INSPIRALLING COMPACT BINARIES IN SCALAR-TENSOR THEORIES OF GRAVITY: EQUATIONS OF MOTION TO 2.5 POST-NEWTONIAN ORDER

Saeed Mirshekari, Clifford Will

Washington University in St. Louis
University of Florida, Gainesville

ePrint: ArXiv:1301.4680 PRD

Seventh Gulf Coast Gravity Meeting
University of Mississippi, Oxford (MS), April 19-20 2013

Outline

- Introduction
  - Scalar-Tensor Theories, What and Why?
  - Outline of the Problem
  - Method and Results
  - DIRE (Direct Integration of Relaxed Einstein equations) Adapted to Scalar-Tensor Theory
  - Formal Structure of Near Zone Fields
  - Matter Source Model (Eardley approach)
  - Two-body Equations of Motion
  - Results
- Summary and Future Work

Scalar-Tensor Theories, What and Why?

- Action in the metric representation

$$I = \frac{1}{16\pi}\int [\phi R - \frac{\omega(\phi)}{\phi} g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}] \times \sqrt{|g|}d^4x + I_{NG}(q_A, g_{\mu\nu})$$

- Field Equations

$$G_{\mu\nu} = \frac{8\pi}{\phi}T_{\mu\nu} + \frac{\omega(\phi)}{\phi^2}\left(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\phi_{,\lambda}\phi^{,\lambda}\right) + \frac{1}{\phi}\left(\phi_{;\mu\nu} - g_{\mu\nu}\Box_g\phi\right)$$

$$\Box_g\phi = \frac{1}{3+2\omega(\phi)}\left(8\pi T - 16\pi\phi\frac{\partial T}{\partial\phi} - \frac{d\omega}{d\phi}\phi_{,\lambda}\phi^{,\lambda}\right)$$

- One of the simplest and most natural ways to modify GR

- Many modern theories which try to unify gravity and microscopic physics or explain dark energy in cosmology, demand a scalar field in addition to the metric tensor.

Outline of the Problem

- Finding Equations Of Motion (EOM) for non-spinning compact object binaries, including black holes, to 2.5PN order.
- Adapting DIRE (Direct Integration of Relaxed Einstein equations) formalism (Wiseman-Pati-Will) to scalar-tensor theory.
- Incorporating compact, self-gravitating bodies (Eardley approach).
- Finding the Lagrangian of the system together with the conserved Energy and Linear Momentum.
- Finding the radiation reaction terms through 2.5PN order and comparing with fluxes

Method: DIRST

(Direct Integration of the Relaxed Scalar-Tensor equations)

- New gothic metric

$$\tilde{\mathfrak{g}}^{\mu\nu} \equiv \varphi\sqrt{|g|}g^{\mu\nu} \equiv \eta^{\mu\nu} - h^{\mu\nu}$$

