A quantum gravitational anisotropic inflationary scenario

(based on work to appear with Brajesh Gupt)

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Motivation

Singularities: Boundaries of spacetime where physical laws break down and evolution stops.

- First discovered in homogenous and isotropic spacetimes. Were believed to be artifacts of simplifying assumptions of the underlying symmetry (Eddington, Tolman 1940's).
- Turned out to be present in more general situations (inclusion of anisotropies) (Raychaudhuri 1955)
- Generic properties of classical spacetime in General Relativity (Penrose, Hawking 1960's).

Classical theory fails to resolve singularities. New physics needed.

- Can a quantum theory of gravity resolve cosmological and black hole singularities?
- What is the new physics near the classical singularities? Does it overcome limitations of models of the early universe (such as the past incompleteness of inflationary scenarios)?

Loop quantum gravity: a brief overview

- A non-perturbative canonical quantization of gravity.
- Phase space variables: Ashtekar-Barbero connection A_a^i (analogous to the vector potential) and the conjugate triad E_i^a (analogous to the electric field).Gravity expressed kinematically as a gauge theory.
- Elementary variables for quantization: Holonomies of connection along curves (fundamental excitations of quantum geometry), fluxes of triad across surfaces.
- Constraints are expressed in terms of holonomies of connection and fluxes of triads and then quantized.
- A key kinematical prediction: Geometrical operators have a discrete spectrum. Differential geometry of the classical theory replaced by a quantum geometry.
- High mathematical precision, thanks to the work of several people: (Ashtekar, Baez, Bombelli, Isham, Gambini, Jacobson, Lewandowski, Marolf, Pullin, Rovelli, Smolin, Thiemann, ...)

Loop quantum cosmology

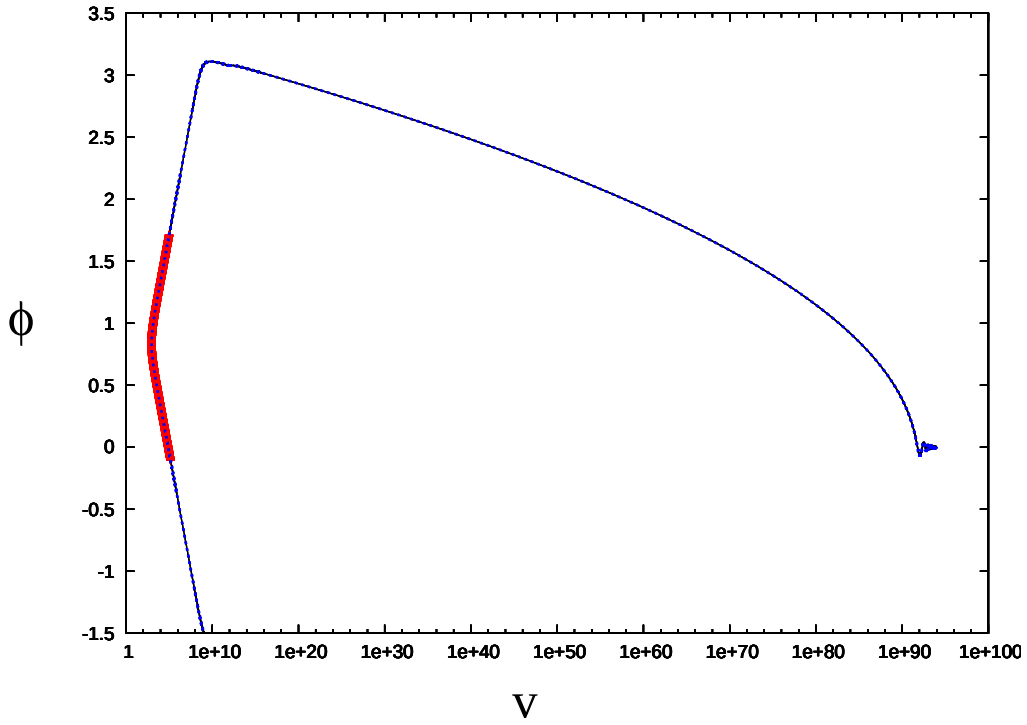
Though a full theory of quantum gravity is not yet available, quantization can be performed for symmetric models (Bojowald (2001); Ashtekar, Bojowald, Lewandowski (2003)).

