Finite Temperature Field Theory

Part III: The Propagator

$$Z = \int D\pi \int D\phi \, e^{\left\{ \int_0^\beta d\tau \int d^3x \left(\sum i\pi_j \frac{\partial \phi}{\partial \tau} - \mathcal{H} + \mu \mathfrak{N} \right) \right\}}$$

Second School and Workshop on Gravity and Theoretical Physics
Univ. Mississippi, Oxford, Jan.8-11, 2007

Empirical Touchstone

$$\langle E \rangle = \frac{\partial \ln Z}{\partial \beta} \qquad C_v = \frac{\partial \langle E \rangle}{\partial T}$$

$$\langle F_j \rangle = \frac{\partial \langle E \rangle}{\partial x_j} \qquad \langle P \rangle = \beta^{-1} \frac{\partial \ln Z}{\partial V}$$

$$d\sigma \sim d\sigma(T) \sim \left(M_{ij}\right)^2$$

$$t_{\frac{1}{2}} \sim t_{\frac{1}{2}}(T) \sim \left(M_{ij}\right)^2$$

$$\langle 0 | T_W : \phi_1 \phi_2 : | 0 \rangle = G(x_1, x_2; t) = \frac{\delta^2 Z}{\delta J(x_1) \delta J(x_2)} \Big|_{J=0}$$

Two Examples

Bosons

$$\mathfrak{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi - \lambda (\phi^* \phi)^2 \qquad \Phi = \frac{(\phi_1 + i\phi_2)}{\sqrt{2}} \quad \pi_i = \frac{\partial \phi_i}{\partial t}$$

$$j^\mu = i\left(\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*\right) \quad Q = N = \int d^3 x j_o = \int d^3 x \left(\phi_2 \pi_1 - \phi_1 \pi_2\right)$$

$$\mathfrak{H} = \tfrac{1}{2}\left[\pi_1^2 + \pi_2^2 + \left(\nabla \phi_1\right)^2 + \left(\nabla \phi\right)^2 + m^2 \phi_1^2 + m^2 \phi_2^2 + \tfrac{1}{4}\lambda\left(\phi_1^2 + \phi_2^2\right)^2\right]$$

$$Z = \int d\pi_1 d\pi_2 d\phi_1 d\phi_2 \exp\{\int_0^\beta d\tau \int d^3 x \left(i\pi_1 \frac{\partial \phi_1}{\partial \tau} + i\pi_2 \frac{\partial \phi_2}{\partial \tau} - \mathfrak{H} + \mu\left(\phi_2 \pi_1 - \phi_1 \pi_2\right)\right)\}$$

Fermions

$$\mathfrak{L} = \overline{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi \qquad \overline{\psi} = \psi^+ \gamma^0$$

$$j_\mu = \overline{\psi}\gamma^\mu \psi \qquad \mathfrak{N} = \int d^3 x \psi^+ \psi \quad \pi = i\psi^+$$

$$Z = \int iD\psi^+ D\psi \exp\left\{\int_0^\beta d\tau \int d^3 x \left[\overline{\psi}\left(-\gamma^0 \frac{\partial}{\partial \tau} + i\vec{\gamma} \cdot \nabla - m + \mu\gamma^0\right)\psi\right]\right\}$$

Perturbative Expansion

$$Z = N\int D\phi e^{S} \qquad S = S_0 + S_I \qquad \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I$$

$$Z = N\int D\phi e^{S_0} \sum_{n=0}^{\infty} \frac{S_I^n}{n!}$$

$$\ln Z = \ln Z_0 + \ln\left(1 + \sum_{n=1} \frac{\int D\phi\, e^{S_0}\, \frac{S_I^n}{n!}}{\int D\phi e^{S_0}} \right) = \ln Z_0 + \ln Z_1$$

Interactions: φ^4

- Add a source term J
- Add an interaction, use free field solution
- Integrate out the momenta, integrate by parts, surface terms->0

$$Z_M[J] = \frac{\int D\phi\, e^{\left\{\int d^4x\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{g}{4!}\phi^4 + iJ\phi\right)\right\}}}{\int D\phi\, e^{\left\{\int d^4x\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{g}{4!}\phi^4\right)\right\}}}$$

$$\Delta_F(x) = \frac{1}{(2\pi)^4}\int \frac{d^4k e^{-ikx}}{k^2 - m^2 + i\varepsilon}$$

$$Z_M[J] = \frac{e^{\left\{i\int \mathfrak{L}\left(-i\frac{\delta}{\delta J(z)}\right)dz\right\}} e^{\left\{-\frac{1}{2}\int J(x)\Delta_F(x-y)J(y)dy\right\}}}{\left[e^{\left\{i\int \mathfrak{L}\left(-i\frac{\delta}{\delta J(z)}\right)dz\right\}} e^{\left\{-\frac{1}{2}\int J(x)\Delta_F(x-y)J(y)dy\right\}}\right]_{J=0}}$$

