Dark Matter Distribution Around Massive Black Holes:

A Fully General Relativistic Approach

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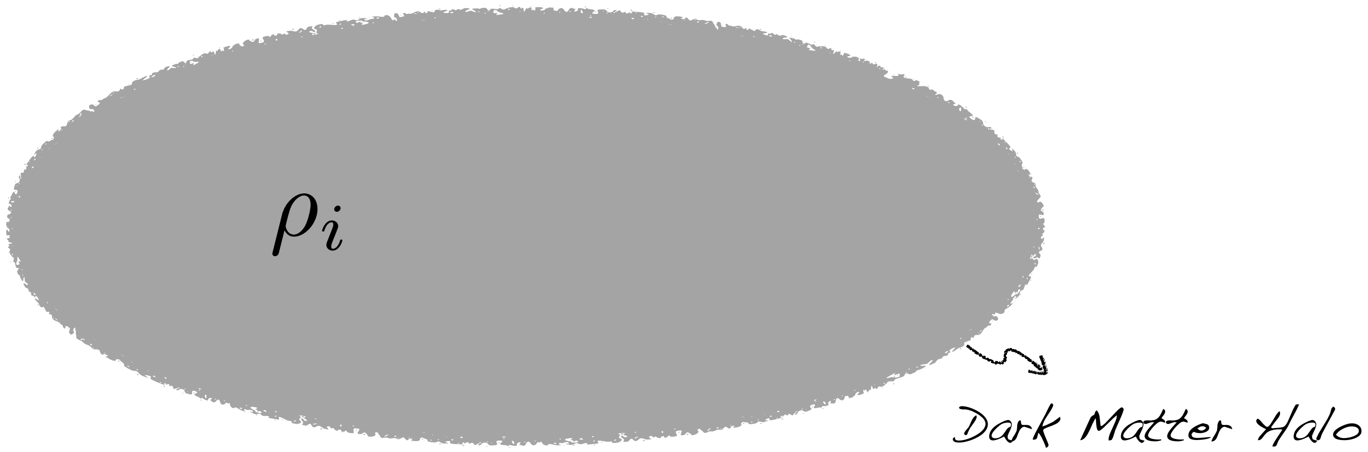
7th Gulf Coast Gravity Meeting
University of Mississippi, MS
April 2013

Question

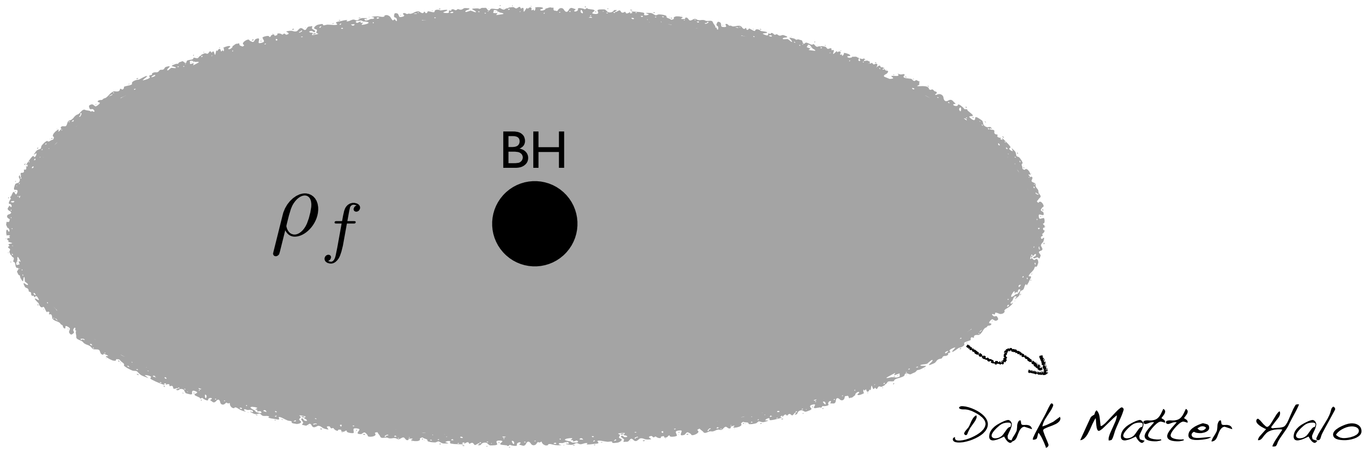
What is the effect of the massive black hole on the dark matter distribution at the galactic centre?