- Complete quantization can be performed i.e. a physical Hilbert space, inner product, observables available. Big Bang is replaced by a big bounce in homogeneous and isotropic models (Ashtekar, Pawlowski, PS 2006)
- Confirmed for various matter models including in the presence of positive or negative cosmological constant, anisotropic models with massless scalar etc. No fine tuning of parameters. No requirements on the equation of state.
- The underlying evolution equation is discrete – a quantum difference equation, resulting from the quantum geometry. For states which correspond to a macroscopic universe at late times, a continuum effective theory capturing the underlying quantum geometry can be derived.

Inflation and isotropic LQC

Inflationary spacetimes are past incomplete (Borde, Guth, Vilenkin 2003)

Turn out to be non-singular in LQC. Complete quantization of isotropic inflationary spacetime with ϕ^2 potential performed. Past singularity is replaced by a bounce when energy density of the universe reaches a universal maxima $\rho = 0.41\rho_{\mathrm{Planck}}$. (Ashtekar, Pawlowski, PS 2013)



Inflation and anisotropic LQC

The initial state of the universe may be highly anisotropic.

- How does an isotropic inflationary universe emerge from a highly anisotropic state?
- Is inflation even possible starting from highly anisotropic initial conditions? (Barrow, Turner 81)
- Does initial anisotropy helps or prevents inflation to occur?

Answers to these questions not possible in General Relativity because of the initial singularity. The anisotropic shear $\sigma^2 \propto a^{-6}$, diverges to infinity as the mean scale factor tends to zero.

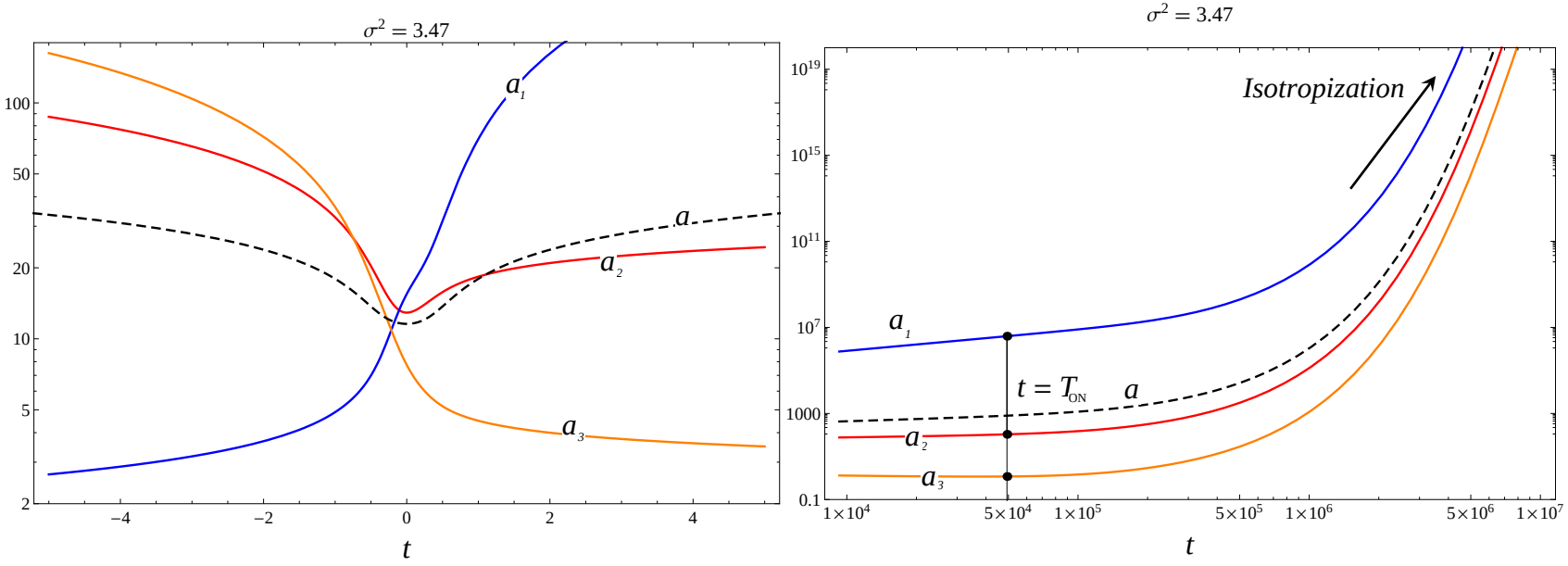
Answers attempted in brane-world models. Presence of anisotropy increases the number of e-foldings $N = \ln(a_{\rm final}/a_{\rm initial})$ if the inflaton is initally rolling down (Maartens, Sahni and Saini 01). However, due to the initial singularity, questions about the evolution from the initial anisotropic state remain unanswered.

