Gravitational Radiation from Eccentric Compact Binaries in Massive Scalar-Tensor Gravity

Michael W. Horbatsch
University of Mississippi

Emanuele Berti, Justin Alsing, Dimitris Manolidis, Clifford Will, Helmut Zaglauer

JA,EB,CW,HZ, PRD **85** (2012) 064041 [arXiv:1112.4903].
HZ, PhD Thesis (1990) Washington U. St. Louis.

2013.04.19

Outline

Scalar-Tensor Gravity — Broad Definition

- A relativistic theory of gravitation which contains light scalars in addition to the usual metric tensor.

Scalar-Tensor Gravity — Several Motivations

- Development of alternative theories leads to better understanding of general relativity and gravitation on both theoretical and experimental levels.
- Direct detection of gravitational waves will test general relativity in the fast-motion-strong-field regime for the first time.

Scalar-Tensor Gravity — Several Motivations

- Strong CP problem in QCD.
- Accelerated expansion of the universe.
- Unification of general relativity with quantum mechanics at high energies.
- Caveat: quantum loop corrections to scalar masses.

Scalar-Tensor Gravity — Several Examples

- Jordan-Fierz-Brans-Dicke Theory and Generalizations
- $f(R)$ Theories (formally equivalent to above)
- Dynamical Chern-Simons Gravity
- Einstein-Dilaton-Gauss-Bonnet Gravity
- Chameleons
- Galileons
- Symmetrons
- Quintessence
- Inflationary Models

Scalar-Tensor Gravity — Action

- Build a relativistic field theory with metric tensor $\tilde{g}_{\mu\nu}$, scalar fields φ^a, and matter fields Ψ^A.
- Action is required to be invariant under space-time and target-space diffeomorphisms.
- In order to incorporate the *weak equivalence principle*, matter fields Ψ^A couple only to the metric $\tilde{g}_{\mu\nu}$ and not to the scalars φ^a.
- In the framework of *effective field theory*, the action is expanded in numbers of space-time derivatives.

Scalar-Tensor Gravity — Action

- The most general action satisfying these requirements has the (Jordan-frame) form:

$$\int d^4x\sqrt{-\tilde{g}}\left(-V(\varphi)+\mathcal{F}(\varphi)\tilde{R}-\frac{1}{2}\tilde{g}^{\mu\nu}\mathcal{G}_{ab}(\varphi)\nabla_\mu\varphi^a\nabla_\nu\varphi^b+\dots\right)$$

$$+S_{\rm matter}[\Psi;\tilde{g}_{\mu\nu}]\,.$$

- The field redefinition $\tilde{g}_{\mu\nu}=A^2(\varphi)g^\star_{\mu\nu}$ transforms the action into the Einstein-frame form:

$$\frac{c^4}{4\pi G_\star}\int\frac{d^4x}{c}\sqrt{-g_\star}\left(-B(\varphi)+\frac{R_\star}{4}-\frac{1}{2}g_\star^{\mu\nu}\gamma_{ab}(\varphi)\nabla_\mu\varphi^a\nabla_\nu\varphi^b+\dots\right)$$

$$+S_{\rm matter}[\Psi;A^2(\varphi)g^\star_{\mu\nu}]\,.$$

Scalar-Tensor Gravity — Field Equations

$$R^{\star}_{\mu\nu} - 2\gamma_{ab}(\varphi)\nabla_\mu\varphi^a\nabla_\nu\varphi^b - 2B(\varphi)g^{\star}_{\mu\nu} \quad = \quad \frac{8\pi G_\star}{c^4}\left(T^{\star}_{\mu\nu} - \frac{1}{2}T_\star g^{\star}_{\mu\nu}\right),$$

$$\Box_\star\varphi^a + \gamma^a{}_{bc}(\varphi)g_\star^{\mu\nu}\nabla_\mu\varphi^b\nabla_\nu\varphi^c - B^a(\varphi) \quad = \quad -\frac{4\pi G_\star}{c^4}\alpha^a(\varphi)T_\star\,,$$

where

$$\alpha_a(\varphi) := \frac{\partial \log A(\varphi)}{\partial\varphi^a}\,, \qquad B_a(\varphi) := \frac{\partial B(\varphi)}{\partial\varphi^a}\,,$$

$$\gamma^a{}_{bc}(\varphi) = \frac{1}{2}\gamma^{ad}(\varphi)\left(\frac{\partial\gamma_{cd}(\varphi)}{\partial\varphi^b} + \frac{\partial\gamma_{bd}(\varphi)}{\partial\varphi^c} - \frac{\partial\gamma_{bc}(\varphi)}{\partial\varphi^d}\right).$$

