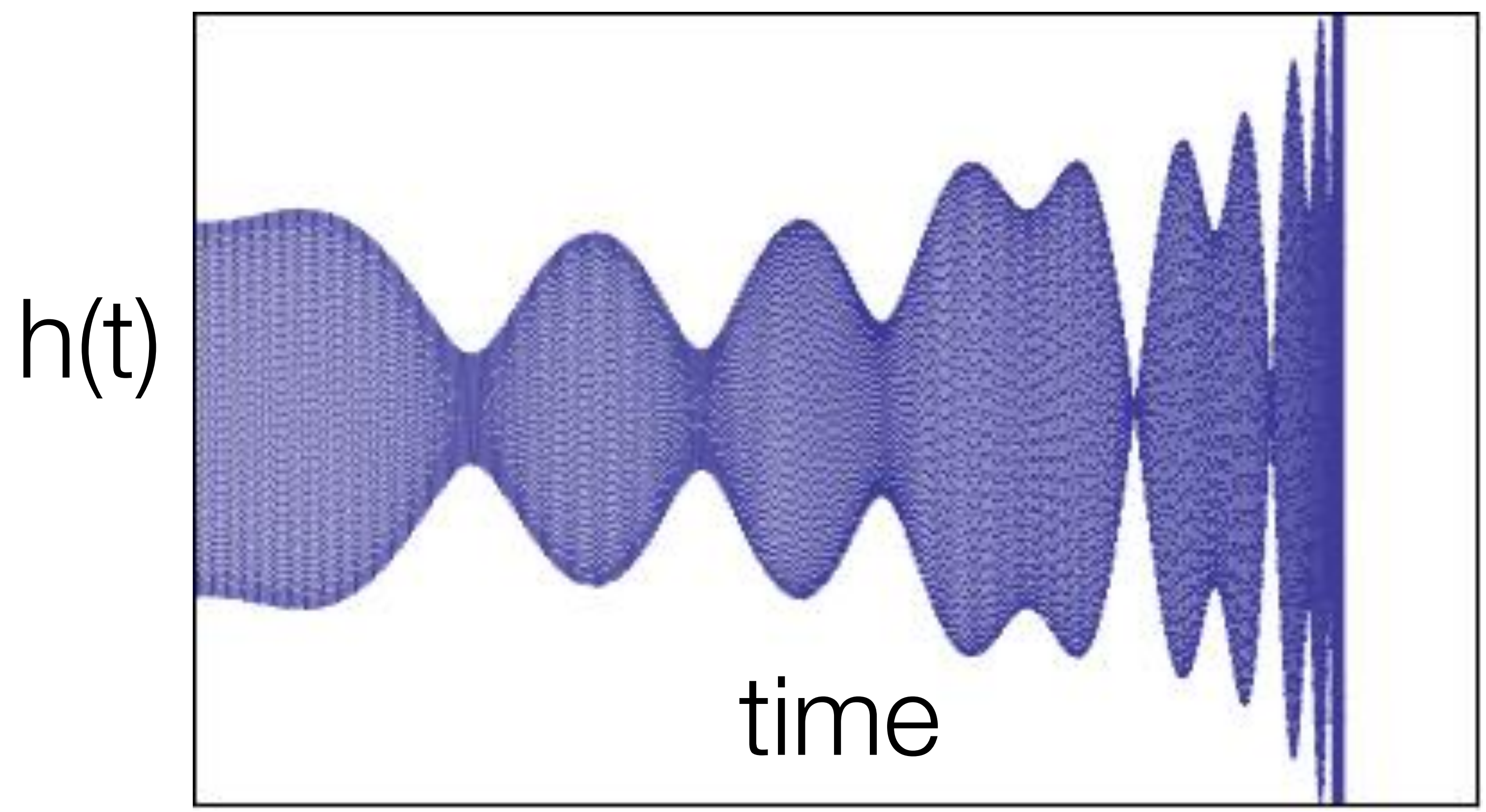
What does a GW from a binary look like?

- Each direction:
    - High-frequency chirping carrier signal (orbit)
- Direction-dependent:
    - Low-frequency amplitude & phase modulation

Example: no modulation



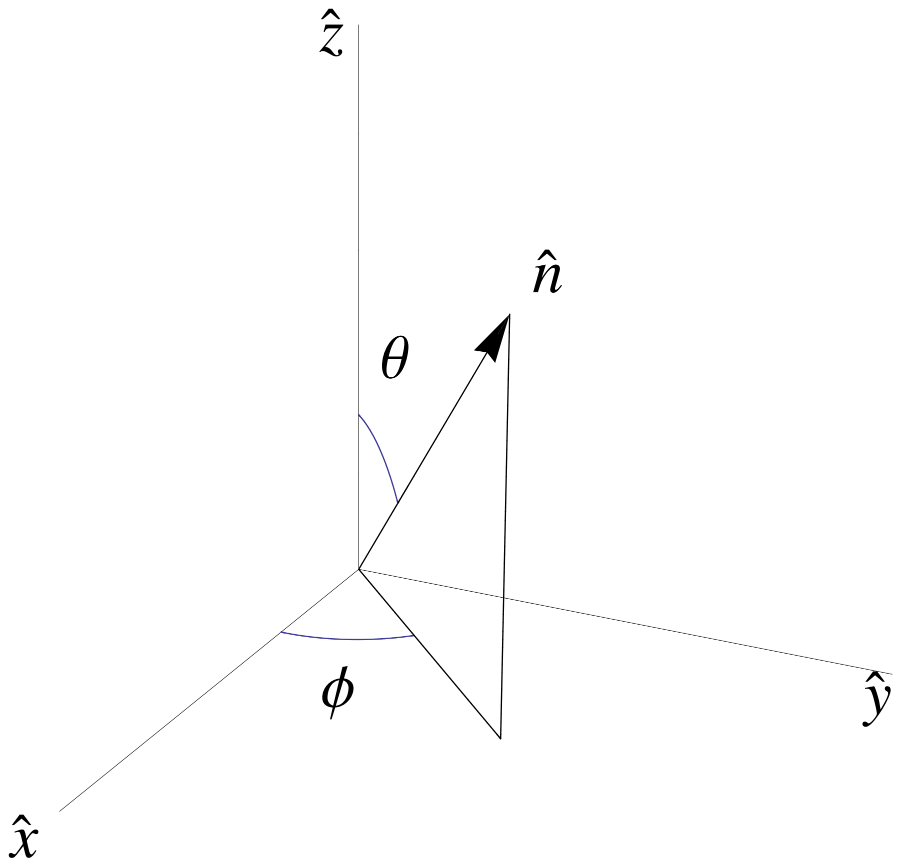
Example: with modulation



Solving the linear wave equation in spherical coord

Generally

- Separate variables
- Outgoing radial solutions at infinity

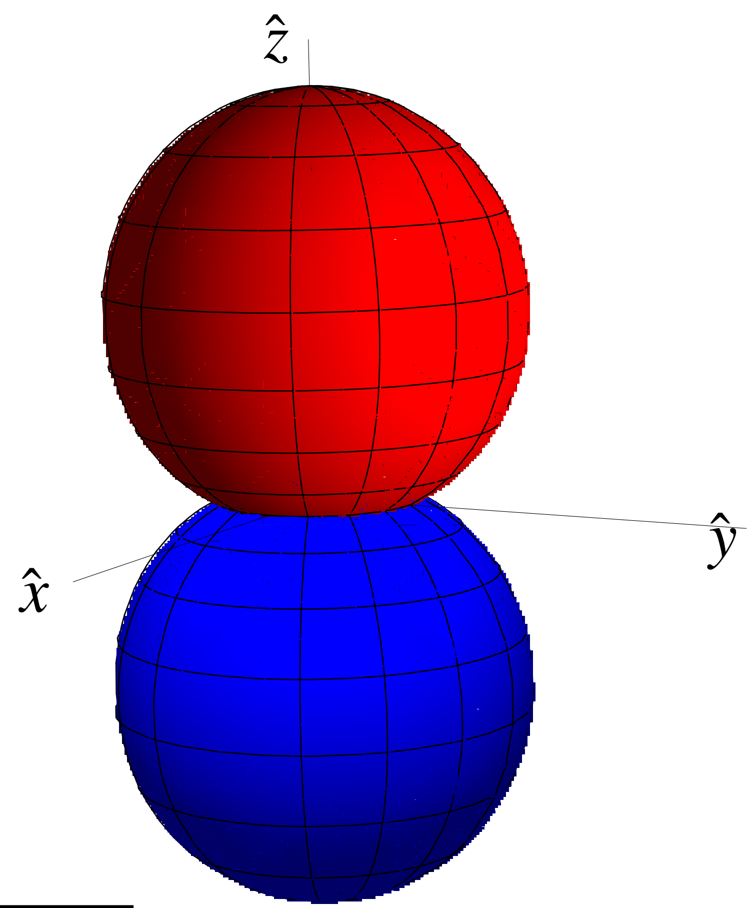


$$h(t,r,\hat{n}) = \sum_{lm} h_{lm}(t,r)\,{}_{-2}Y_{lm}(\hat{n})$$

$$h_{lm}(t,r) \simeq \frac{H_{lm}(t-r)}{r}$$

Angular equation

- Orthonormal angular basis ("spherical harmonics")
- "Beaming"



$$
\begin{aligned}
{}_{-2}Y_{2-2} &= \sqrt{\frac{5}{64\pi}}(1-\cos\theta)^2 e^{-2i\phi}\,, \\
{}_{-2}Y_{22} &= \sqrt{\frac{5}{64\pi}}(1+\cos\theta)^2 e^{+2i\phi}\,.
\end{aligned}
$$

Gravitational waves from nonprecessing binaries

Quasicircular binary

- Reflection-symmetric

$$P(x,y,z) = (x,y,-z)$$
$$h(t,P\mathbf{r}) = h(t,\mathbf{r})^*$$
$$\leftrightarrow (-1)^l h_{l,-m}(t)^* = h_{lm}(t)$$

- Periodic source

$$h_{lm}(t) = |h_{lm}|e^{-im\Phi(t)}$$

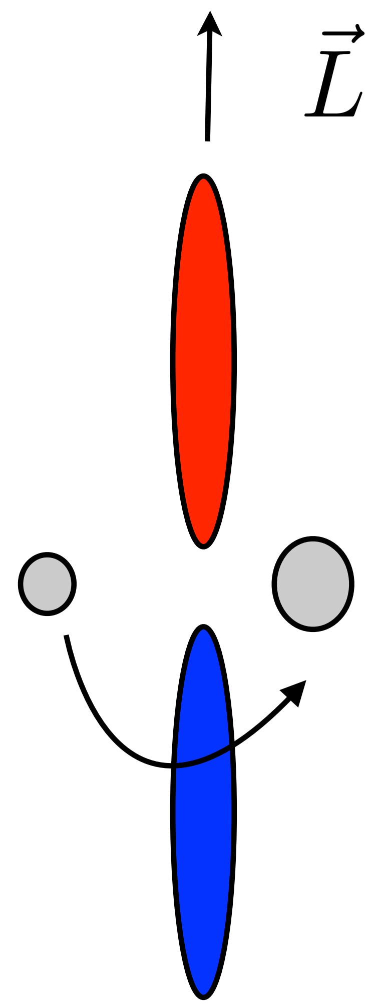
- Dominant $h_{2,\pm 2}(t)$

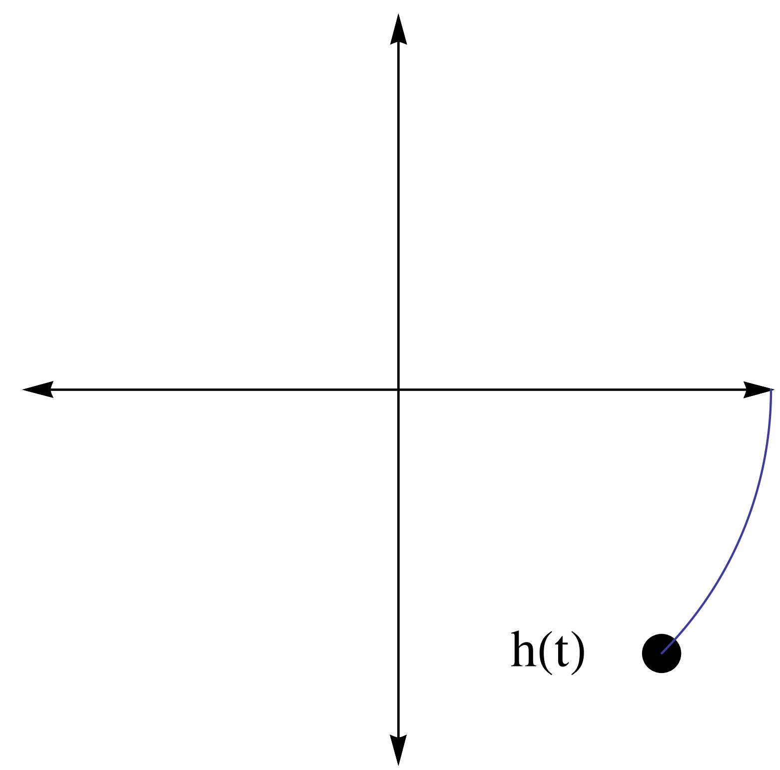
Polarization (definition via fourier)

- Right handed

$$\tilde{h}(\omega < 0) = 0$$

  - m>0 : right-handed**. Polarized beams!**
- Obvious projection to right, left

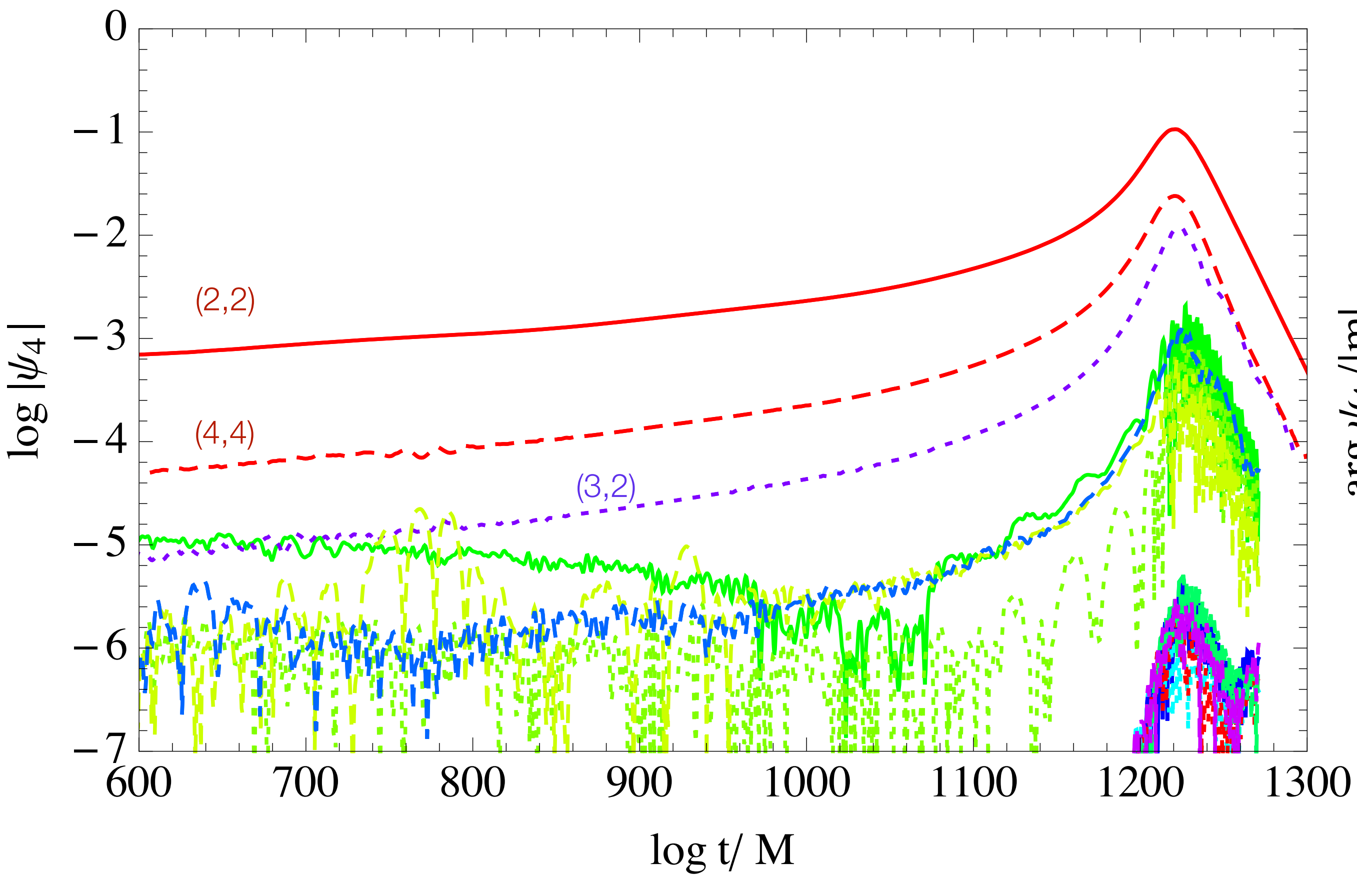


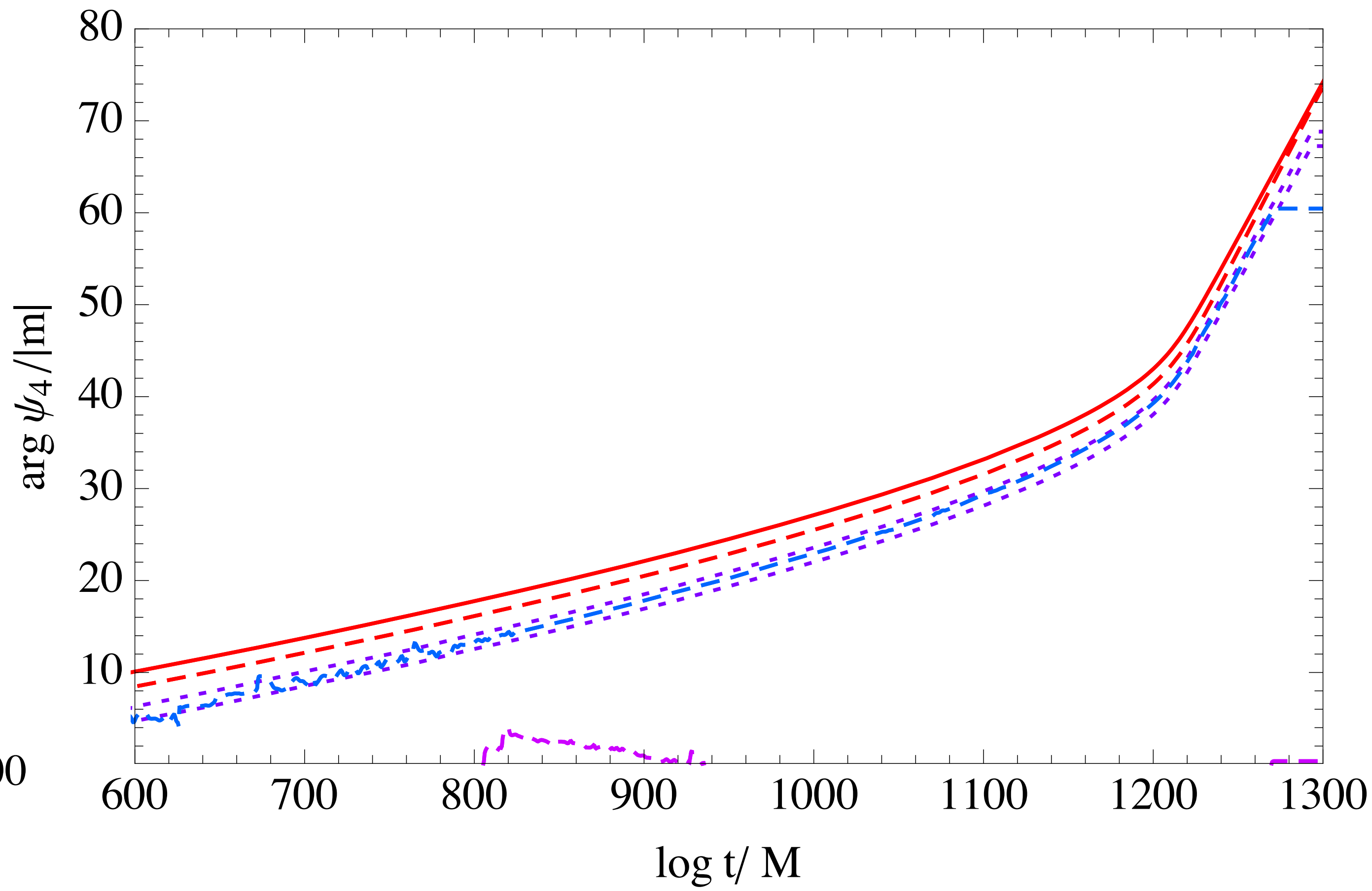


Phase and amplitude: all modes + strong field

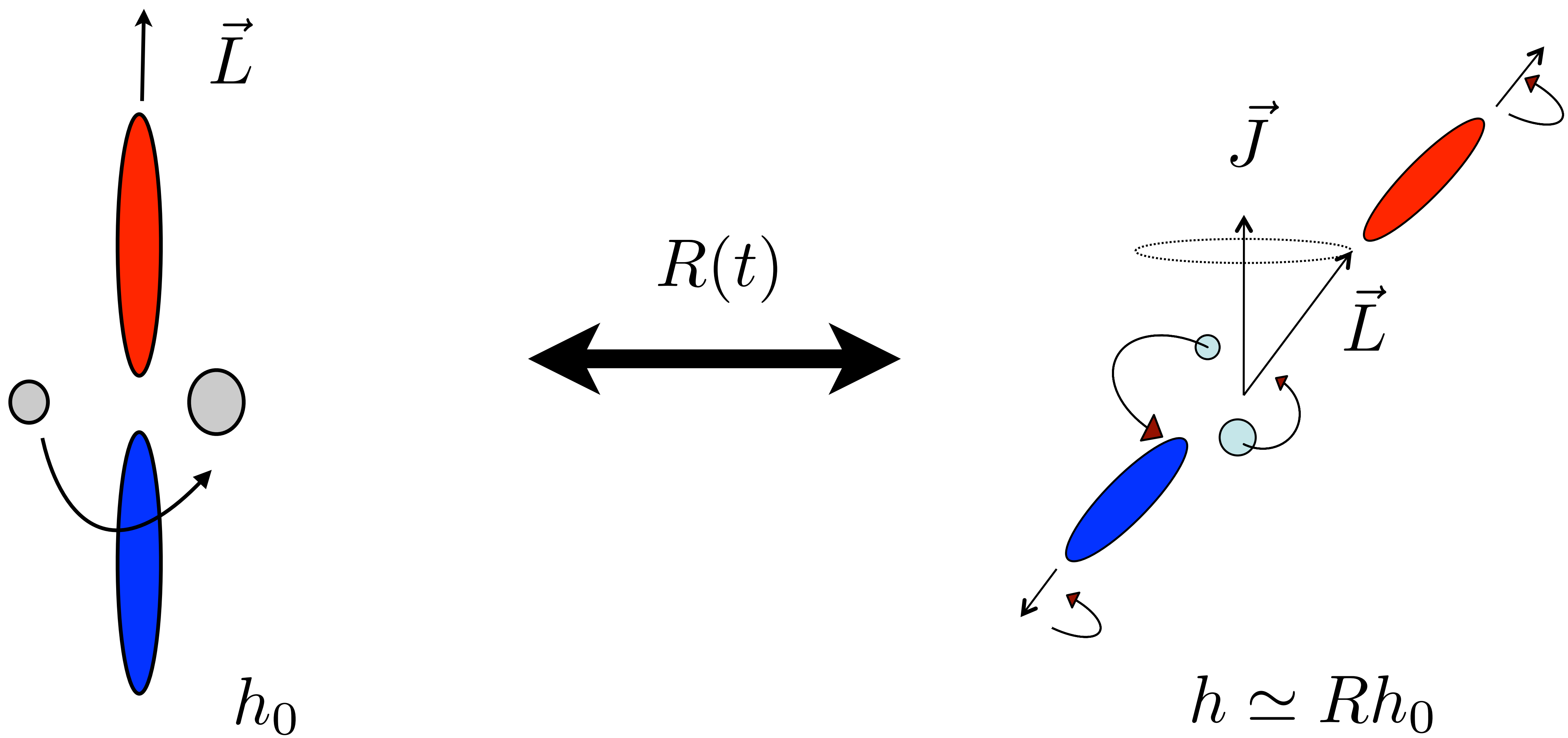
- Harmonics

$$\psi_4(t,\hat{n}) = \sum_m \psi_{lm} Y^{(-2)}_{lm}(\hat{n})$$





Comoving Frame



Radiation from precessing binary

~ rotation x (radiation from nonprecessing)