$$h^{\mu\nu}_{,\nu} = 0$$ Gauge Condition

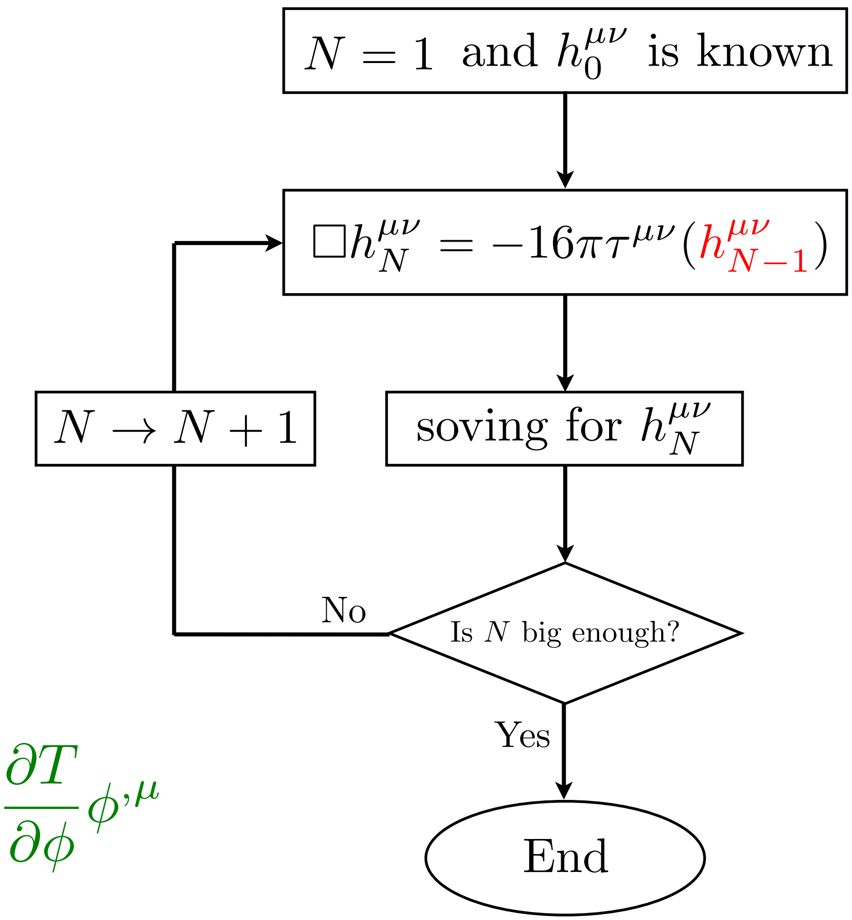
$$\varphi \equiv \frac{\phi}{\phi_0}$$

- Relaxed ST Field Equations

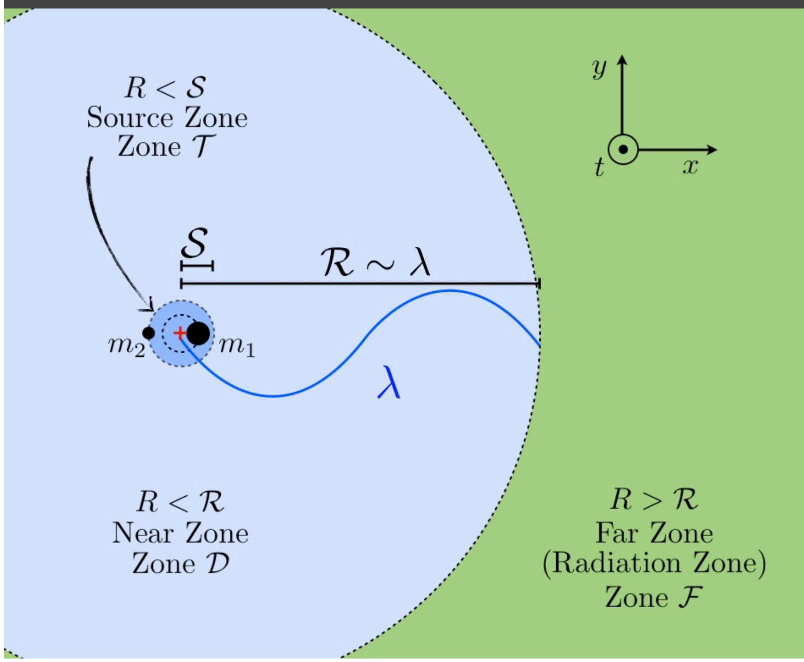
$$\Box_\eta h^{\mu\nu} = -16\pi\tau^{\mu\nu}$$