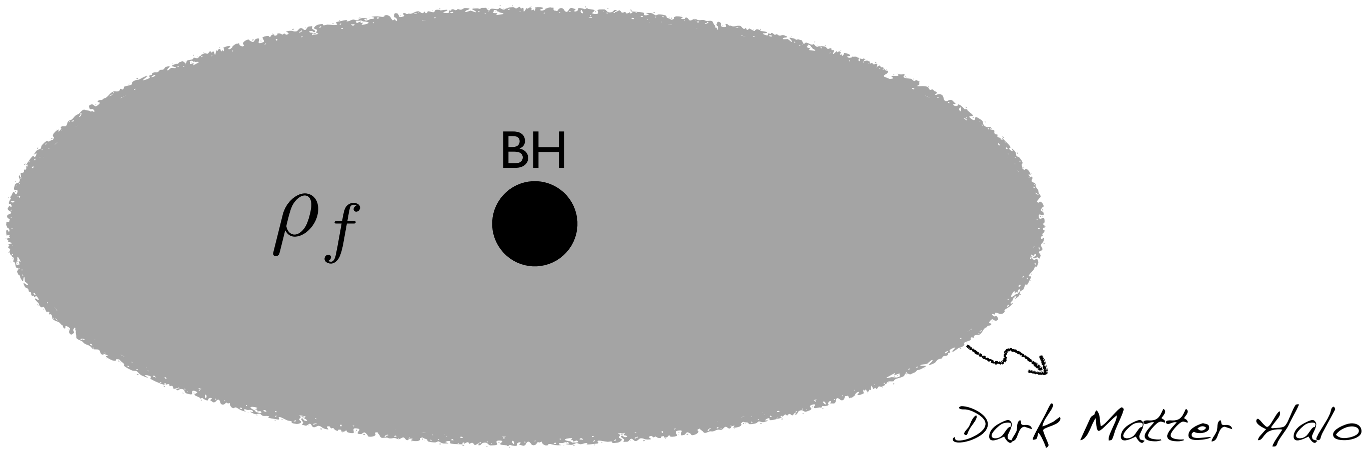
Why do we care?

- If BH formation increases the DM density locally, it also boosts the number of secondary particles and improves the prospects for **indirect DM detection**.
- If formation of a rotating BH makes the DM distribution non-spherical, then its gravity might affect the **precession of stars' orbital planes** near SgrA*.



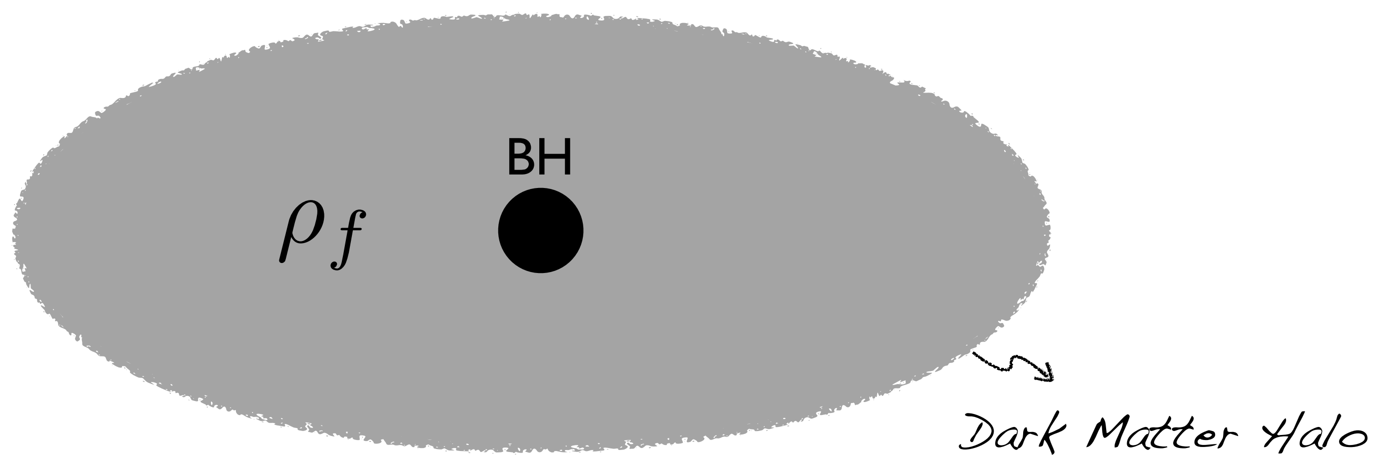
ρ_i
Dark Matter Halo


BH
ρ_f
Dark Matter Halo



Assumption:

The growth of the massive black hole is adiabatic (slow).



