Scalar wigs and galactic dark matter halos

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Phys. Rev. D 84, 083008 (2011): *Are black holes a serious threat to scalar field dark matter models?*
Phys. Rev. Lett. 109, 081102 (2012): *Schwarzschild black holes can wear scalar wigs.*

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Introduction

- Scalar fields have been proposed as candidates to describe the dark matter component of the universe[1], as opposed to other more accepted candidates, like WIMPs.

- Since super-massive black holes seem to exist at the center of most galaxies, scalar field configurations can be viable as dark matter halos only if they can "survive" in the presence of a central black hole.

- This idea may seem to be in conflict with the no-hair theorems. However, a scalar field may only look like hair for practical reasons, without violating these theorems. We will see that some configurations can last around the black hole for cosmological timescales. We call them "scalar wigs."

[1] Turner, 1983; Peebles, 2000; Matos et al, 2000; Hu et al, 2000; Arbey, 2001; Harko et al, 2011; Briscese, 2011; etc.

Scalar field on a Schwarzschild background

We solve for a massive Klein-Gordon scalar field with mass parameter μ on a Schwarzschild space-time:

$$\nabla^\alpha \nabla_\alpha \phi - \mu^2 \phi = 0 \ .$$

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right) ,$$

An expansion in spherical harmonics of the form

$$\phi(t, r, \theta, \varphi) = \frac{1}{r} \sum_{\ell, m} \psi_{\ell m}(t, r) Y^{\ell m}(\theta, \varphi) \ ,$$

gives

$$\left\{ \left(1 - \frac{2M}{r}\right)^{-1} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial r}\left[\left(1 - \frac{2M}{r}\right)\frac{\partial}{\partial r}\right] + \frac{\ell(\ell+1)}{r^2} + \frac{2M}{r^3} + \mu^2 \right\} \psi_{\ell m}(t, r) = 0 \ .$$

Separation of variables

We use the ansatz:

$$\psi_{\ell m}(t,r) = e^{i\omega_{\ell m} t}\, u_{\ell m}(r) \ ,$$

and obtain an eigenvalue ODE with eigenvalues ω^2,

$$\left\{-\left(1-\frac{2M}{r}\right)\frac{d}{dr}\left[\left(1-\frac{2M}{r}\right)\frac{d}{dr}\right]+\left(1-\frac{2M}{r}\right)\left(\frac{\ell(\ell+1)}{r^2}+\frac{2M}{r^3}+\mu^2\right)\right\}u(r)=\omega^2 u(r) \ .$$

Changing to tortoise coordinates,

$$r^* := r + 2M\ln(r/2M-1) \ ,$$

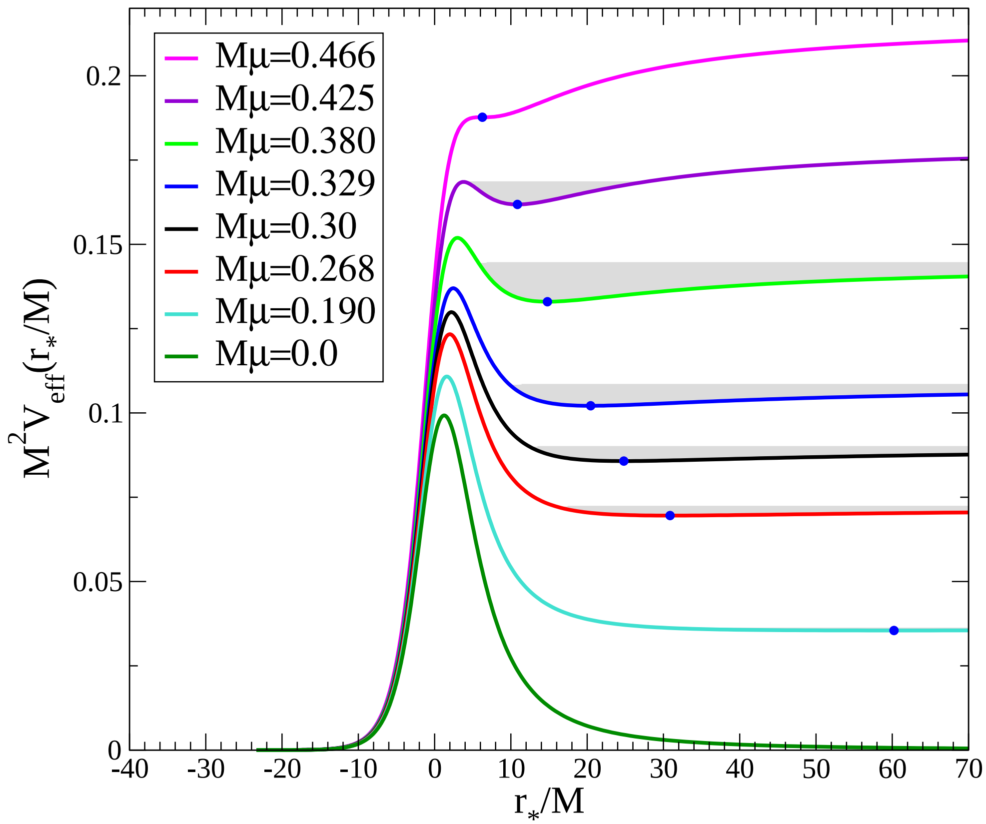
we get a Schrödinger-like equation,

$$\left[-\frac{d^2}{dr^{*2}}+V_{\rm eff}(r^*)\right]u(r^*)=\omega^2 u(r^*) \ ,$$

with effective potential

$$V_{\rm eff}(r^*) = \left(1-\frac{2M}{r(r^*)}\right)\left(\frac{\ell(\ell+1)}{r(r^*)^2}+\frac{2M}{r(r^*)^3}+\mu^2\right) \ .$$

