Dilaton Black Holes

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Einstein – Hilbert Action

$$S = \frac{1}{16} \int d^4x \sqrt{-g}\, R$$

Field Equations

$$R_{\mu\nu} = 0$$

Schwarzschild Metric

$$ds^2 = (1 - \frac{2M}{r})\, dt^2 - (1 - \frac{2M}{r})^{-1}\, dr^2 - r^2(d\theta^2 + \sin^2(\theta)\, d\phi^2)$$

Einstein – Hilbert – Maxwell Action

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g}\, \left[R - F^2\right] \ ,$$

Field Equations

$$\begin{cases} \nabla^i(F_{ij}) = 0 \\ \\ \nabla_{[k} F_{ij]} = 0 \ , \end{cases} \qquad \text{(Maxwell)}$$

$$R_{ij} = 2T_{ij}^{EM} \quad \text{(Einstein)} \ ,$$

$$T_{ij}^{EM} = \left[F_{ik} F_j^k - \frac{1}{4} g_{ij} F^2 \right] \ .$$

Reissner-Nordstrom Metric

$$ds^2 = (1 - \frac{2M}{r} + \frac{Q^2}{r^2})\, dt^2 - (1 - \frac{2M}{r} + \frac{Q^2}{r^2})^{-1}\, dr^2 - r^2(d\theta^2 + \sin^2(\theta)\, d\phi^2)$$

Kerr – Newman Metric

$$ds^2 = -\frac{\Delta\ \sin^2\theta}{\Psi}\,(dt)^2 + \Psi\,(d\varphi - \omega\, dt)^2 + \rho^2\ \left[\frac{(dr)^2}{\Delta} + (d\theta)^2\right]\ ,$$

$$\begin{aligned}
\Delta &= r^2 - 2\, M\, r + \alpha^2 + Q^2 \\
\rho^2 &= r^2 + \alpha^2\ \cos^2\theta \\
\Psi &= -\frac{\Delta - \alpha^2\ \sin^2\theta}{\rho^2} \\
\omega &= -\alpha\ \sin^2\theta\,[1 + \Psi^{-1}]
\end{aligned}$$

Affect of the Dilaton

- **Violation of the Equivalence Principle**
- **Scalar component of gravity, *dilaton***
- **Solar tests rule out such possibilities in the weak field regime**
- **Regions of strong gravitational fields not yet tested**
- **Strong gravitational fields exist around very dense, compact sources**

Phys. Rev. D 55, 814(1997)

Low Energy String Action

$$S = \frac{1}{16\,\pi} \int d^4x \,\sqrt{-g}\, \left[R - \frac{1}{2}\,(\nabla \phi)^2 - e^{-a\,\phi}\, F^2 \right]$$

Field Equations

$$\begin{cases} \nabla^i (e^{-a\,\phi}\, F_{ij}) = 0 \\ \nabla_{[k} F_{ij]} = 0 \;, \end{cases} \qquad \text{(Maxwell)}$$

$$\nabla^2 \phi = -a\, e^{-a\,\phi}\, F^2 \qquad \text{(dilaton)} \;,$$

$$R_{ij} = \frac{1}{2}\, \nabla_i \phi\, \nabla_j \phi + 2\, T^{EM}_{ij} \quad \text{(Einstein)}$$

Reissner – Nordstrom – Dilaton Metric

$$ds^2 = \lambda^2\, dt^2 - \frac{1}{\lambda^2}\, dr^2 - R^2(d\theta^2 + \sin^2(\theta)\, d\phi^2)$$