Scalar-Tensor Gravity — Scalar Masses and Couplings

- For an isolated system with asymptotically constant scalar φ_∞, it is natural to carry out an expansion of the form

$$B(\varphi) = B^\infty + B_a^\infty(\varphi^a - \varphi_\infty^a) + \frac{1}{2}(m_s^2)_{ab}(\varphi^a - \varphi_\infty^a)(\varphi^b - \varphi_\infty^b) + \ldots ,$$

$$\alpha_a(\varphi) = \alpha_a^\infty + (\beta_{ab}^\infty + \gamma^{c\infty}{}_{ab}\alpha_c^\infty)(\varphi^b - \varphi_\infty^b) + \ldots ,$$

$$\gamma_{ab}(\varphi) = \gamma_{ab}^\infty + (\gamma_{abc}^\infty + \gamma_{bac}^\infty)(\varphi^c - \varphi_\infty^c) + \ldots ,$$

where the eigenvalues of m_s^2 are the scalar masses, and α^∞ and β^∞ are the leading-order scalar-matter coupling parameters.

Scalar-Tensor Gravity — Constraints on Simple Single-Scalar Models

- **Brans-Dicke Model:** constant scalar-matter coupling $\alpha(\varphi) \to \alpha_\infty$:

$$\alpha_\infty^2 < 1.2 \cdot 10^{-5} \text{ for } m_s = 0$$

from Cassini mission.

- **Quadratic Model:** linear scalar-matter coupling $\alpha(\varphi) \to \alpha_\infty + \beta_\infty(\varphi - \varphi_\infty)$:

$$\beta_\infty \gtrsim -5 \text{ for } m_s = 0$$

from binary pulsar timing.

Binary Systems — Effective Point Particle Theory

- Consider a bound binary system of compact objects with comparable masses $M_{1,2}$, radii $R_{1,2}$, and orbital separation $D \gg R_{1,2}$.

- In order to describe the orbital dynamics, it is useful to 'integrate out' length scales smaller than $R_{1,2}$, obtaining a *point-particle effective field theory* with matter action of the form

$$S_{\mathrm{matter}} = \sum_{A=1}^{2} \int_{\Gamma_A} \mathcal{L}_A \, ds_A \,, \qquad \mathcal{L}_A = -cM_A(\varphi) + \ldots \,,$$

where Γ_A is the worldline of body A, and $\mathcal{L}_A$ is the point-particle Lagrangian of body A, which is expanded in numbers of derivatives.

Binary Systems — Energy Flux

- Consider an observer at a distance $\mathcal{D} \gg D$ from the binary. Our goal is to calculate the energy flux seen by this observer in the limit $D/\mathcal{D} \to 0$.

- This flux receives contributions from both tensor and scalar modes, and has a multipolar expansion of the form

$$\begin{aligned} F_\varphi &= F_\varphi^{\mathrm{mon}} + F_\varphi^{\mathrm{dip}} + F_\varphi^{\mathrm{quad}} + F_\varphi^{\mathrm{oct}} + \ldots\,, \\ F_g &= F_g^{\mathrm{quad}} + F_g^{\mathrm{oct}} + \ldots\,. \end{aligned}$$

- Each term in the above series has a Post-Newtonian expansion in powers of v/c.

Flux Formulas — Hierarchy of Length Scales

- For a widely-separated binary, we have:

$$R_{1,2} \ll D \ll \frac{D}{v/c} \sim \lambda_{\mathrm{GW}} .$$

- We will additionally assume that:

$$R_{1,2} \ll D \ll \frac{\hbar}{m_s c} \sim \lambda_s .$$

- We make no assumptions about:

$$\zeta := \frac{m_s c^2}{\hbar\omega} \sim \frac{\lambda_{GW}}{\lambda_s} .$$

Flux Formulas — Quantum-Classical Intuition

- In order for a highly-excited oscillator of frequency ω to radiate a particle of mass m_s, we must have:

$$\hbar\omega \gtrsim m_s c^2 \, .$$

- An eccentric Kepler trajectory is a non-harmonic oscillator with frequencies ω, 2ω, 3ω, etc.
- In the circular limit, the oscillator becomes harmonic, with a single frequency ω.