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$$\rho_i \xrightarrow{\text{Eddington's Method}} f_i \xrightarrow{\text{Adiabatic Invariants}} f_f \xrightarrow{\text{integration in the phase space}} \rho_f$$

Newtonian Analysis

$$\rho(\mathbf{r}) = \int f(\mathbf{r}, \mathbf{v}) \mathrm{d}^3\mathbf{v}$$

Change of variables: $\mathbf{v} \rightarrow E \, , \, L \, , \, L_z$

$$\rho(r) = \frac{4\pi}{r^2} \int_{\Phi(r)}^{E_{\max}} \mathrm{d}E \int_0^{L_{\max}} L \mathrm{d}L \frac{f(E, L)}{\sqrt{2E - 2\Phi(r) - L^2/r^2}}$$

Adiabatic Invariants:

$$\begin{cases} I_r(E, L) \equiv \oint v_r \mathrm{d}r \\ I_\theta(L, L_z) \equiv \oint v_\theta \mathrm{d}\theta \\ I_\phi(L_z) \equiv \oint v_\phi \mathrm{d}\phi \end{cases} \Rightarrow \; L \text{ and } L_z \text{ remain constant.}$$

$$\underbrace{I_{r,i}(E_i, L)}_{\text{DM}} = \underbrace{I_{r,f}(E_f, L)}_{\text{DM+BH}} \quad \Longrightarrow \quad E_i = E_i(E_f, L)$$

$$\boxed{f_f(E_f, L) = f_i(E_i(E_f, L), L)}$$

Relativistic Analysis

Mass current: $$J^\mu(x) \equiv \int f^{(4)}(p)\frac{p^\mu}{\mu}\sqrt{-g}\mathrm{d}^4p$$

$$J^\mu \equiv \rho u^\mu$$

$$u_\mu u^\mu = -1$$

$$\Rightarrow \rho = (-J_\mu J^\mu)^{1/2}$$

$$= \sqrt{-g_{00}}\, J^0 \quad \text{if} \quad J^i = 0 \text{ (in spherical symmetry)}$$

Change of variables: $$p^0,\ p^r,\ p^\theta,\ p^\phi \quad \rightarrow \quad \underbrace{\mathcal{E},\ C,\ L_z,\ \mu}_{\text{constants of motion}}$$

Spherical symmetry limit: $$C \ = \ L^2$$

Kerr metric in Boyer-Lindquist coords:

$$\mathrm{d}s^2 = -\left(1 - \frac{2Gmr}{\Sigma^2}\right)\mathrm{d}t^2 + \frac{\Sigma^2}{\Delta}\mathrm{d}r^2 + \Sigma^2\mathrm{d}\theta^2 - \frac{4Gmra}{\Sigma^2}\sin^2\theta\mathrm{d}t\mathrm{d}\phi + \left(r^2 + a^2 + \frac{2Gmra^2\sin^2\theta}{\Sigma^2}\right)\sin^2\theta\mathrm{d}\phi^2$$

$$a \equiv J/m$$
$$\Sigma^2 \equiv r^2 + a^2\cos^2\theta$$
$$\Delta \equiv r^2 + a^2 - 2Gmr$$

$$\mathcal{E} \equiv -u_0 = -g_{00}u^0 - g_{0\phi}u^\phi$$
$$L_z \equiv u_\phi = g_{0\phi}u^0 + g_{\phi\phi}u^\phi$$
$$C \equiv \Sigma^4\left(u^\theta\right)^2 + \sin^{-2}\theta L_z^2 + a^2\cos^2\theta(1-\mathcal{E}^2)$$
$$\mu^2 = -g_{\mu\nu}p^\mu p^\nu$$

$$\sqrt{-g}\,\mathrm{d}^4p = \frac{2\mu^3}{\Sigma^2\Delta|u_r||u^\theta|\sin\theta}\mathrm{d}\mathcal{E}\mathrm{d}C\mathrm{d}L_z\mathrm{d}\mu$$

Schwarzschild BH: $$\rho(r) = -\frac{4}{r^4\sqrt{1-2Gm/r}\sin\theta}\int f(\mathcal{E},L)\frac{\mathcal{E}}{u^r u^\theta}\mathrm{d}\mathcal{E}\mathrm{d}L^2\mathrm{d}L_z$$