where

$$\lambda^2 \,=\, (1 - r_+/r)\,(1 - r_-/r)^{(1-a^2)/(1+a^2)}$$

$$R \,=\, r\,(1 - r_-/r)^{a^2/(1+a^2)}$$

The free parameters r_+ and r_- are related to M and Q by

$$M \,=\, \frac{r_+}{2} + \left(\frac{r_+ r_-}{1+a^2}\right)^{1/2}$$

$$R \,=\, \left(\frac{r_+ r_-}{1+a^2}\right)^{1/2}$$

Kerr – Newman – Dilaton Metric

$$
\begin{aligned}
g_{tt} &= \tilde{\Psi} \\
&\approx -\left[1 - \frac{2M}{r}\left(1 + \frac{Q^2\, a^2}{6M^2}\right)\right] \\
g^{tt} &= \frac{1}{\tilde{\Psi}} - \frac{\tilde{\omega}^2\, \tilde{\Psi}}{\Delta\delta} \\
&\approx -\left[1 + \frac{2M}{r}\left(1 + \frac{Q^2\, a^2}{6M^2}\right)\right] \\
g_{\varphi\varphi} &= \tilde{\Psi}\, \tilde{\omega}^2 - \frac{\Delta\delta}{\tilde{\Psi}} \\
&\approx r^2\, \delta \\
g^{\varphi\varphi} &= -\frac{\tilde{\Psi}}{\Delta\, \delta} \\
&\approx \frac{1}{r^2 \delta} \\
g_{t\varphi} &= \tilde{\Psi}\, \tilde{\omega} \\
&\approx \frac{-2\alpha\, \delta\, M}{r}\left(1 + \frac{Q^2\, a^2}{6\, M^2}\right) \\
g^{t\varphi} &= \frac{1}{\Delta\, \delta}\, \tilde{\Psi}\, \tilde{\omega} \\
&\approx \frac{-2\alpha\, M}{r^3}\left(1 + \frac{Q^2\, a^2}{6\, M^2}\right) \\
g_{rr} &= \frac{\rho^2}{\Delta} \\
&\approx 1 + \frac{2M}{r}\left(1 + Q^2\, \frac{a^2}{6\, M^2}\right) \\
g^{rr} &= \frac{\Delta}{\rho^2} \\
&\approx 1 - \frac{2\, M}{r}\left(1 + \frac{Q^2 a^2}{6\, M^2}\right) \\
g_{\theta\theta} &= \rho^2 \\
&\approx r^2 \\
g^{\theta\theta} &= \rho^{-2} \\
&\approx r^{-2}\, ,
\end{aligned}
$$

Corrections to the Gyromagnetic Ratio

$$\mu_{phys} = g \, \frac{Q_{phys} \, J_{phys}}{2 \, M_{phys}} \ ,$$

$$\begin{aligned} \mu_{phys} &= \alpha \, Q \, \left[1 + \frac{a^2 \, Q^2}{6 \, M^2} \right] \\ J_{phys} &= \alpha \, M \, \left[1 + \frac{a^2 \, Q^2}{6 \, M^2} \right] \\ M_{phys} &= M \, \left[1 + \frac{a^2 \, Q^2}{6 \, M^2} \right] \end{aligned}$$

$$g = 2 \, \left[1 - \frac{a^2 \, Q_{phys}^2}{6 \, M_{phys}^2} \right]$$

Black Hole Statistical Mechanics

Phys. Rev. D 46, 2334(1992).

Schwarzschild Black Holes

Microcanonical Density of States

$$\Omega(E,V) = \sum_{n=1}^{\infty} \Omega_n(E,V)$$

where

$$\Omega_n(E,V) = \left[\frac{V}{(2\pi)^3}\right]^n \frac{1}{n!} \prod_{i=1}^{n} \left\{ \int_{m_0}^{\infty} dm_i \, \rho_{B.H.}(m_i) \int_{-\infty}^{\infty} d^3 p_i \right\}$$
$$\times \delta(E - \sum_{i=1}^{n} E_i) \, \delta^3(\sum_{i=1}^{n} \vec{p}_i)$$

This can be reduced to

$$\Omega_n(E,V) \simeq \left[\frac{cV}{(2\pi)^3}\right]^n \frac{1}{n!} e^{4\pi[E-(n-1)m_0]^2} e^{4\pi(n-1)m_0^2}$$

Total Entropy of the System

$$S(E,V) \equiv \ln \Omega(E,V) \simeq \ln \Omega_N(E,V)$$
$$\simeq N \ln \left[\frac{cV}{(2\pi)^3}\right] - \ln \Gamma(N+1) + s(E)$$

with

$$s(m) = 4\pi m^2$$

Inverse Temperature

$$\beta = \frac{dS}{dE} = \frac{\partial S}{\partial N}\frac{\partial N}{\partial E} + \frac{\partial S}{\partial E}$$