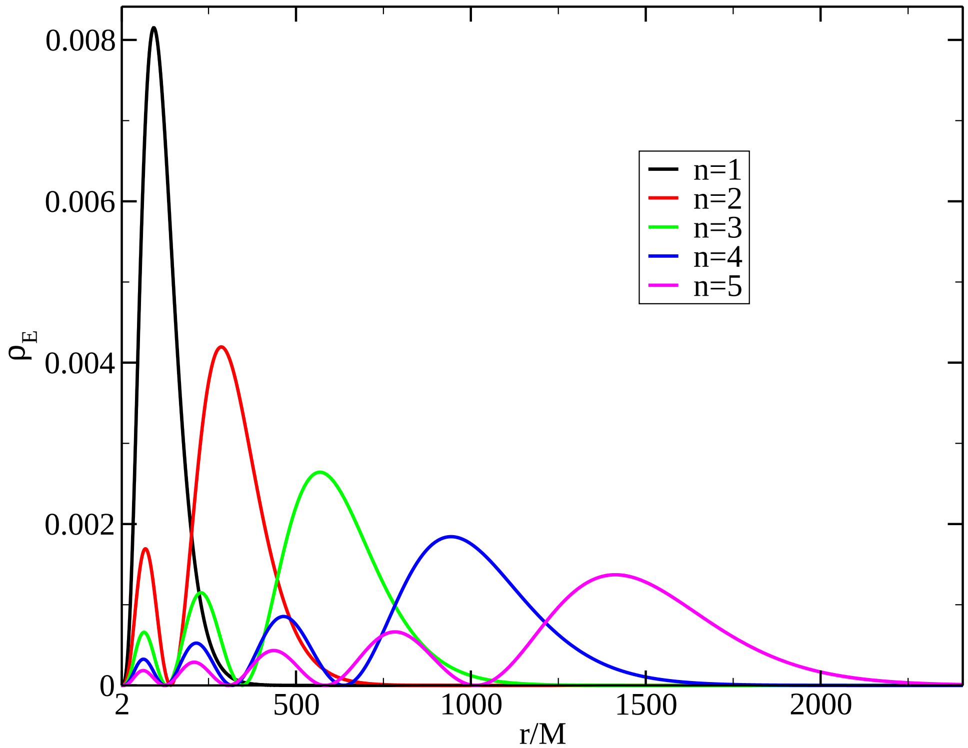
Effective potentials

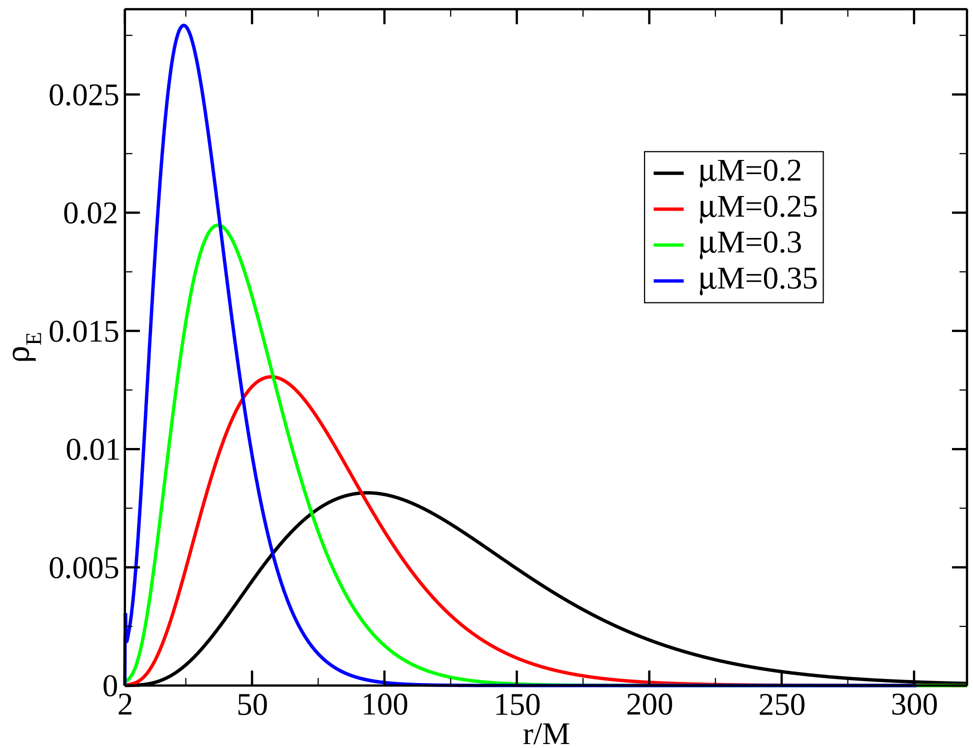


- Continuous eigenvalue spectrum ω^2, no bound states.
- Discrete spectrum of resonant modes ω_n^2.