$$u^\theta = r^{-2}\sqrt{L^2 - \frac{L_z^2}{\sin^2\theta}}$$

$$u^r = \sqrt{V(r)}$$

$$V(r) = \mathcal{E}^2 - (1-\frac{2m}{r})(1+\frac{L^2}{r^2})$$

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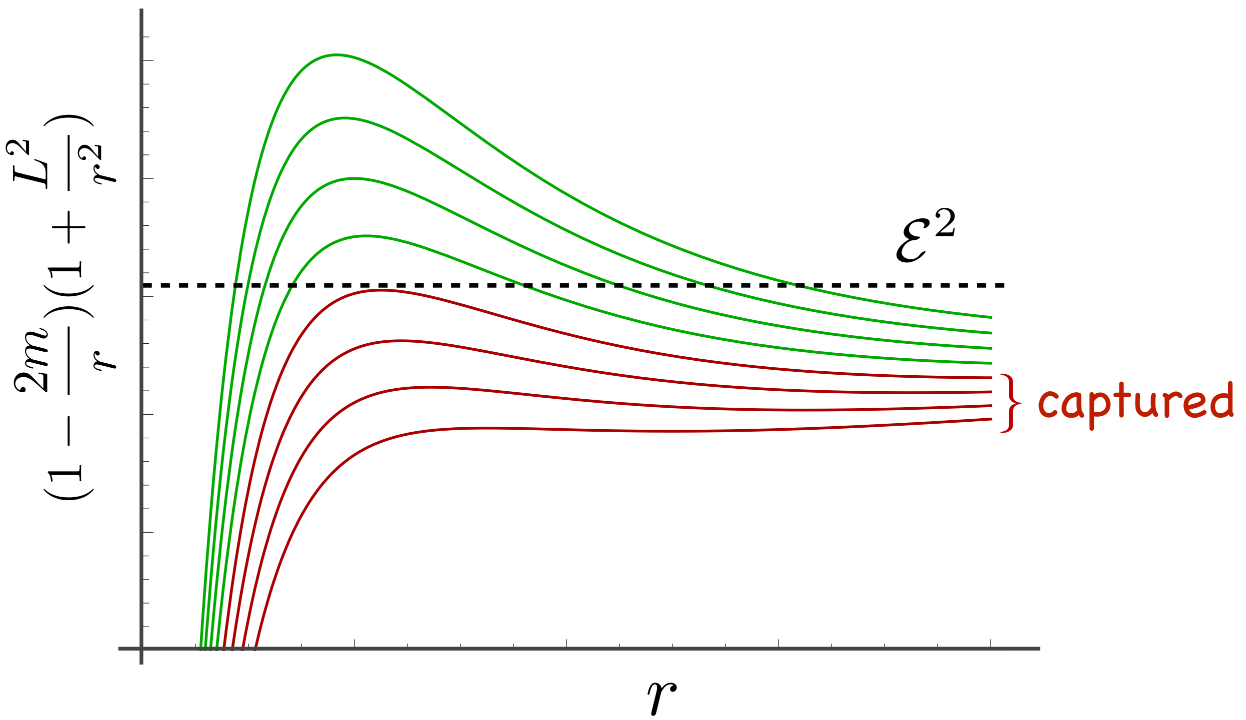

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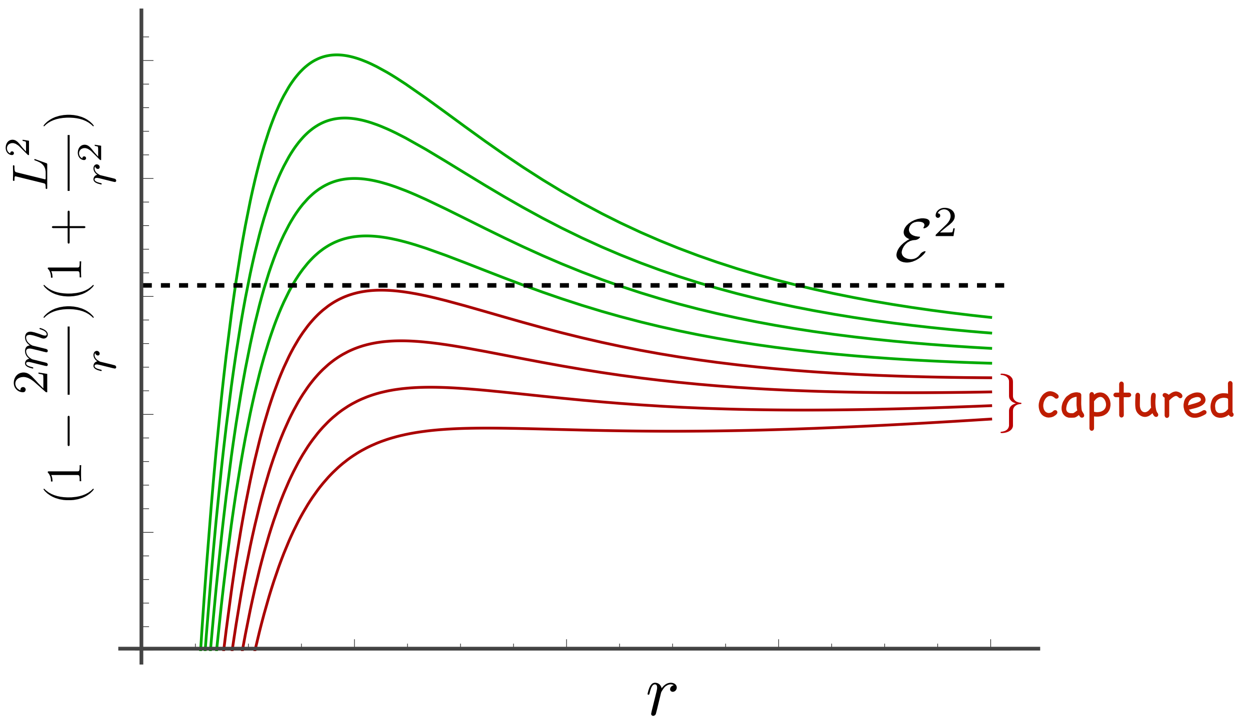