Dilaton Black Holes

Hawking Entropy and Temperature

$$s(m,Q) = \pi m^2 \left[1+\sqrt{1-\frac{(1-a^2)Q^2}{m^2}}\;\right]^2 \left(1-\frac{(1+a^2)Q^2}{m^2[1+\sqrt{1-\frac{(1-a^2)Q^2}{m^2}}]^2}\right)^{\frac{2a^2}{1+a^2}}$$

$$\beta_{Hawking}(m,Q) = 4\pi m \left[1+\sqrt{1-\frac{(1-a^2)Q^2}{m^2}}\;\right] \left(1-\frac{(1+a^2)Q^2}{m^2[1+\sqrt{1-\frac{(1-a^2)Q^2}{m^2}}]^2}\right)^{\frac{a^2-1}{a^2+1}}$$

Electromagnetic and Dilatonic Waves

Waves on the Perturbed K-N-D Black Hole

Phys. Rev. D 56, 4948(1997)

$$\phi(t,r,\theta,\varphi) = \sum_{p,n} g^p\, Q^n\, \phi^{(p,n)}$$
$$\phi_i(t,r,\theta,\varphi) = \sum_{p,n} g^p\, Q^n\, \phi_i^{(p,n)} \ , \quad i = 0,1,2$$
$$G(t,r,\theta,\varphi) = \sum_{p,n} g^p\, Q^n\, G^{(p,n)}$$

For $p = 0$ **the static solutions are**

$$\begin{aligned} \phi^{(0,n)} &= \phi^{(0,n)}(r,\theta) \\ \phi_i^{(0,n)} &= \phi_i^{(0,n)}(r,\theta) \ , \quad i = 0,1,2 \\ G^{(0,n)} &= G^{(0,n)}(r,\theta) \end{aligned}$$

and for $p = 1$

$$\begin{aligned} \phi^{(1,n)}(t,r,\theta,\varphi) &= k_d\, e^{i\,\omega\, t + i\, m\, \varphi}\, \phi^{(1,n)}(r,\theta) \\ \phi_i^{(1,n)}(t,r,\theta,\varphi) &= k_{EM}\, e^{i\,\omega\, t + i\, m\, \varphi}\, \phi_i^{(1,n)}(r,\theta) \ , \quad i = 0,1,2 \\ G^{(1,n)}(t,r,\theta,\varphi) &= k_G\, e^{i\,\omega\, t + i\, m\, \varphi}\, G^{(1,n)}(r,\theta) \end{aligned}$$

Wave Detection

Free outgoing waves

$$\phi_0^{(1,0)} \sim \frac{e^{-i\,\omega\,r}}{r^3}$$
$$\phi_2^{(1,0)} \sim \frac{e^{-i\,\omega\,r}}{r}$$

Correction terms

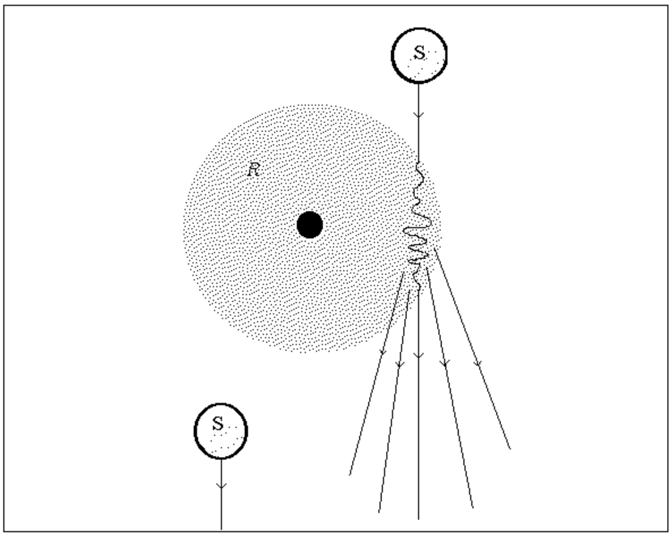
$$r\,\phi_0^{(1,1)} \sim \phi_2^{(1,1)} \sim e^{-i\,\omega\,r}/r^3$$