Resonant modes as initial data

- Initial energy density distributions, ρ_E, corresponding to different resonant modes n and mass parameters μ:



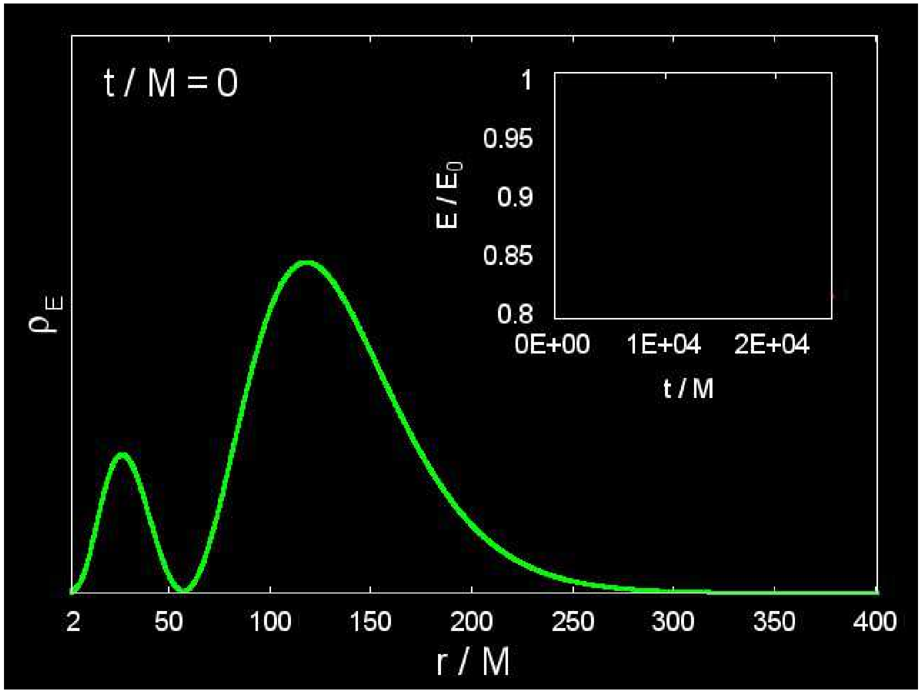


(we are back in Schwarzschild coordinates)

