Characteristics of Kantowski Sachs space time in Loop Quantum Cosmology

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Classical Kantowski Sachs spacetime

- Homogenous and anisotropic cosmological model with spatial topology $R \times S^2$
- Metric: $ds^2 = -N(t)^2 dt^2 + g_{xx}(t) dx^2 + g_{\Omega\Omega}(t)(d\theta^2 + \sin^2\theta d\phi^2)$
- Interior of Schwarzschild black hole can be treated as vacuum Kantowski Sachs space time for which,

$$N(t)^2 = \left(\frac{2m}{t} - 1\right)^{-1},\ g_{xx} = \left(\frac{2m}{t} - 1\right),\ g_{\Omega\Omega} = t^2. \tag{1}$$

- Loop Quantum gravity is a non-perturbative canonical quantization of gravity. It's cosmological offshoot is loop quantum cosmology.
- Singularity at the center of Schwarzschild black hole resolved in effective Loop Quantum Cosmological (LQC) model of Kantowski Sachs space time[1].

[1] Bohmer, C. G., Vandersloot, K. (2007). Loop quantum dynamics of the Schwarzschild interior. Physical Review D, 76(10), 104030.

Ashtekar variables for Kantowski Sachs space time

The LQC description of Kantowski Sachs(KS) is given in terms of triads, which are related to metric coeffecients as,

$$p_b = L\sqrt{g_{xx}g_{\Omega\Omega}}, \quad p_c = g_{\Omega\Omega} \tag{2}$$

and their corresponding conjugate variables b and c. Here L is the length of the fiducial cell. Volume of the fiducial cell $V = 4\pi p_b\sqrt{p_c}$.

For classical Schwarzschild BH interior:

- Horizon: $p_b = 0$, $p_c = 4m^2$,
- Center of BH: $p_b = 0$, $p_c = 0$.

The Hamiltonian in these variables is

$$H = \frac{-N}{2G\gamma^2}\left(2bc\sqrt{p_c} + \left(b^2 + \gamma^2\right)\frac{p_b}{\sqrt{p_c}}\right) + N4\pi p_b\sqrt{p_c}\rho_m, \tag{3}$$

LQC Hamiltonian and energy density

In the effective description of LQC, KS Hamiltonian with matter can be written as

$$H = \frac{-N}{2G\gamma^2}\left(2\frac{\sin b\delta_b}{\delta_b}\frac{\sin c\delta_c}{\delta_c}\sqrt{p_c} + \left(\frac{\sin^2 b\delta_b}{\delta_b^2} + \gamma^2\right)\frac{p_b}{\sqrt{p_c}}\right) + N4\pi p_b\sqrt{p_c}\rho_m, \tag{4}$$

where,

$$\delta_b = \sqrt{\frac{\Delta}{p_c}}, \quad \delta_c = \frac{\sqrt{\Delta p_c}}{p_b} \tag{5}$$

with $\gamma = 0.2375$ and $\Delta = 4\sqrt{3}\pi\gamma l_p^2$.

In Loop Quantum Gravity, the Hamiltonian is constrained to be zero, which gives

$$\rho_m = \frac{1}{8\pi G\gamma^2\Delta}\left(2\sin b\delta_b \sin c\delta_c + \sin^2 b\delta_b + \frac{\gamma^2\Delta}{p_c}\right) \tag{6}$$

$$\Rightarrow |\rho_m| \leq \frac{3}{8\pi G\gamma^2\Delta} + \frac{1}{8\pi G p_c} \tag{7}$$

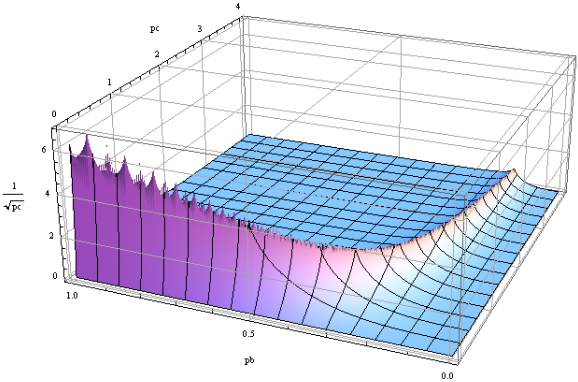
ρ_m is not bounded as $p_c \to 0$, indicating the need for inverse triad correction.