Expand the Exponential

$$Z_M[J] = \frac{\left(1 - \frac{ig}{4!}\int\left(-i\frac{\delta}{\delta J(z)}\right)^4 dz\right) e^{\left\{-\frac{i}{2}\int J(x)\Delta_F(x-y)J(y)dxdy\right\}}}{\left[e^{\left\{i\int \mathfrak{L}\left(-i\frac{\delta}{\delta J(z)}\right)dz\right\}} e^{\left\{-\frac{i}{2}\int J(x)\Delta_F(x-y)J(y)dxdy\right\}}\right]_{J=0}}$$

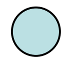
$$-i\frac{\delta}{\delta J(z)} e^{\left\{-\frac{1}{2}\int J(x)\Delta_F(x-y)J(y)dy\right\}} = -\int \Delta_F(z-y)J(x)dx e^{-\frac{i}{2}\int J\Delta J dxdy}$$

$$\left(-i\frac{\delta}{\delta J(z)}\right)^2 e^{\left\{-\frac{1}{2}\int J(x)\Delta_F(x-y)J(y)dy\right\}} = \left\{ i\Delta_F(0) + \left[\int \Delta_F(z-x)J(x)dx\right]^2 e^{-\frac{i}{2}\int J\Delta J dxdy} \right\}$$

Diagramatic Ideas

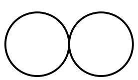
$$\Delta_F(x-y)$$

$$\Delta_F(0)$$

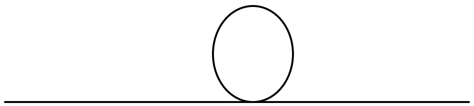


$$\left(-i\frac{\delta}{\delta J(z)}\right)^4 e^{\left\{-\frac{1}{2}\int J(x)\Delta_F(x-y)J(y)dy\right\}} =$$

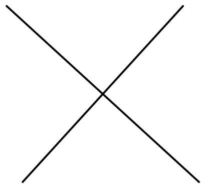
$$= \left\{-3\left(\Delta_F(0)\right)^2 + 6i\Delta_F(0)\left[\int \Delta_F(z-x)J(x)dx\right]^2 + \left[\int \Delta_F(z-x)J(x)dx\right]^4\right\} e^{-\frac{i}{2}\int J\Delta J dxdy}$$



Zero J's

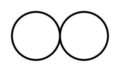
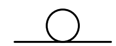
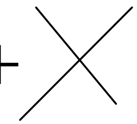


2 J's



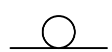
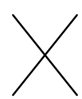
4 J's

Canceling a diagram

$Z_M[J] = \Big[1-\frac{ig}{4!}\int -3$  $+6i$  $+$  $\Big]e^{-\frac{i}{2}\int J\Delta_F Jdxdy}$

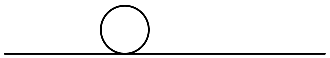
$\Big/ \; 1-\frac{ig}{4!}\int\Big(-3$  $\Big)dz$

$$\frac{\big(1-(-a+b+c)\big)}{(1-a)} = (1-(-a+b+c))(1+a) = 1-b-c$$

$Z_M = \Big[1-\frac{ig}{4!}\int\Big(6i$  $+$  $\Big)dz\Big]e^{-\frac{i}{2}\int J\Delta_F Jdxdy}$