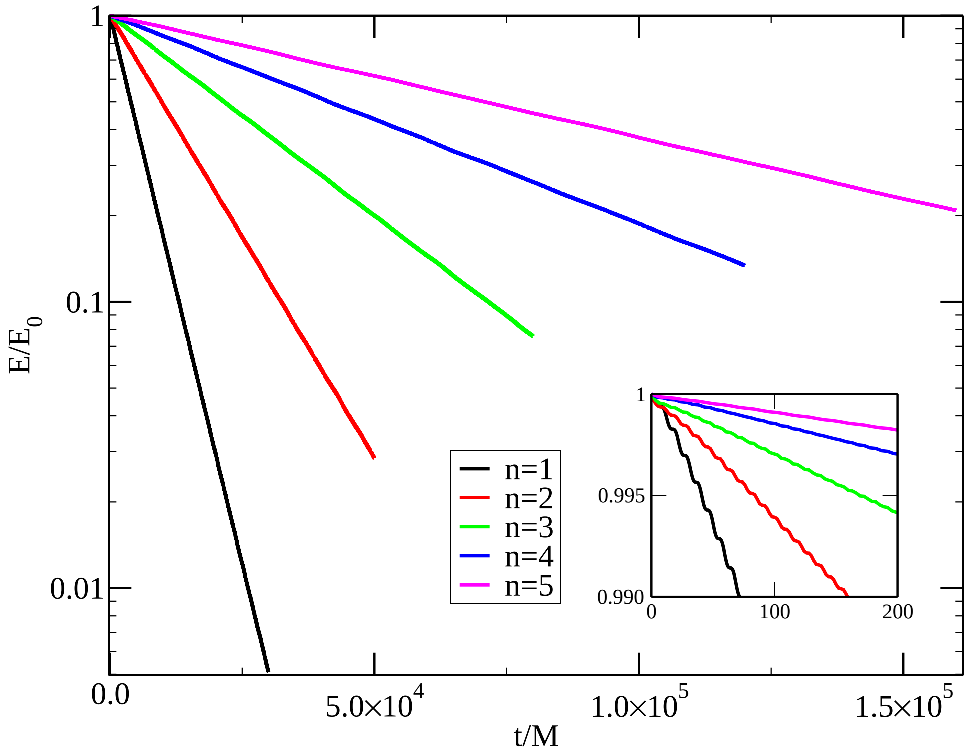
Numerical evolution

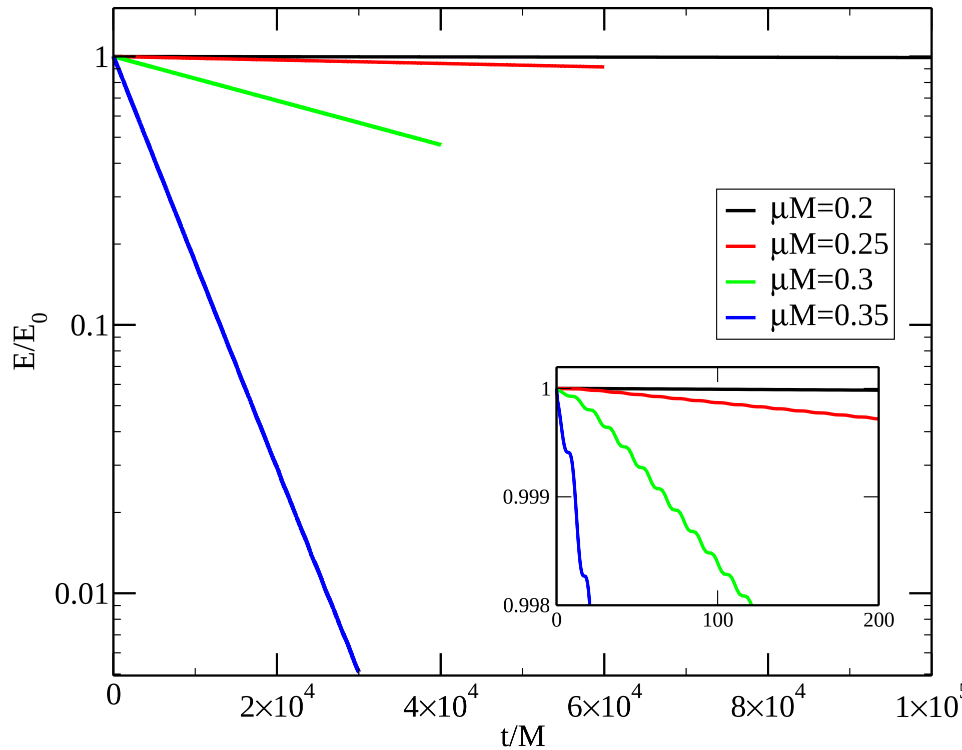
- We evolve numerically the original PDE using initial data as shown in the previous slide:

$$\left\{ \left(1-\frac{2M}{r}\right)^{-1}\frac{\partial^2}{\partial t^2}-\frac{\partial}{\partial r}\left[\left(1-\frac{2M}{r}\right)\frac{\partial}{\partial r}\right]+\frac{\ell(\ell+1)}{r^2}+\frac{2M}{r^3}+\mu^2\right\}\psi_{\ell m}(t,r)=0\ .$$