Can LQC provide some insights on these issues?

Inflation and anisotropic LQC

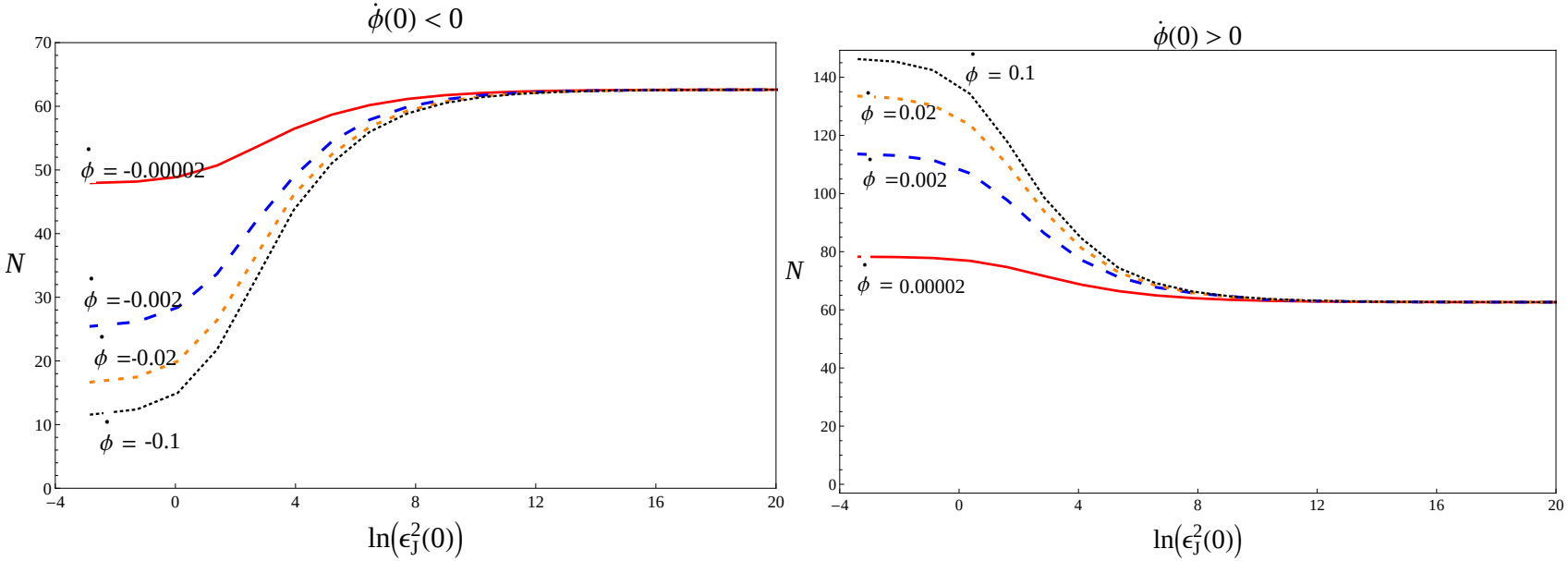
In contrast to the classical theory, in LQC, the energy density, expansion and shear scalars are bounded above by universal values: $\rho_{\rm max} = 0.41\rho_{\rm Planck}$, $\theta_{\rm max} = 2.78/l_{\rm Planck}$, $\sigma^2_{\rm max} = 11.57/l^2_{\rm Planck}$
Indicates geodesic completeness independent of any energy conditions (PS 11)

Irrespective of the initial conditions, there is non-singular evolution, and isotropization takes place before or just after the accelerated expansion begins.