Two Point Function

$$G(x_1, x_2) = -\frac{\delta^2 Z[J]}{\delta J(x_2) J(x_1)}\Big|_{J=0}$$

$= i$ $-\frac{g}{2}$ 

The order "g" effect on the free field propagator

Renormalized Mass

$$-\tfrac{1}{2}g\Delta_F(0)\int\Delta_F(x_1-z)\Delta(x_2-z)dz =$$

$$= -\frac{g\Delta_F(0)}{2(2\pi)^8}\int\frac{e^{-ip(x_1-z)}}{p^2-m^2+i\varepsilon}\frac{e^{-ip(x_2-z)}}{q^2-m^2+i\varepsilon}d^4p\,d^4q\,d^4z$$

$$G(x_2,x_1) = \frac{1}{(2\pi)^4}\int\frac{e^{-ip(x_1-x_2)}}{p^2-m^2+i\varepsilon}\left[1+\frac{\tfrac{1}{2}ig\Delta_F(0)}{p^2-m^2+i\varepsilon}\right]d^4p$$

$$= \frac{i}{(2\pi)^4}\int\frac{e^{-ip(x_1-x_2)}}{p^2-m^2-\tfrac{1}{2}ig\Delta_F(0)+i\varepsilon}d^4p$$

Thermal Diagrams

Conversion rules from Feynman diagrams

- **Each propagator gets a factor of T**
- **Each vertex a factor of β**
- **Each mass shifted by μ**
- **Each frequency from momentum shifted by ω**
- **Overall factor of Vβ**

Applications

- SSB
- Lamb shift
- Casimir Force
- White Dwarf Mass
- Ultra Dense Matter
- Quark Gluon Plasma
- Instantons
- Phase defects-strings, monopoles
- Green’s Function-noncommutative geometry
- RGE-DMRG
- Gravitational Field Effects
- Global Topology

High Temperature SSB

Scalar – Vector Boson Theory

$$\mathfrak{L} = \left(D_\mu \phi\right)^+ \left(D^\mu \phi\right) + \mu^2 \phi^+ \phi - \lambda(\phi^+ \phi)^2 - \tfrac{1}{4} l^{\mu\nu} l_{\mu\nu} - \tfrac{1}{4} f_a^{\mu\nu} f_{\mu\nu}^a$$

$$D_\mu = \partial_\mu + \frac{ig}{2} \tau_a A_\mu^a + \frac{ig'}{2} B_\mu$$

$$f_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g \varepsilon_{abc} A_\mu^b A_\nu^c$$

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

High T gives a tachyon->negative renormalized masses…unless…

Goldstone Boson

Each continuous broken symmetry yields a new particle. Higgs-shift the ground state- potential is T dependent, there is a phase transition that breaks the symmetry

$$P = \frac{-\pi^2}{45} T^4 - \frac{\mu^2}{12} T^2 + \frac{\lambda^2}{32\pi^2} \ln T^2 + \ldots$$

Critical temperature shifts ground state when symmetry is broken

Casimir Force

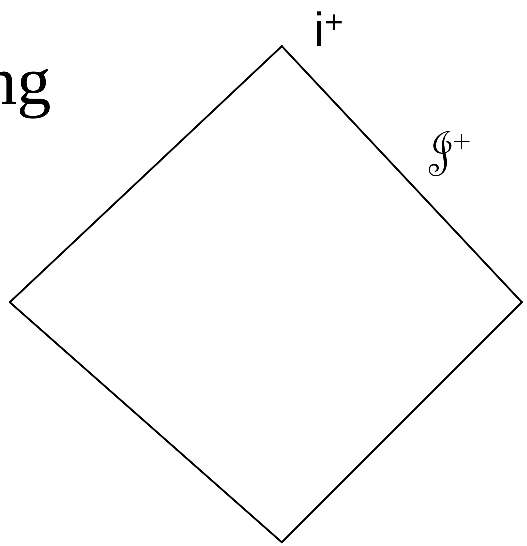
Boundary Conditions influence the number of ground state
Modes available giving rise to vacuum pressure

- **Casimir 1949 van der Waals interaction $1/d^4$**
- **Schwinger 1953, Milton and Schwinger 1979 T**
- **Attractive plates**
- **Repulsive hollow sphere**
- **Effect more pronounced at higher T-more modes available**
- **WEC violated!**
- **Compact topological spacetimes should exhibit different vacuua Gott**

Quantum Fields in Curved Spacetime

Couple to gravity through covaraint derivative-find shift in E,
Conformal diagram-periodicity, weak field
(Davies, Wald, Birrwll, Unruh…..

$$\langle 0 | : T_{\mu\nu} : | 0 \rangle - \text{spacelike point splitting}$$



$$\mathrm{u} = \mathrm{t} - \mathrm{x}$$

$$\mathrm{v} = \mathrm{t} + \mathrm{x}$$

$$\mathrm{ds}^2 = du dv$$

$$u' = 2\tan^{-1} u$$

$$v = 2\tan^{-1} v$$

- Curved regions give rise to finite T
- Accelerated observers see particles
- Unruh- mirror boundary condition ->Casimir like calculation