Inverse triad correction

Inverse triad correction for Kantowski Sachs space time

$$\frac{1}{\sqrt{p_c}} = \frac{2|p_b|}{\gamma\sqrt{\Delta}l_p^2}\left(\sqrt{\left|\frac{2p_b\sqrt{p_c}}{\gamma\sqrt{\Delta}l_p^2}+1\right|}-\sqrt{\left|\frac{2p_b\sqrt{p_c}}{\gamma\sqrt{\Delta}l_p^2}-1\right|}\right)^2 . \qquad (8)$$

The inverse triad correction substitutes $\frac{1}{\sqrt{p_c}}$ with a function that does not blow up as p_c or p_b tend to zero.

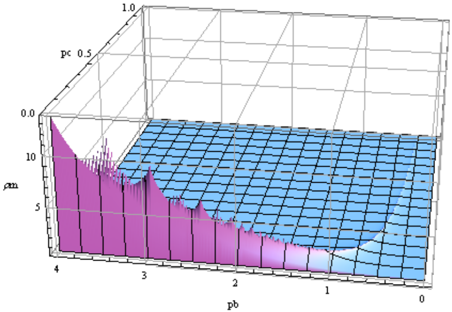


Energy density

With the inverse triad correction, the bound on energy density becomes,

$$|\rho_m| \leq \frac{3}{8\pi G\gamma^2\Delta} + \frac{p_b}{4\pi G\gamma\sqrt{\Delta p_c} l_p^2}\left(\sqrt{\left|\frac{2p_b\sqrt{p_c}}{\gamma\sqrt{\Delta} l_p^2}+1\right|} - \sqrt{\left|\frac{2p_b\sqrt{p_c}}{\gamma\sqrt{\Delta} l_p^2}-1\right|}\right)^2 . \tag{9}$$

The first term is same as the bound on ρ_m in isotropic and Bianchi I cosmologies. The second term remains bounded except when $p_b \to \infty$ and $p_c \to 0$ simultaneously.



Bound on expansion scalar

Convergence or divergence of geodesic congruences can be understood by looking at the expansion scalar.

$$\Theta = \frac{\dot{V}}{V} \tag{10}$$

$$= \frac{\dot{p}_b}{p_b} + \frac{1}{2}\frac{\dot{p}_c}{p_c} \tag{11}$$

Using Hamilton's equations of motion for $\dot{p}_b$ and $\dot{p}_c$,

$$|\Theta| \leq \frac{3}{2\gamma\sqrt{\Delta}} \approx \frac{2.78}{l_p} \tag{12}$$

Hence the expansion scalar is bounded in LQC description of Kantowski Sachs. The value of bound is same as that for isotropic and Bianchi I cosmologies.

The bound goes to infinity as $\Delta \to 0$.

Bound on shear scalar

The shear scalar in Kantowski Sachs cosmology is given by

$$\sigma^2 = \frac{1}{6}\left(\frac{\dot{g}_{xx}}{g_{xx}} - \frac{\dot{g}_{\Omega\Omega}}{g_{\Omega\Omega}}\right)^2 \tag{13}$$

$$= \frac{2}{3}\left(\frac{\dot{p}_b}{p_b} - \frac{\dot{p}_c}{p_c}\right)^2 \tag{14}$$

Following bound was obtained numerically

$$|\sigma^2| \leq \frac{3.3581}{\gamma^2\Delta} = \frac{11.52}{l_p^2} \tag{15}$$

The above bound is slightly lower than the one for Bianchi I[2] (11.57).

[2] Gupt, B., Singh, P. (2012). Contrasting features of anisotropic loop quantum cosmologies: the role of spatial curvature. Physical Review D, 85(4), 044011.

Conclusion

- Inverse triad corrections are important in loop quantum cosmological Kantowski Sachs space time.
- Energy density, expansion scalar and shear scalar are bounded for finite values of triads.
- Boundedness of studied quantities indicates singularity resolution for generic matter. Future study would be directed to find if Kantowski Sachs space time is never singular.