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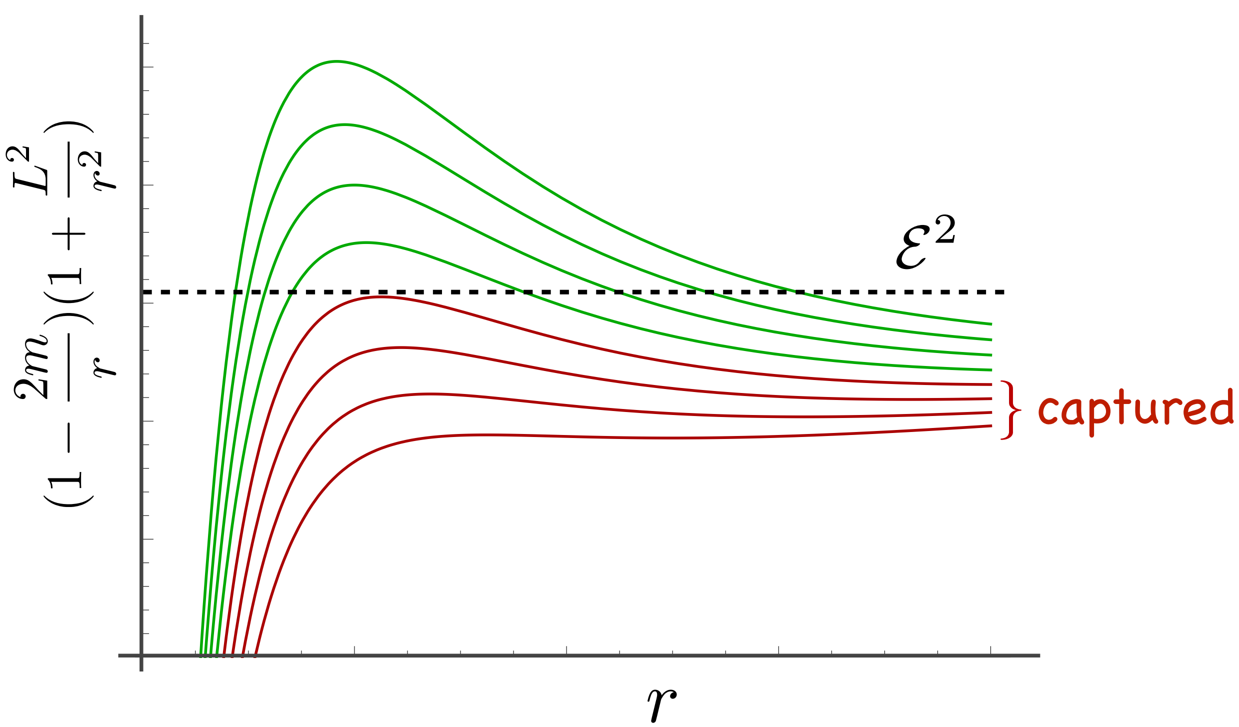