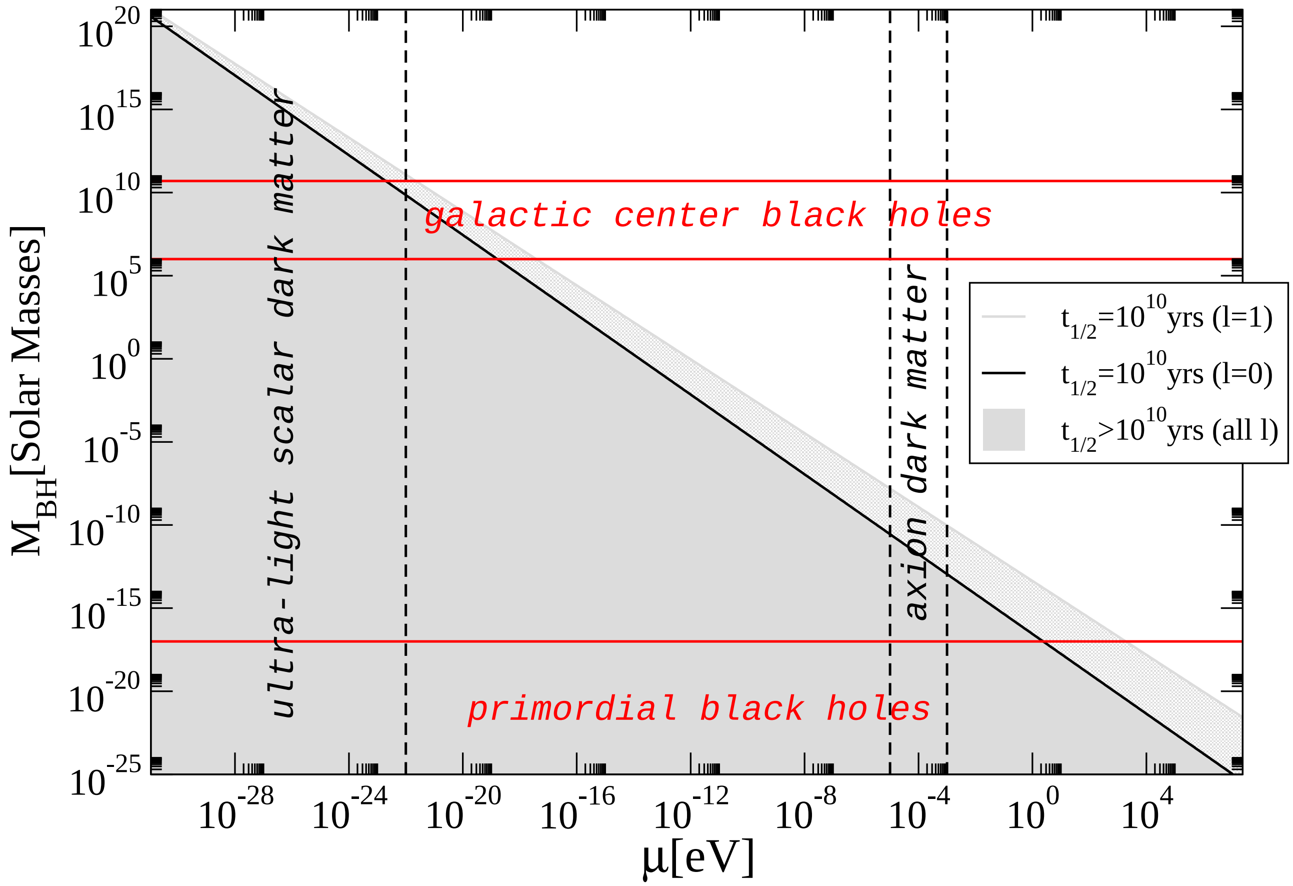
Energy decay





- The energy decays as $E(t) = E_0\ \exp\left(-k\ t\right)$
- The half-life time is $t_{1/2} = \ln(2)/k$
- For very small μ the numerical evolutions have problems, but in those cases one can use an analytical approximation (Detweiler, 1980) to calculate $t_{1/2}$.

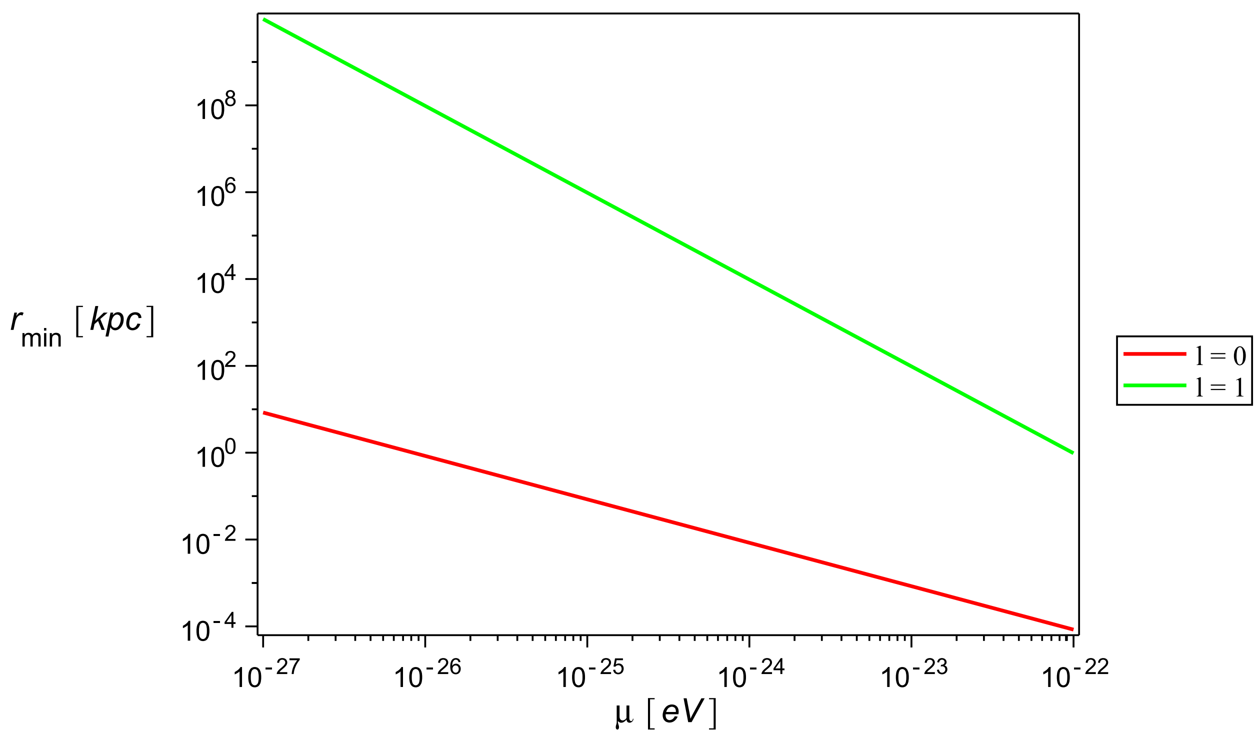
Regions in (M, μ) where $t_{1/2} > 10^{10}$ years



We see that even the less lasting mode $(l = 0, n = 1)$ has a half-life larger than the age of the universe for values of μ and M that are consistent with scalar field dark matter and black holes at galactic centers, respectively.