These imply a correction to the energy flux,

$$\frac{d^2E}{dt\,d\Omega} \simeq \lim_{r\to\infty} \frac{r^2}{2\,\pi} \left|\phi_2^{(1,0)} + e^{i\,\gamma}\,Q\,\phi_2^{(1,1)}\right|^2$$

Average over $\gamma \in (0, 2\,\pi)$

$$\frac{d^2E}{dt\,d\Omega} \simeq \lim_{r\to\infty} \frac{r^2}{2\,\pi} \int_0^{2\,\pi} \frac{d\gamma}{2\,\pi} \left|\phi_2^{(1,0)} + e^{i\,\gamma}\,Q\,\phi_2^{(1,1)}\right|^2$$
$$\simeq \lim_{r\to\infty} \frac{r^2\,k_{EM}^2}{2\,\pi}\,|\phi_2^{(1,0)}|^2 + \frac{a^2\,Q^2}{128\,\pi^2\,r_d^4}\,k_d^2\,\frac{(F^{\rm out})^2}{1+\omega^2 M^2}\,|Z_-^{\rm out}|^2$$



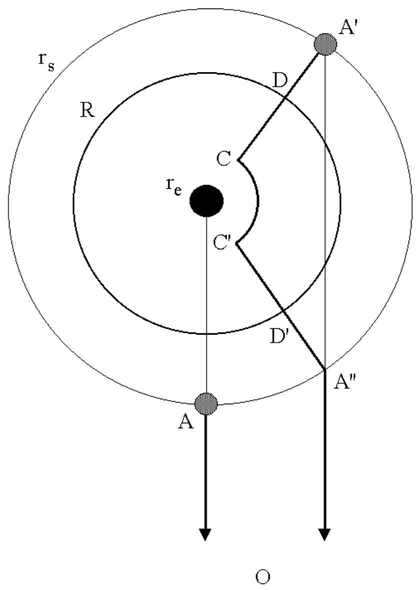
Absorption of E-M Radiation by Dilaton Field

Phys. Rev. D 58, 044015(1998)

Geometrical Optics Effects

Rate of decrease of the intensity

$$\frac{I_{(r)}}{I_{(-\infty)}} = \frac{A_{(-\infty)}}{A_{(r)}} \sim \frac{D^4}{M^2}\left[1 - \frac{2\,a^4 + 4\,a^2 - 5}{1 + a^2}\,\frac{Q^2}{8\,M^2}\right]\frac{1}{r^2}$$



The Binary System

Phys. Rev. D 60, 104017(1999)

$$ds^2 = -\Psi\,(dt - \omega\, d\varphi)^2 + \frac{\Delta\,\sin^2\theta}{\Psi}\,(d\varphi)^2 + \rho^2\,\left[\frac{(dr)^2}{\Delta} + (d\theta)^2\right]$$

where

$$\begin{aligned}
\Delta &= r^2 - 2\,M\,r + \alpha^2 + Q^2 \\
\rho^2 &= r^2 + \alpha^2\,\cos^2\theta \\
\Psi &= -(\Delta - \alpha^2\,\sin^2\theta)/\rho^2 \\
\omega &= -\alpha\,\sin^2\theta\,[1 + \Psi^{-1}]
\end{aligned}$$

Dilaton Waves

In the large r regime

$$\Phi^{out/in} = A_\phi^{out/in}\, S_\phi^{out/in}\, \frac{e^{\mp i\,\bar\omega\, r_*}}{r^{n_{out/in}}}$$

$$n_{out} = 3$$

$$S_\phi^{out} = \tfrac{\bar\lambda}{C}\,\mathcal{L}_1\, S_0 - \mathcal{L}_1^\dagger\, S_2 - 2\,\alpha\,\bar\omega\, f_{-1}$$

$$A_\phi^{out} = -i\tfrac{\sqrt{2}}{12}\,\tfrac{a^2}{\bar\omega^2}\, A_2^{out}$$

Near Horizon $r \sim r_+$

$$\Phi^{out/in} = A_\phi^{out/in}\, S_\phi^{out/in}\, \Delta^{n_{out/in}}\, e^{\mp i\, k\, r_*}$$