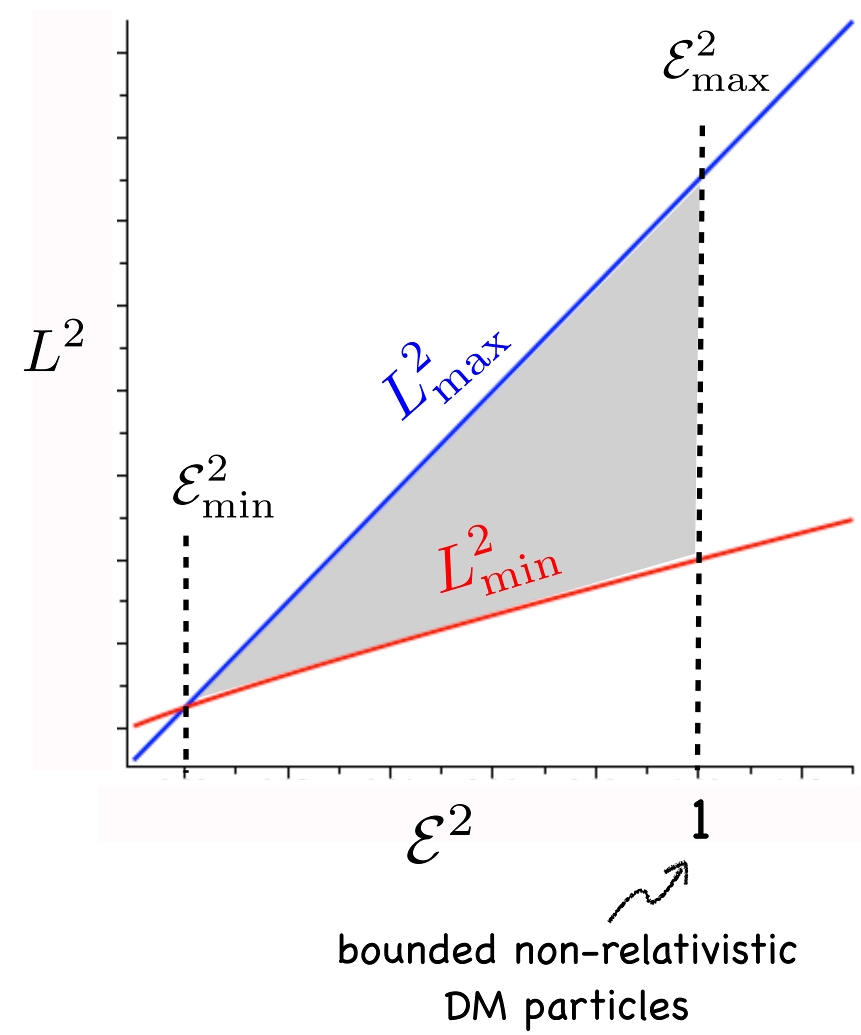
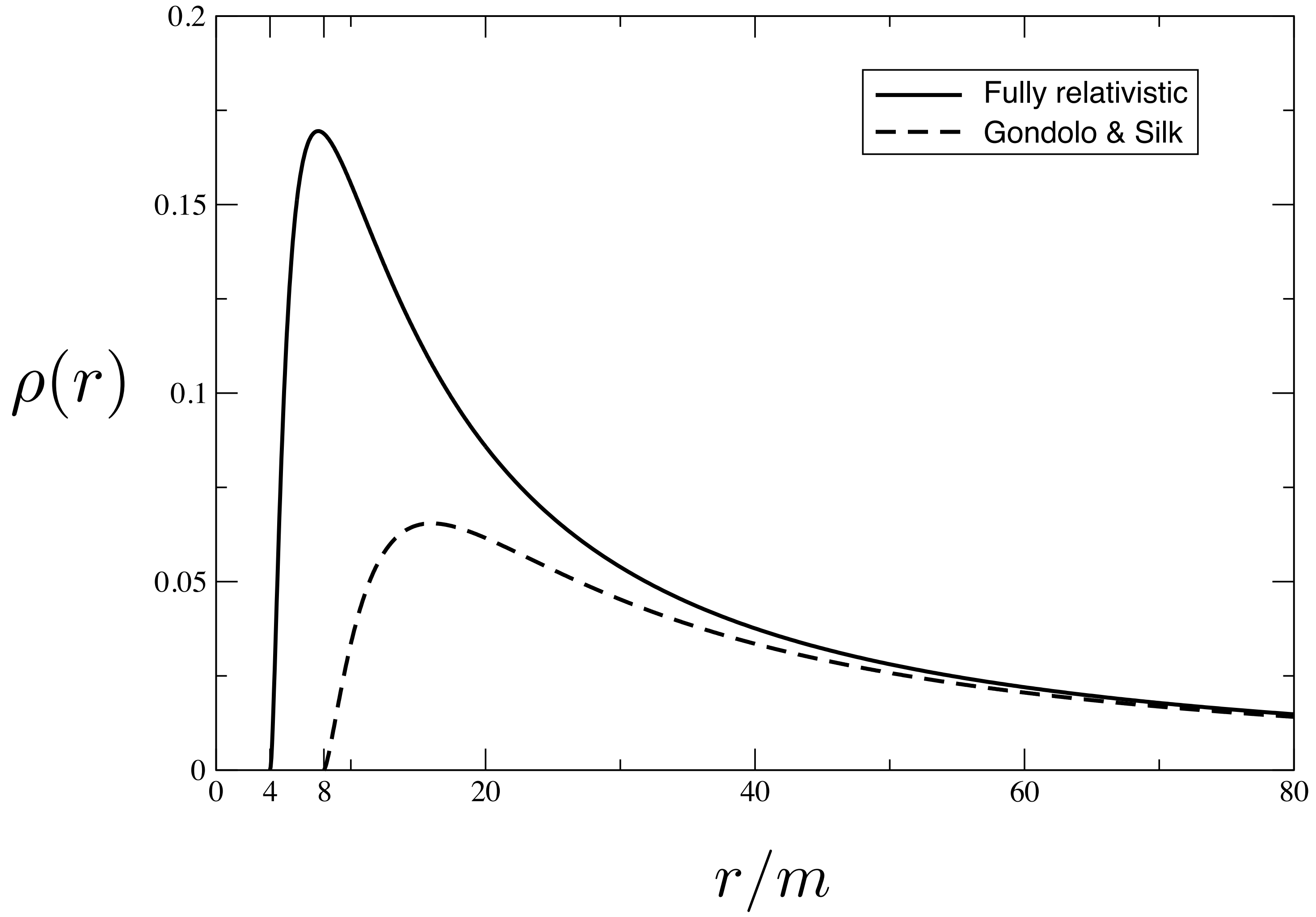