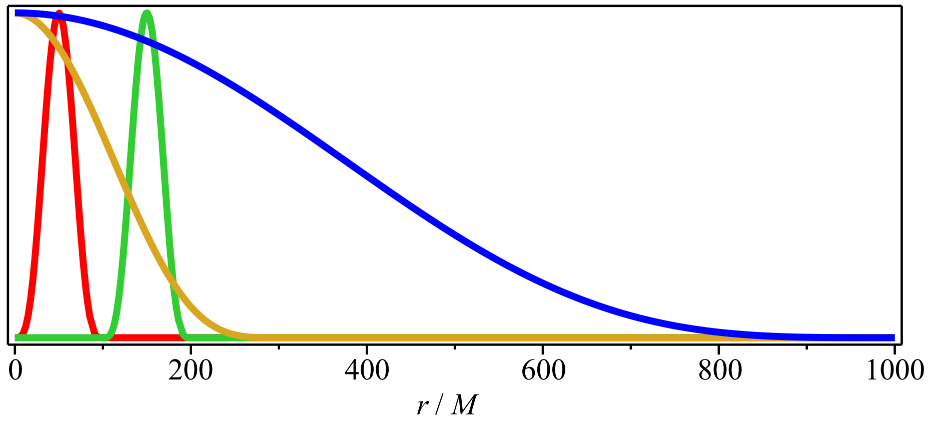
Mode size

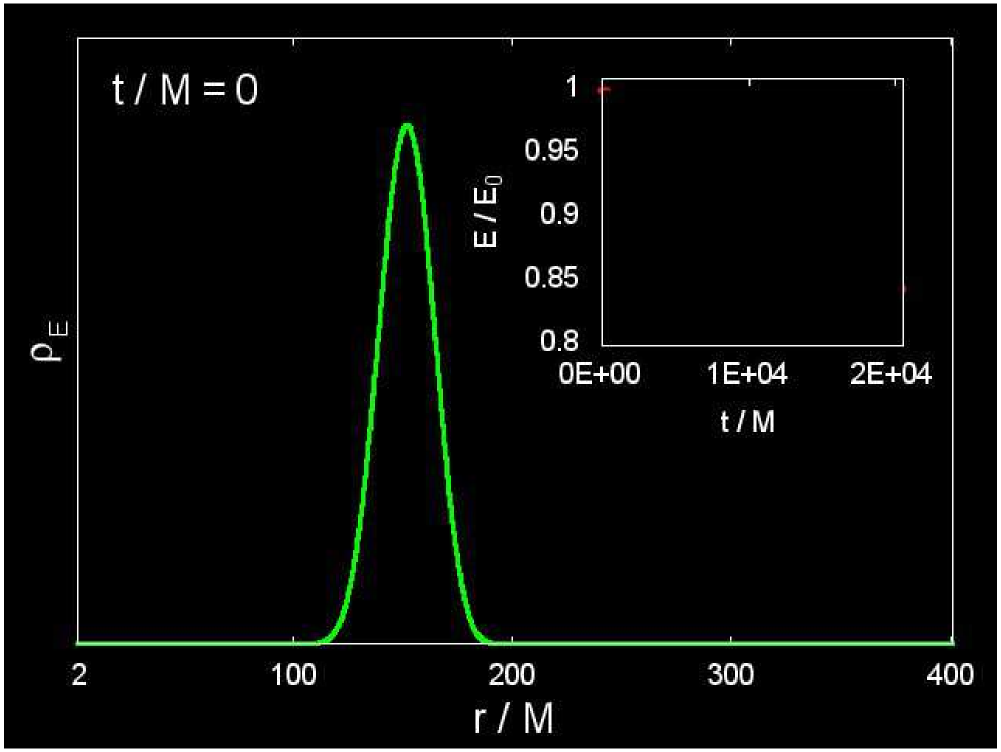
- To get an idea of the "size" of the resonant modes, we calculate the position $r_{\rm min}$ of the effective potential's local minimum, which actually gives a lower bound for the scalar field spatial extension.



- This is consistent with typical galaxy sizes. (note that the spatial extention of the modes may be much larger than this lower bound).