Schmidt et al 2011
ROS et al 2011 [arxiv: 1109.5224]
Boyle et al 2012
Ochsner and ROS 2012 [arxiv:1205.2287]

Corotating frame algorithm

- Construct tensor
  - If reflection-symmetric,trivial

$$\langle L_{(a}L_{b)}\rangle = \frac{\langle \psi_4^*(\hat{n},t)\mathcal{L}_{(a}\mathcal{L}_{b)}\psi_4(\hat{n},t)\rangle_{\rm angles}}{\langle |\psi_4|^2\rangle_{\rm angles}}$$

$$= \frac{\sum_{lmm'}\psi^*_{4,lm'}\psi_{4,lm}\langle lm'|\mathcal{L}_{(a}\mathcal{L}_{b)}|lm\rangle}{\int d\Omega|\psi_4|^2}$$

- Find eigensystem
  - Largest: V

$$\hat{V} = (\cos\alpha\sin\beta, \sin\alpha\sin\beta, \cos\beta)\ .$$

- Construct frame
  - Minimal rotation

$$\gamma(t) = -\int_0^t \cos\beta\frac{d\alpha}{dt}dt$$

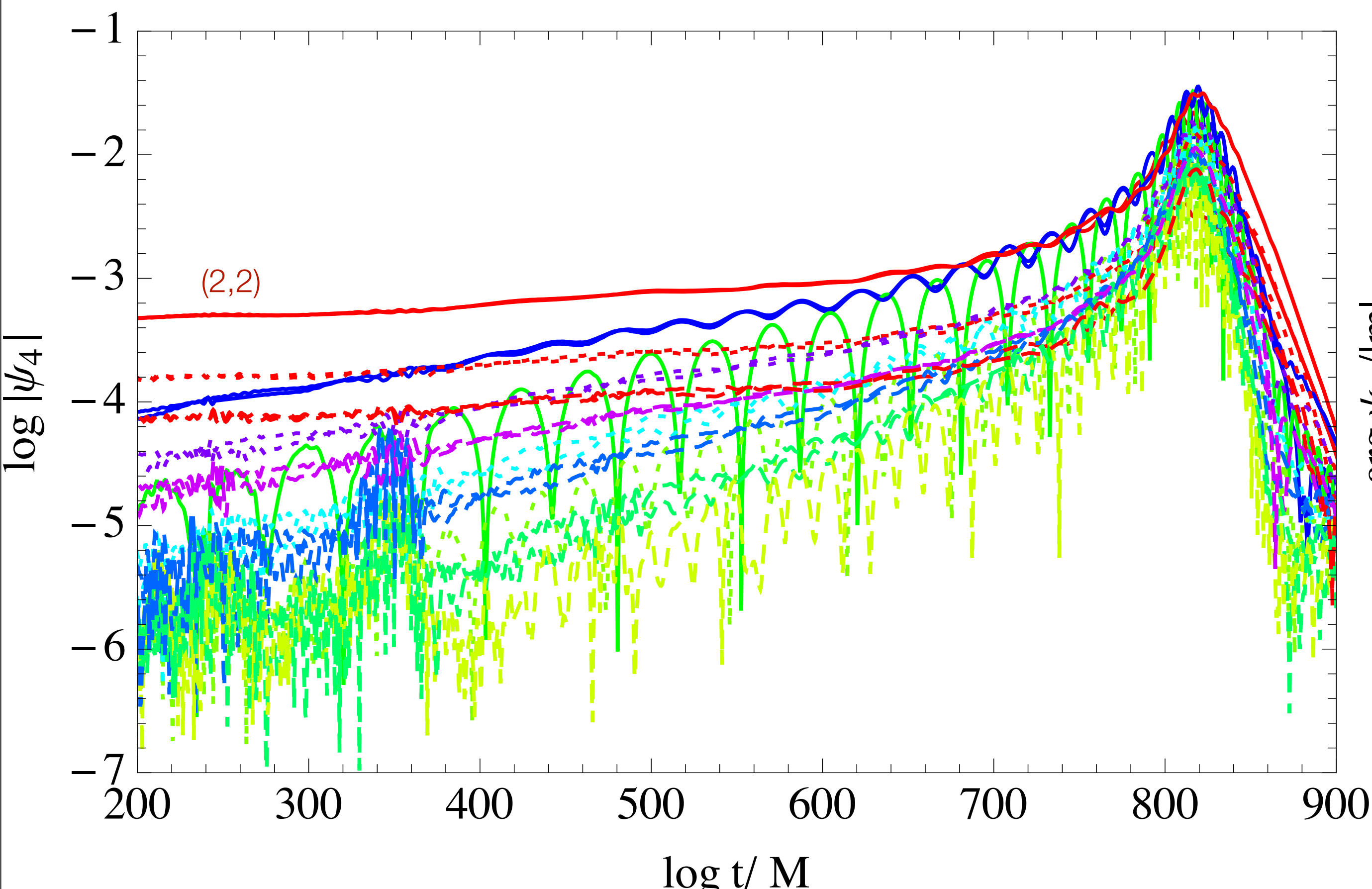
- Transform to (noninertial) basis

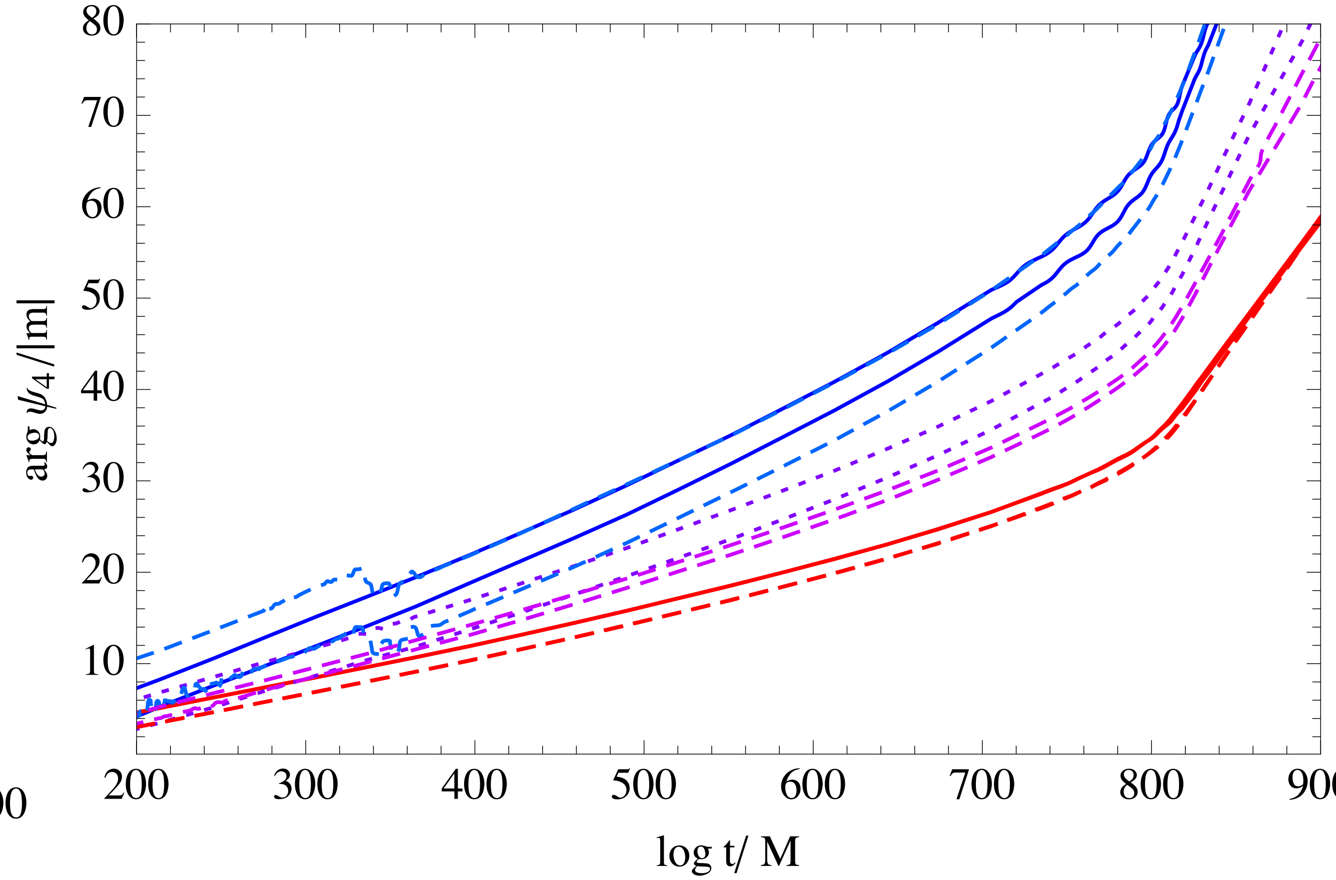
$$\begin{aligned}\psi^{ROT}_{4,l,m} &= \sum_{\bar{m}} D^l_{m\bar{m}}(R(\alpha,\beta,\gamma)^{-1})\psi_{4,l\bar{m}} \\ &= \sum_{\bar{m}} e^{i\bar{m}\gamma}d_{\bar{m}m}(\beta)e^{im\alpha}\psi_{4,l\bar{m}}\end{aligned}$$

Simulation-frame radiation

Precession-induced “mode mixing”

$$h_{lm} = \sum_{m'} D^{l}_{mm'}(\alpha,\beta,\gamma) h^{\mathrm{ROT}}_{lm'}$$





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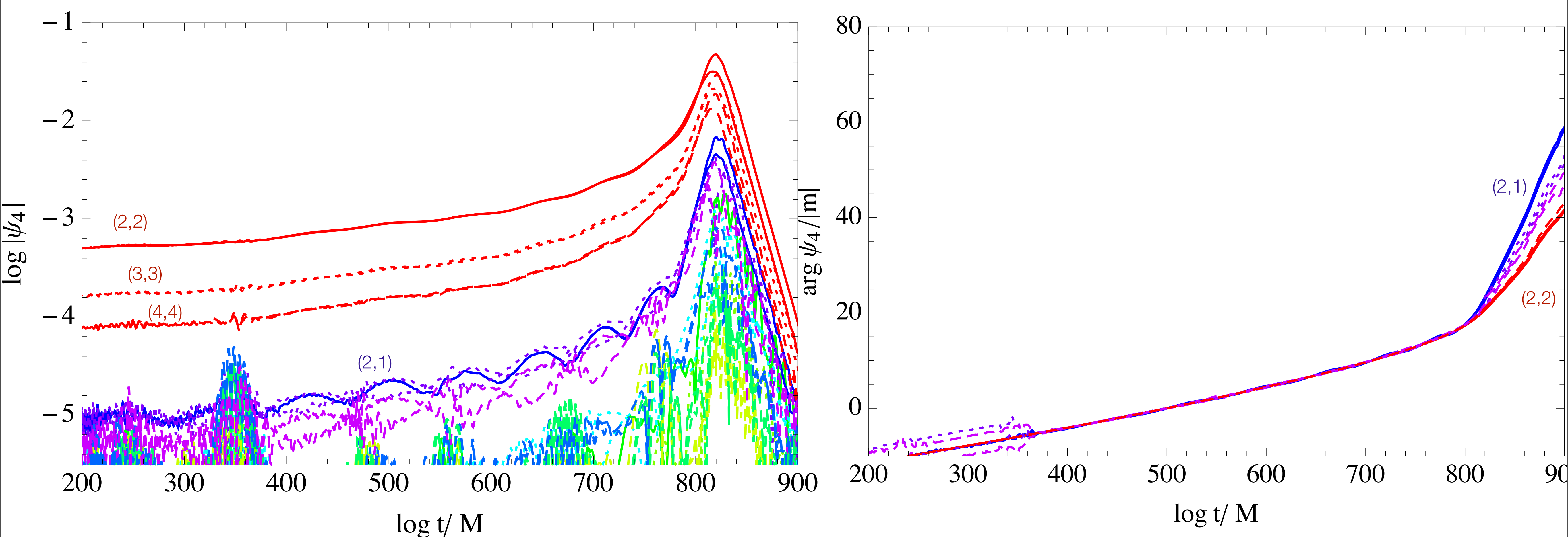
Corotating-frame radiation

- Corotating-frame harmonics
  - Inspiral: recover common phase evolution

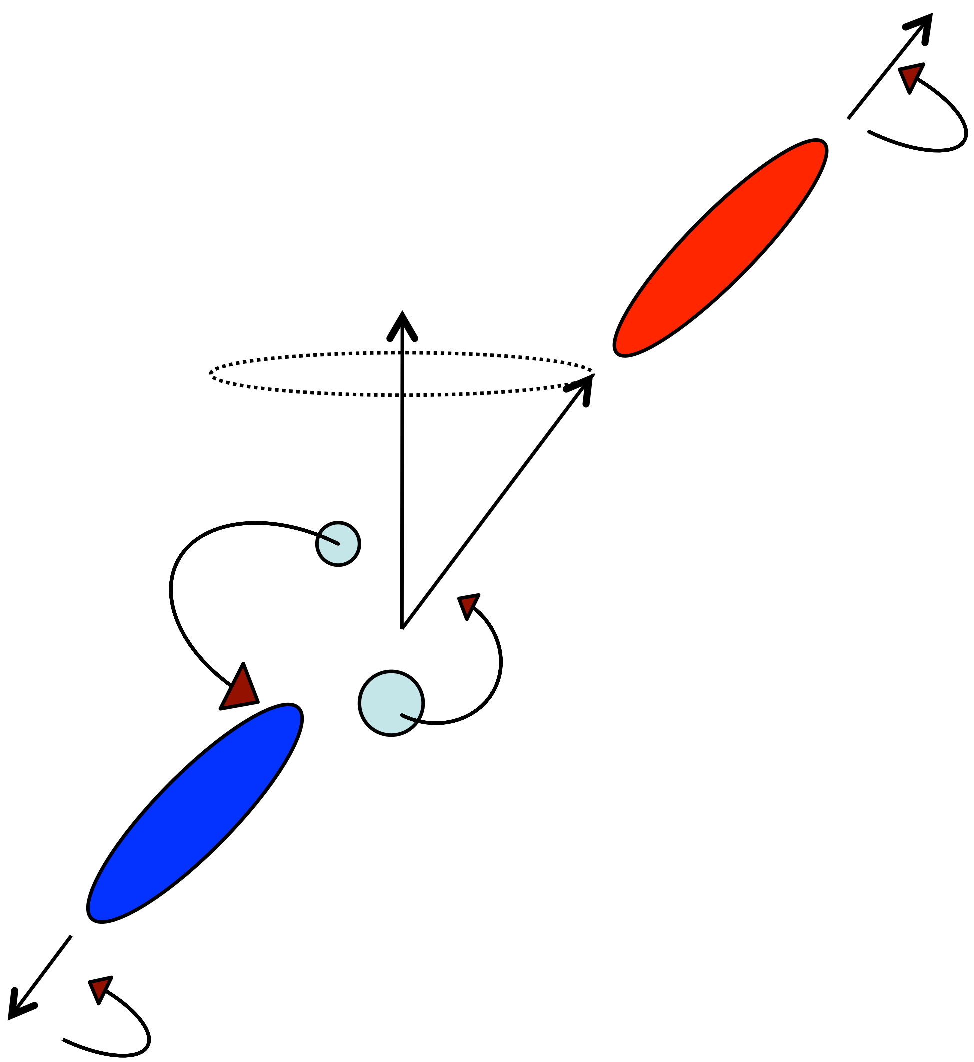
$$\psi_{lm} \simeq |\psi_{lm}(t)| e^{-im\Phi(t)}$$

  - Post-merger: recover exponential decay



Polarization (versus time)

- Left- and right-handed radiation **easy** to distinguish
  - Constrains opening angle of precession
  - Sets lower bound on (transverse) spin
  - Often: separation of timescales

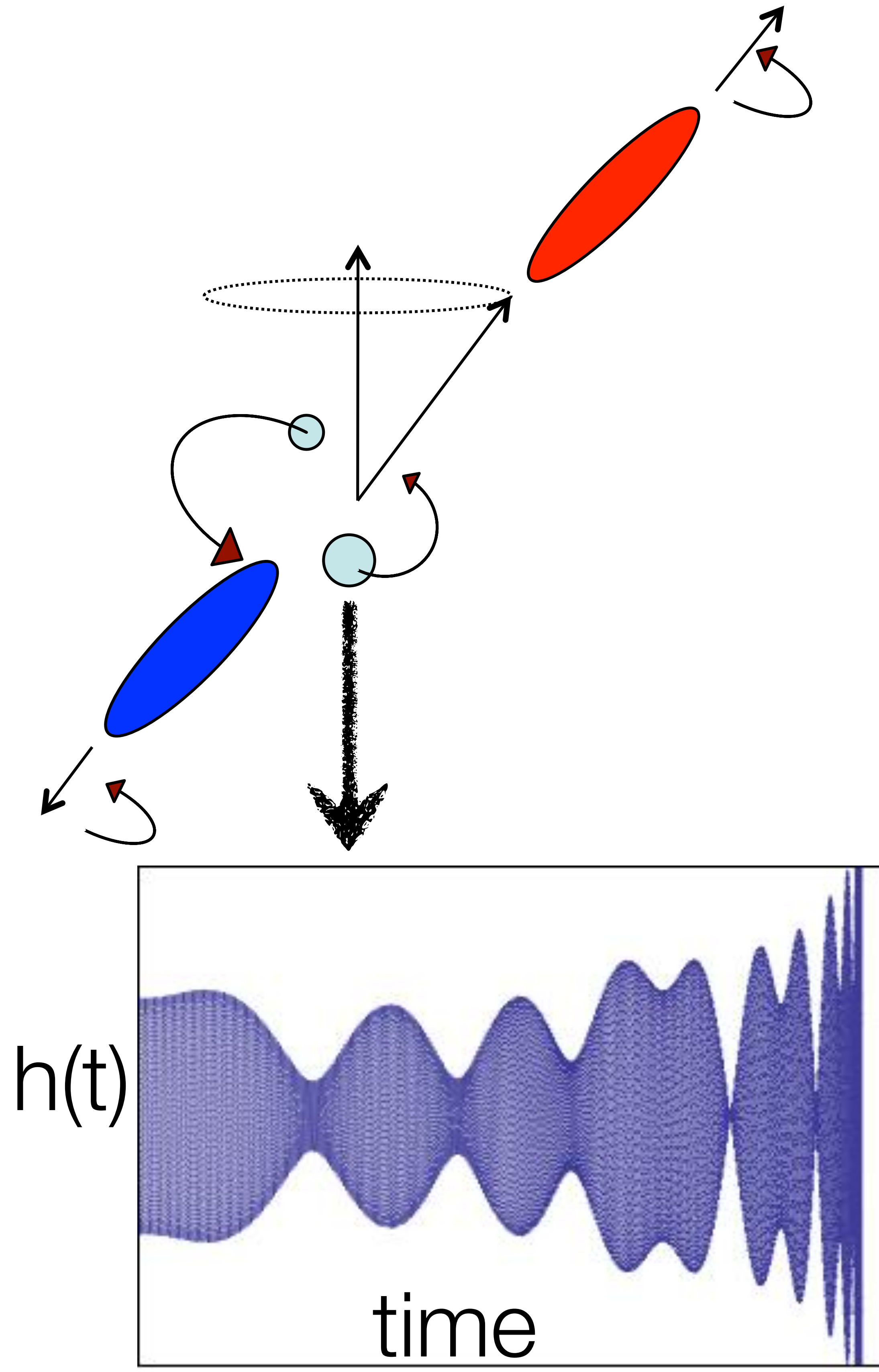


R

L

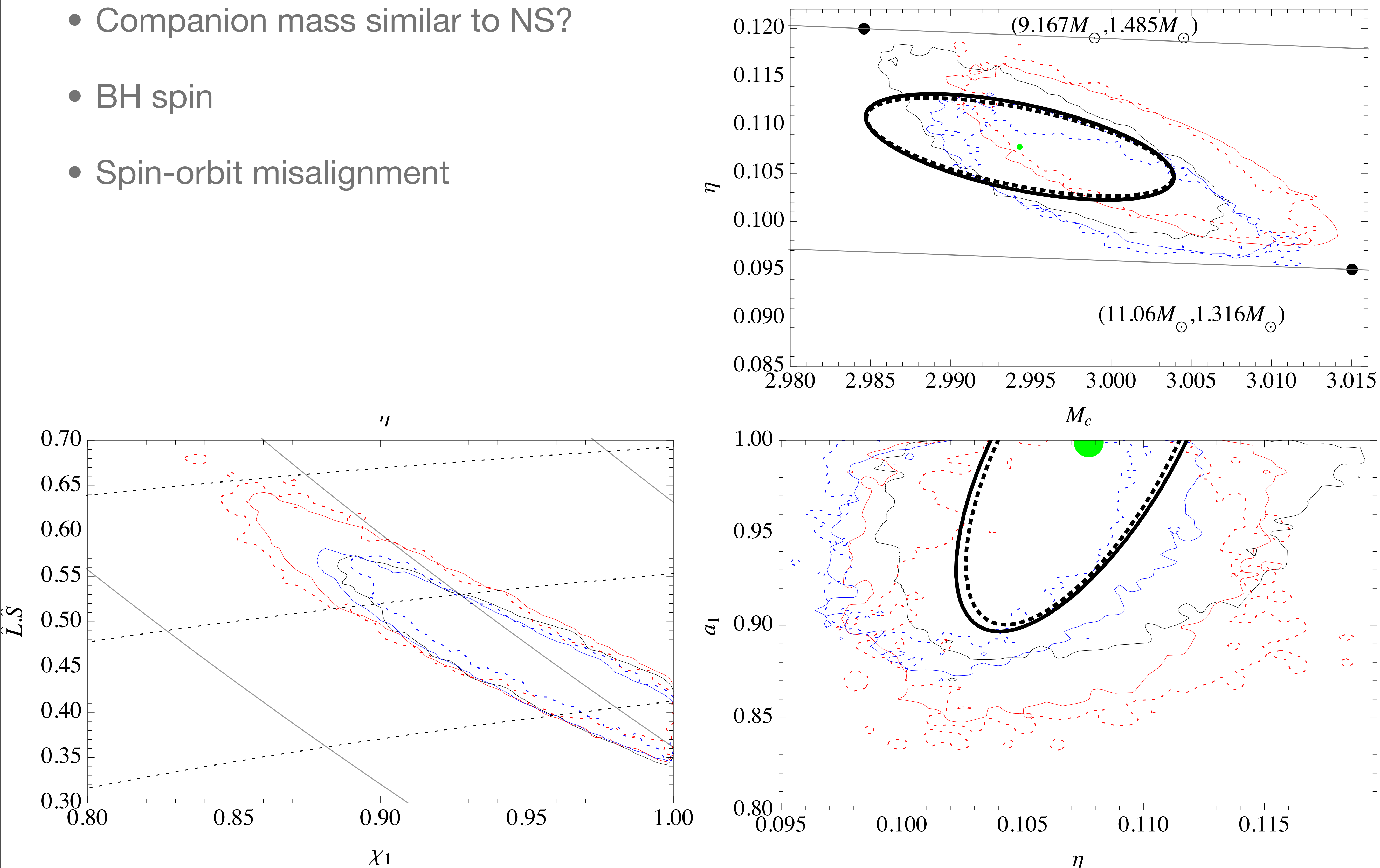
Polarization for alignment and precession

- Polarization **easy** to measure
  - “Only see what we see” = at 100 Hz !
- Measure **spin-orbit misalignment**
  - via simple geometry + polarization
  - Traces strength whatever misaligns them
    - SN kicks
    - Stellar dynamics [binary collisions]
  - Measure BH spin
    - Insight into SN, massive star physics



More accurate mass, spin measurements

- Companion mass similar to NS?
- BH spin
- Spin-orbit misalignment



Summary

- Why use gravitational waves for astronomy?
  - Rich data, addressing previously inaccessible phenomena
- Gravitational wave astronomy 101
  - Precise measurements, modulo systematics and degeneracies
  - Robust constraints on astrophysics
- Gravitational waves astronomy with precessing binaries
  - Richer signals, tracing different phenomena
- The future
  - Look to the sky, to new scales and new phenomena