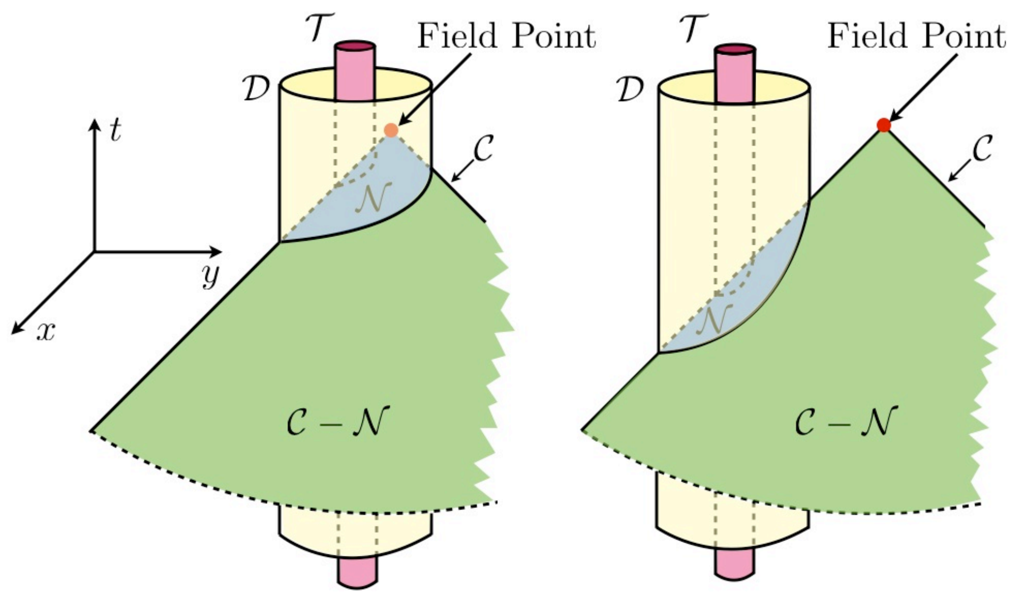
$$\Box_\eta \varphi = -8\pi\tau_s$$

$$T^{\mu\nu}_{;\nu} = \frac{\partial T}{\partial \phi}\phi^{,\mu}$$



Near-zone vs Wave-zone





$$h^{\mu\nu}_{\mathcal{N}}(t,x) = 4\int_{\mathcal{N}} \frac{\tau^{\mu\nu}(t-|x-x'|,x')}{|x-x'|} d^3x'$$

Matter Source Model (Eardley approach)

- Mass of a self-gravitating body is a function of G, which is a function of scalar field ϕ.
- In Eardley approach we consider bodies as point masses but still let the mass to be a function of scalar field.
- Stress-energy tensor

$$T^{\mu\nu}(x^\alpha) = (-g)^{-\frac{1}{2}} \sum_A M_A(\phi)\delta^3(x - x_A)u_A^\mu u_A^\nu (u_A^0)^{-1}$$

- Defining dimensionless "sensitivities" as

$$s_A \equiv \left(\frac{d\ln M_A(\phi)}{d\ln\phi}\right)_{\phi=\phi_0}$$

- BH & NS sensitivities:

$s_{BH} = 0.5, s_{NS} \approx 0.1$ (depending on EOS)

Metric for 2.5PN EOM

$$
\begin{aligned}
g_{00} &= -1 + \left(\frac{1}{2}N + \Psi\right) + \left(\frac{1}{2}B - \frac{3}{8}N^2 - \frac{1}{2}N\Psi - \Psi^2\right) \\
&\quad + \left(\frac{5}{16}N^3 - \frac{1}{4}NB + \frac{1}{2}K^jK^j + \frac{3}{8}N^2\Psi - \frac{1}{2}B\Psi + \frac{1}{2}N\Psi^2 + \Psi^3\right) + \mathcal{O}(\epsilon^4) \\
g_{0i} &= -K^i + \left(\frac{1}{2}N + \Psi\right)K^i + \mathcal{O}(\epsilon^{7/2})\,, \\
g_{ij} &= \delta^{ij}\left\{1 + \left(\frac{1}{2}N - \Psi\right) - \left(\frac{1}{8}N^2 + \frac{1}{2}B + \frac{1}{2}N\Psi - \Psi^2\right)\right\} + B^{ij} + \mathcal{O}(\epsilon^3) \\
(-g) &= 1 + (N - 4\Psi) - (B + 4N\Psi - 10\Psi^2) + \mathcal{O}(\epsilon^3)
\end{aligned}
$$

$$
\begin{aligned}
&N \equiv h^{00} \sim O(\epsilon)\,, \\
&K^i \equiv h^{0i} \sim O(\epsilon^{3/2})\,, \\
&B^{ij} \equiv h^{ij} \sim O(\epsilon^2)\,, \\
&B \equiv h^{ii} \equiv \sum_i h^{ii} \sim O(\epsilon^2)\,, \\
&\Psi \equiv \varphi - 1 \sim \mathcal{O}(\epsilon)
\end{aligned}
$$