Hawking Radiation

For compact geometries, drop surface terms, measure over all metrics,

$$S = \frac{-1}{16\pi}\int \left(R - 2\Lambda\right) d^4x \qquad Z = \int D[g]e^{-S}$$

$$Z = -4\pi M^2$$

$$\langle E \rangle \sim T \sim M$$

$$S = \frac{A}{4} \sim R^2$$

Lovelock Geometries

$$\mathcal{L}_k = 2^{-k}\,\delta^{c_1 d_1 \ldots . c_k d_k}_{a_1 b_1 \ldots . a_k b_k}\, R^{a_1 b_1}_{c_1 d_1} \cdots R^{a_k b_k}_{c_k d_k}$$

$$\delta^{c_1 d_1 \ldots . c_k d_k}_{a_1 b_1 \ldots . a_k b_k} = \begin{vmatrix} \delta^{c_1}_{a_1} & \ldots & \delta^{c_1}_{a_k} \\ \vdots & & \vdots \\ \delta^{d_k}_{a_1} & \ldots & \delta \end{vmatrix}$$

$$\delta^{aaaa}_{bbbb} = \varepsilon_{bbbbb}\varepsilon^{aaaa}$$

- **2nd derivative**
- **Conservative**
- **Ghost free**
- **…..exactly solvable for f(r)**

String Generated Gravity Models
Pure Gravity Sector- to 1st Order

$$I = \int d^d x \sqrt{-g} \left\{ \frac{R}{\kappa} + \alpha \left(R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \right) \right\}$$

String tension

Euler Lagrange Equations

$$G_{\mu\nu} = \alpha \, \kappa \, T_{\mu\nu}$$

$$T_{\mu\nu} = \frac{1}{2} g_{\mu\nu} \left(R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} - 4 R_{\alpha\beta} R^{\alpha\beta} + R^2 \right) - 2 R R_{\mu\nu} + 4 R_{\alpha\beta} R^{\alpha\beta}_{\mu\nu} - 2 R_{\mu\alpha\beta\gamma} R^{\alpha\beta\gamma}_{\nu}$$

$$R^{\lambda}_{\mu\nu\kappa} = \frac{\partial \Gamma^{\kappa}_{\mu\nu}}{\partial x^{\kappa}} - \frac{\partial \Gamma^{\lambda}_{\mu\kappa}}{\partial x^{\nu}} + \Gamma^{\eta}_{\mu\nu} \Gamma^{\lambda}_{\lambda\eta} - \Gamma^{\eta}_{\mu\kappa} \Gamma^{\lambda}_{\nu\eta}$$

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\rho} \left(\frac{\partial g_{\rho\mu}}{\partial x^{\nu}} + \frac{\partial g_{\rho\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\rho}} \right)$$

Role of Student if no MAPLE Solve for $g_{\mu\nu}$

Negative Entropy: Negenetropy

Consider b bits of digital data representing a highly organized system, then the system is organized instead of randomized so that the negative of its entropy is a measure of the level of organization

Construct a conservation law such that the sum of the Entropy and the negentropy is a constant.