The future

General relativity

Astrophysics

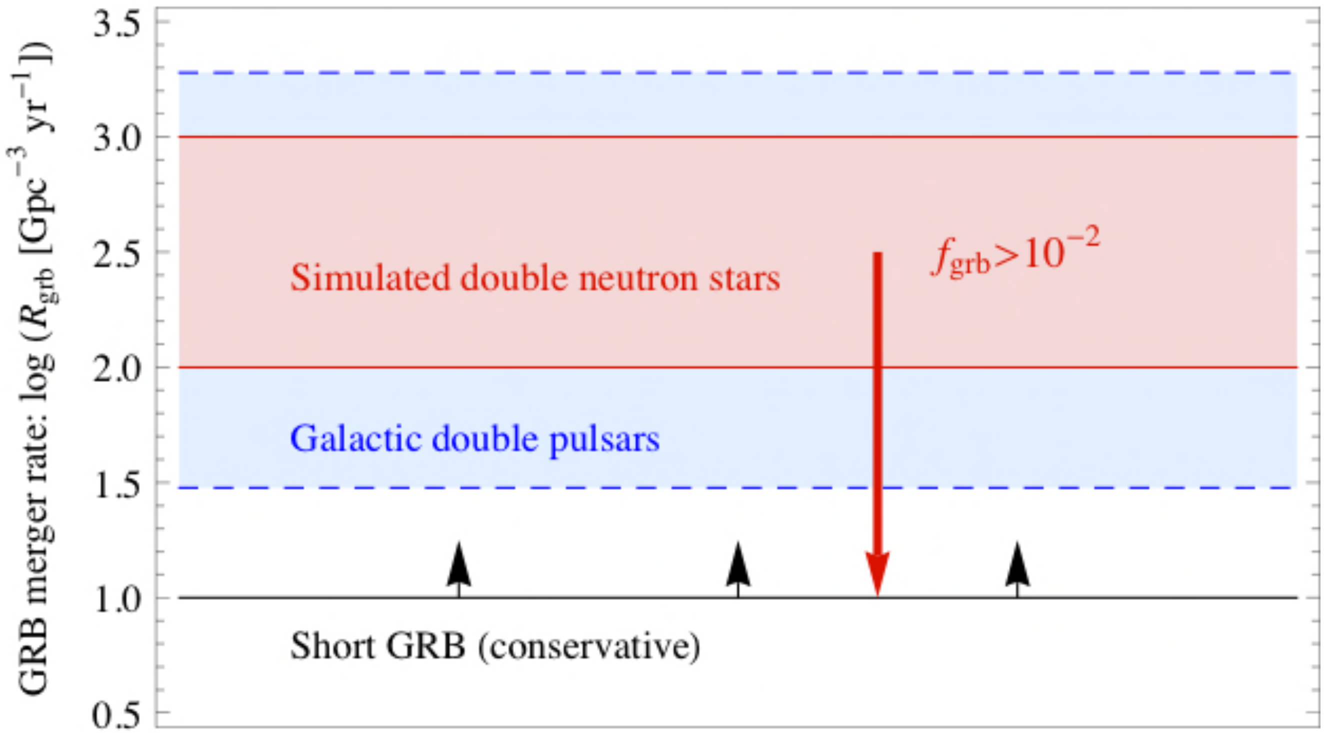
Data analysis

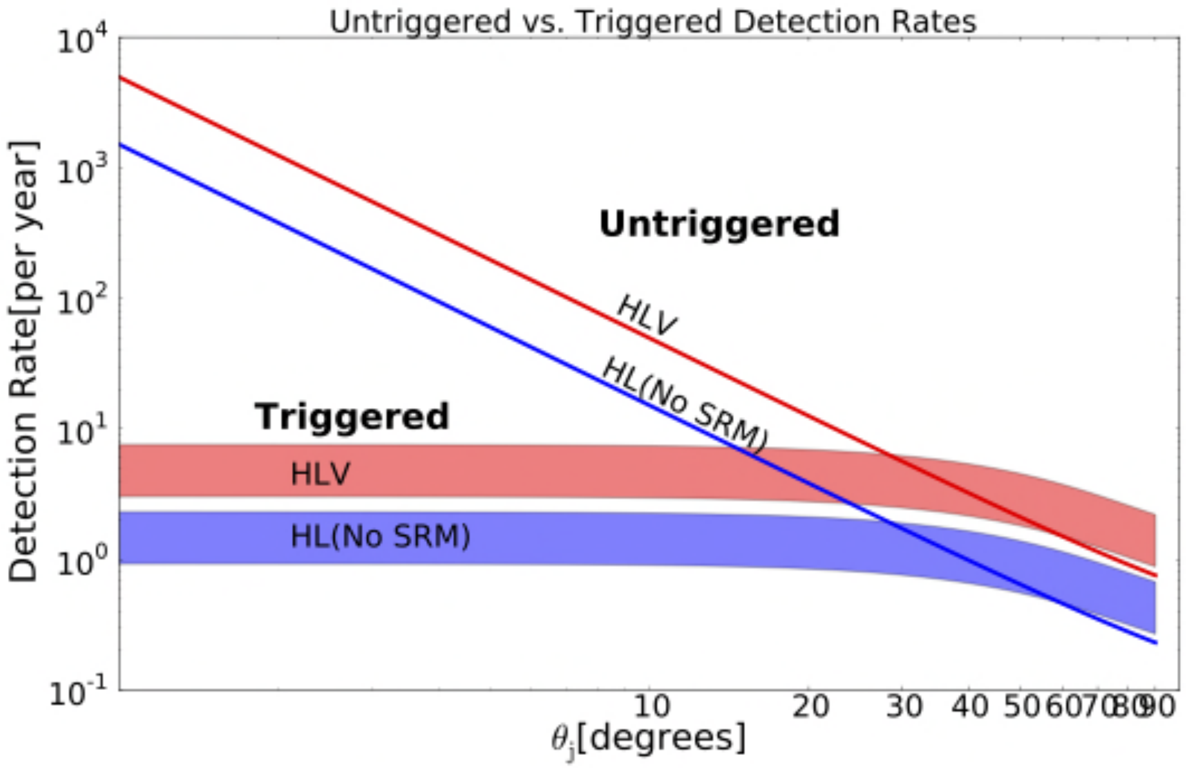
Plan

- Interpreting gravitational waves from the expected first GW detections
  - Establish infrastructure for inference (GR/GW + astrophysics)
  - Timely and correct inference
- Connect the EM and GW sky
  - Short GRBs: on/off, output/input, geometry, hosts
  - Joint population inference
- Prepare for the unexpected and exciting: Dynamical formation and eccentric mergers
- Other mass scales: Supermassive black holes
  - EM, GW, and EM+GW


GAIA
Fermi
Palomar

Short GRB rates





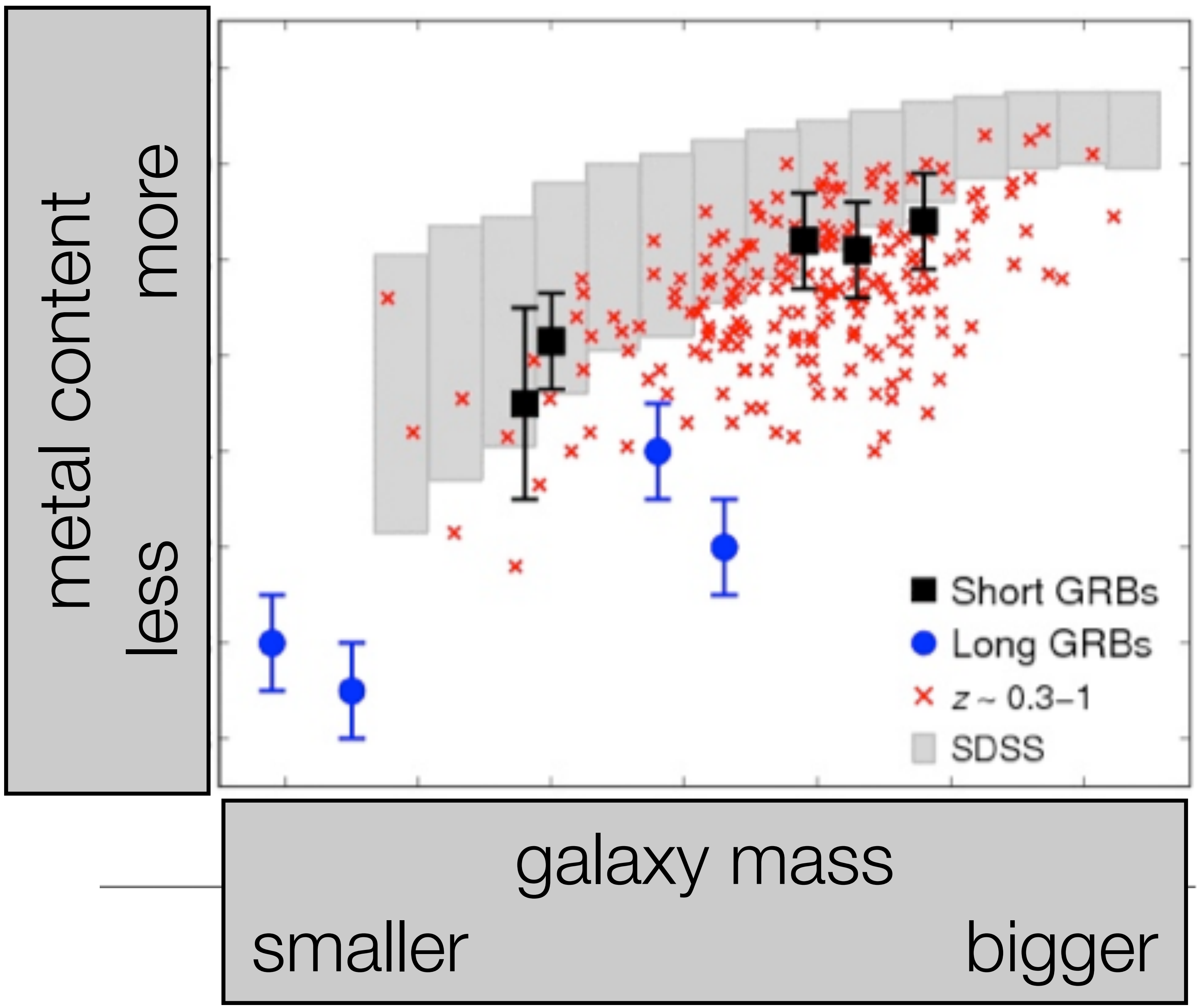
Belczynski, ROS et al 2007

Chen and Holz 2012

Also: Coward et al 2011; Petrillo, Dietz, Cavaglia 2012 (102.0804)

What do we learn about **formation** from EM?

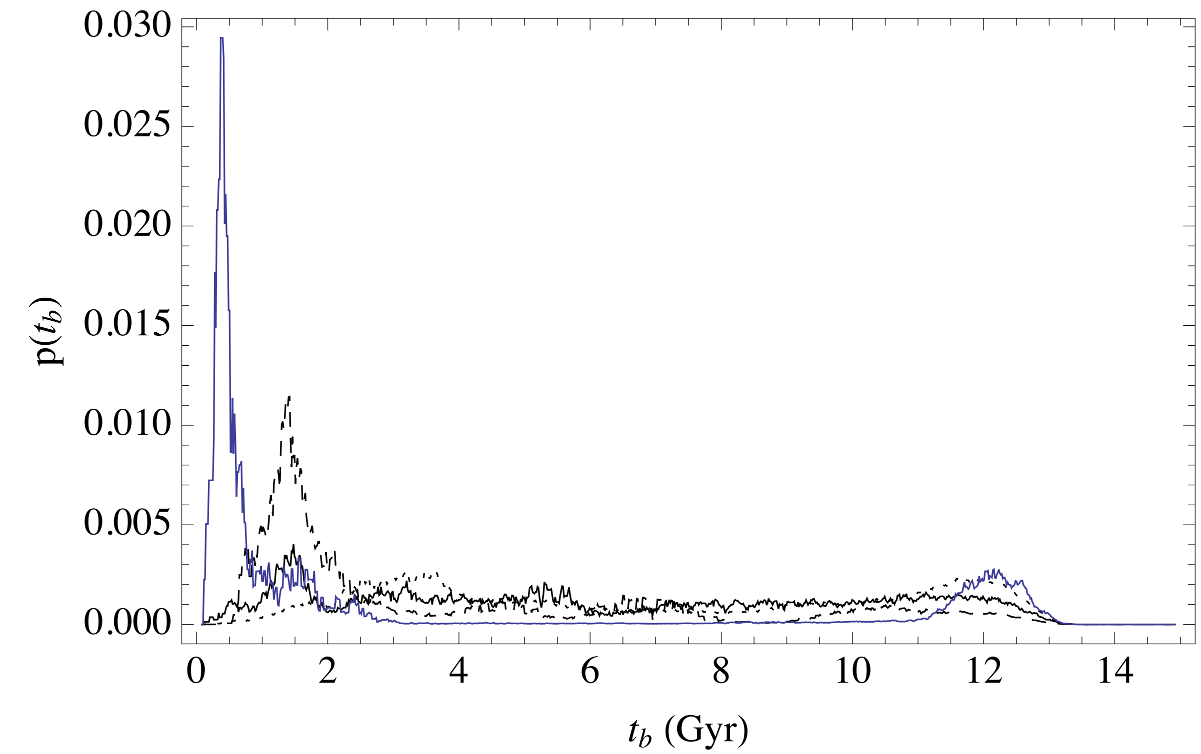
- Progenitor composition

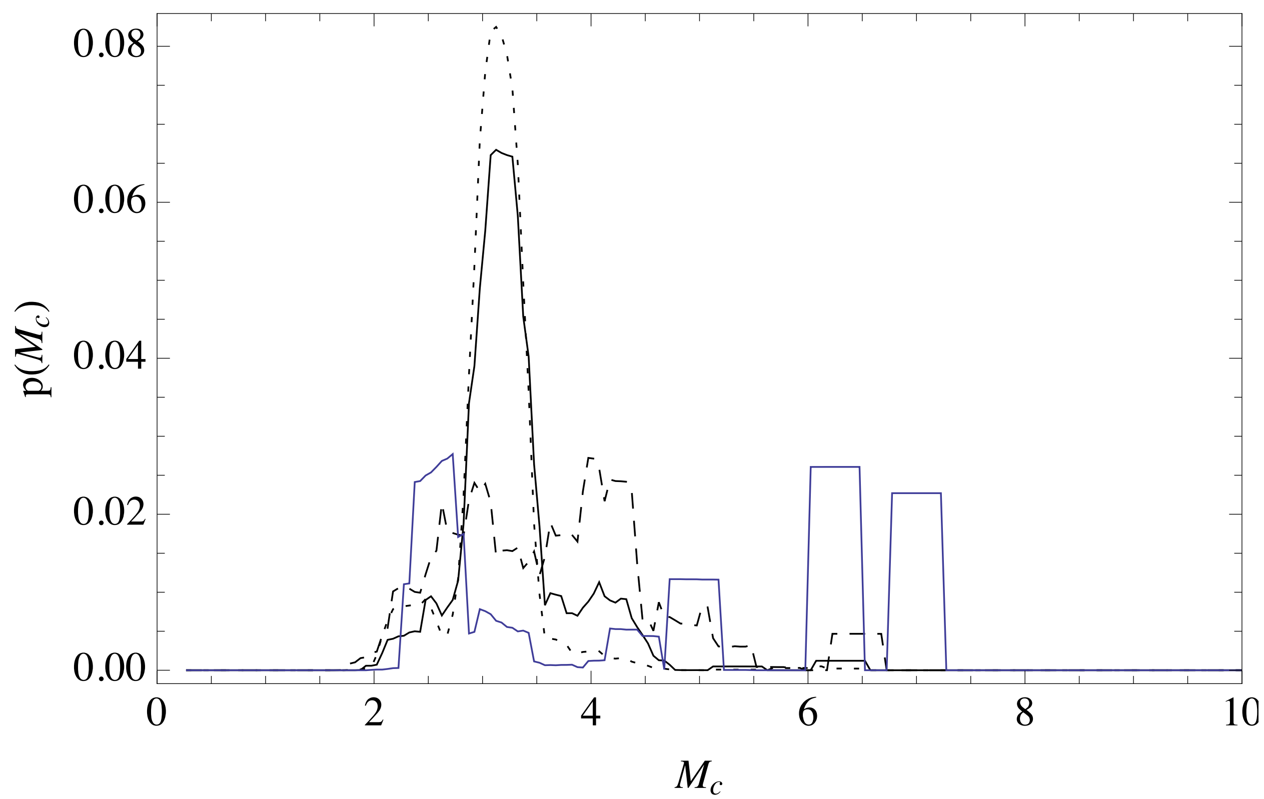


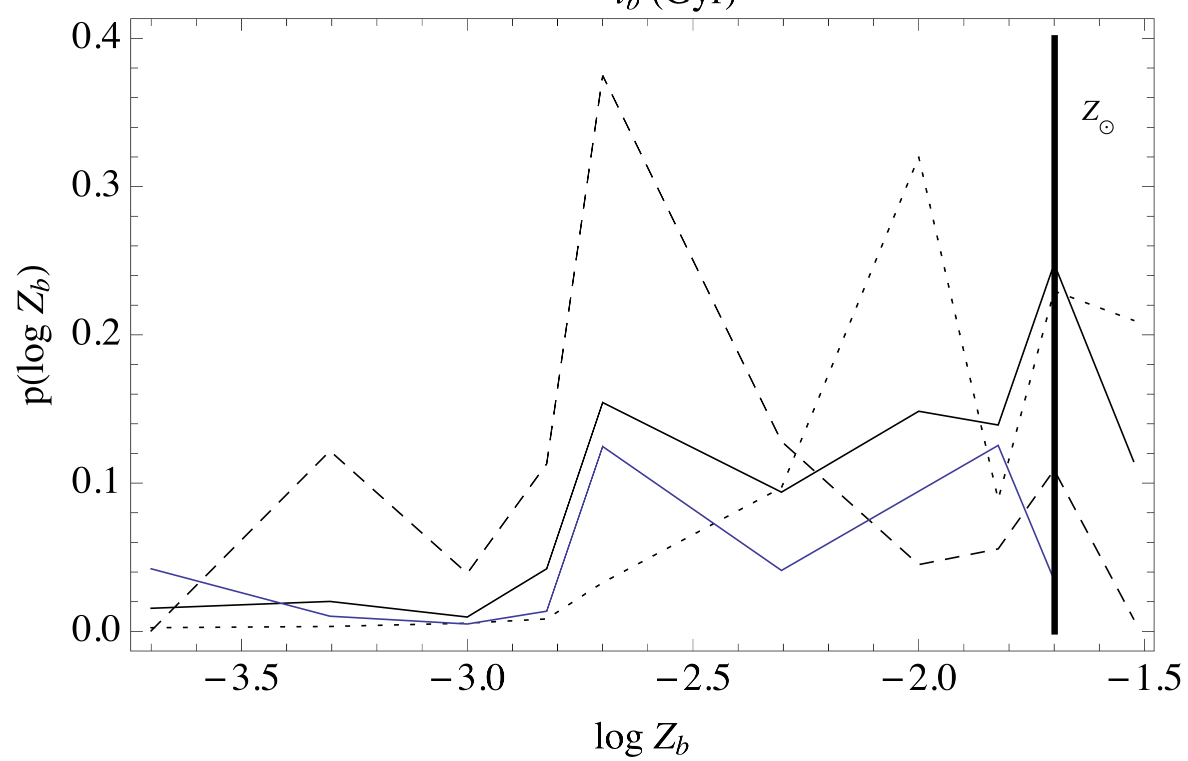
Berger 2009

What do we learn with EM : BH-NS binaries

- **Model degeneracy**
  - Where did detected mergers come from?
    - Recent star formation, like us?
    - Ancient star formation, unlike us?







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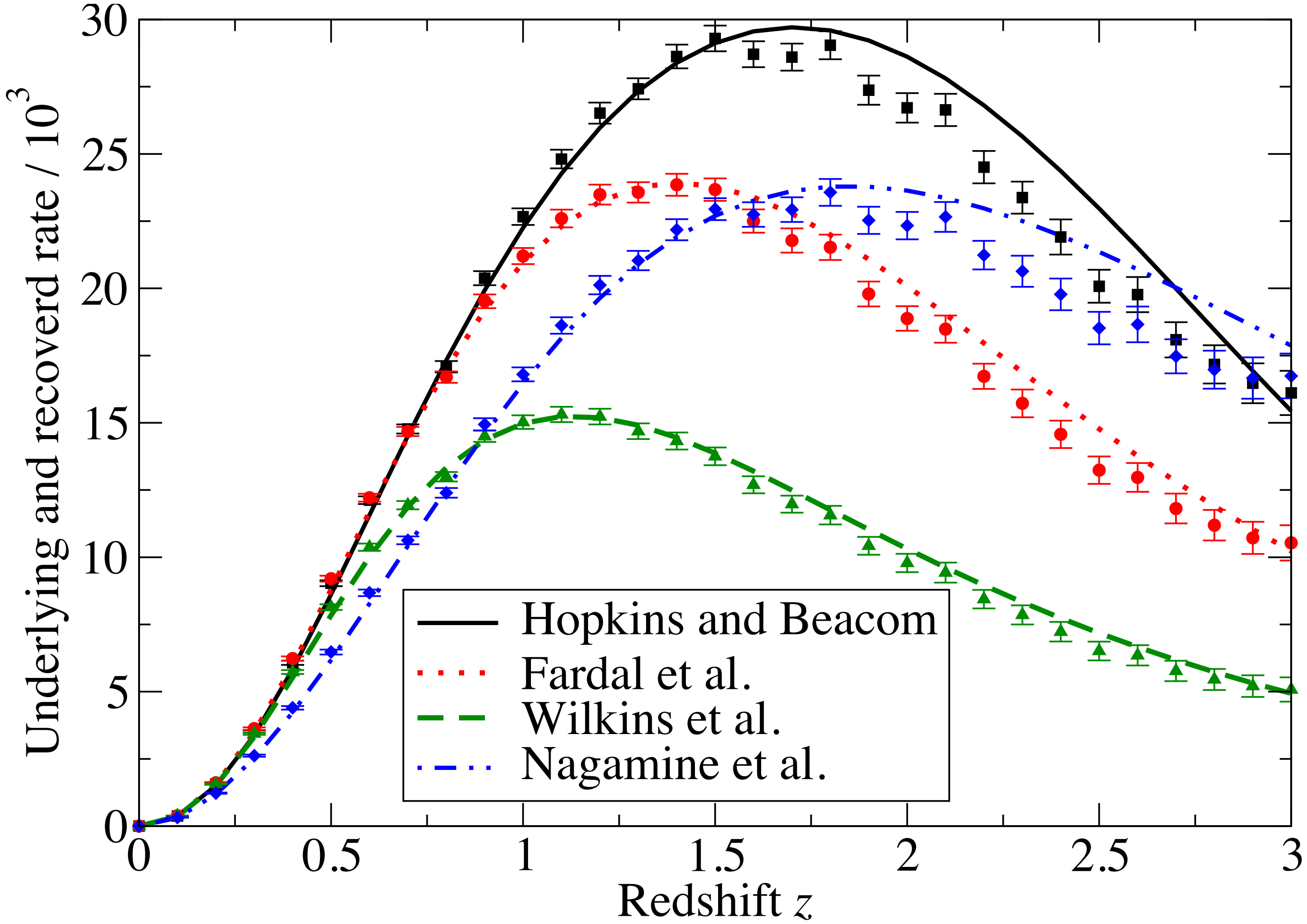
Long-term vision

- Ground-based next-generation:
  - Sensitive to high redshift
  - Many events/yr

Source	BNS	NS-BH	BBH
Rate ($\mathrm{Mpc}^{-1}\ \mathrm{Myr}^{-1}$)	0.1–6	0.01–0.3	2×10^{-3}–0.04
Event Rate (yr^{-1}) in aLIGO	0.4–400	0.2–300	2–4000
Event Rate (yr^{-1}) in ET	$\mathcal{O}(10^3$–$10^7)$	$\mathcal{O}(10^3$–$10^7)$	$\mathcal{O}(10^4$–$10^8)$

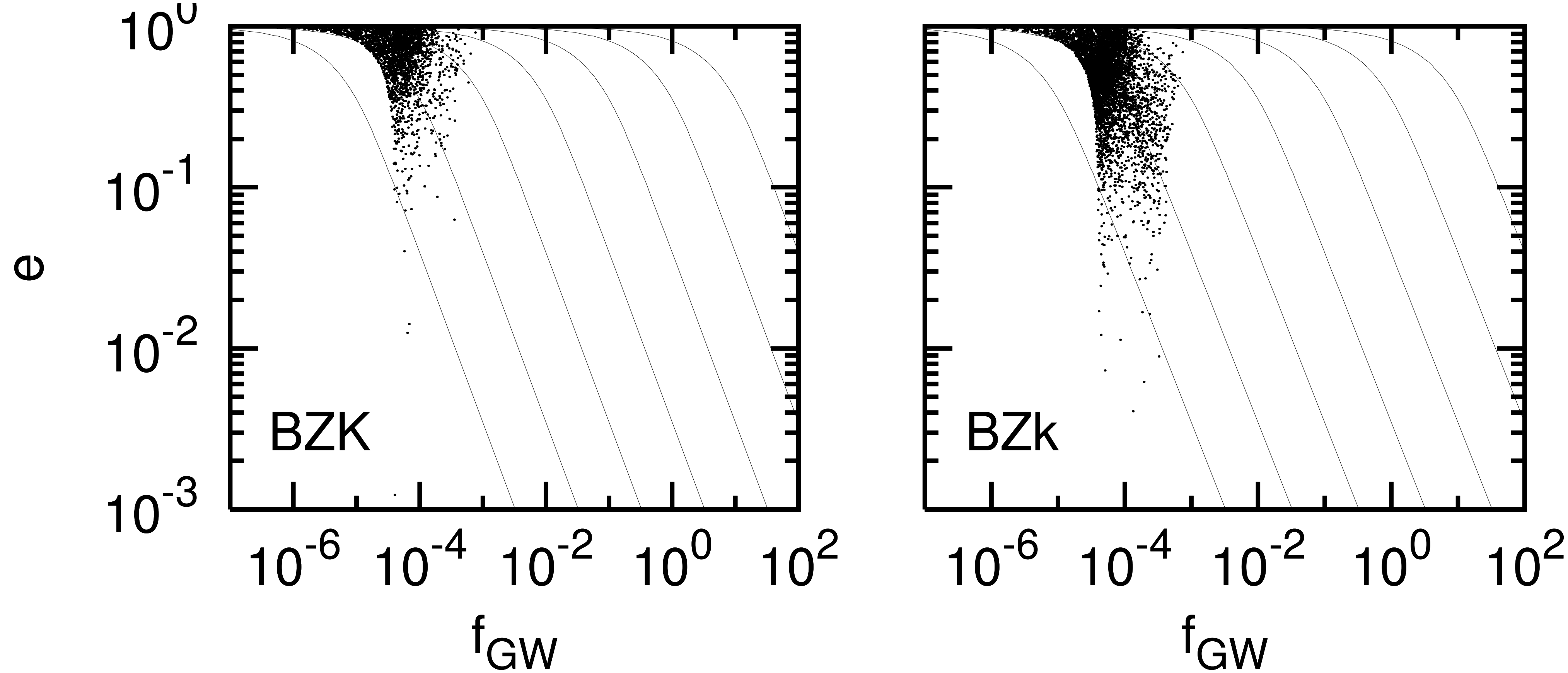
  - Enable high-precision tests [cosmology, astrophysics,nuclear matter]



ET design study

Eccentric mergers?