Flux Formulas — Dipole

$$F_\varphi^{\rm dip} = \frac{G_\star}{3c^3}\left(\frac{G_{12}^\star M_1 M_2}{D^2}\right)^2 \mathcal{D}_\varphi^2 \times$$

$$\left\{ \frac{1+e^2/2}{(1-e^2)^{5/2}} - \zeta^2 - 2\sum_{k=1}^{K}(k^2-\zeta^2)\Big(J_k'^2(ke) + (e^{-2}-1)J_k^2(ke)\Big)\right\}.$$

- $G_{12}^\star$ is the gravitational coupling between the two bodies.
- $\mathcal{D}_\varphi$ is related to the scalar dipole moment of the binary.
- $K := \lfloor\zeta\rfloor$ is the largest integer with the property $K\hbar\omega \leq m_s c^2$.
- e is the orbital eccentricity.
- $J_k(z)$ are Bessel functions of the first kind.
- In the circular limit, $\{\cdots\} \to (1-\zeta^2)\Theta(1-\zeta)$.

Flux Formulas — Quadrupole

$$
\begin{aligned}
&\sim \frac{G_\star}{c^3}\left(\frac{G_{12}^\star \Gamma M_1 M_2}{D^2}\right)^2 (D\omega/c)^2 \left\{ \frac{1+\frac{73}{24}e^2+\frac{37}{96}e^4}{(1-e^2)^{7/2}} - \frac{[(6+e^2)\sqrt{1-e^2}+6-2e^2]}{6\sqrt{1-e^2}(1+\sqrt{1-e^2})^2}\zeta^2 + \frac{(3-e^2)}{48}\zeta^4 \right. \\
&-\frac{1}{2}\sum_{k=1}^{\lfloor\zeta\rfloor}\left(k-\frac{\zeta^2}{k}\right)^2 \left[\frac{(1-e^2)^3}{e^4}k^2 J_k^2(ke) + \frac{1-e^2+e^4/3}{e^4}J_k^2(ke) + \frac{1-e^2}{e^2}J_k'^2(ke)\right. \\
&\qquad\qquad \left.\left. + \frac{(1-e^2)^2}{e^2}k^2 J_k'^2(ke) + \frac{(1-e^2)(3e^2-4)}{e^3} k J_k(ke) J_k'(ke)\,,\right]\right\}
\end{aligned}
$$

where ω is the orbital frequency, and Γ depends on the mass ratio M_1/M_2 and sensitivities $\alpha_{1,2}$.

Flux Formulas — Monopole

$$
\begin{aligned}
\sim \frac{G_\star}{c^3}\left(\frac{G^\star_{12}M_1M_2}{D^2}\right)^2 (D\omega/c)^2 \Bigg\{ & (\Gamma + 3\Lambda/\mathcal{G})^2 \frac{e^2(1+e^2/4)}{(1-e^2)^{7/2}} \\
& + \frac{2\Gamma(\Gamma+3\Lambda/\mathcal{G})e^2}{\sqrt{1-e^2}(1+\sqrt{1-e^2})}\zeta^2 + \frac{e^2\Gamma^2}{4}\zeta^4 \\
& - 4\sum_{k=1}^{\lfloor\zeta\rfloor} k^2 J_k^2(ke)\left[\Gamma\left(1+\frac{\zeta^2}{2k^2}\right)+\frac{3\Lambda}{\mathcal{G}}\right]^2 \Bigg\},
\end{aligned}
$$

where ω is the orbital frequency, $\mathcal{G}$, Λ, and Γ depend on the mass ratio M_1/M_2, sensitivities $\alpha_{1,2}$, and second-order sensitivities $\beta_{1,2}$.

Summary

- Scalar-tensor theories of gravity contain uncharted territory at second order in the derivative expansion of the action.

- The effective field theory framework is useful not only in particle physics, but also in classical gravitational physics.

- Scalar gravitational wave fluxes emitted from an *eccentric* binary system of compact objects were calculated analytically in massive scalar-tensor theory.

- In this calculation, no assumptions have been made about the size of $\zeta = m_s c^2/\hbar\omega$. The fluxes are non-analytic at $\zeta \in \mathbb{N}$.

- Work in progress: obtaining bounds on $(m_s, \alpha_\infty, \beta_\infty)$ from binary pulsars

- Of particular interest is the eccentric WD-NS binary PSR J1141-6545 with $e \sim 0.172$.