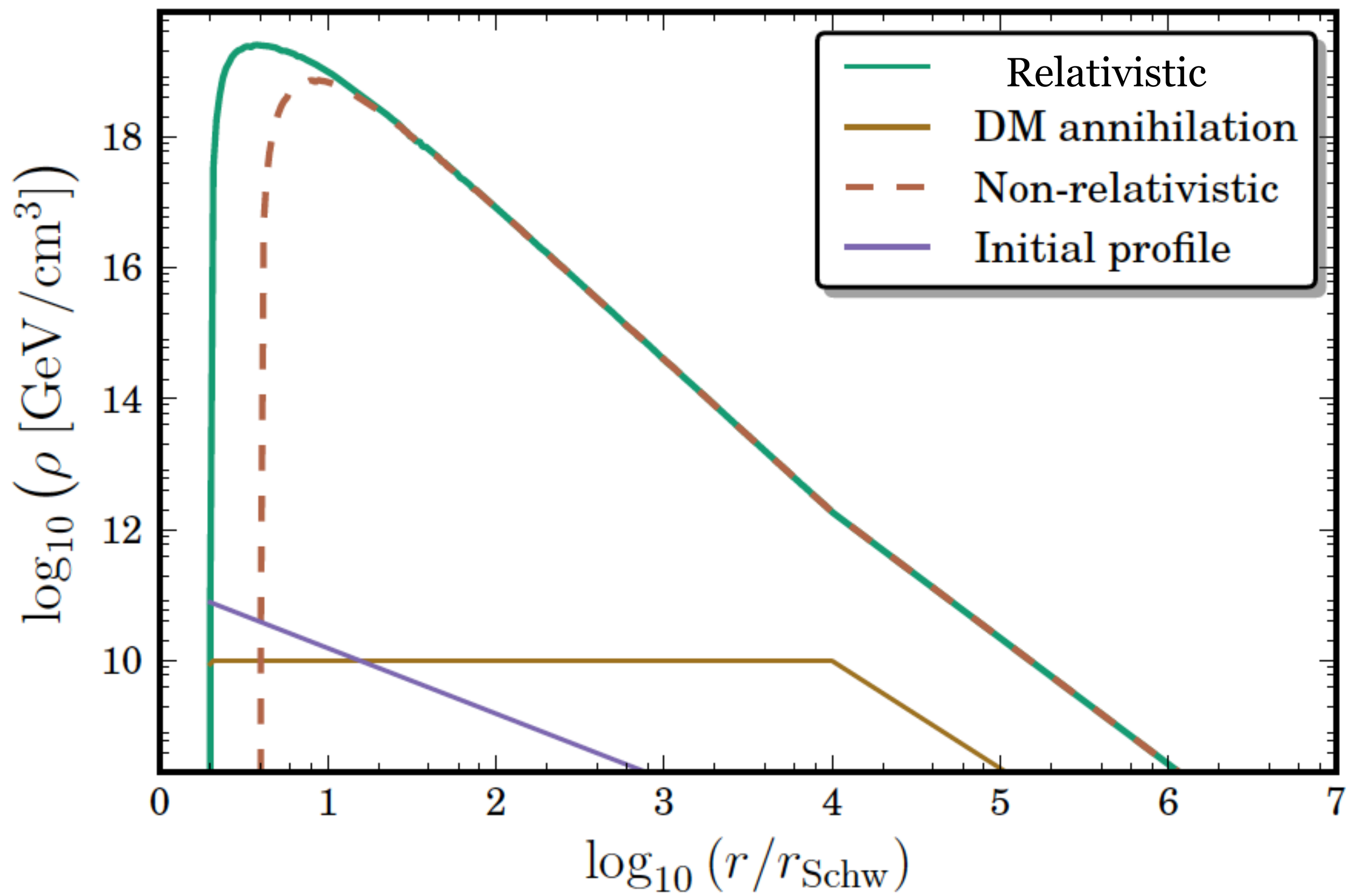
constant distribution function: $f(\mathcal{E}, L) = f_0 = \mathrm{const}$