- GW signal "In band" : Only see what we see!
  - Eccentricity **rapidly** lost: e ~ 1/f
    - Residual eccentricity implies something exciting happened lately?
    - Binary evolution and clusters: **historically** assume low-e at merger
      - Astrophysical initial orbits ~ hours vs detection frequency ~ 100 Hz



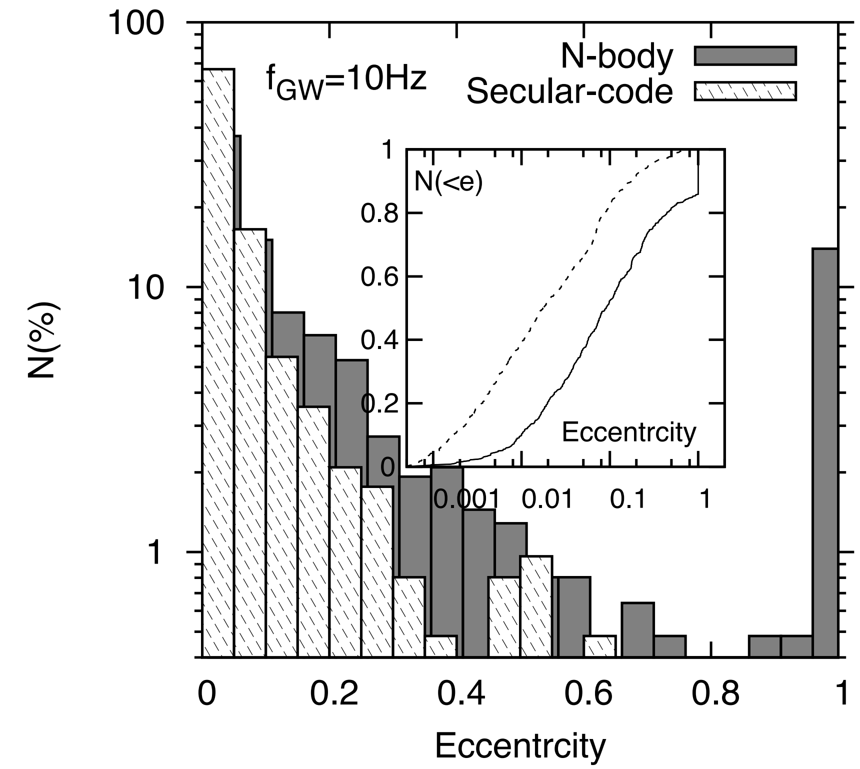
[Kowalska et al 2011]

Eccentric mergers!

- Hierarchical triples + “Kozai”

Antonini 2013; Antognini 2013; Pejcha 2013

  - Potentially **many** triples with large eccentricity at merger



Antonini 2013

- Dense environments (nuclear or GC): capture via GW
  - Binary-single: cross-section boosted by “resonances”. [Samsing] Rare but plausible (0.1%?)
  - Random spin orientations

O’Leary et al 2009; Kocsis and Levin 2012
Tsang 2013
Lee, Ramirez-Ruiz, Van den ven 2010
Samsing et al 2013

Conclusion: Vision and opportunities

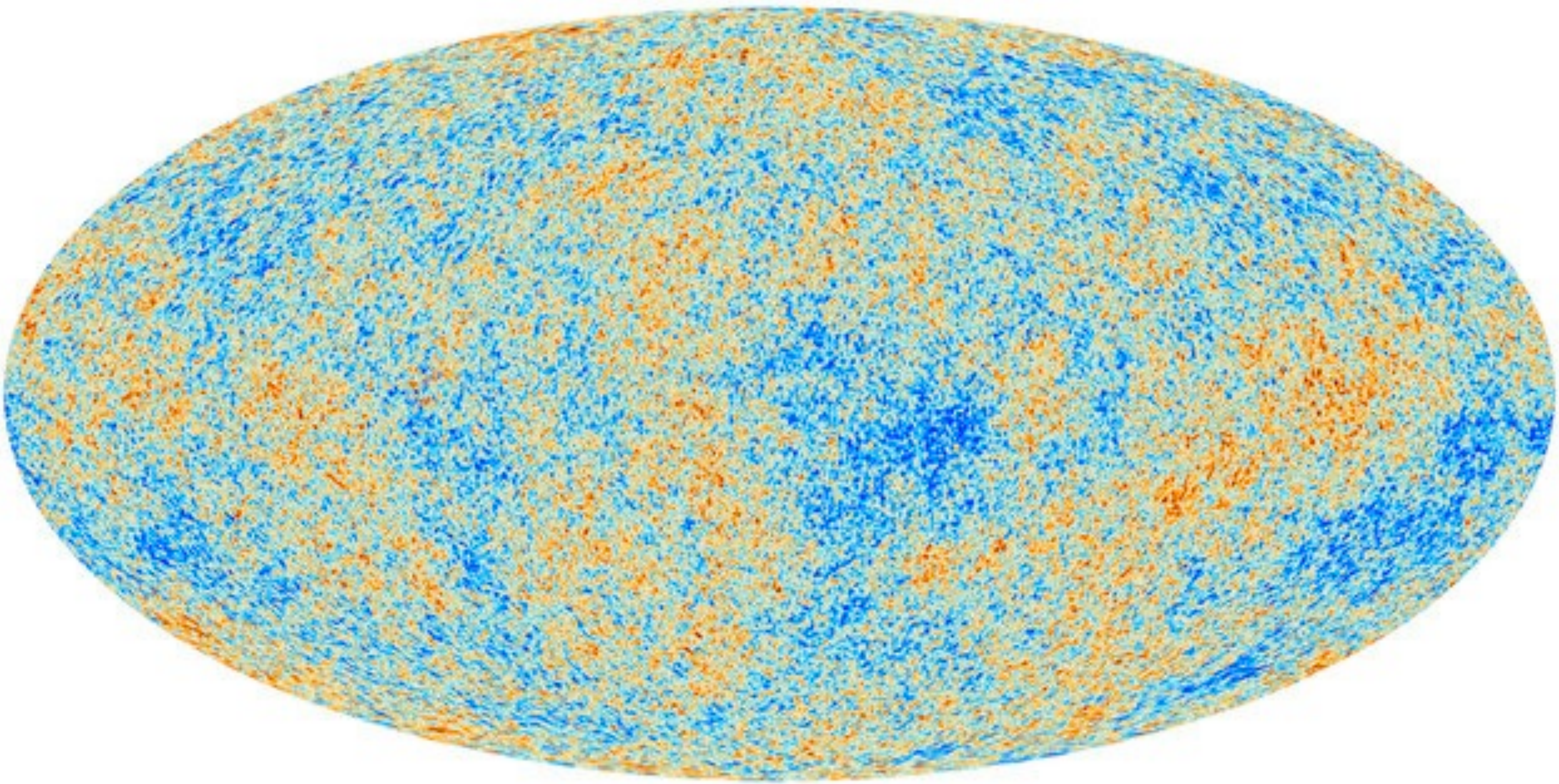
- Detections soon

- Complex signal **informative**: challenge & opportunity
  - Calibrating and applying precessing full-GR models for astrophysics
  - Unique opportunities with violent events (eccentric mergers)

- Compact binary parameter distributions
  - Constrain nearly-unconstrained astrophysics
  - Immensely powerful constraints

- Connecting GW and EM sky

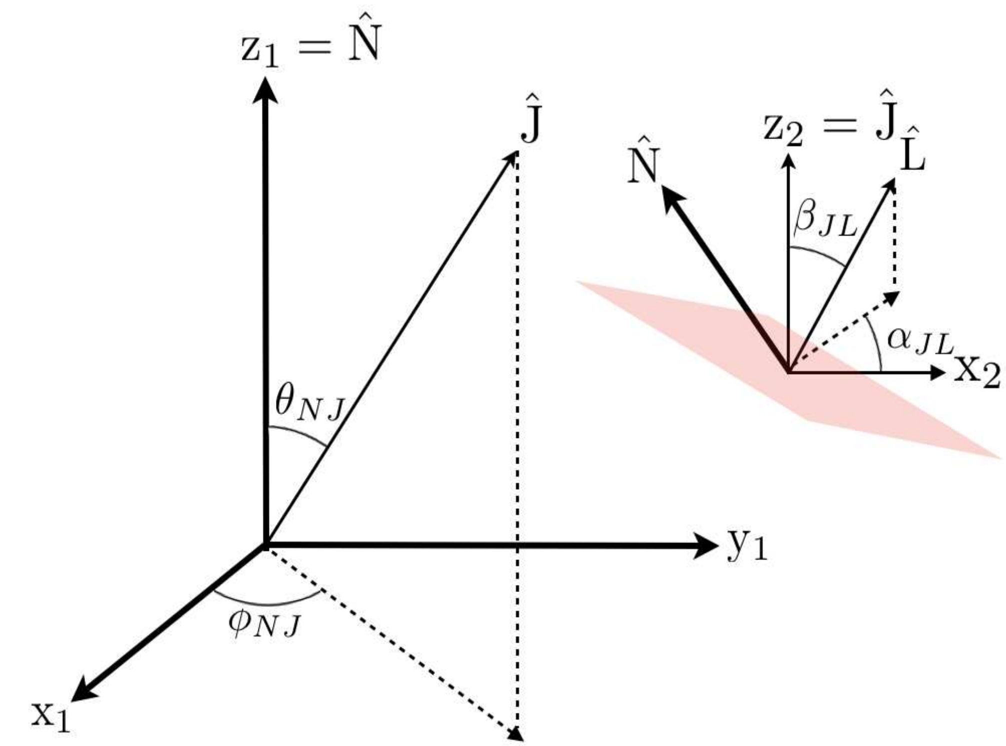
Precision measurement enables discovery

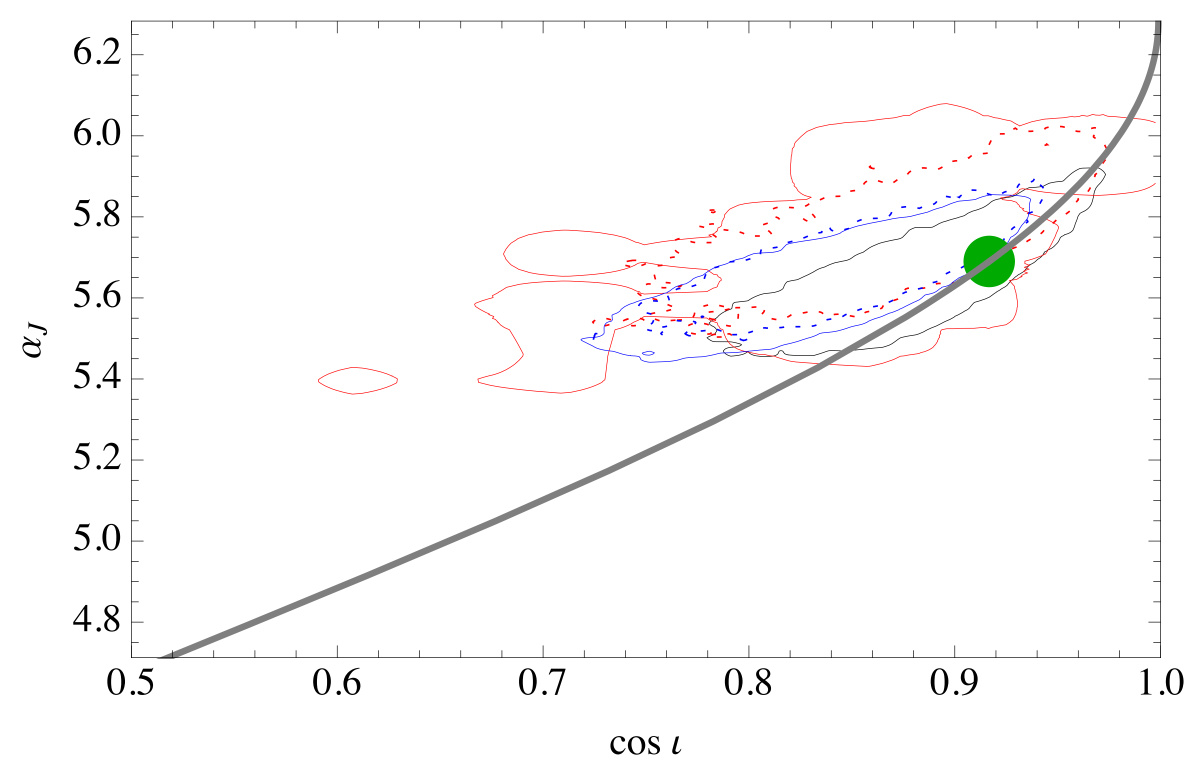


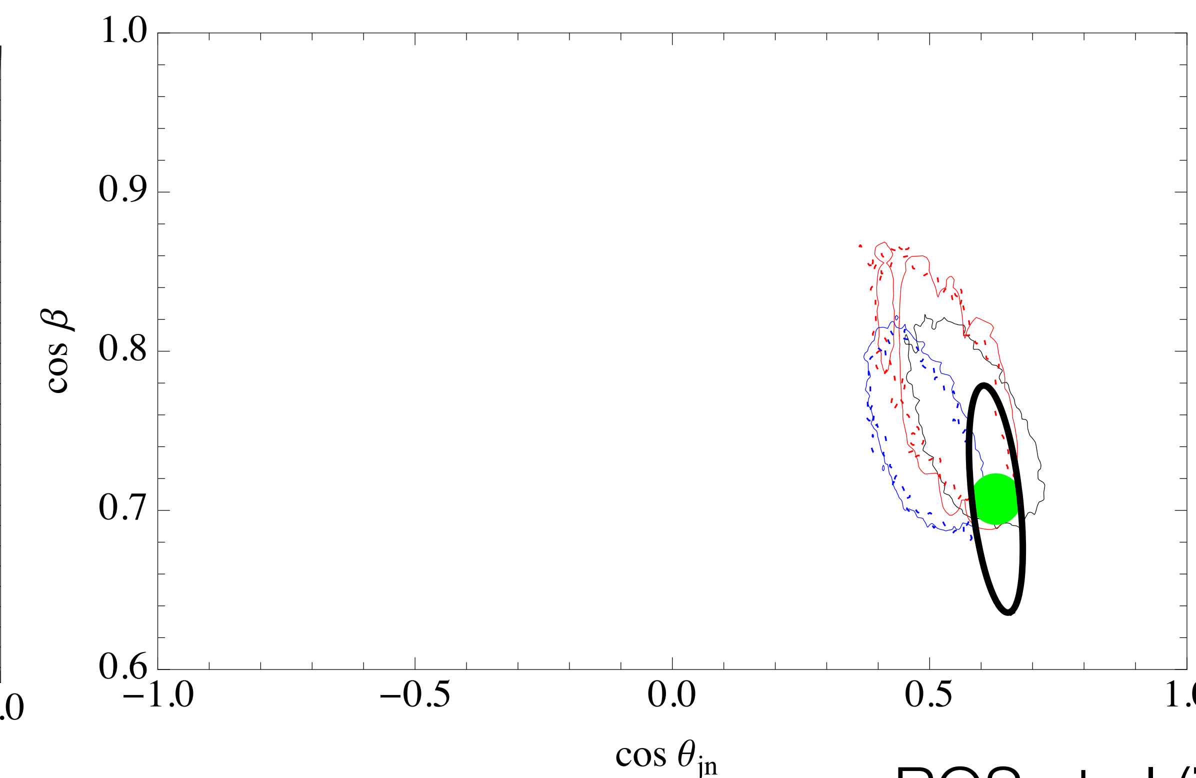
Planck, 2013

Measuring the precession cone

- Cone shape and **phase of precession** measurable
  - Application: check if EM radiation correlates with instantaneous orbital plane, versus other directions

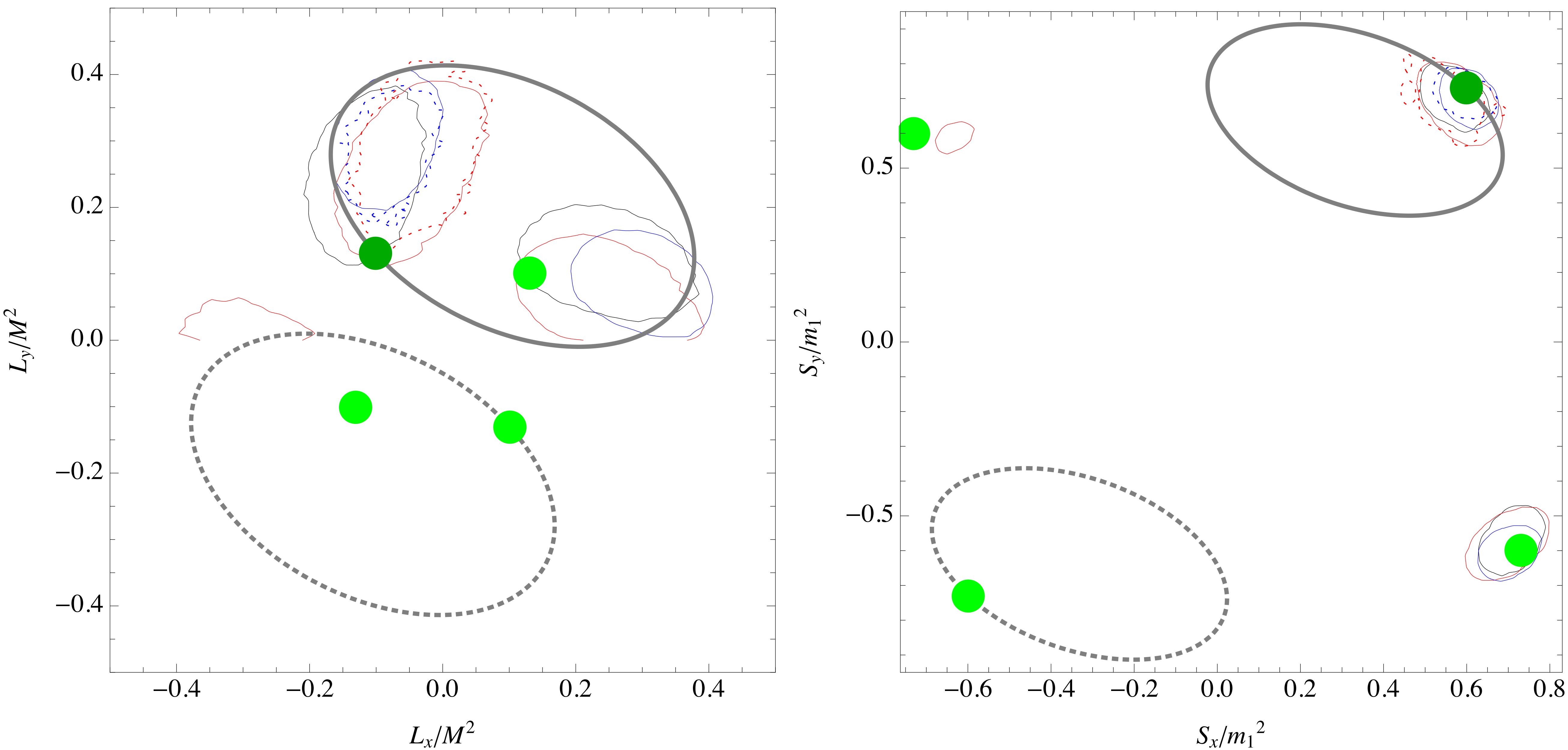






ROS et al (in prep)

Measure spin direction

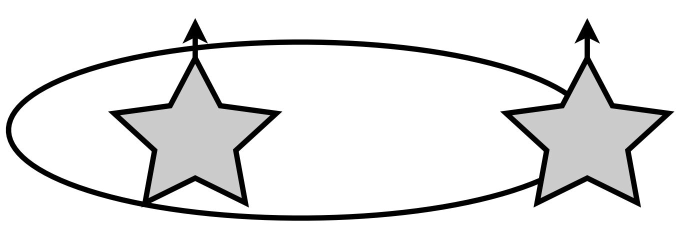


Bonus slides

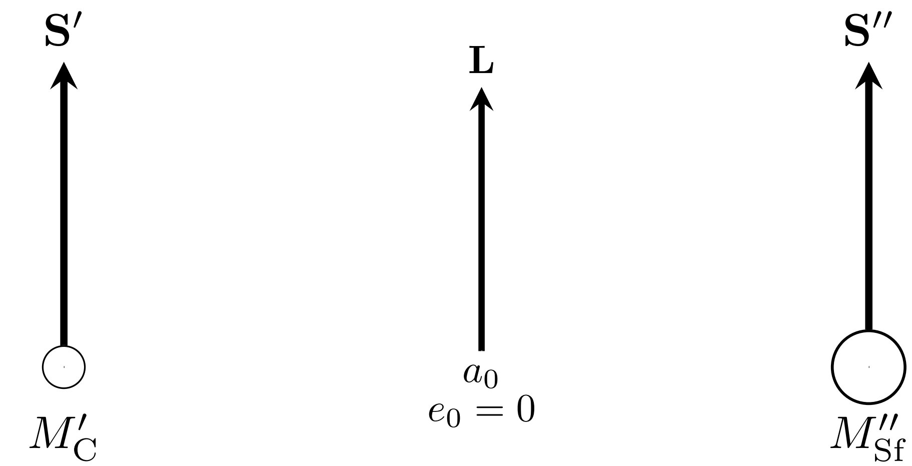
What physics will we have access to?

- Formation order?