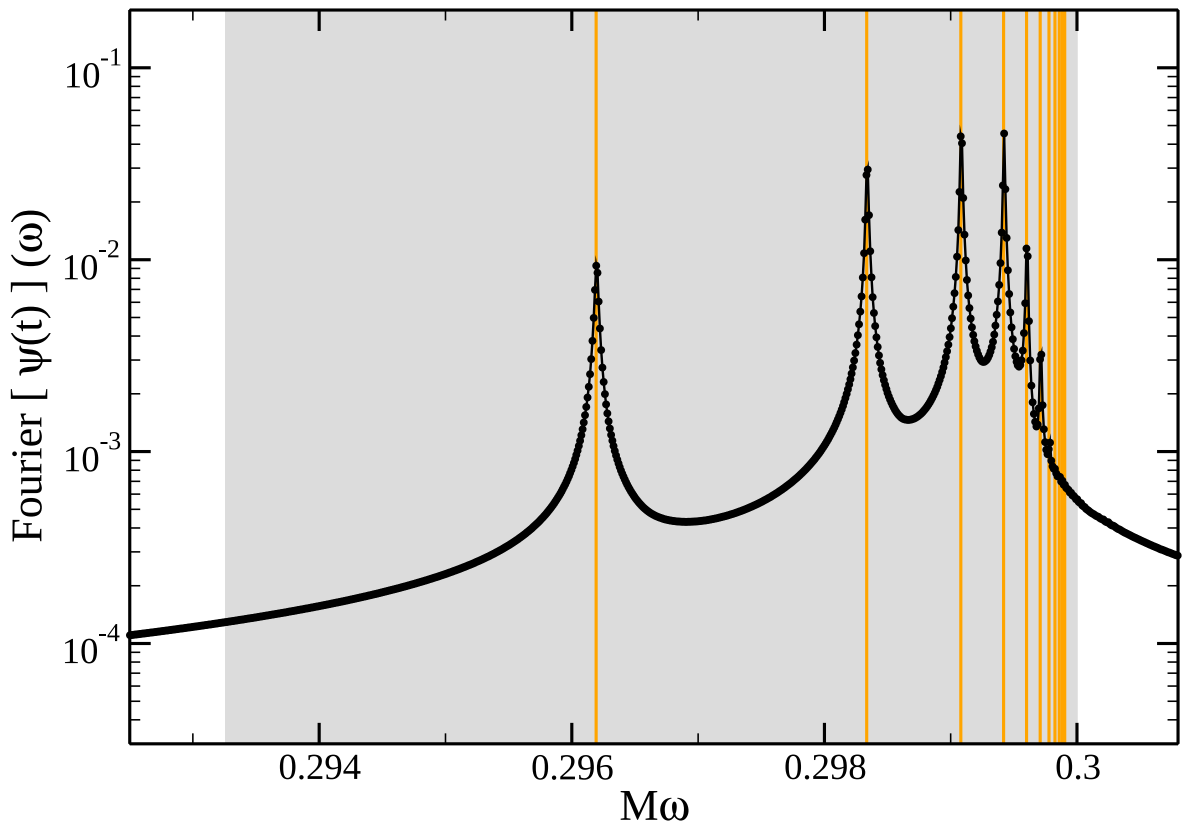
Evolution of more general initial data





Evolution of more general initial data

Fourier transform in time of $\psi(r,t)$:

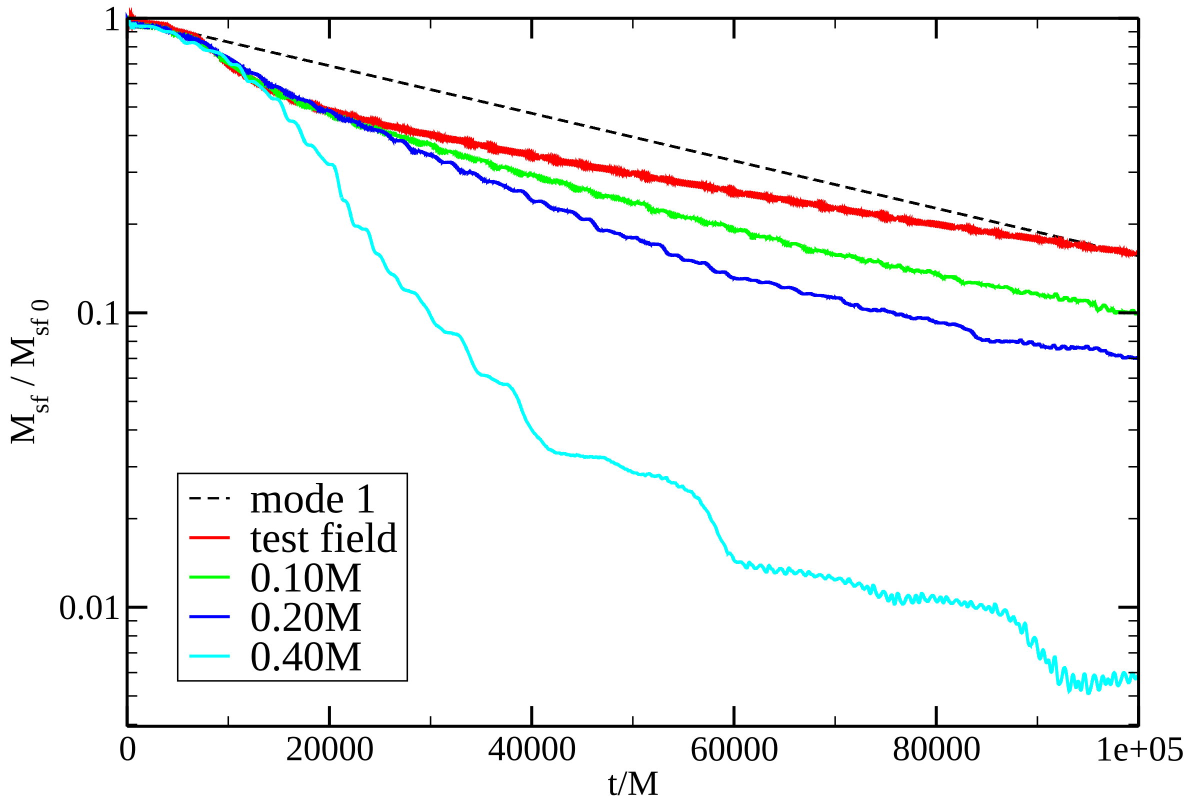


- Generic initial scalar field distributions evolve, after some time, as a combination of the resonant modes.