$$n_{out/in} = 0$$

$$S_\phi^{out/in} = \tfrac{1}{C}\,\left[\cos\theta\,\mathcal{L}_1^\dagger + \sin\theta\right]\, S_2 = s^{(1)}_{out/in}\,\alpha\,\bar\omega + \mathcal{O}(\alpha^2\,\bar\omega^2)$$

$$A_\phi^{out/in} = \tfrac{\sqrt{2}}{16}\,\tfrac{a\,\alpha^2}{\alpha\,\bar\omega}\,\tfrac{A_0^{in}}{M\, r_+^3}$$

Energy Transfer

Flux of Energy for a Given Wave Mode

For $r \to \infty$

$$\left.\frac{dE}{dt\, d\Omega}\right|_{\infty} = r^2\, T^r{}_t$$

For $r \sim r_+$

$$\left.\frac{dE}{dt\, d\Omega}\right|_{+} = \frac{2\, M\, r_+\, \bar{\omega}}{\bar{\omega} - \bar{\omega}_+}\, T_{ij}\, \tilde{l}^i\, \tilde{l}^j$$

Electromagnetic Field

$$\begin{aligned} T_{ij} = & \Big[|\phi_0|^2\, n_i\, n_j + |\phi_2|^2\, l_i\, l_j + 2\, |\phi_1|^2\, \left(l_{(i}\, n_{j)} + m_{(i}\, m^*_{j)} \right) \\ & -4\, \phi_0^*\, \phi_1\, n_{(i}\, m_{j)} - 4\, \phi_1^*\, \phi_2\, l_{(i}\, m_{j)} + 2\, \phi_0^*\, \phi_2\, m_i\, m_j \Big] + \text{c.c.} \end{aligned}$$

Dilaton Field

$$|T_{ij}| = \frac{Q^2}{2}\, |\partial_i \Phi\, \partial_j \Phi|$$

$$dE_\phi^{out/in}(out) \sim \left(\alpha\, \bar{\omega}\;\, A_2^{out} \right)^2\, \frac{Q^2}{r^4}$$

$$dE_\phi^{out/in}(in) \sim \left(\alpha\, \bar{\omega}\;\, A_0^{in} \right)^2\, \frac{Q^2}{r^4}$$

$$T^r{}_t|_\infty = \begin{cases} \dfrac{1}{2\,\pi}\, \dfrac{|\Phi_2|^2}{4\, r^4} & \text{(outgoing modes)} \\[2ex] -\dfrac{1}{8\,\pi}\, |\Phi_0|^2 & \text{(ingoing modes)} \end{cases}$$

$$\Gamma(r \to \infty) \equiv \frac{dE_\phi^{out/in}}{dE^{out/in}}$$

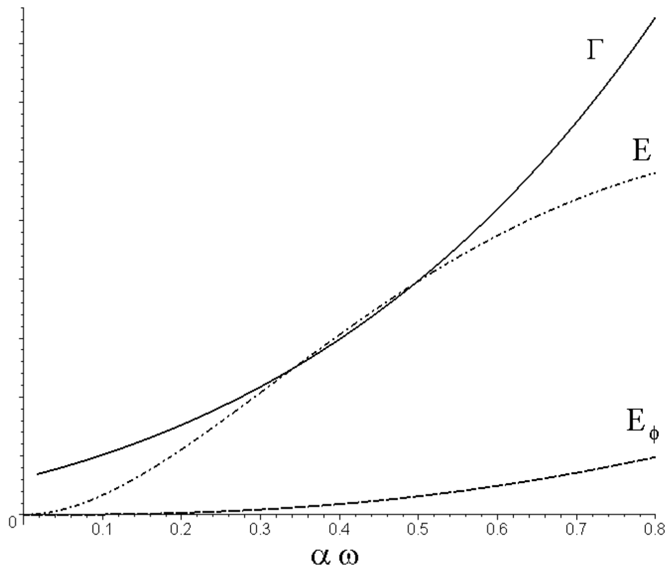


FIG. 1: Large r ($r \to \infty$)