Inertial frame



In frame of orbit



Gerosa et al 2013

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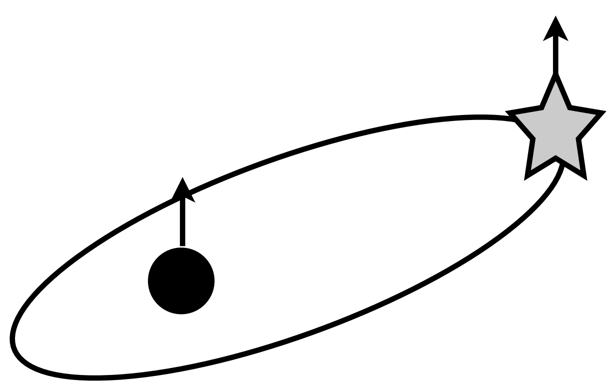
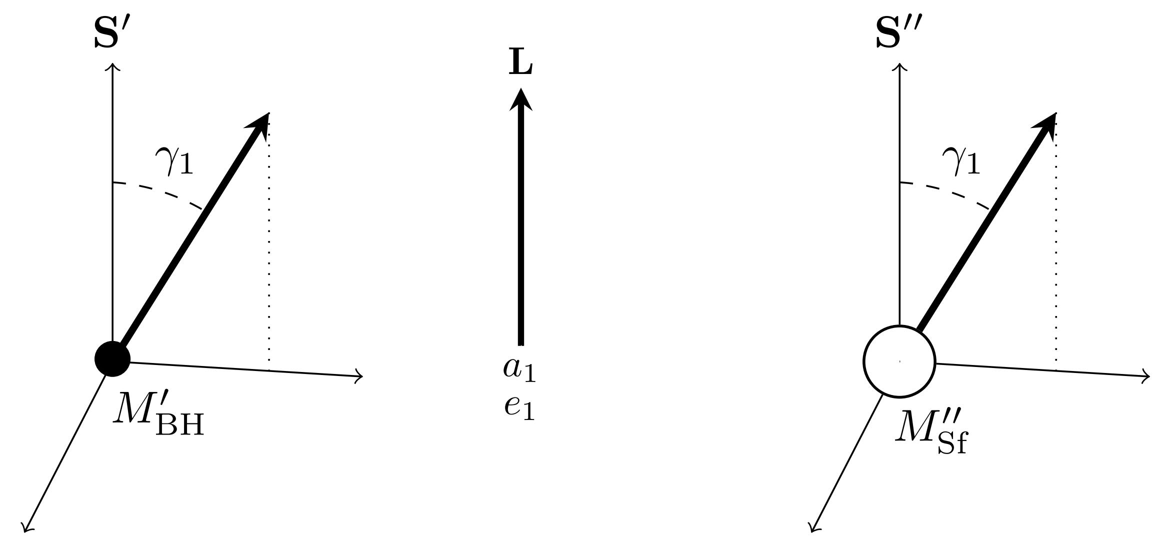
- Formation order?

Inertial frame

In frame of orbit





Gerosa et al 2013

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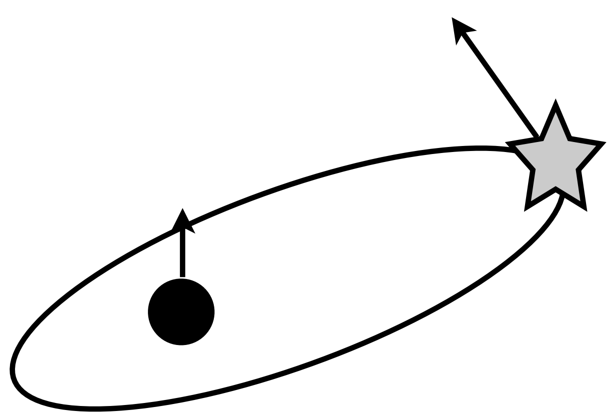
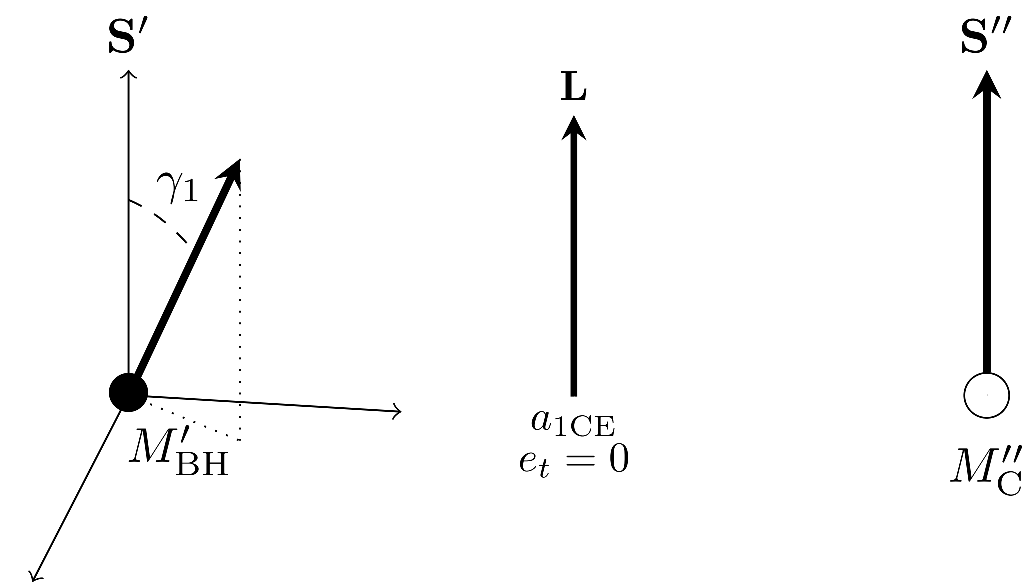
Inertial frame



In frame of orbit

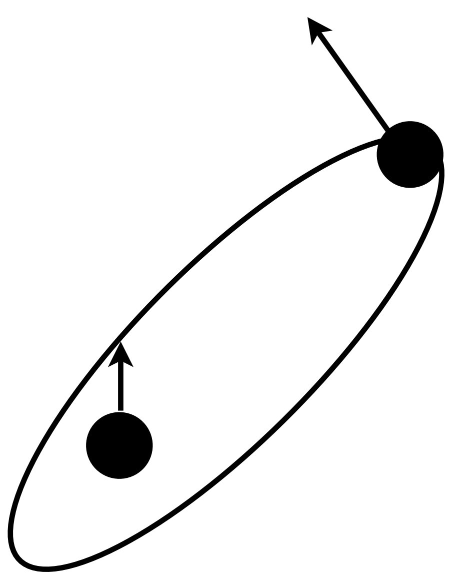


Gerosa et al 2013

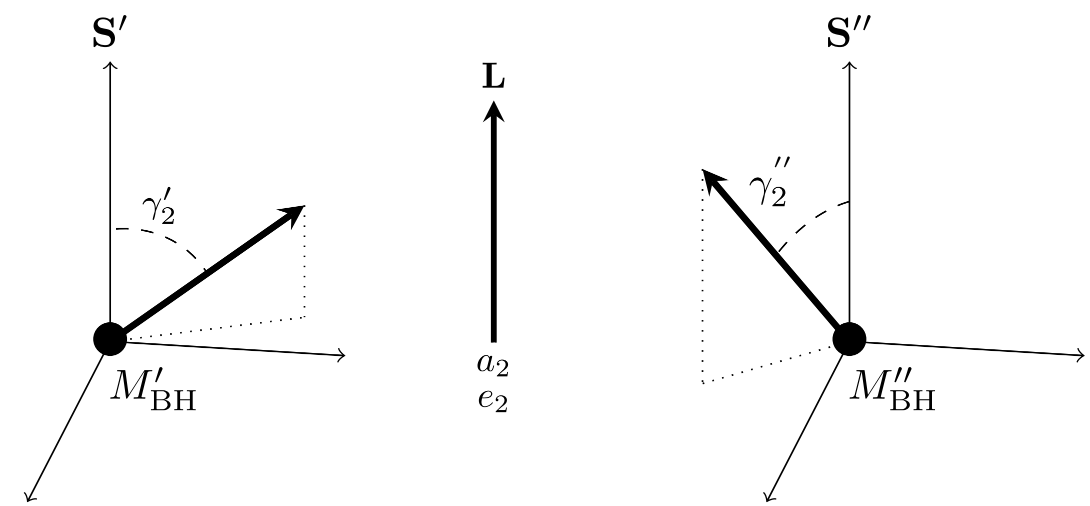
What physics will we have access to?

- Formation order?

Inertial frame



In frame of orbit



- **Key feature**:
  - first-born BH has larger misalignment [need not be most massive]
  - distinguishable gravitational waves

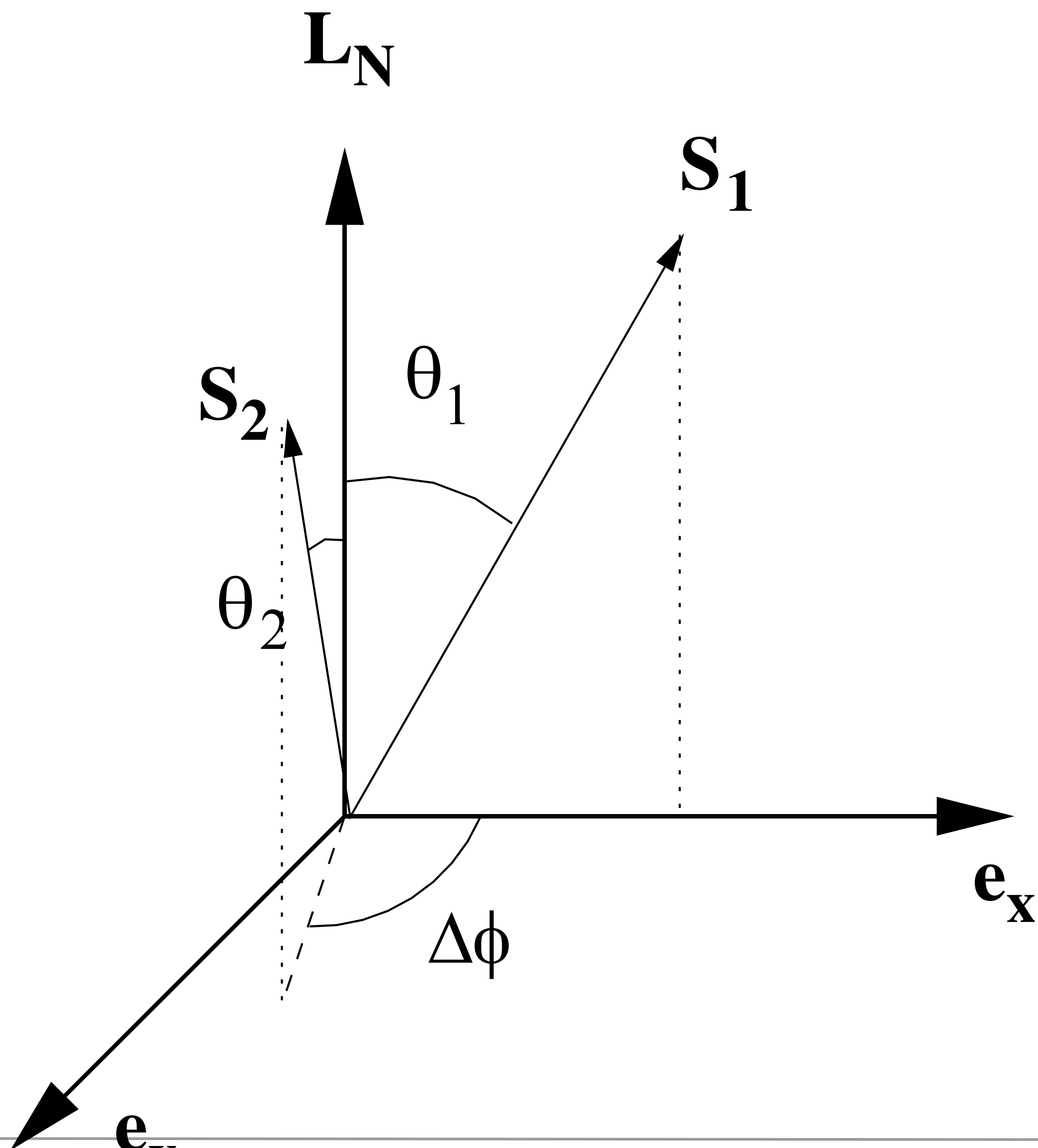
Gerosa et al in prep

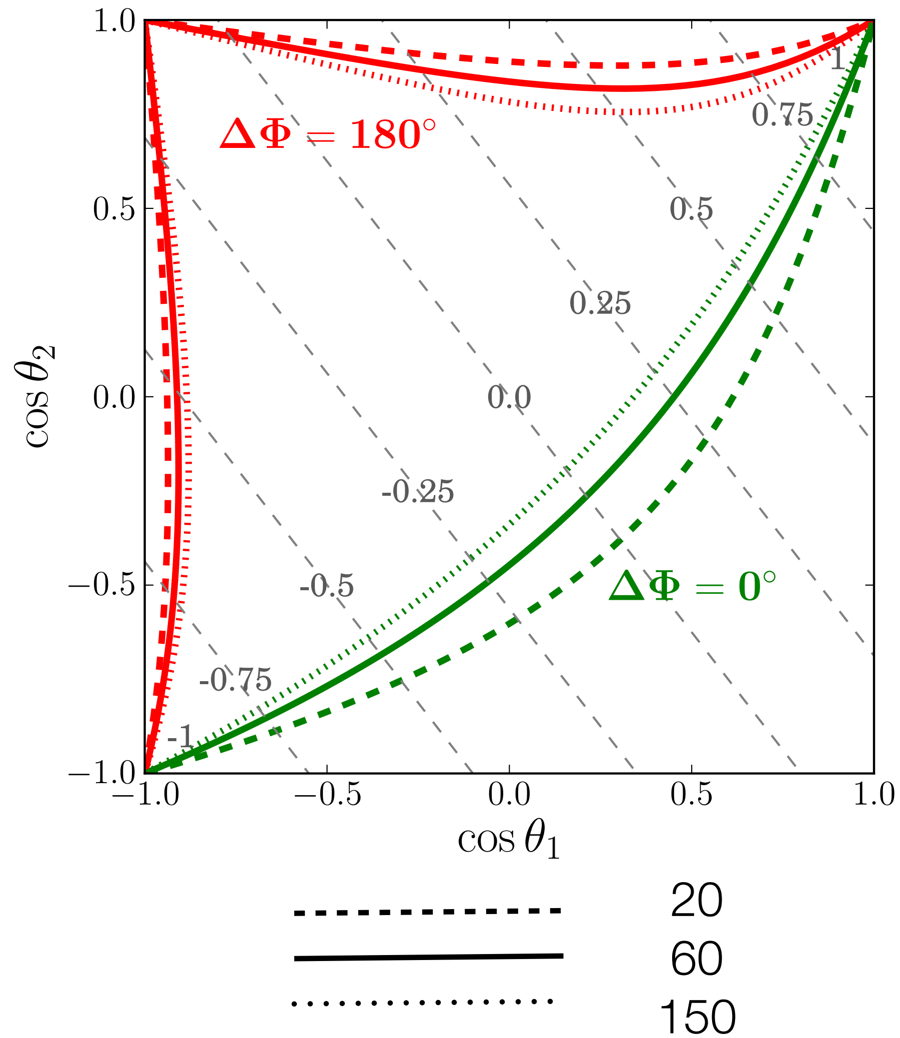
Resonances

- Spin vectors relative to L

$$\cos\theta_1 = \hat{\mathbf{S}}_1 \cdot \hat{\mathbf{L}}, \qquad \cos\theta_2 = \hat{\mathbf{S}}_2 \cdot \hat{\mathbf{L}},$$

$$\cos\theta_{12} = \hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2, \quad \cos\Delta\Phi = \frac{\hat{\mathbf{S}}_1 \times \hat{\mathbf{L}}}{|\hat{\mathbf{S}}_1 \times \hat{\mathbf{L}}|} \cdot \frac{\hat{\mathbf{S}}_2 \times \hat{\mathbf{L}}}{|\hat{\mathbf{S}}_2 \times \hat{\mathbf{L}}|}.$$

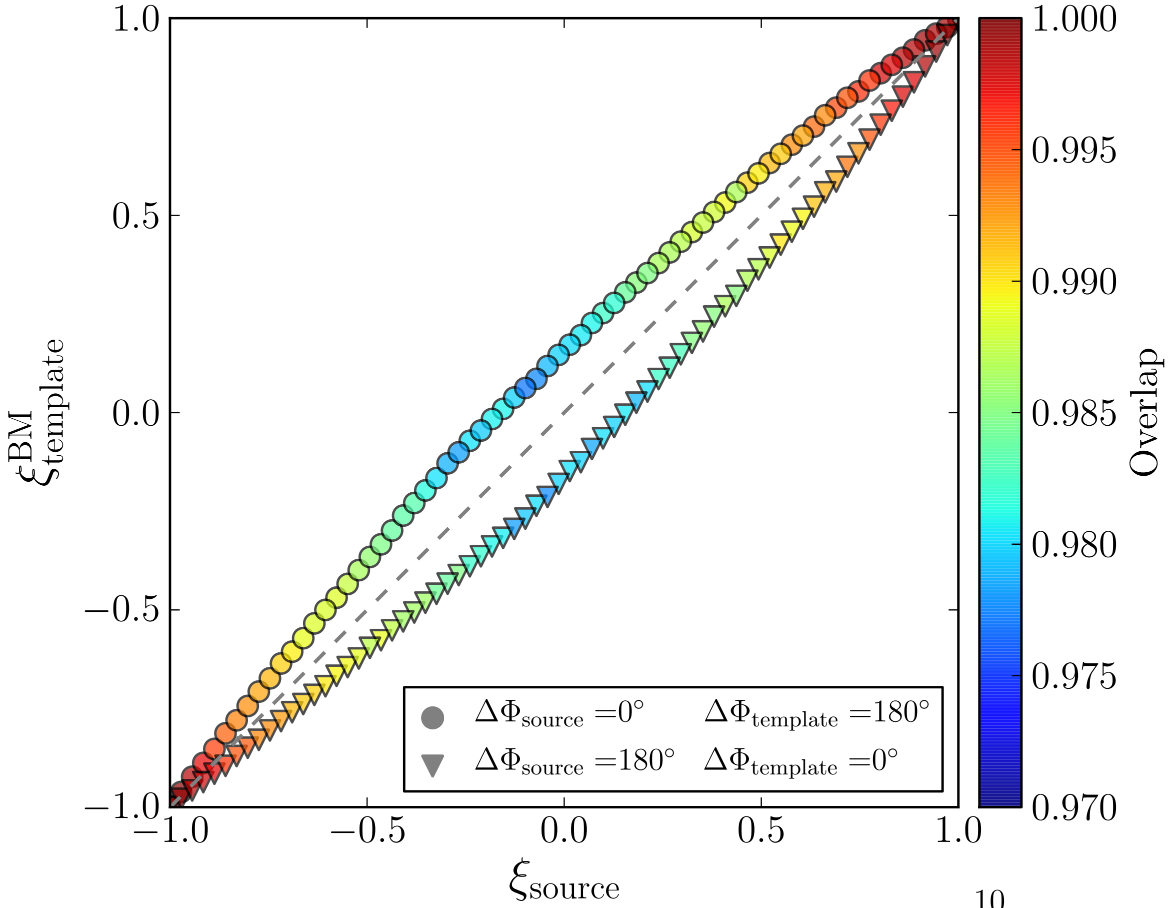


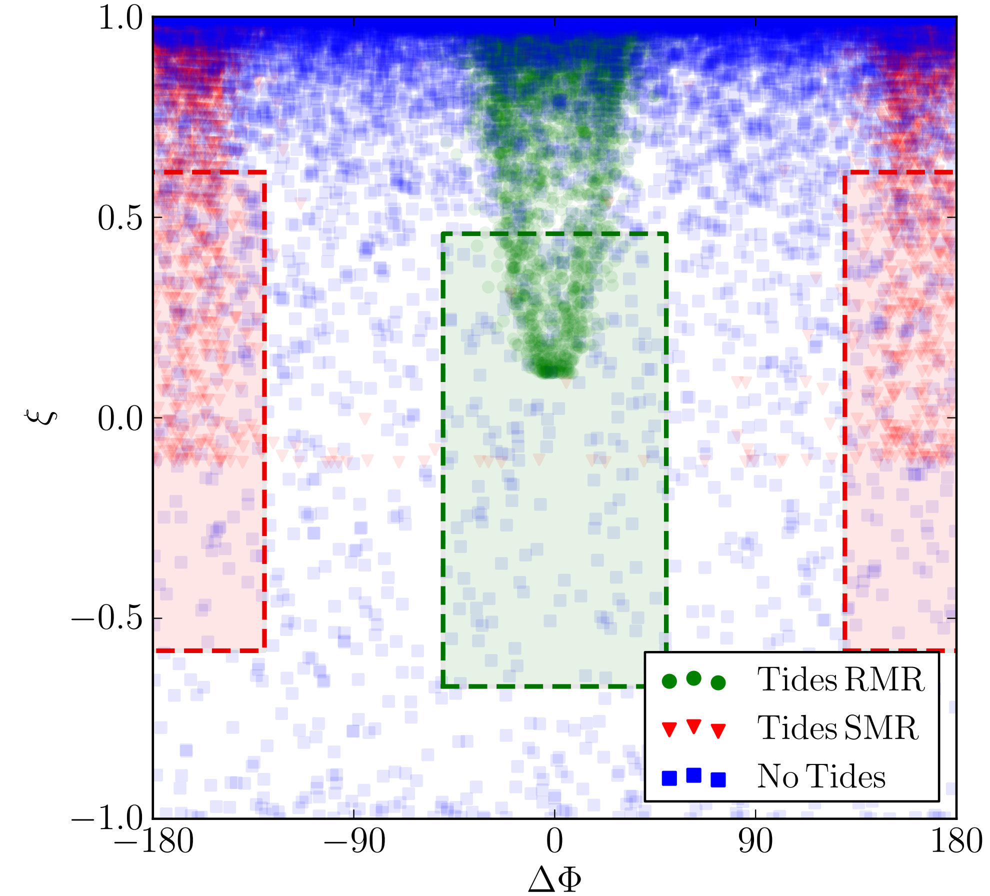


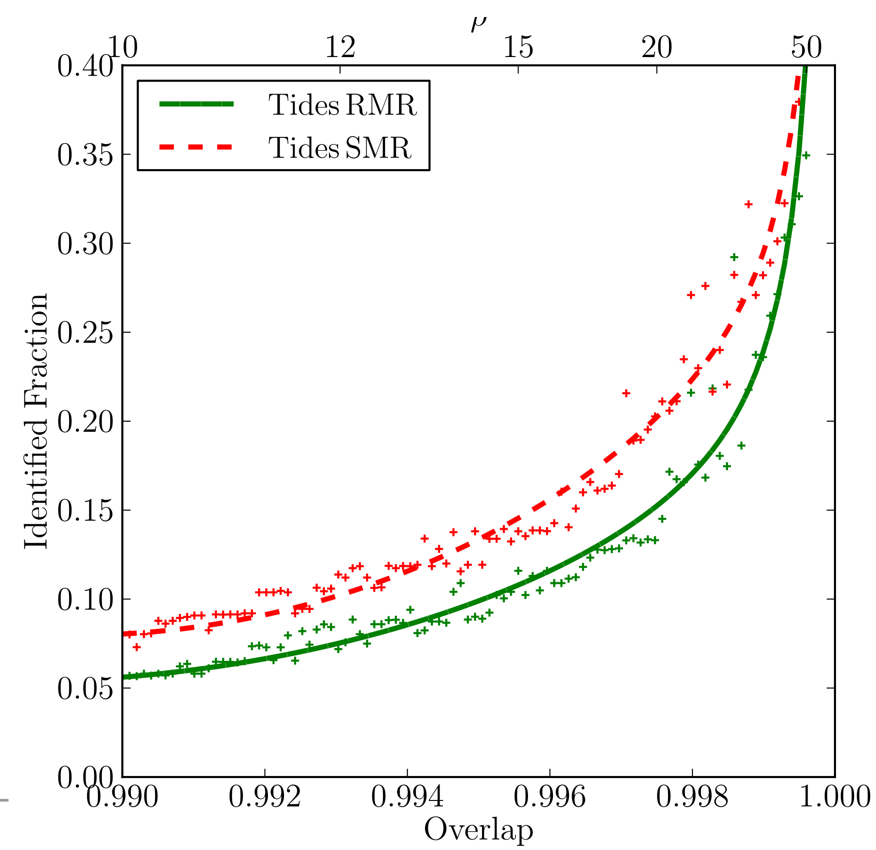
$$\mathbf{S_0} = (1+q)\mathbf{S_1} + \left(1 + \frac{1}{q}\right)\mathbf{S_2}$$

$$\xi \equiv \frac{\mathbf{S_0} \cdot \hat{\mathbf{L}}}{M^2}$$

Fraternal twins are distinguishable







Student project opportunities

- Data analysis and parameter estimation
  - Scale up alternative algorithm
  - Detection algorithm for precessing binaries

- Strong-field waveforms
  - Modeling effort in NR/strong GR (Berti, SXS, GT)
  - New DOF: Eccentricity

- Tidal disruption and EM counterparts
  - Where are the missing short GRBs at high redshift?
  - Coincident parameter measurements: EM & GW and short GRBs [Farr++...]
  - Mining host galaxy correlations

- Astrophysics
  - Compare O(1000) existing model predictions
  - Limit of many detections? ("Analytic" fisher matrix)
  - Astrophysics ("dirty mergers"; predicting final BH spins)

Interpreting stellar mass binary mergers

- General relativity
  - Precessing IMR models and systematics
  - “**Inverse problem**”: dynamics, given waveform
  - Investigate robust features (attractors/cycles)

- Parameter estimation
  - Attention to detail: convergence, systematics
  - **Provable results**: circumvent ergodicity/non-MCMC
  - **Low latency**: efficient computation

- Astrophysics
  - Controlled, useful exploration of model space
  - EM+GW: Host galaxy correlations

When will we detect gravitational waves

- Order of magnitude estimate : NS-NS detection soon!