Metric for 2.5PN EOM (Example)

- Metric components to their first (non-constant) order

$$g_{00} = -1 + 2G(1-\zeta)U + 2G\zeta U_s$$
$$g_{0i} = -4G(1-\zeta)V^i$$
$$g_{ij} = \delta_{ij}\left[1 + 2G(1-\zeta)U - 2G\zeta U_s\right]$$

$$U = \int_{\mathcal{M}} \frac{\rho^*(t,x')}{|x-x'|} d^3x'$$
$$U_s = \int_{\mathcal{M}} \frac{(1-2s(x'))\rho^*(t,x')}{|x-x'|} d^3x'$$
$$V^i = \int_{\mathcal{M}} \frac{v^i(t,x')\rho^*(t,x')}{|x-x'|} d^3x'$$

$$G \equiv \frac{1}{\phi_0}\frac{4+2\omega_0}{3+2\omega_0}$$
$$\zeta \equiv \frac{1}{4+2\omega_0}$$
$$\omega_0 \equiv \omega(\phi_0)$$

- We have completed the metric up to 2.5PN

2-body EOM to 2.5PN Order

- Christoffel symbols
- Conversion to the baryonic density ρ^*
- Final continuum equations of motion

$$a^i = \frac{dv^i}{dt} = -\Gamma^j_{\alpha\beta} v^\alpha v^\beta + \Gamma^0_{\alpha\beta} v^\alpha v^\beta v^j - \frac{1}{M_A (u^0)^2} \frac{dM_A}{d\phi} (\phi^{,j} - \phi^{,0} v^j)$$