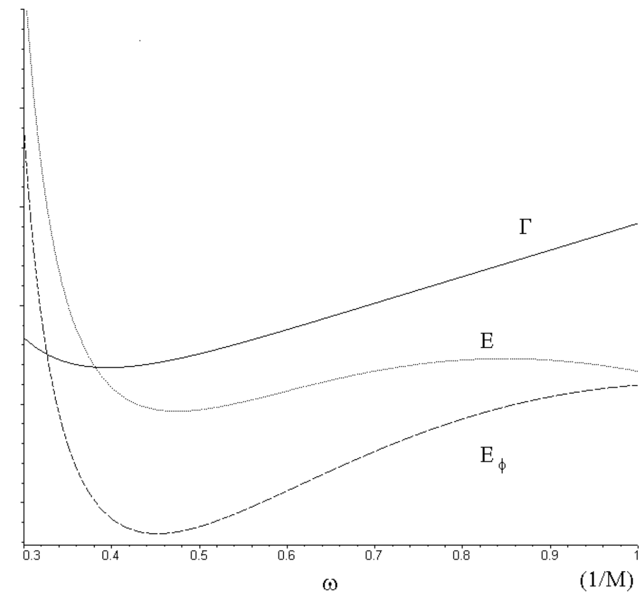


FIG. 2: Near horizon ($r \sim r_+$)

$$E_\phi^{in/out} \sim \int_{r_m}^{R} dE_\phi^{in/out} \sim \frac{1}{r_m^3} - \frac{1}{R^3}$$

Electromagnetic Waves Propagating Around Dilatonic Stars

Phys. Rev. D 70, 044026(2004)

Janis-Newman-Winicour Solution

Invariant Line Element

$$-\left[\frac{2\,R - r_0\,(\mu-1)}{2\,R + r_0(\mu+1)}\right]^{1/\mu} dt^2 + \left[\frac{2\,R + r_0\,(\mu+1)}{2\,R - r_0(\mu-1)}\right]^{1/\mu} dR^2 + r^2(d\theta^2 + \sin^2(\theta) d\varphi^2)$$

where

$$r^2 = \frac{1}{4}\left[2\,R + r_0(\mu+1)\right]^{1+1/\mu}\left[2\,R - r_0(\mu-1)\right]^{1-1/\mu}$$

and

$$\mu = (1 + 4\,\kappa\,A^2/r_0^2)^{1/2}$$

Field Equations

$$G_{\mu\nu} = -\kappa\,T_{\mu\nu}$$

$$\Box\Phi = 0$$

Energy - Momentum Tensor

$$T_{\mu\nu} = \Phi_{,\,\mu}\,\Phi_{,\,\nu} - \frac{1}{2}\,g_{\mu\nu}\,\Phi_{,\,\alpha}\,\Phi_{,\,\beta}\,g^{\alpha\beta}$$

Dilaton

$$\Phi = \frac{A}{\mu}\,\ln\left[\frac{2\,R - r_0(\mu-1)}{2\,R + r_0(\mu+1)}\right]$$

Invariant Line Element Re-expressed

$$ds^2 = \frac{\Delta}{\rho^2}\, dt^2 - \frac{\rho^2}{\Delta}\, dr^2 - \rho^2\, d\Omega^2$$

where

$$d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$$

$$\rho^2 = (r - r_-)^{1-a}\; (r - r_+)^{1+a}$$

$$\Delta = (r - r_-)\; (r - r_+)$$

$$a = 1/\mu$$

$$r_- = +\frac{r_0}{2}\,\frac{1-a}{a}$$

$$r_+ = -\frac{r_0}{2}\,\frac{1+a}{a}\quad .$$

where a, r_- and r_+ are treated as independent. The limits

$$r_- \longrightarrow 0 \qquad \text{and} \qquad a \longrightarrow -1$$

are taken separately to obtain the Schwarzschild limit.