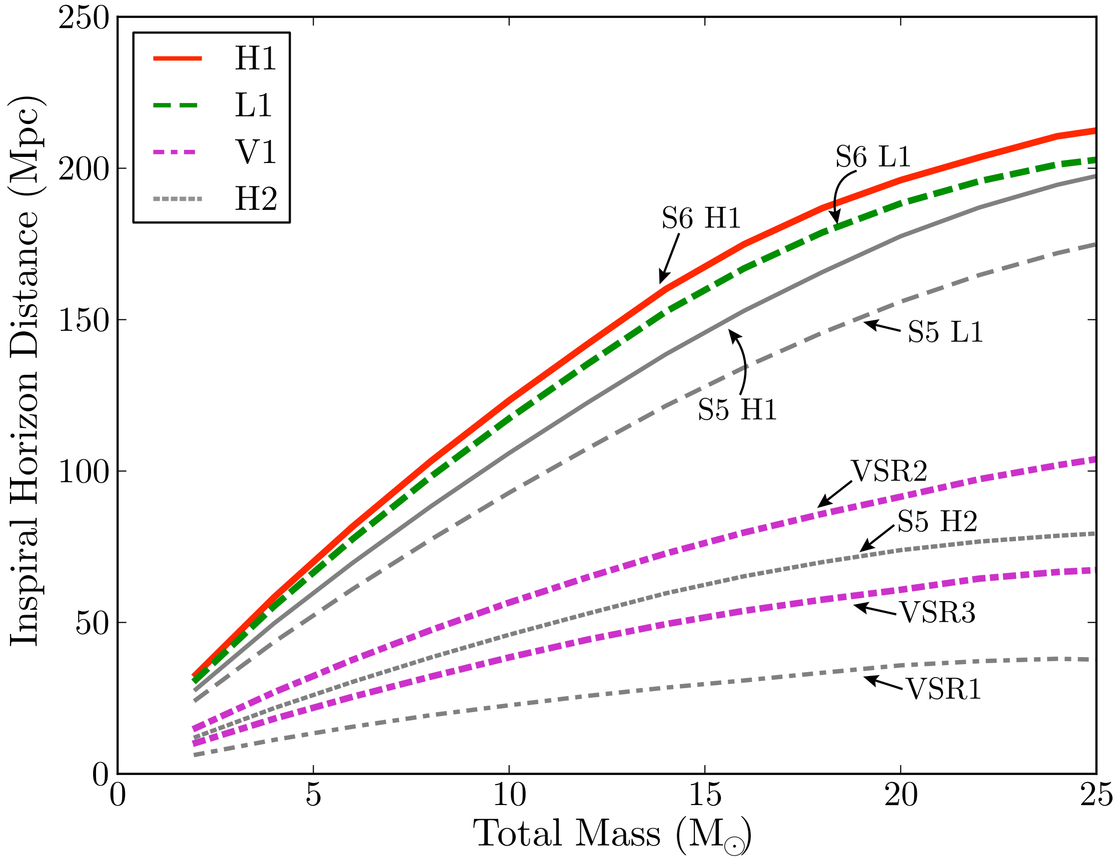
[Abadie et al 2010, arXiv:1003.2480]

- **Galactic NS-NS:** > few mergers/Myr
- **LIGO range**: ~ 200 Mpc [for NS-NS] [ROS & Kim 2010,ApJ 715 230]
- **Galaxy density**: 1/100 Mpc^3
- **Detection rate:** > few/yr

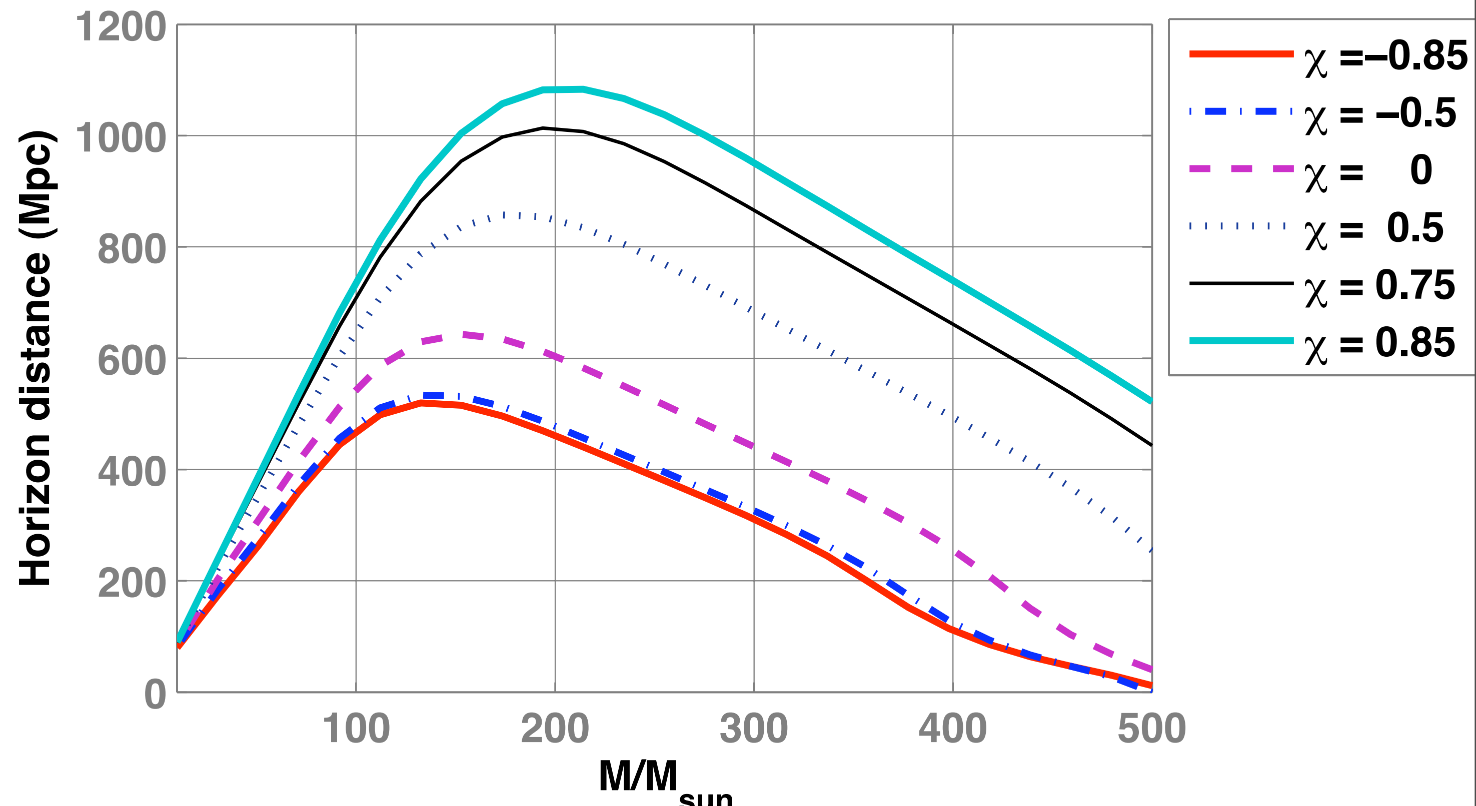
- Other compact object binaries (=NS or black hole) too!

What will GW detectors find?

- Gravitational wave detectors sensitive to source mass (& spin)
  - Low mass: (horizon) range ~ mass $^{5/6}$



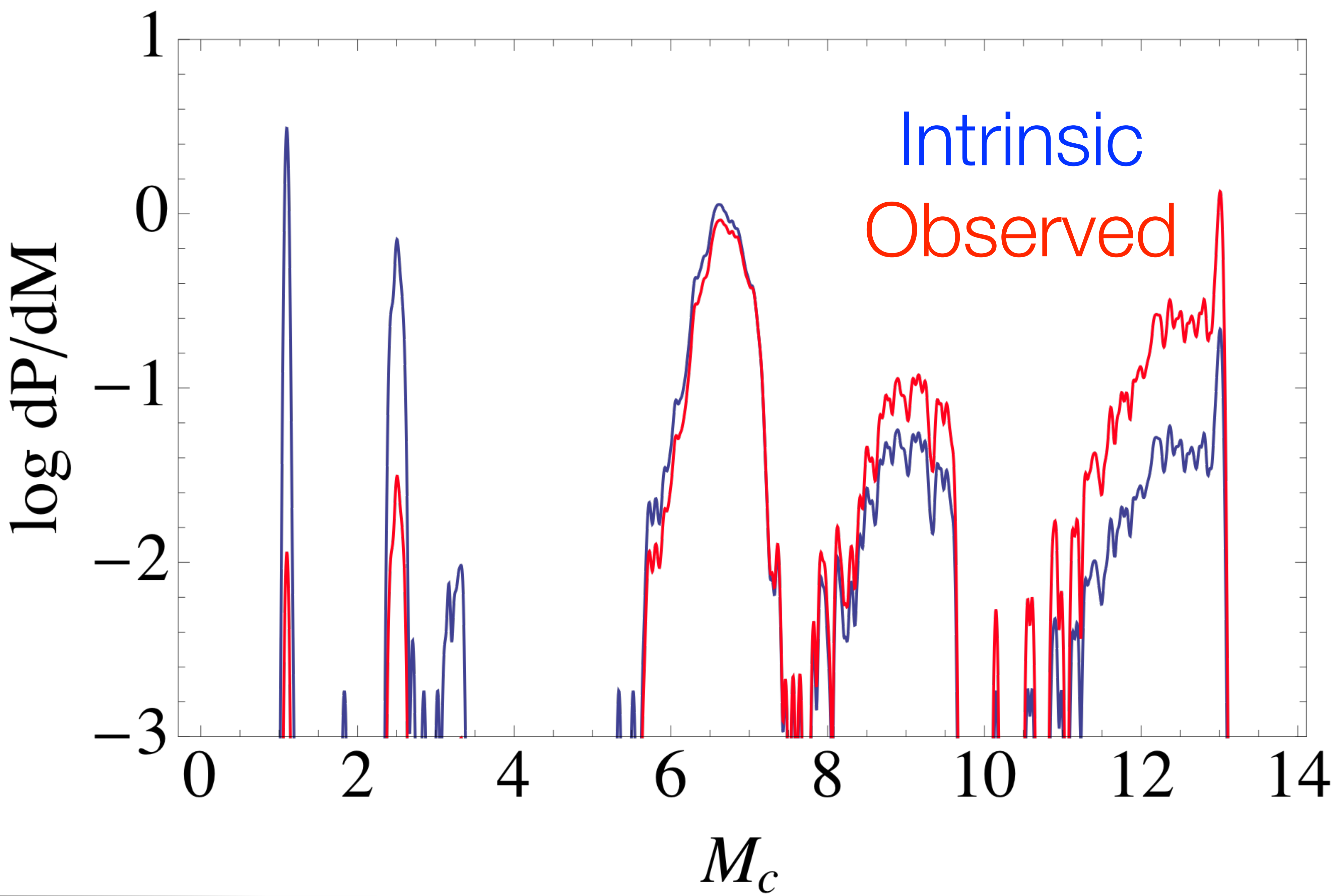
Abadie et al, PRD [arxiv:1111.7314]



Ajith et al [arxiv:0909.2867]

What will GW detectors find?

- Detection **strongly** favors large masses (+BH spin)
- Be careful: include rare, massive events!



What will we see?

- How many?
  - Star formation rate + efficiency
  - Insert known range vs mass

$$\frac{dN}{dt} \simeq 1 M_\odot/yr \times \frac{10^{-2}}{\mathrm{Mpc}^3}\frac{4\pi}{3}\left\langle\left(\frac{D_H(M)}{2.26}\right)^3\right\rangle \frac{P_{form}}{m_{av,*}}$$

$$\simeq 3.6\times 10^4 M_\odot/yr \frac{P_{\rm form}}{m_{av,*}}(D_{bns}/100\mathrm{Mpc})^3\left\langle\left(\frac{M}{M_{bns}}\right)^{15/6}\right\rangle$$

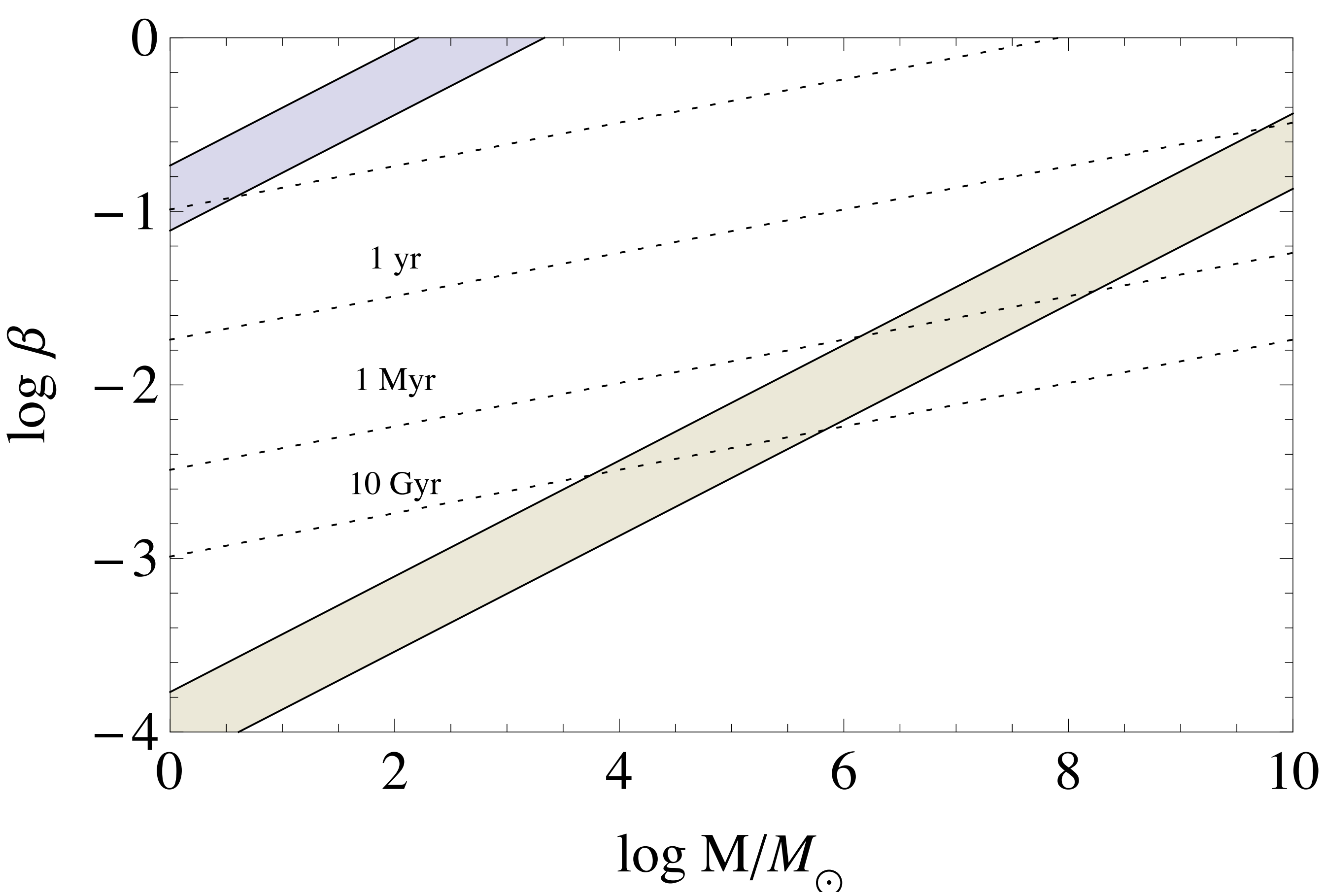
Key factors for theorists

Eccentric mergers and dynamical scenarios

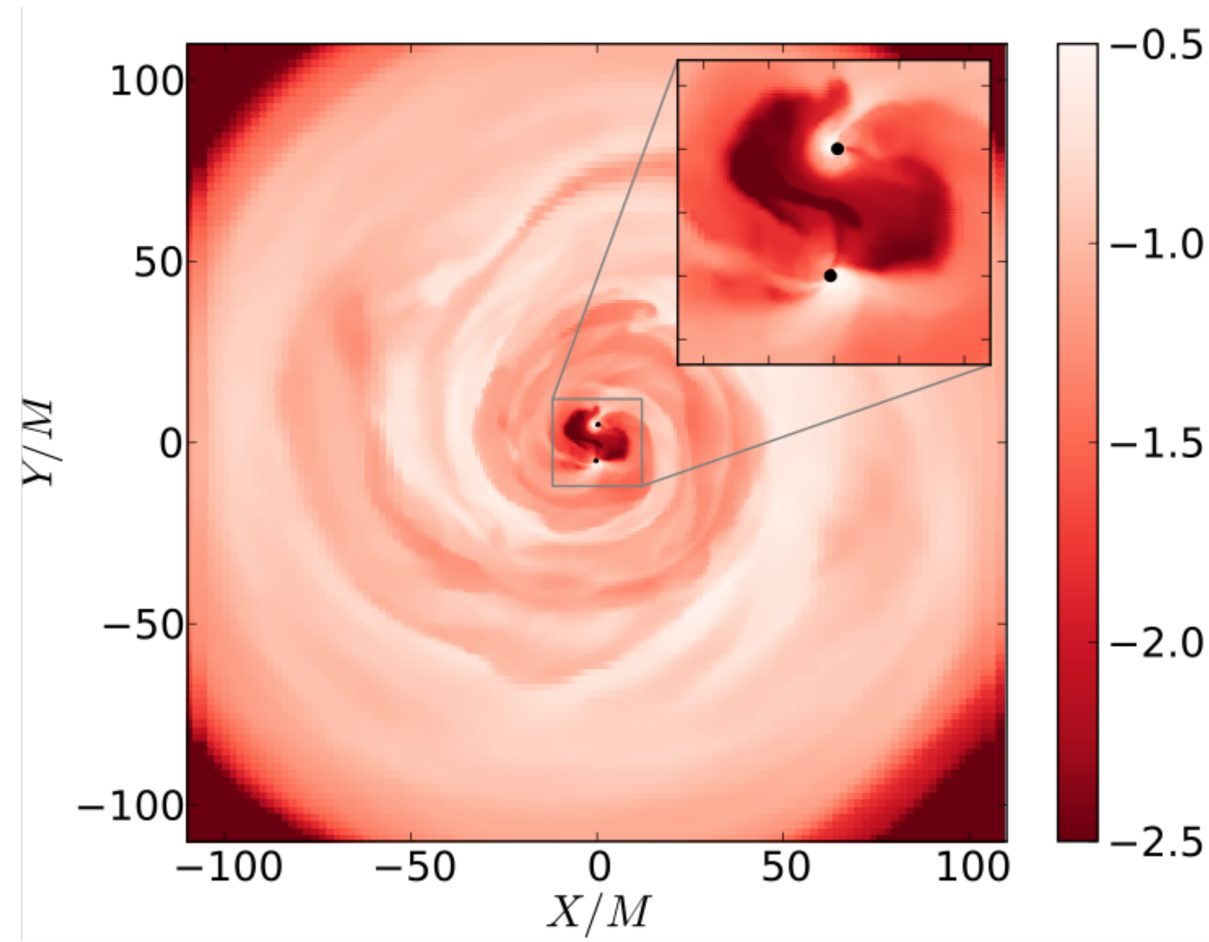
- General relativity and parameter estimation
  - Hamiltonian dynamics [point particle; multi-encounter models with matter]
  - Robust non-quasicircular waveforms at moderate and low eccentricity
- Parameter estimation
  - Structurally different algorithms + insight
- Astrophysics
  - Formation scenarios; “inverse problem” (dynamical vs not)

Supermassive black hole binaries and pulsar timing

Inspiral timescale

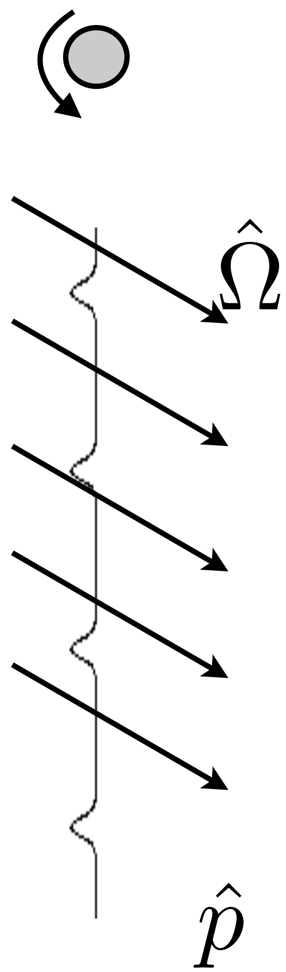


Binary environment?



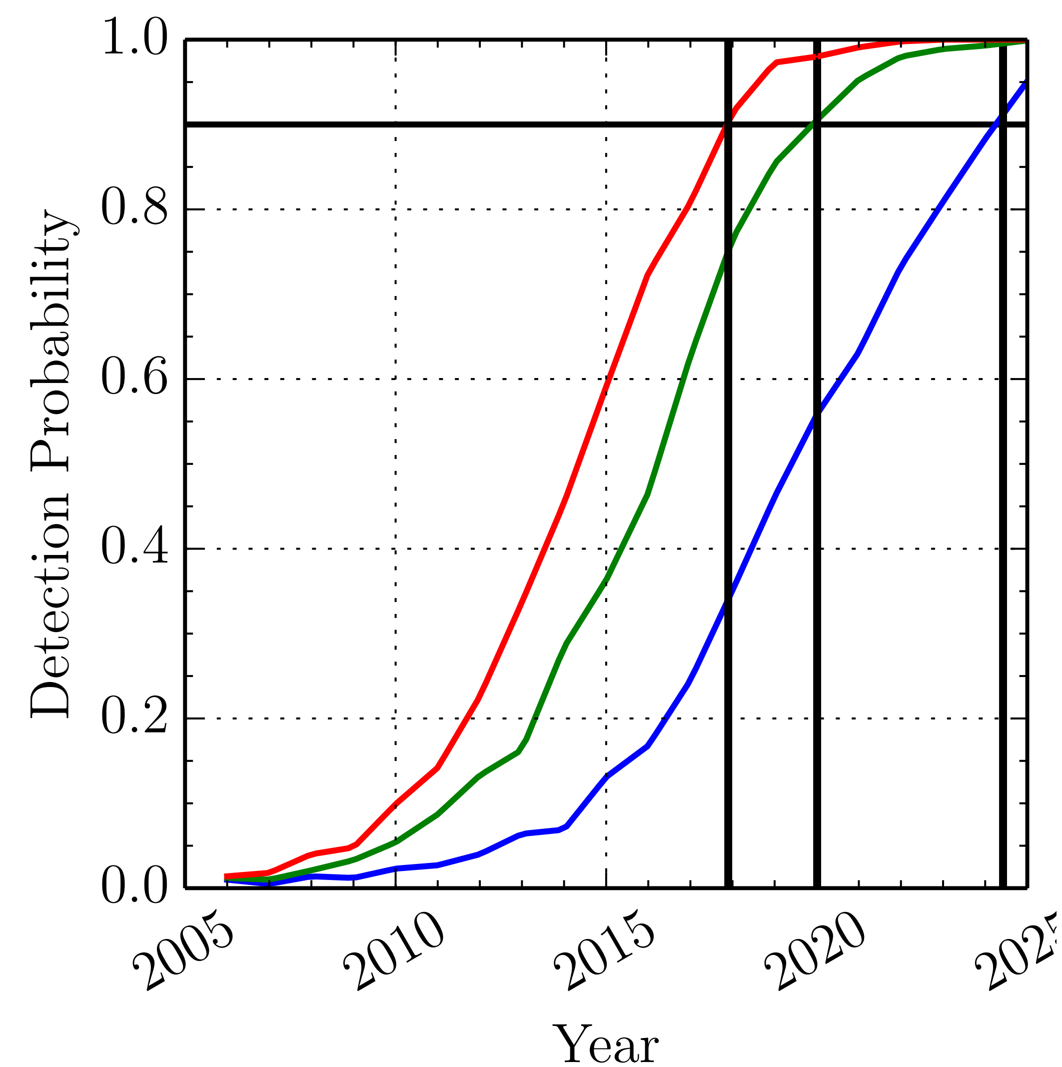
Modulated EM signal?