- 2-body equations of motion

$$a^i_A = \frac{1}{m_A} \int_A \rho^* a^i d^3x$$

RESULTS: EOM: Newtonian Order

$$a_N = G\left(1 - \zeta + \zeta(1 - 2s_1)(1 - 2s_2)\right)\frac{m_2 N_i}{r^2}$$

acceleration of body-1

$$G \equiv \frac{1}{\phi_0}\frac{4+2\omega_0}{3+2\omega_0}$$

$$\zeta \equiv \frac{1}{4+2\omega_0}$$

$\omega_0 \to \infty$

$$G \to \frac{1}{\phi_0}$$

$$\zeta \to 0$$

$$a_N = \frac{1}{\phi_0}\frac{m_2 N_i}{r^2}$$

$ST \to GR$

BBH

$\omega_0 \not\to \infty$

$$s_1 = \frac{1}{2}$$

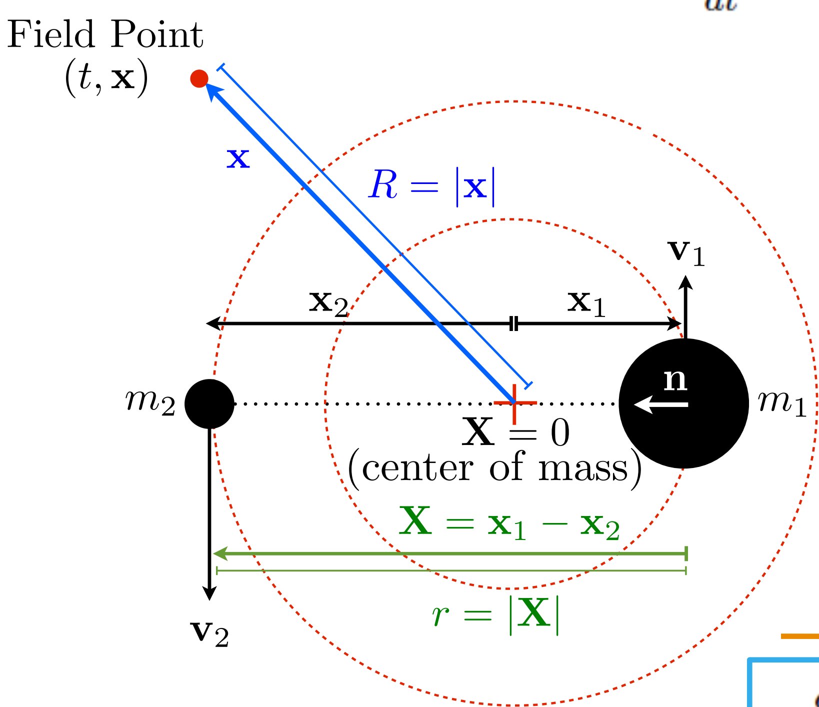
$$s_2 = \frac{1}{2}$$

$$a_N = G(1-\zeta)\frac{m_2 N_i}{r^2} = \frac{1}{\phi_0}\frac{m_2 N_i}{r^2}$$

$BH_{ST} \equiv BH_{GR}$

Full EOM: Up to 2.5PN
i.e. 5 orders beyond the Newtonian order: (v/c)^5

$$\frac{d^2\mathbf{x}}{dt^2} = -\frac{G\alpha m}{r^2}\mathbf{n} + \frac{G\alpha m}{r^2}\left[\mathbf{n}(A_{PN} + A_{2PN}) + \dot{r}\mathbf{v}(B_{PN} + B_{2PN})\right]$$
$$+\frac{8}{5}\eta\frac{(G\alpha m)^2}{r^3}\left[\dot{r}\mathbf{n}(A_{1.5PN} + A_{2.5PN}) - \mathbf{v}(B_{1.5PN} + B_{2.5PN})\right],$$



$$\boxed{\alpha} = \frac{3+2\omega_0}{4+2\omega_0} + \frac{(1-2s_1)(1-2s_2)}{4+2\omega_0},$$
$$\boxed{m} \equiv m_1 + m_2, \boxed{\eta} \equiv m_1 m_2/m^2, \boxed{\mathbf{v}} \equiv \mathbf{v}_1 - \mathbf{v}_2;$$

Full EOM: Up to 2.5PN
i.e. 5 orders beyond the Newtonian order

$$\boxed{A_{PN}} = -(1+3\eta+\bar{\gamma})v^2 + \frac{3}{2}\eta\dot{r}^2 + 2(2+\eta+\bar{\gamma}+\bar{\beta}_+ - \psi\bar{\beta}_-)\frac{G\alpha m}{r}\,, \quad \boxed{B_{PN}} = 2(2-\eta+\bar{\gamma})\,,$$

$$\boxed{A_{1.5PN}} = \frac{5}{2}\zeta\mathcal{S}_-^2\,, \qquad \boxed{B_{1.5PN}} = \frac{5}{6}\zeta\mathcal{S}_-^2\,.$$

$$\begin{aligned}\boxed{A_{2PN}} = & -\eta(3-4\eta+\bar{\gamma})v^4 + \frac{1}{2}\left[\eta(13-4\eta+4\bar{\gamma}) - 4(1-4\eta)\bar{\beta}_+ + 4\psi(1-3\eta)\bar{\beta}_-\right]v^2\frac{G\alpha m}{r} - \frac{15}{8}\eta(1-3\eta)\dot{r}^4 \\ & + \frac{3}{2}\eta(3-4\eta+\bar{\gamma})v^2\dot{r}^2 + \left[2+25\eta+2\eta^2+2(1+9\eta)\bar{\gamma}+\frac{1}{2}\bar{\gamma}^2 - 4\eta(3\bar{\beta}_+ - \psi\bar{\beta}_-) + 2\bar{\delta}_+ + 2\psi\bar{\delta}_-\right]\dot{r}^2\frac{G\alpha m}{r} \\ & - \left[9+\frac{87}{4}\eta+(9+8\eta)\bar{\gamma}+\frac{1}{4}(9-2\eta)\bar{\gamma}^2 + (8+15\eta+4\bar{\gamma})\bar{\beta}_+ - \psi(8+7\eta+4\bar{\gamma})\bar{\beta}_-\right. \\ & \left. \quad + (1-2\eta)(\bar{\delta}_+ - 2\bar{\chi}_+) + \psi(\bar{\delta}_- + 2\bar{\chi}_-) - 24\eta\frac{\bar{\beta}_1\bar{\beta}_2}{\bar{\gamma}}\right]\left(\frac{G\alpha m}{r}\right)^2\,,\end{aligned}$$