Combined modes oscillations

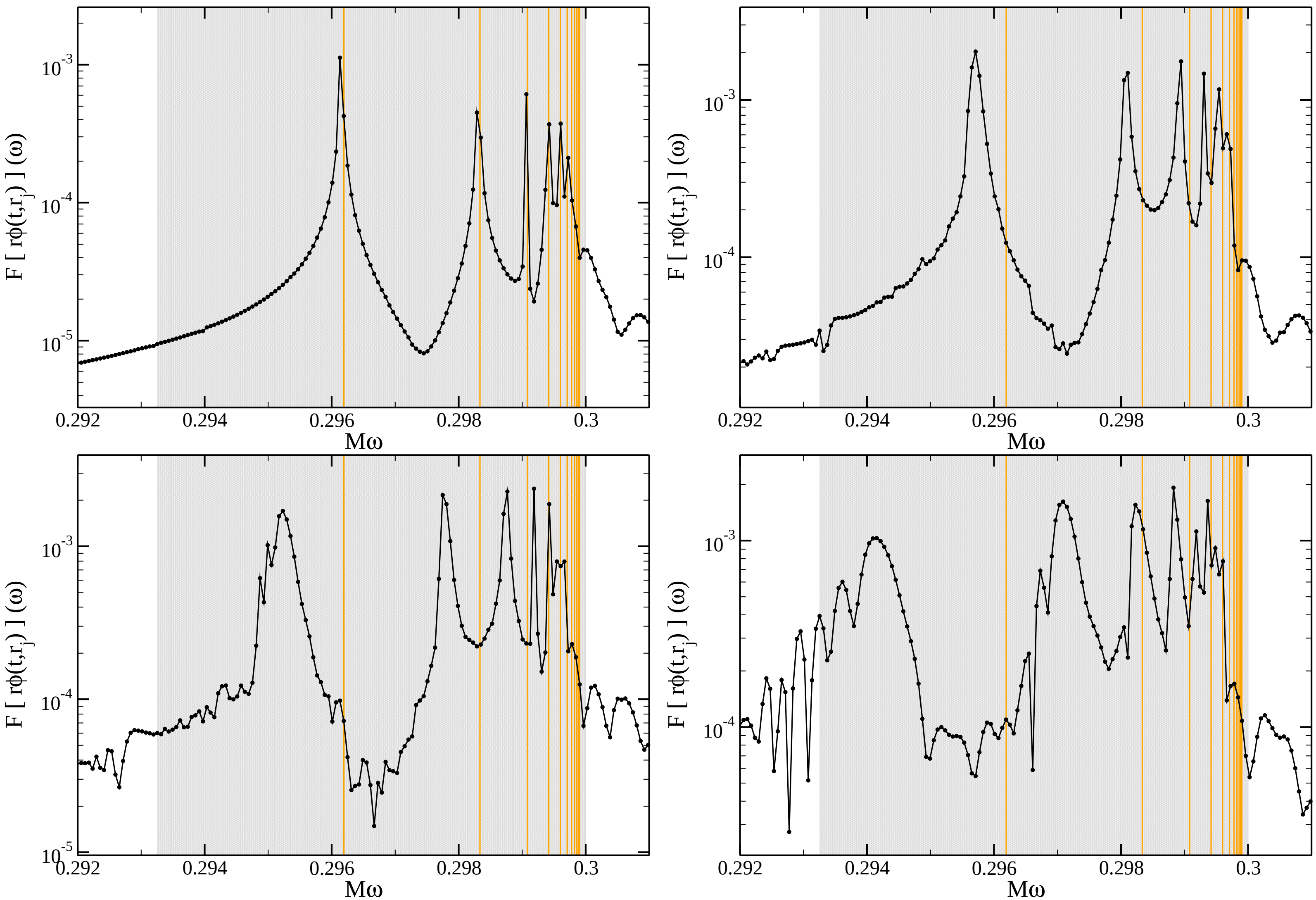
- We saw that the density profile, $\rho_{n\ell}$, of each resonant mode $\phi_{n\ell}$ is quasi static.
- However, a combination of different modes will give rise to an oscillating density profile (due to the non-linear dependence)
- For example, combining two modes with frequencies ω_1 and ω_2, the energy density will show oscillations with frequency $\omega_r = |\omega_1 - \omega_2|$.
- If galactic halos are actually described as scalar fields, these oscillations might be observable, for example, by observing changes in rotation curves (maybe over a few years) with frequency ω_r.

Self gravitating scalar field (work in progress)



Evolutions with $M_{\rm sf0} = 0.01 M_{\rm BH}$, $0.10 M_{\rm BH}$, $0.20 M_{\rm BH}$ and $0.40 M_{\rm BH}$.

Self gravitating scalar field (work in progress)



Evolutions with $M_{\mathrm{sf0}} = 0.01 M_{\mathrm{BH}}$, $0.10 M_{\mathrm{BH}}$, $0.20 M_{\mathrm{BH}}$ and $0.40 M_{\mathrm{BH}}$.

Analytical results for small μ (work in progress)

- At late times, the scalar field around the black hole behaves as a combination of the resonant modes.
- It is possible to calculate the modes amplitudes at late times for any given initial configuration in the small μ approximation.
- We can then determine a priori "how much" of the initial state "remains" at long times given arbitrary initial data (without the need of numerical evolutions).

Conclusions

- Resonant scalar field modes last for cosmological timescales when the parameters (M and μ) are within accepted values for super-massive black holes and scalar field dark matter.

- The "size" of these modes is also compatible with what is believed to be the size of dark matter halos.

- Generic scalar field distributions evolve at late times as a combination of resonant modes.

- Characteristic frequencies of combined resonant modes might be observable.

- Preliminary self gravitating evolutions are showing similar results.