Pulsar timing



$$z = \frac{1}{2}\frac{\hat{p}^i\hat{p}^j}{1+\hat{\Omega}\cdot\hat{p}}\left[h_{ij}^{\mathrm{E}} - h_{ij}^{\mathrm{P}}\right]$$

Earth Term   Pulsar term



Stochastic background detection
Siemens 2013, Midwest Relativity Meeting

Research program

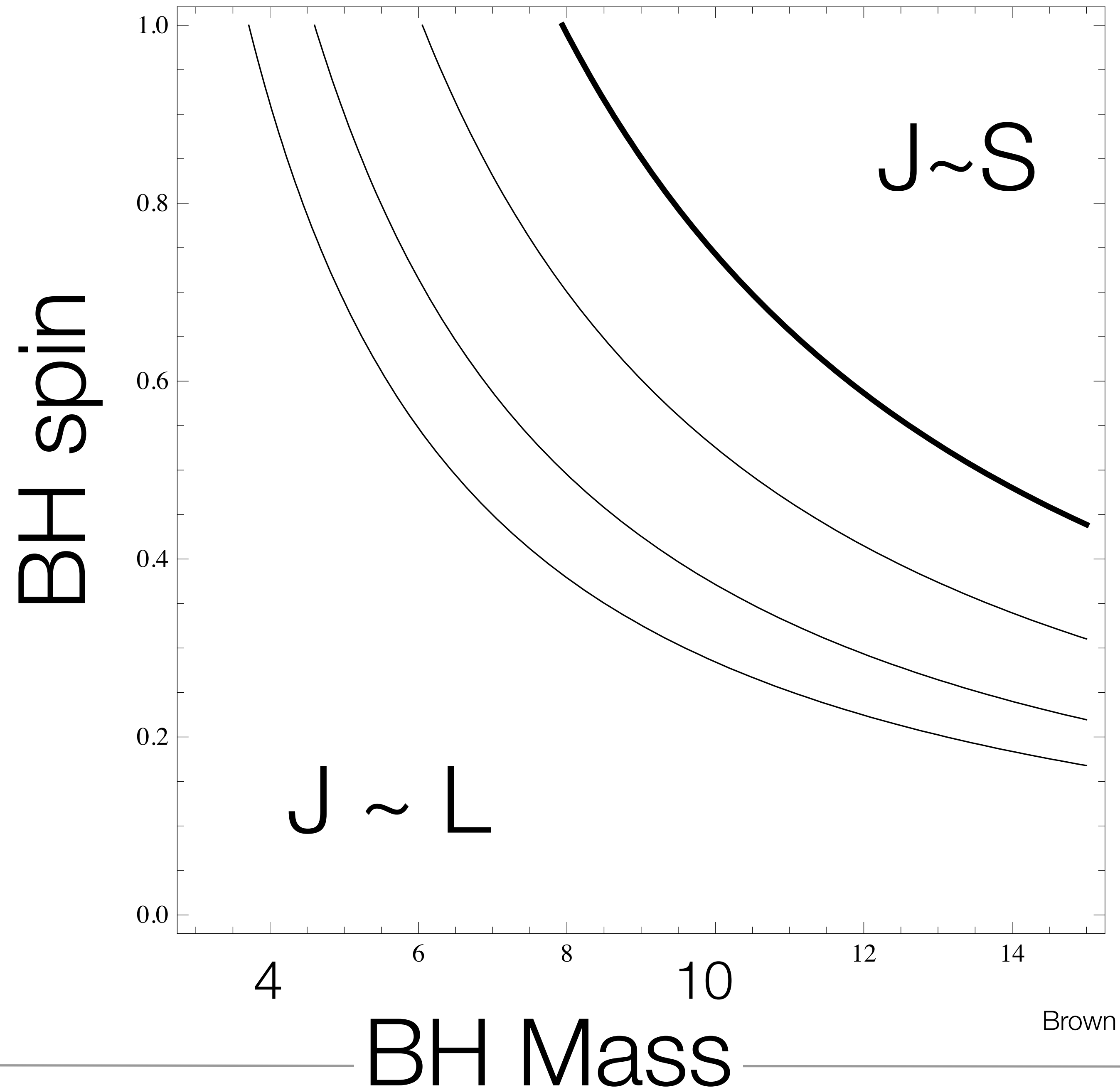
- Supermassive black holes
  - Science:
    - Search pipelines and parameter constraints
    - Targeted astrophysics using posterior of all data (fore- and back-ground)
    - EM counterparts
  - Rationale:
    - Vast interest (galaxy evolution; how structure forms; black holes)
    - Multiband accessibility: GW [PTA & eventually LISA] & EM
  - Plan:
    - GW:
      - Existing algorithms (PTA: Ellis/Siemens; ...) with astro priors/insight
      - Tailor LIGO experience to unique PTA challenges
    - GW+EM
      - Build on prior & RIT experience

BONUS SLIDES

How do **precessing** binaries evolve?

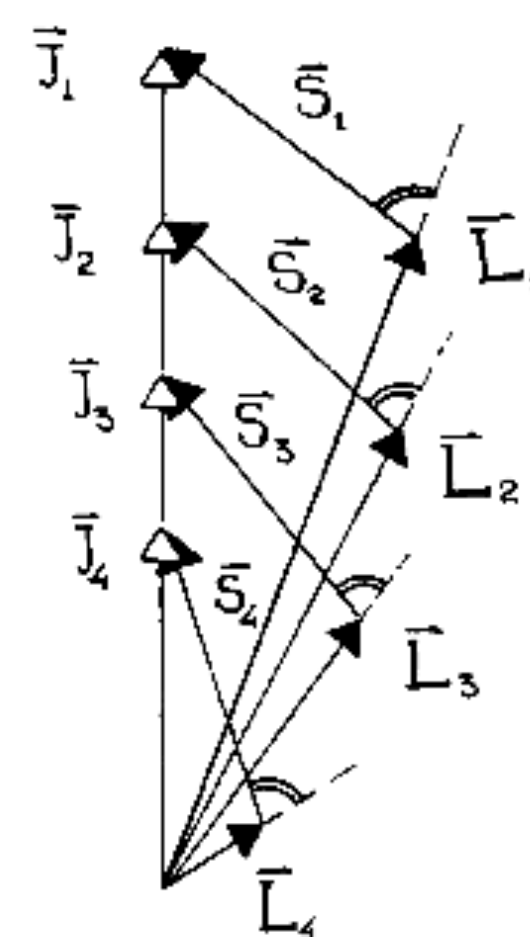
- Orbit shrinks
  - Spin angular momentum more significant
  - L<S easier if unequal mass or high spin

$$J = L + S_1 + S_2$$



Brown et al 2012

Example: BH-NS (40 Hz)



Apostolatos et al 1994

A helpful way to represent the radiation

- Want $\tilde{h}(f|\lambda)$ and $\dfrac{d\tilde{h}(f|\lambda)}{d\lambda}$

- Stationary-phase ("F2"): **example calculation**

$$h(t) = {}^{(-2)}Y_{2,2}(\hat{n})h_{2,2}(t) + {}^{(-2)}Y_{2,-2}(\hat{n})h_{2,-2}(t)$$

$$h_{2,2}(t) \propto \frac{\eta v^2}{d_L} e^{-2i\Phi(t)}$$

$$\tilde{h}_{2,2}(\omega) \propto \frac{\eta v^2}{d_L}\frac{1}{\sqrt{id^2\Phi/dt^2/\pi}} e^{i\Psi(\omega)}$$

$\Psi(\omega)$: power series in v
set by stationary phase condition
(=Legendre transform of Φ)

$$\Psi = \omega t - 2\Phi$$

$$\frac{d\Psi}{d\omega} = t \qquad \frac{dt}{d\omega} = \frac{dt}{dv}\frac{dv}{d\omega}$$

Representing GW from precessing binaries

- Time-domain signal

$$\begin{aligned} h_+(t) - i h_\times(t) &= e^{-2i\psi} \sum_{lm} h_{lm}(t)\ {}_{-2}Y_{l,m}(\theta,\phi) \\ &= e^{-2i\psi} \sum_{lm'} \sum_{m} \underbrace{D^l_{m',m}(\alpha(t),\beta(t),\zeta(t))}_{R(t)} h^{\mathrm{ROT}}_{l,m}(t)_{-2}Y_{l,m'}(\theta,\phi) \end{aligned}$$

- Fourier-transform term-by-term

$$\begin{aligned} X(t) &\equiv D^l_{m',2}(R(t)) \times \frac{\eta v^2}{d_L} e^{-i2\Phi(t)} \times {}^{(-2)}Y_{l,m'}(\theta,\phi) \\ \tilde{X}(\omega) &\simeq D^l_{m',2}(R(t(\omega))) \times \frac{\eta v^2}{d_L} \frac{e^{i\Psi(\omega)}}{\sqrt{i d^2\Phi/dt^2/\pi}} \times {}^{(-2)}Y_{l,m'}(\theta,\phi) \end{aligned}$$

- Regroup terms: “carrier+sideband” [restricted to (l,m)=(2,2) + (2,-2)]

What's in the paper

- Time domain form

$$h_+ = \frac{2M\eta}{D} v^2 \mathrm{Re}\left[\sum_m z_m e^{im\alpha} e^{2i(\Phi-\zeta)}\right]$$

$$z_m = {}_{-2}Y_{2,m}(\beta,0)\frac{4\pi}{5}\left[e^{-2i\psi}\ {}_{-2}Y_{2m}(\theta,0) + e^{2i\psi}\ {}_{-2}Y_{2-m}(\theta,0)\right] .$$

- Kinematics

$$\gamma \equiv \frac{|\mathbf{S}_1|}{|\mathbf{L}|} = \left(\frac{m_1\chi}{m_2}\right) v \ ;$$

$$\Gamma_J \equiv |\mathbf{J}|/|\mathbf{L}| = \sqrt{1 + 2\kappa\gamma + \gamma^2} \ .$$

- Precession angles

$$\alpha(v) = \eta\left(2 + \frac{3m_2}{2m_1}\right)\int v^5\ \Gamma_J \left(\frac{dt}{dv}\right)\mathrm{d}v$$

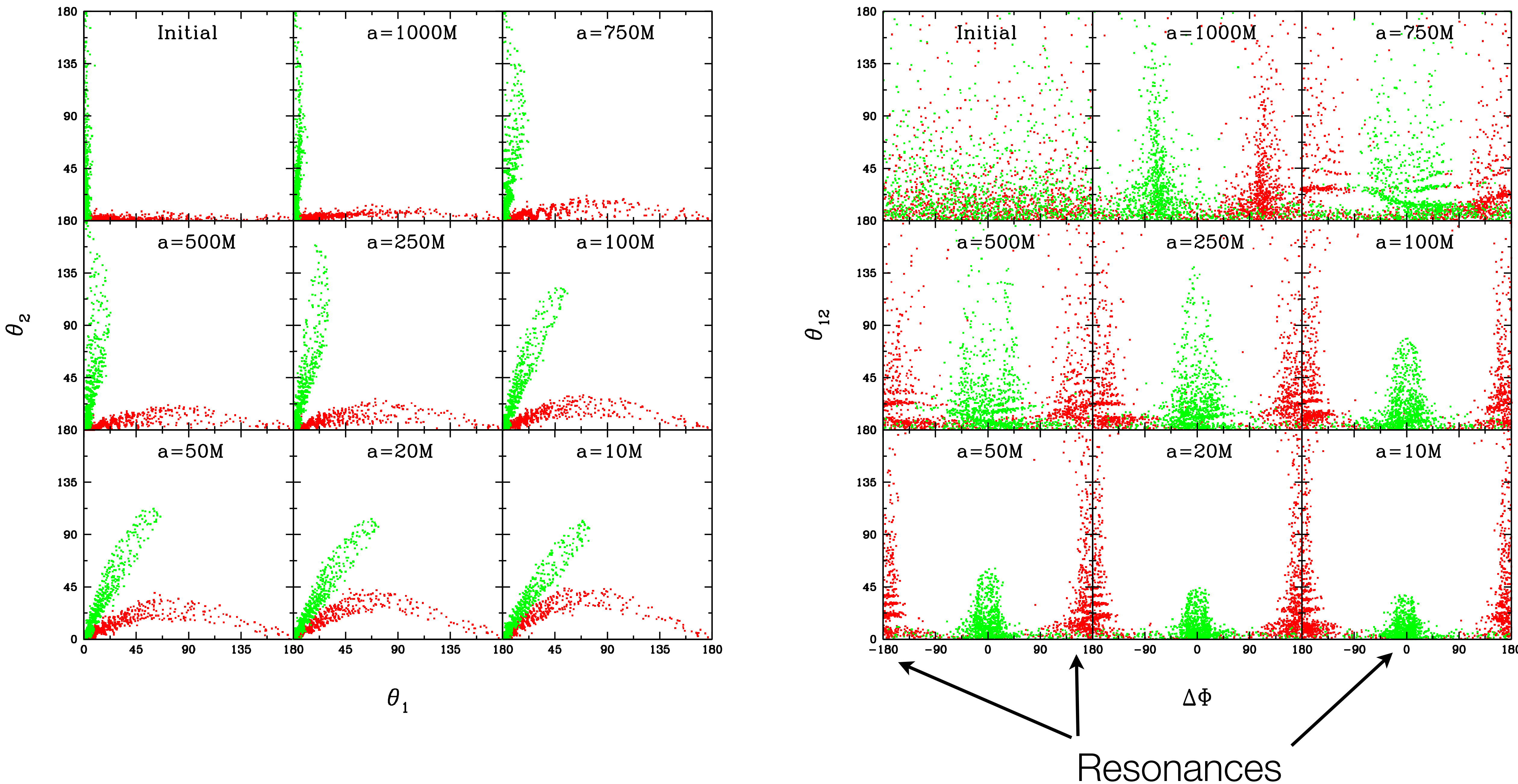
$$\zeta(v) = \eta\left(2 + \frac{3m_2}{2m_1}\right)\int v^5 (1+\kappa\gamma)\left(\frac{dt}{dv}\right)\mathrm{d}v \ .$$

- Frequency domain form

$$\bar{h}_+(f) \simeq \frac{2\pi\mathcal{M}_c^2}{D}\sqrt{\frac{5}{96\pi}}(\pi\mathcal{M}f)^{-7/6}\sum_m z_m e^{i(\Psi-2\zeta)+im\alpha}$$

Resonances

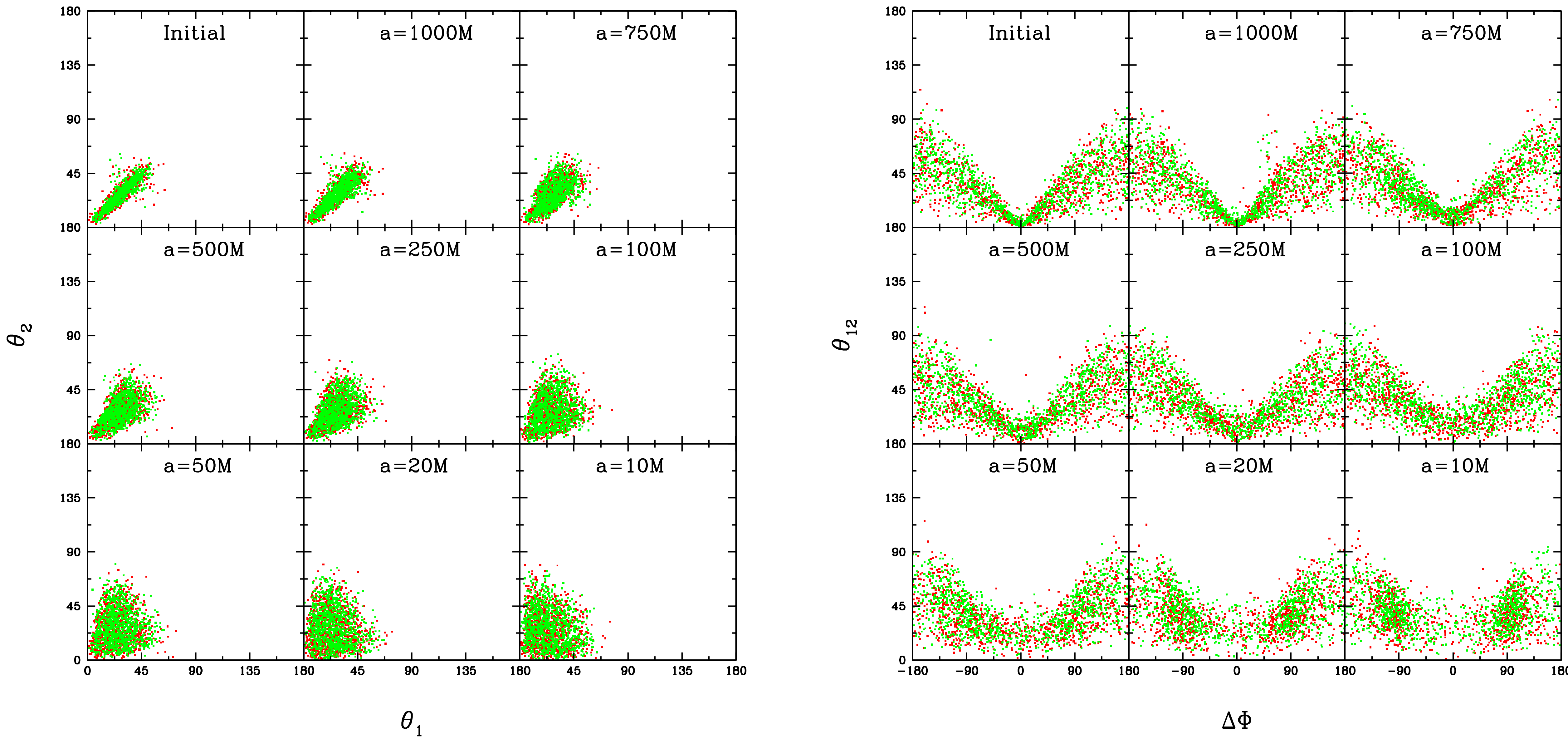
- Resonances occur if an initial asymmetry exists



Thursday, February 27, 2014

Resonances

- But not without one

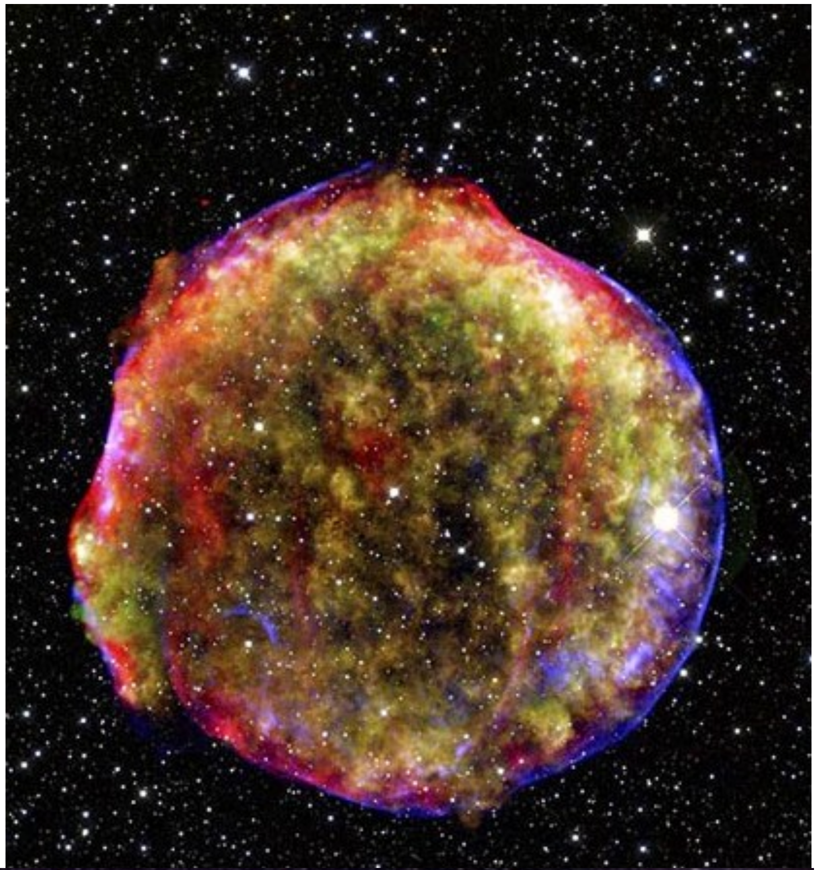


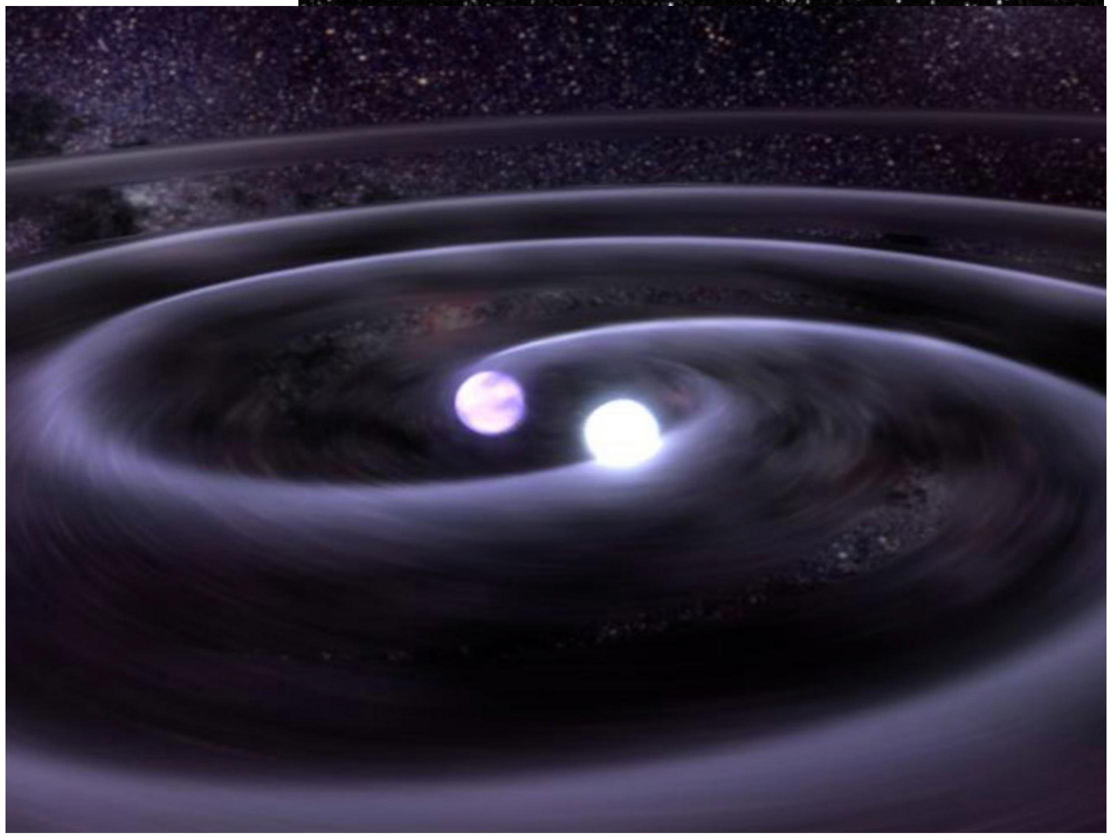
Binary population synthesis

Key fact: stars evolve and **expand**

Typical life of interacting binary system

- Star 1 ages, expands. Potential mass transfer 1->2
- Star 1 explodes
- Star 2 ages, expands. Potential mass transfer 2->1
- Star 2 explodes
- Compact binary left behind
  - **May** merge via GW....