Dilaton Field

$$\Phi = \sqrt{1-a^2}\, \ln\left(\frac{r-r_-}{r-r_+}\right)$$

The relevant dilatonic quantity is the spatial derivative

$$\partial_r \Phi = \frac{\sqrt{1-a^2}\,(r_- - r_+)}{(r-r_-)\,(r-r_+)} \ ,$$

which determines the energy-momentum tensor

$$T_{\mu\nu} = \delta^r_\mu\, \delta^r_\nu\, (\partial_r \Phi)^2 - \frac{1}{2}\, g_{\mu\nu}\, g^{rr}\, (\partial_r \Phi)^2$$

Non-vanishing components diverge at $r = r_+$,

$$T^t{}_t \sim T^r{}_r \sim T^\theta{}_\theta \sim T^\phi{}_\phi \sim \frac{1-a^2}{(r-r_+)^{2+a}}$$

Quasi-Schwarzschild Regime

$$\begin{cases} r_- = 0 \\ \\ a = -1 + 2\,x^2 \ , \end{cases}$$

Approximate Line Element

$$ds^2 \simeq \left[1 - (1-2\,x^2)\,\frac{r_h}{\rho}\right] dt^2 - \left[1 + (1-2\,x^2)\,\frac{r_h}{\rho}\right] d\rho^2 - \rho^2\, d\Omega^2$$

Ingoing and Outgoing Solutions

$$
\begin{aligned}
\phi_0 &= \frac{A}{\sin(\theta)}\,\frac{e^{+i\,\sigma}}{r-r_h}\left[1+B\left(1-\frac{r_h}{r}\right)^{2\,\tilde{x}}\right] \\
&\simeq \frac{A\,(1+B)}{\sin(\theta)}\,\frac{e^{+i\,\sigma}}{r-r_h}\left[1+\frac{2\,\tilde{x}}{1+B}\,\ln\left(1-\frac{r_h}{r}\right)\right] \\
\phi_2 &= -\frac{A}{\sin(\theta)}\,\frac{e^{-i\,\sigma}}{r}\left[1-B\left(1-\frac{r_h}{r}\right)^{2\,\tilde{x}}\right] \\
&\simeq \frac{A\,(B-1)}{\sin(\theta)}\,\frac{e^{-i\,\sigma}}{r}\left[1-\frac{2\,\tilde{x}}{1-B}\,\ln\left(1-\frac{r_h}{r}\right)\right] \ ,
\end{aligned}
$$

$$
A = A_{lm}(\omega) \qquad\qquad B = B_{lm}(\omega)
$$

Darkened Stars

Energy ratios – Schwarzschild case

$$\Gamma_{\rm Schw} \simeq \left[\frac{\phi_2^{(0)}(r_o)}{\phi_2^{(0)}(r_e)}\right]^2 \sim \left(\frac{r_e}{r_o}\right)^2$$

Energy ratios – JNW case

$$\Gamma_{\rm JNW} \sim \left(\frac{r_e}{r_o}\right)^2 \left[\frac{1-\tilde{x}\,\ln\left(1-\frac{r_h}{r_o}\right)}{1-\tilde{x}\,\ln\left(1-\frac{r_h}{r_e}\right)}\right]^2$$

$$\begin{aligned}\Gamma_{\rm JNW} &\sim \Gamma_{\rm Schw}\left[1+2\,\tilde{x}\,\ln\left(1-\frac{r_h}{r_e}\right)\right]\\ &\simeq \Gamma_{\rm Schw}\left[1-2\,\tilde{x}\,\frac{r_h}{r_e}\right]\end{aligned}$$

Freely Falling Source

Perturbed E-M Field Tensor

$$F_{\mu\nu} = F^{(0)}_{\mu\nu} + Q\, F^{(1)}_{\mu\nu}$$

$$F^{(0)}_{\mu\nu} = 0$$

Maxwell–Dilaton Equation

$$\partial_\nu \left(\sqrt{-g}\, e^{-q\,\Phi}\, F^{(1)\,\mu\nu} \right) = 4\,\pi\, \sqrt{-g}\, j^\mu$$