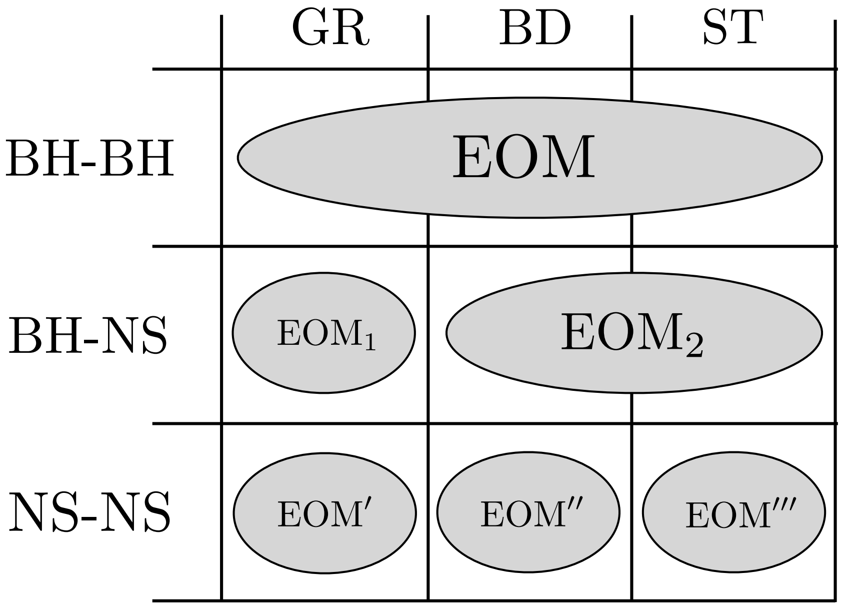
$$\begin{aligned}\boxed{B_{2PN}} = & \frac{1}{2}\eta(15+4\eta+8\bar{\gamma})v^2 - \frac{3}{2}\eta(3+2\eta+2\bar{\gamma})\dot{r}^2 \\ & - \frac{1}{2}\left[4+41\eta+8\eta^2+4(1+7\eta)\bar{\gamma}+\bar{\gamma}^2 - 8\eta(2\bar{\beta}_+ - \psi\bar{\beta}_-) + 4\bar{\delta}_+ + 4\psi\bar{\delta}_-\right]\frac{G\alpha m}{r}\,,\end{aligned}$$

Full EOM: Up to 2.5PN
i.e. 5 orders beyond the Newtonian order

$$\boxed{\begin{matrix}\tau_+\\ \tau_-\end{matrix}}\begin{matrix}\equiv \frac{1}{2}(\tau_1+\tau_2)\,,\\ \equiv \frac{1}{2}(\tau_1-\tau_2)\,.\end{matrix} \qquad \boxed{\begin{matrix}\mathcal{S}_-\\ \mathcal{S}_+\end{matrix}}\begin{matrix}\equiv -\alpha^{-1/2}(s_1-s_2)\,,\\ \equiv \alpha^{-1/2}(1-s_1-s_2)\,,\end{matrix} \qquad \boxed{\psi}\equiv \frac{m_1-m_2}{m_1+m_2}=\pm\sqrt{1-4\eta}\,.$$