Common envelope shrinks orbit the most

- Phenomenology:

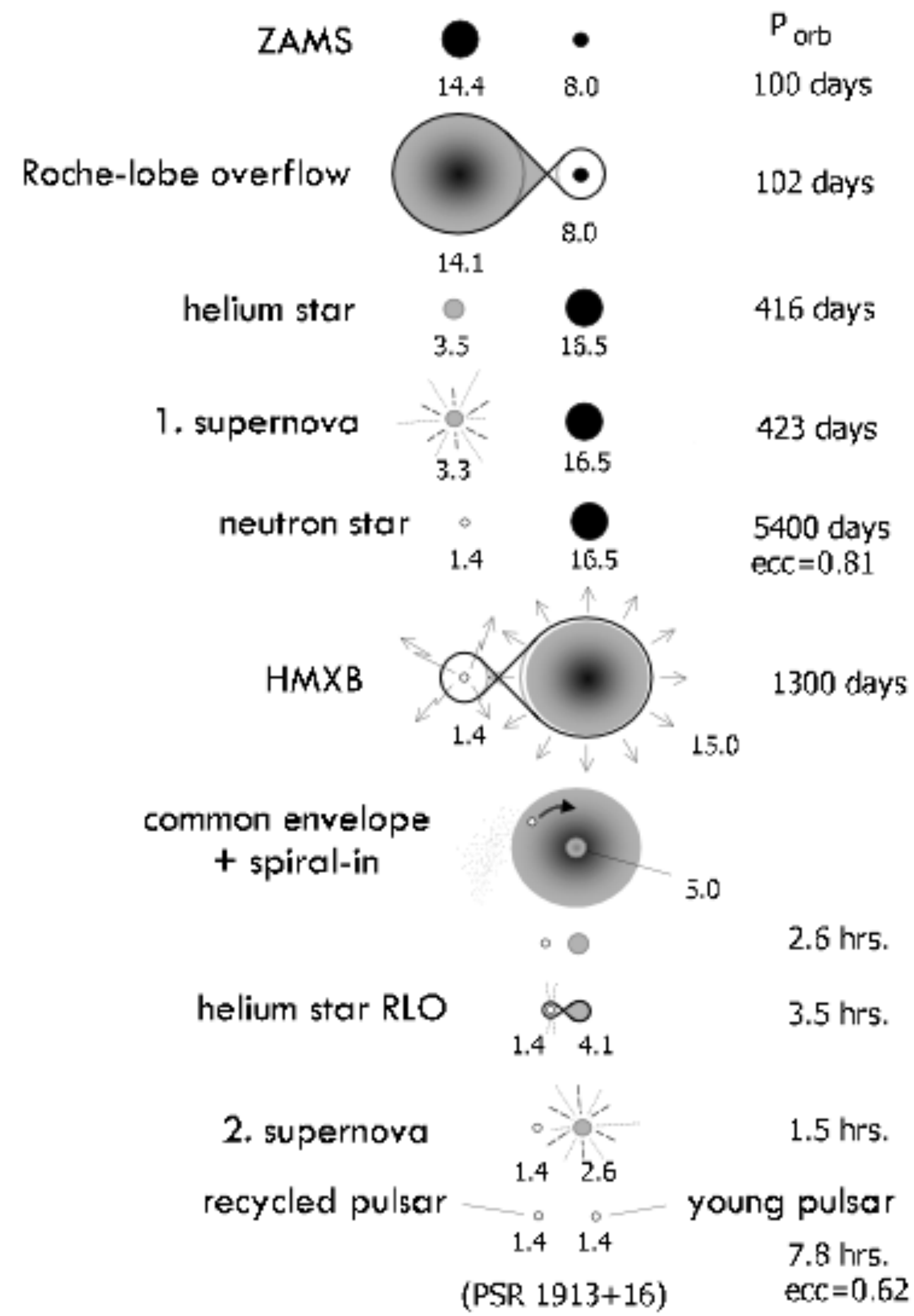
$$E_{bind} = \alpha \Delta E_{orb}$$

  - [**Hard** to model via hydro]

- Critical for most mergers

- Shrinkage model depends on stellar (envelope) structure
  - New tables-> new results

Loveridge et al; Xu et al   Dominik et al 2012, Belczynski et al 2012, etc

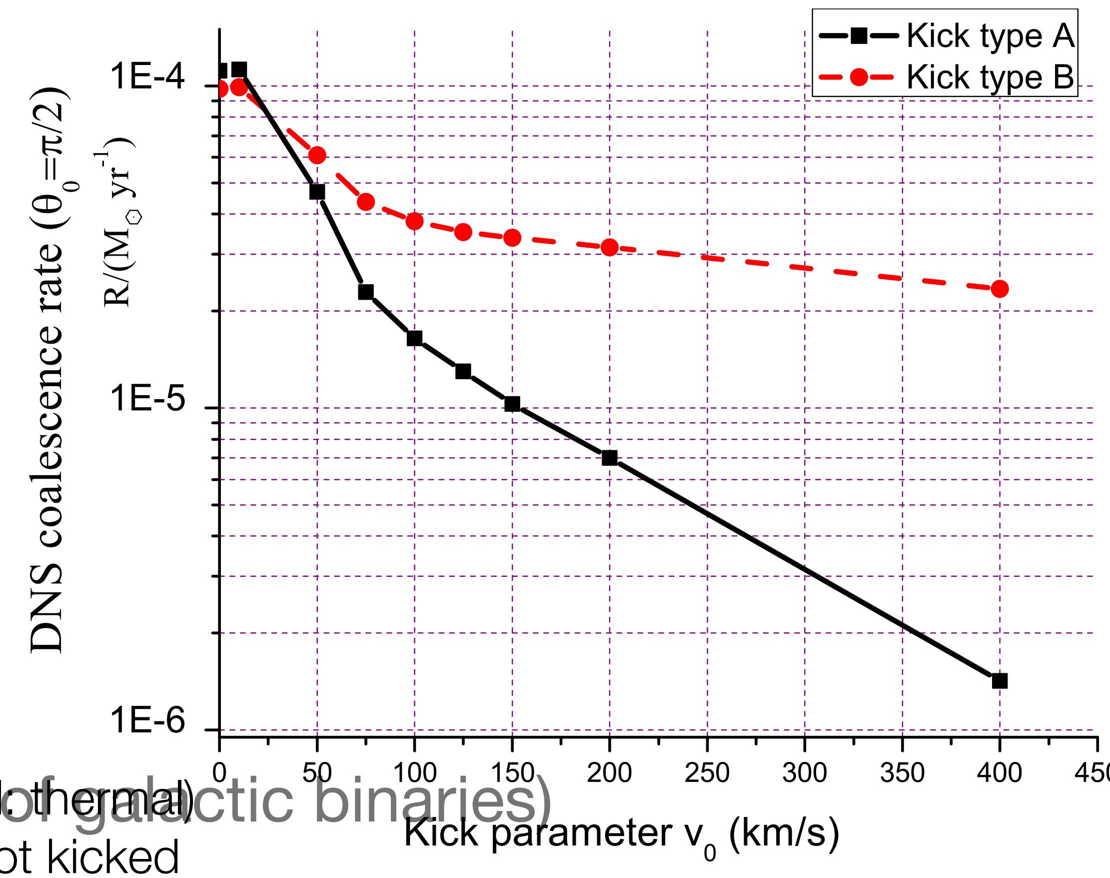


Voss and Tauris

What physics will we have access to?

- Kicks **strongly** influence
  - merger rate
    - disruption vs pushed closer
  - (present-day) eccentricity, proper motion (of galactic binaries)

Postnov and Kuranov [0710.4465]



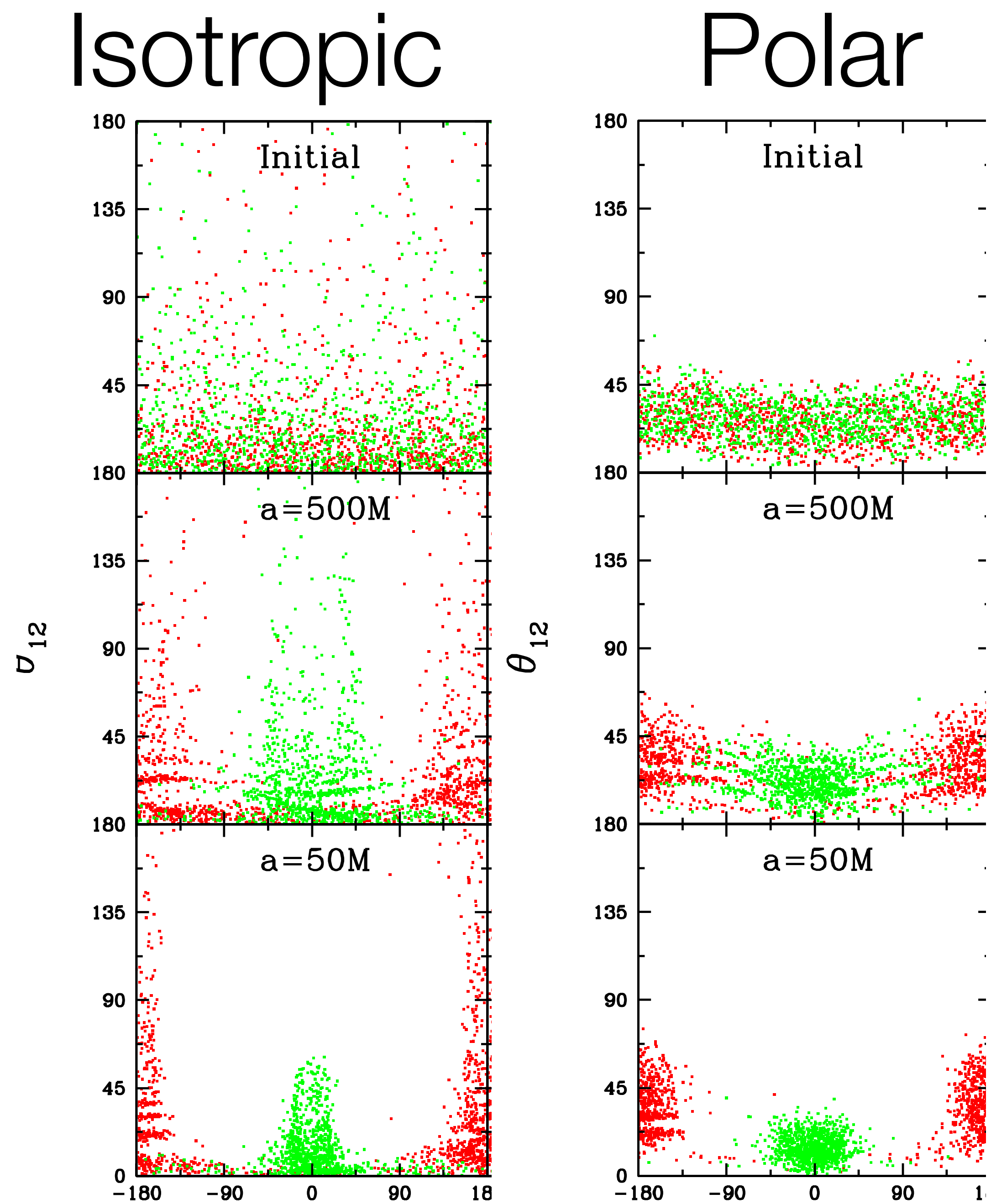
A: all NS kicked (Maxwell; thermal)
B: low-mass (ECS) NS not kicked

Willems et al 2008, Kalogera et al 0712.2540, Fragos et al 2010, Wong et al 2012, Corongiu et al 2007, ...

What physics will we have access to?

- Kicks **strongly** influence
  - merger rate
    - disruption vs pushed closer
  - (present-day) eccentricity, proper motion (of galactic binaries)
  - spin-orbit misalignment distribution

Willems et al 2008, Kalogera et al 0712.2540, Fragos et al 2010, Wong et al 2012, Corongiu et al 2007, ...



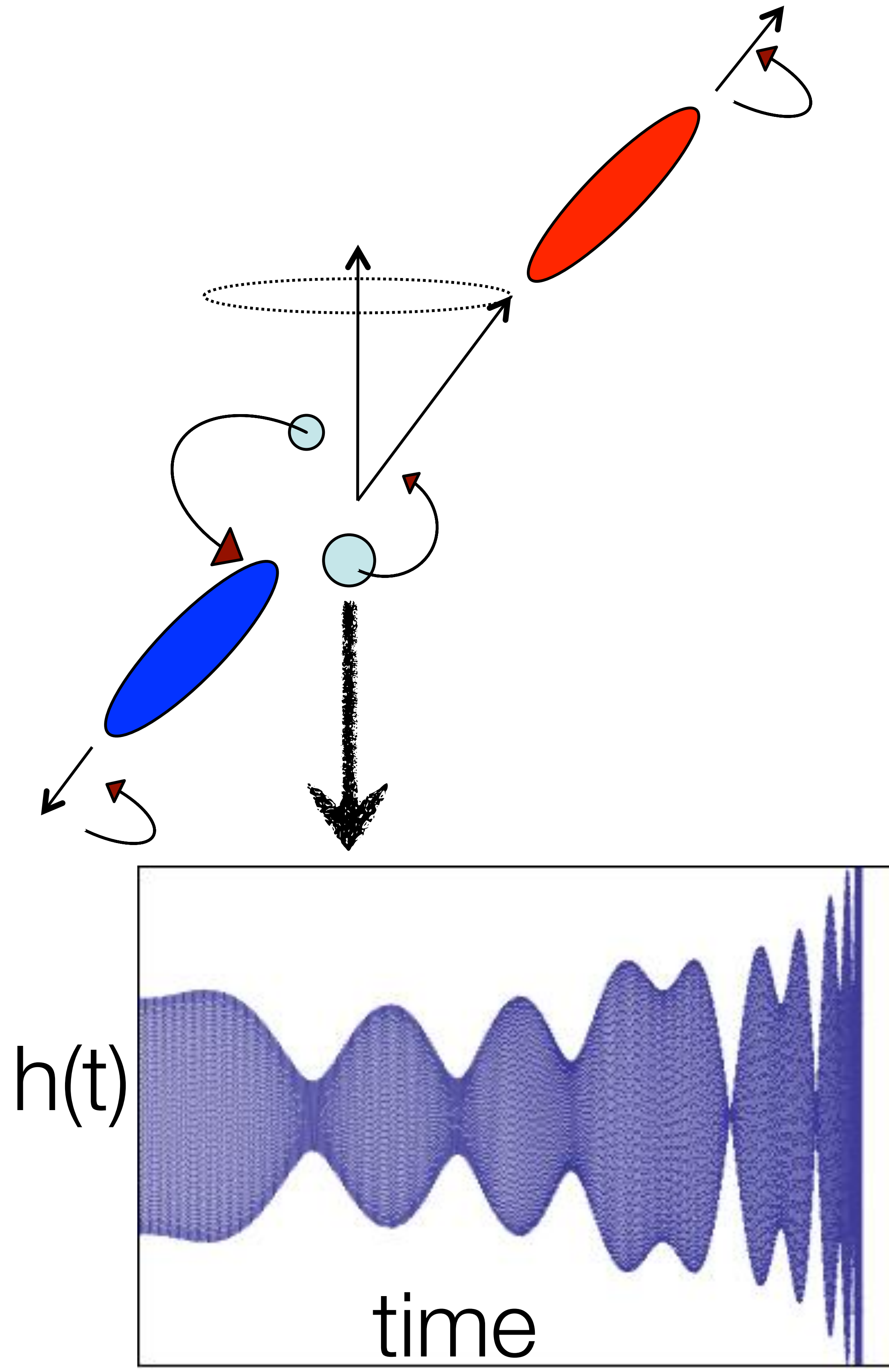
- if interpreted properly: birth != merger

Gerosa et al (in prep)

What physics will we have access to?

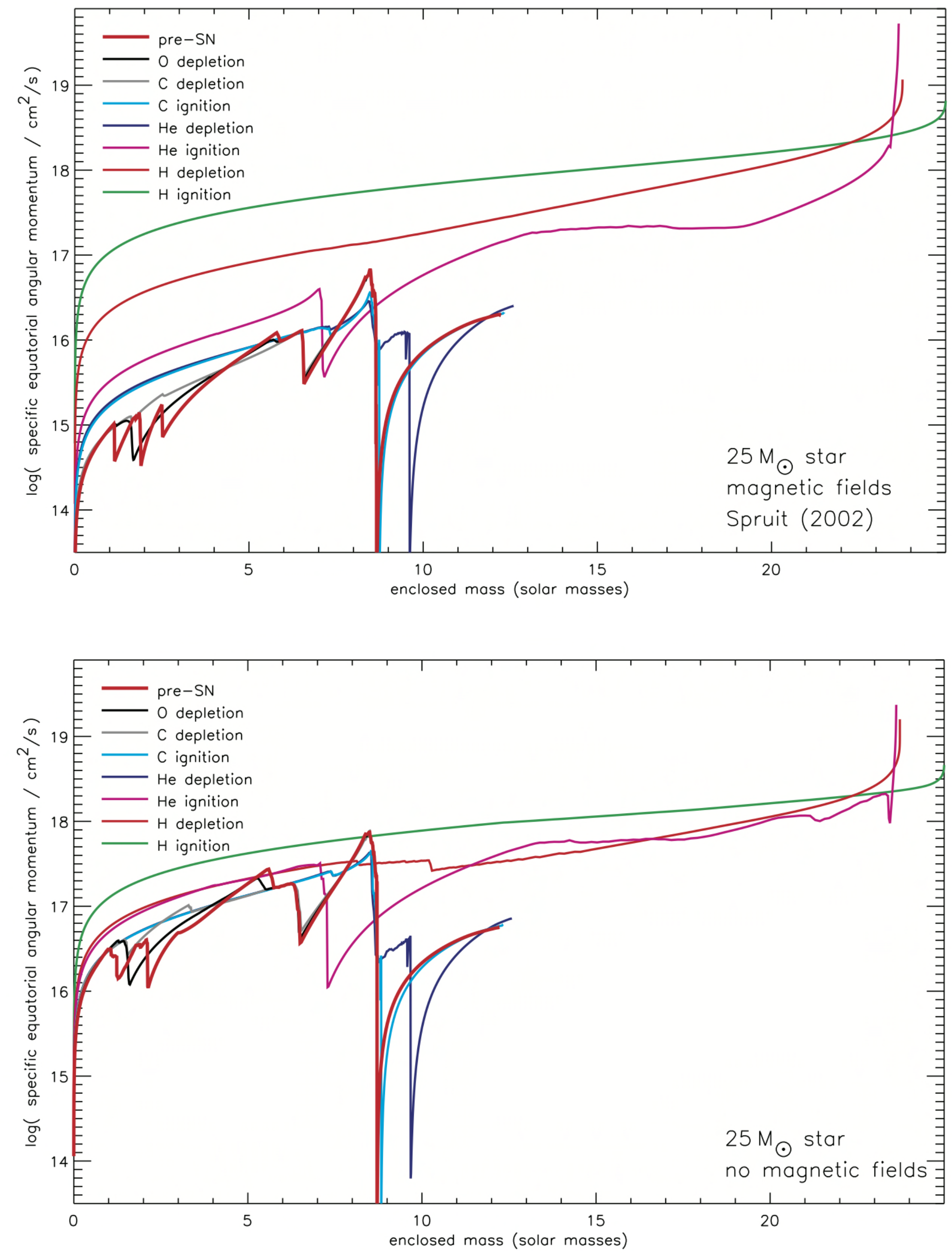
- Precession

- Measure **spin-orbit misalignment**
  - via simple geometry + polarization
  - Traces strength whatever misaligns them
    - SN kicks
    - **Stellar dynamics** [binary collisions]
  - Measure BH spin
    - Insight into SN, massive star physics



What physics will we have access to?

- Black hole spins **suggests** massive star evolution
    - Weakly (BH spin measurements poor)
    - Complement NS constraints on birth spin



NS progenitor, Heger et al 2005 ApJ 626 350

Future directions

- Relate observed (merger) properties to formation
  - Evolution (post-Newtonian resonances); inverting
- Identify, quantify features to connect to observations
  - Favored delay times set by stellar structure?
  - Masses, spins, spin-orbit: Fisher matrix limit
- More physics
  - Modified CE termination condition (Ivanova et al)
  - Rotation (Langer; MESA)

Conclusions

- Detections likely: greater sensitivity, higher predictions
- Expected nursery: **low-metallicity star formation**
  - Need detailed understanding of all low-Z star formation, ever

- EM signals complement GW detections
  - GW: collision input
  - EM : collision output (mod precursor)

- Simulations needed
  - Connect EM, GW signal
  - Relate observables

Statistical challenges: Comparing distributions

Frequently must compare distributions

- Convergence of distribution estimate
- Compare model family members to each other (and random samples)

One diagnostic: KL divergence

- Definition

$$H_p = -\int dx p \ln p$$

$$D_{KL}(p|q) \equiv \int dx p \ln p/q$$

- Natural local "radius squared" coordinate on manifold of distributions $U = \{p(x|\lambda)\}$

$$\simeq \frac{\delta\lambda_a \delta\lambda_a}{2} \left\langle \frac{\partial^2}{\partial\lambda_a \partial\lambda_b} \ln p \right\rangle$$

Fisher matrix, if p is gaussian!

Validating scaling of parallel tempering

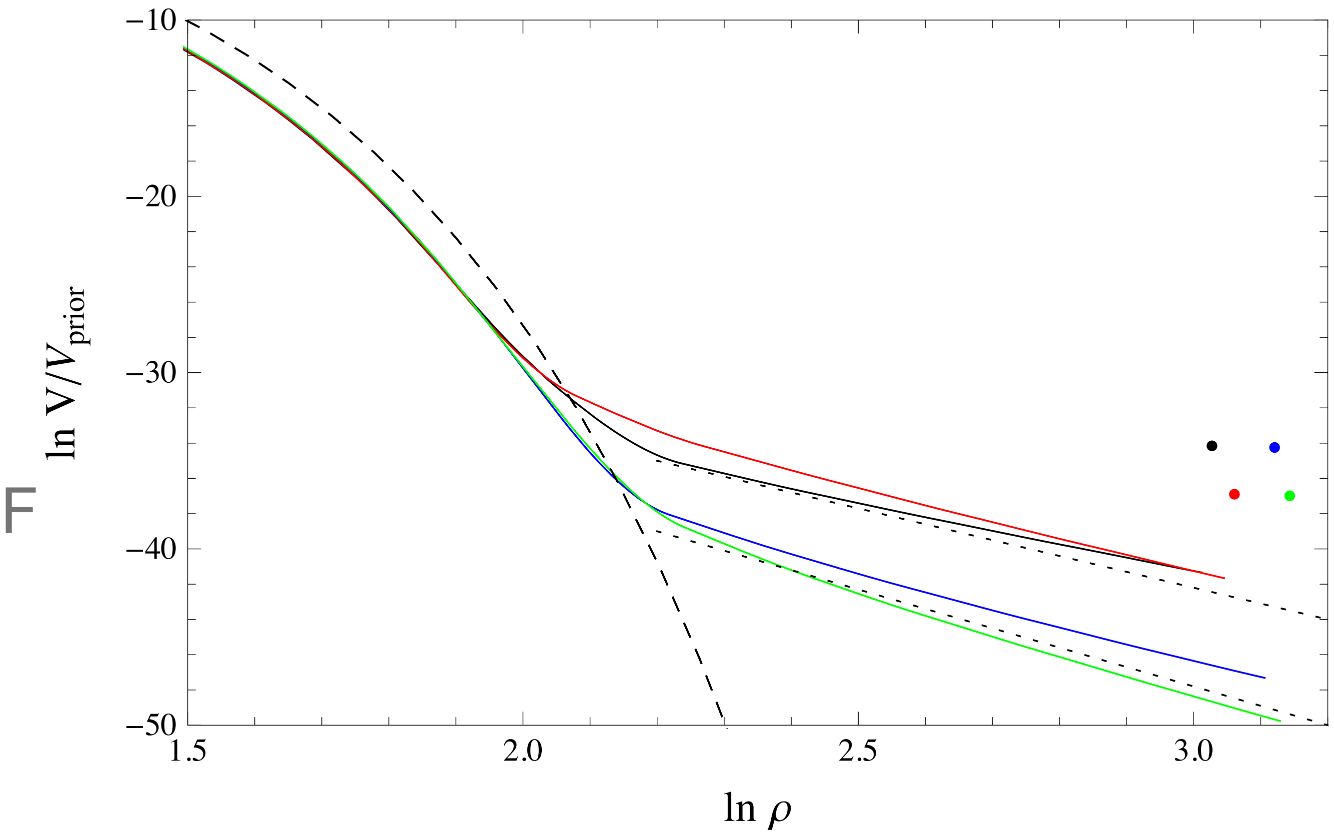
- High evidence:
  - Partly better fit ("louder"; Z up):
  - Partly via stronger constraints ("tighter"; Z down)

$$Z \propto e^{\rho^2/2}$$

$$Z \propto |\Gamma|^{1/2}/\rho^{D_{eff}}$$

- Isolate second effect
  - Identify # of parameters measured
  - Test if constraints possible on new DOF

$$V/V_{prior} \equiv \frac{Z(d|H_1)}{\max_\lambda \mathcal{L}(\lambda|\{d\})} \ .$$



ROS et al 2013