E-foldings and anisotropy

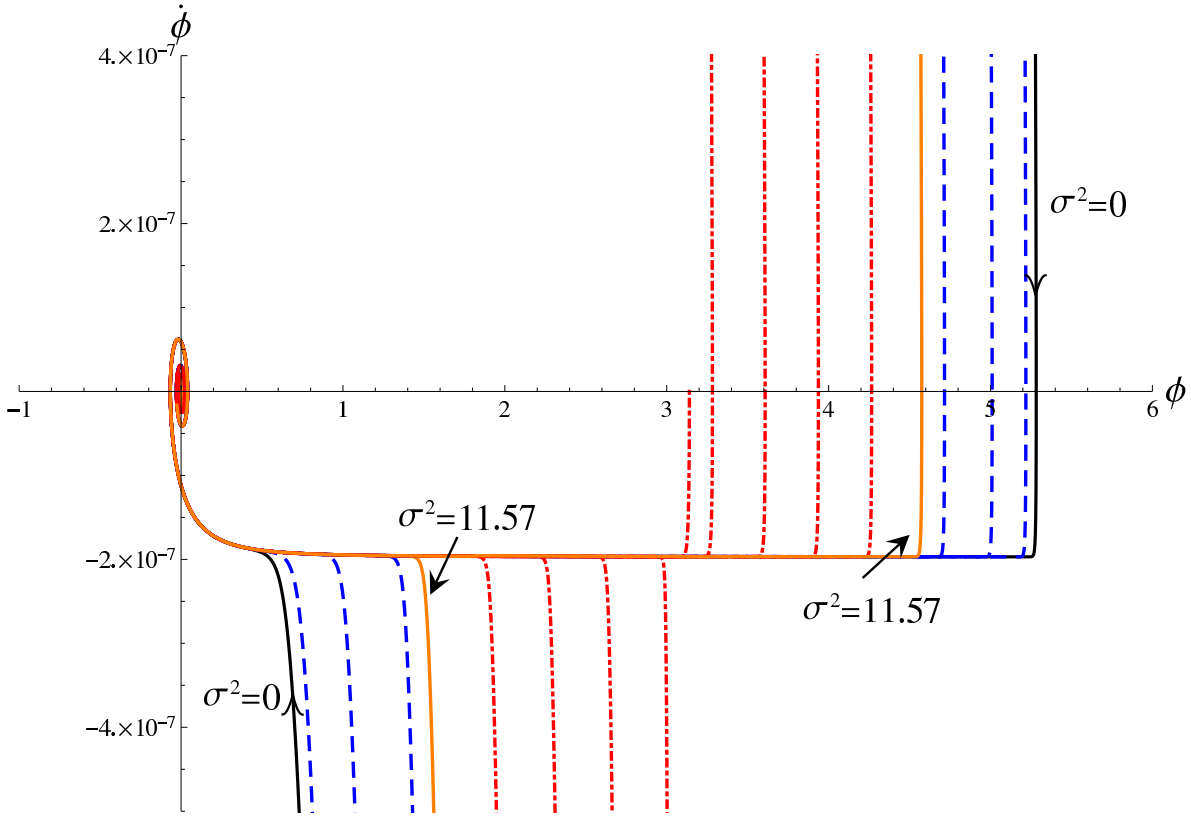
Anisotropy increases Hubble friction in an expanding universe.
(In GR: $H^2 = \frac{8\pi G}{3}\rho + \frac{1}{6}\sigma^2; \ddot{\phi} + 3H\dot{\phi} = -V'(\phi)$)
Leads to an opposite effect on the amount of inflation depending whether the inflaton is initially rolling up or down.



For large values of initial anisotropy in the universe, (denoted by $\epsilon_J^2 = \frac{\sigma^2}{4\pi G\rho}$), number of e-foldings becomes independent of initial conditions the field velocity.

Attractor behavior

At late times, all trajectories in Bianchi-I inflationary spacetime LQC lead to the isotropic slow roll inflation.



Conclusions

- Though we do not yet have a complete theory of loop quantum gravity, known apparatus and techniques can be applied to homogeneous models.
- All of the models which have been quantized so far turn out to be non-singular. Energy density and anisotropic shear are bounded above, leading to a bounce which is a direct result of the underlying discreteness of quantum geometry.
- Onset of slow-roll inflation turns out to be very robust even from highly anisotropic initial conditions given in the Planck regime. Isotropic slow roll inflation an attractor in the LQC phase space.
- Anisotropic shear can significantly effect the number of e-foldings. Presence of anisotropy can result in sufficient inflation, even for highly unfavorable isotropic initial conditions. Can significantly broaden the window for potential observable effects in CMB in certain models (eg. Agullo, Ashtekar, Nelson 2013)