Explicit Form of the E–M Field Tensor

$$F^{(1)}_{\mu\nu} = \begin{bmatrix} 0 & f_{01}\, Y^m_l & f_{02}\,\partial_\theta Y^m_l & -f_{02}\,\sin(\theta)\,\partial_\phi Y^m_l \\ * & 0 & f_{12}\,\partial_\theta Y^m_l & f_{12}\,\partial_\phi Y^m_l \\ * & * & 0 & 0 \\ * & * & 0 & 0 \end{bmatrix}$$

Form of $f_{ij}(r,t)$

$$f_{12}(r,t) = \hat{f}_{12}(r)\, \exp(i\,\omega\,t)$$

Equation satisfied by $\hat{f}_{12}$

$$\frac{d^2\hat{f}_{12}}{dr^2} + \frac{r_h\,(3+2\,\tilde{x})}{r\,(r-r_h)}\,\frac{d\hat{f}_{12}}{dr} + \left[\frac{\omega^2\,r^2}{(r-r_h)^2} - \frac{r_h\,(2\,r-3\,r_h)}{r^2\,(r-r_h)^2}\,(1+2\,\tilde{x}) - \frac{l\,(l+1)}{r\,(r-r_h)}\right]\hat{f}_{12}$$
$$= S(r)$$

where the source term is given by

$$S(r) = -\frac{e^{i\,\omega\,T(r)}}{(r-r_h)^2}\,\left[1-2\,\tilde{x}\,\ln\left(1-\frac{r_h}{r}\right)\right]$$

and

$$T(r) = r_h\left[2\,\frac{r^{1/2}}{r_h^{1/2}} - \frac{2\,r^{3/2}}{3\,r_h^{3/2}} + \ln\frac{(r/r_h)^{1/2}-1}{(r/r_h)^{1/2}+1}\right]$$

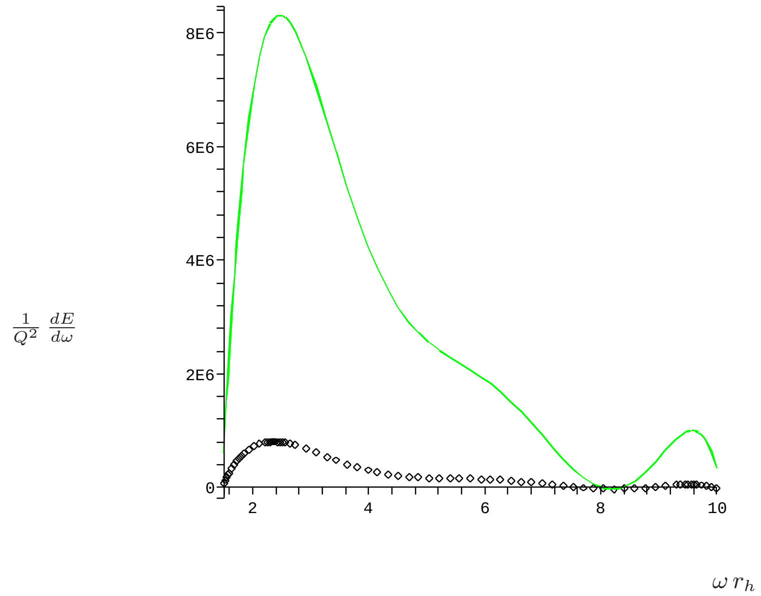


FIG. 3: Energy distribution for a particle of unit charge falling into a four-dimensional black hole of mass M.

Frequency Distribution of the Energy

$$\left\langle \frac{dE}{d\omega} \right\rangle = \frac{l\,(l+1)}{2\,\pi}\,\hat{f}_{12}\,\hat{f}^*_{12}$$

Energy of the Radiation

$$\Delta E \simeq C_{(4)}\,\frac{Q^2}{M}$$

$$C_{(4)} = \text{Area under the curve}$$

Summary

- **Physical characteristics of black holes (mass, charge, angular momentum) are affected by the presence of a dilaton field.**

- **Dilatonic waves absorb energy.**

- **Propagation of electromagnetic waves affected by the dilaton.**

- **Star light dimmed if dilatonic field is present – implications for ‘dark energy‘ scenario.**

- **Enhancement effect in frequency spectrum if dilatonic field is present – should be detectable if $dE/d\omega$ can be accurately measured.**