Parameter	Definition	Parameter	Definition
Scalar-tensor parameters		**Equation of motion parameters**	
G	$\phi_0^{-1}(4+2\omega_0)/(3+2\omega_0)$	**Newtonian**	
ζ	$1/(4+2\omega_0)$	α	$1-\zeta+\zeta(1-2s_1)(1-2s_2)$
λ_1	$(d\omega/d\varphi)_0\zeta^2/(1-\zeta)$	**post-Newtonian**	
λ_2	$(d^2\omega/d\varphi^2)_0\zeta^3/(1-\zeta)$	$\bar{\gamma}$	$-2\alpha^{-1}\zeta(1-2s_1)(1-2s_2)$
Sensitivities		$\bar{\beta}_1$	$\alpha^{-2}\zeta(1-2s_2)^2\left(\lambda_1(1-2s_1)+2\zeta s_1'\right)$
s_A	$[d\ln M_A(\phi)/d\ln\phi]_0$	$\bar{\beta}_2$	$\alpha^{-2}\zeta(1-2s_1)^2\left(\lambda_1(1-2s_2)+2\zeta s_2'\right)$
s_A'	$[d^2\ln M_A(\phi)/d\ln\phi^2]_0$	**2nd post-Newtonian**	
s_A''	$[d^3\ln M_A(\phi)/d\ln\phi^3]_0$	$\bar{\delta}_1$	$\alpha^{-2}\zeta(1-\zeta)(1-2s_1)^2$
		$\bar{\delta}_2$	$\alpha^{-2}\zeta(1-\zeta)(1-2s_2)^2$
		$\bar{\chi}_1$	$\alpha^{-3}\zeta(1-2s_2)^3\left[(\lambda_2-4\lambda_1^2+\zeta\lambda_1)(1-2s_1)-6\zeta\lambda_1 s_1'+2\zeta^2 s_1''\right]$
		$\bar{\chi}_2$	$\alpha^{-3}\zeta(1-2s_1)^3\left[(\lambda_2-4\lambda_1^2+\zeta\lambda_1)(1-2s_2)-6\zeta\lambda_1 s_2'+2\zeta^2 s_2''\right]$

How can we distinguish gravitation theories in different types of binaries?



However, in this case all the deviations from general relativity depend on a single parameter Q.

$$Q \equiv \zeta(1-\zeta)^{-1}(1-2s_1)^2\,,$$

Summary and Future Work

- SUMMARY
  - We use DIRE adapted to scalar-tensor theory (DIRST), coupled with the Eardley approach for treating gravitating bodies.
  - Equations of motion for non-spinning compact objects, including black holes, to 2.5 post-Newtonian order, in a general class of massless scalar-tensor theories of gravity have been obtained. [see the next talk on massive ST by Michael Horbatsch]
  - A central question which has been answered is this: there is no measurable difference between GR and ST for binary black holes at 2.5PN order. $S_{BH} \to \frac{1}{2} \quad M \to (\frac{3+2\omega}{4+2\omega})M$
  - BH-NS systems differ from GR depending upon a single parameter Q
  - We have shown the conservation of energy and linear momentum in EOM.
  - We obtained the 1PN and 2PN Lagrangian for the motion. $\frac{dL}{dx_i} = \frac{d}{dt}\frac{dL}{dv_i}$

Summary and Future Work

- FUTURE WORK
  - DIRE procedure in far zone and calculating 2PN waveform and energy flux in ST. [a project by Ryan Lang and Clifford Will]
  - Gravitational-wave data analysis questions and testability of ST
  - At which PN order Eardley approach breaks down?

INSPIRALLING COMPACT BINARIES IN SCALAR-TENSOR THEORIES OF GRAVITY: EQUATIONS OF MOTION TO 2.5 POST-NEWTONIAN ORDER

Saeed Mirshekari, Clifford Will

ePrint: ArXiv:1301.4680 PRD

Washington University in St. Louis
University of Florida, Gainesville

Seventh Gulf Coast Gravity Meeting
University of Mississippi, Oxford (MS), April 19-20 2013