Hernquist profile density: $\rho_i(r) = \rho_H(r) = \dfrac{\rho_0}{(r/a)(1+r/a)^3}$

If DM particles self-annihilate:

$$\rho(r) = \frac{\rho_{\rm core}\,\rho_f(r)}{\rho_{\rm core} + \rho_f(r)}$$



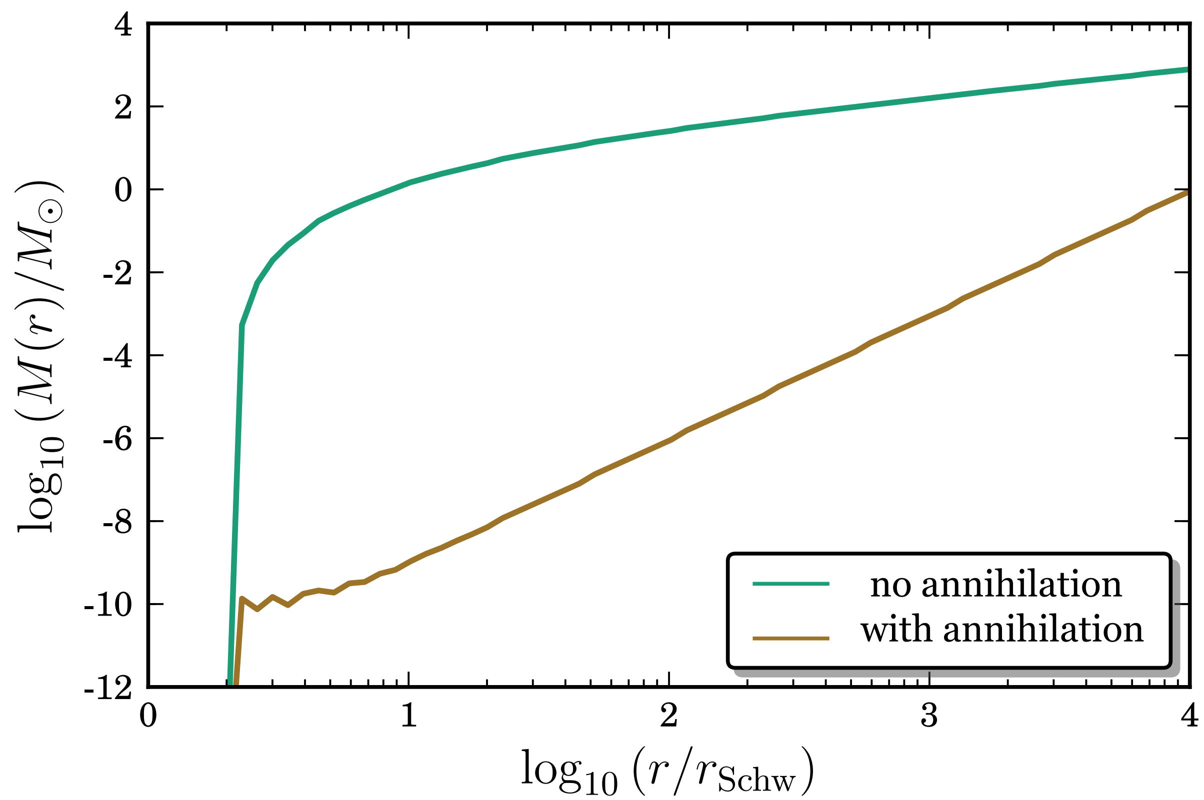
Periastron precession with a DM spike:

$$M(r) = 4\pi \int \rho(r) r^2 \mathrm{d}r$$

$$M(r) \sim m_0 \left(\frac{r}{r_0}\right)^q$$

$$\Delta\omega_{\mathrm{DM}} = -\pi q \left(\frac{m_0}{m}\right)\left(\frac{a}{r_0}\right)^q (1-e^2)^{1/2} f_{\mathrm{q}}(e)$$

$$1 \leq f_{\mathrm{q}}(e) \leq 2$$



$$m_0 = \begin{cases} 10^3\ M_\odot \ , & q=1 \quad \text{no self-annihilation} \\ 1\ M_\odot \ , & q=3 \quad \text{with self-annihilation} \end{cases} \quad , \quad r_0 = r_{\mathrm{Sch}} \times 10^4 = 10^4 \times (2m)$$

For S2 star $(a = 4.6\ \mathrm{mpc},\ e = 0.88,\ P = 15.5\ \mathrm{years})$:

$$\Delta\omega_{\mathrm{GR}} = 44\ \mathrm{arc\ sec/yr}$$

$$\Delta\omega_{\mathrm{DM}} = 7.5\ \mathrm{arc\ sec/yr}$$, (no annihilation)

summary:

- We have developed a fully relativistic approach for adiabatic growth of BH in DM distribution.
- Significant differences with results of G&S (1999) have been found: In particular ρ vanishes at r=4m not 8m, and it is substantially larger at small r than what G&S found (The profile is more cuspy).
- The pericenter precession caused by the DM spike is potentially detectable if DM does not self annihilate.

Future work:

- How will the enhancement of the DM density due to relativistic considerations boost the prospect for the indirect detection of DM?
- Considering a rotating BH: How non-spherical does the DM distribution become?

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