$$SK = S + NE \qquad \frac{d(SK)}{dt} = 0$$

Cosmological Particle Creation
Robertson-Walker

$$ds^2 = dt^2 - a^2(t)dx^2 \quad conformal\ time\ d\eta = \frac{dt}{a}$$

$$t\int dt = \int a d\eta$$

$$ds^2 = a^2(\eta)\left(d\eta^2 - dx^2\right)$$

$$\langle N \rangle \sim T$$

$$S \sim \ln\langle N \rangle \sim R^2$$

$$SK = S + NE$$

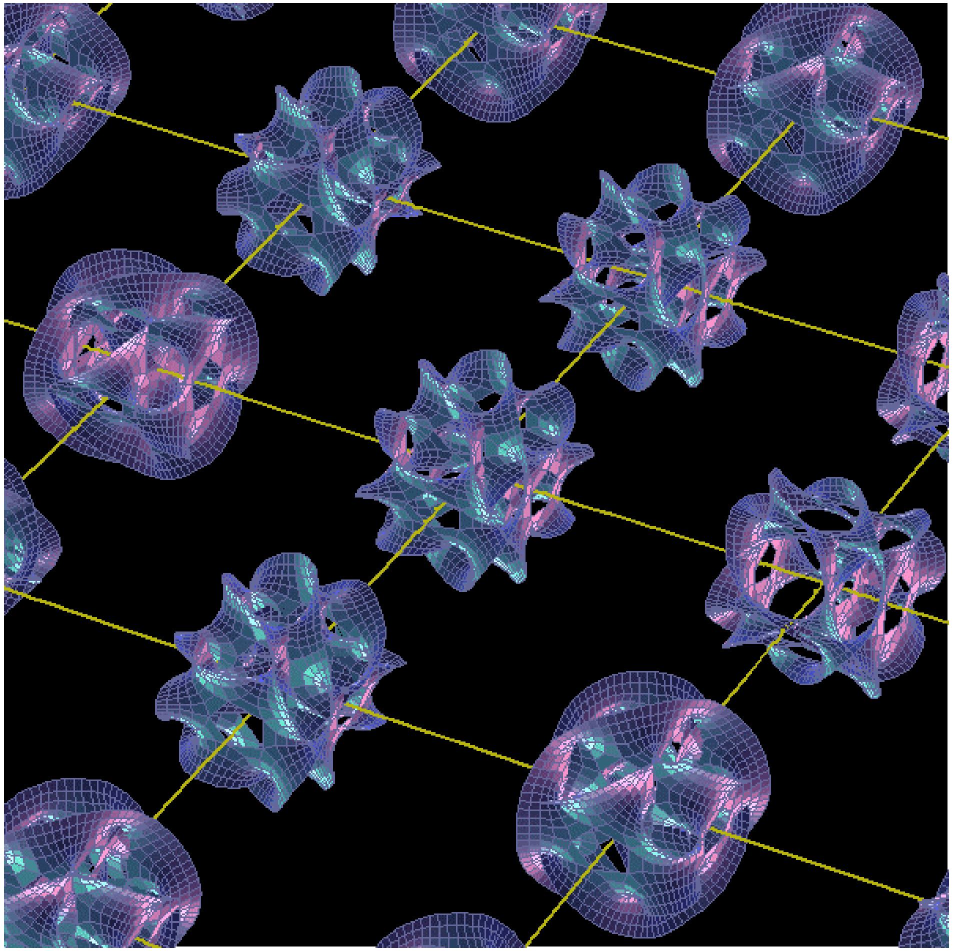
$$NE = SK - S$$

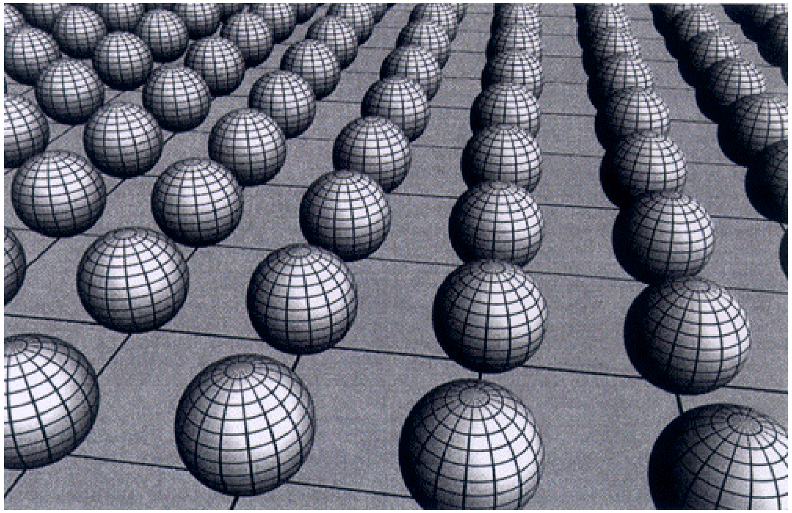
$$n_{\text{int}Unit} = \frac{NE}{10^{37}}$$

$$n_{UFOperators} = \frac{n_{\text{int}Unit}}{250/sauc} \quad N_{sentient} = \frac{n_{UFOperators}}{n_{stars*}} = 10^{12}/universe$$

Conclusions

- All processes require thermal corrections
- Combine knowledge from Univ. Mississippi workshop + imagination
- Solve the problems discovered by older sentient beings





Boundary Conditions
In compact dimensions

Fiber Bundles
Compact Dimensions