Time Jitter Due to Gravitational Solitons
Peter L. Biermann$^{1, 2, 3, 4, 5}$ and Benjamin C. Harms$^1$

$^1$Department of Physics and Astronomy, 
The University of Alabama, Box 870324, 
Tuscaloosa, AL 35487-0324, USA

$^2$MPI for Radioastronomy, Bonn, Germany

$^3$Karlsruhe Institute of Technology (KIT) - Institut für Kernphysik, Germany

$^4$Department of Physics, University of Alabama at Huntsville, AL, USA

$^5$Department of Physics & Astronomy, University of Bonn, Germany

ArXiv:1205.4016, 1302.0040

Gulf Coast Gravity Conference
April 2013

plbiermann@mpifr-bonn.mpg.de
bharms@bama.ua.edu
Outline of Talk

Motivation

Our Model

Effective Metric Tensor Elements

Calculation of Time Shifts

Comparison to Experimental Limits

Conclusions
Motivation

Energy output from gravitational waves

Supermassive black holes began forming at a redshift of $z \sim 50$

We propose that these black holes resulted in the production of gravitational solitons.

Equality to value of dark energy

The present dark energy density can be accounted for by assuming:

1. a reasonable number density of black holes created at $z \sim 50$.
2. an original mass of $M_{BH} \simeq 3 \times 10^6 M_\odot$ for the black holes formed.
3. a release of $\sim M_{BH} c^2 / 2$ in the form of gravitational solitons which maintain their amplitudes through interaction with the background.
4. the ensemble of these solitons constitutes dark energy.
Our Model

**Background field** = gravitons with Planck spectrum at maximal energy density and peaked at the Planck energy.

**Equilibrium** with respect to gravitational collapse maintained due to equality of free-fall time scale and pressure wave time scale at the Planck length.

**FIG. 1:** The strong- and weak-gravity branes

\[ \psi(r) \]

Planck brane               TeV brane
Formation of first super-massive black holes produced a coherent burst of soliton-like gravitational waves which combine to give a total energy density of order

\[ \rho \sim \frac{1}{2} N_{BH,0} M_{BH} c^2 (1 + z_*)^3. \]
Stimulated emission of gravitons due to interaction of soliton on weak-gravity-brane with strong-gravity brane

**FIG. 2:** The stimulated emission of gravitons in a shell

**Metric**

\[
 ds^2 = -e^{(u/l)^m t/\psi} c^2 dt^2 + e^{(1-b(u^n)/2 t/\alpha \tau_H} e^{-\left(\frac{u}{\tau_H}\right)^k} \left(1-\frac{t}{\phi}\right) dx_i dx^i + e^{(u/p)^p t/\beta} du^2
\]

\[i = 1, 2, 3, \quad \tau_H = \text{Hubble time}, \quad l = \text{Planck length}, \quad u = \text{coordinate in 5th dimension}\]
Effective Metric Tensor Elements

Increase in dark energy is due to the large number of individual soliton fronts emanating from the creation and mergers of black holes in the early universe.

Each individual soliton front carries a large energy, taking into account the original rest mass energy of the black hole, and subsequent increase in energy due to solitons interacting with the background Planck sea. This energy $E_s$ can be estimated by

$$E_s = \frac{1}{2} M_{BH} c^2 \left( \frac{1 + z_*}{1 + z} \right)^3.$$  

For $M_{BH} = 10^{6.5} M_\odot$ and $z_* = 50$ this energy is today ($z = 0$) $10^{65.6}$ erg.
The energy density per soliton is thus $10^{-23.7}$ erg cm$^{-3}$. The overall change in dark energy, if it were to just relax without any re-creation, would be about $10^{-25.0}$ erg cm$^{-3}$ s$^{-1}$.

These simplistic numbers give a front coming through every 20 seconds, but the transient time is of the order of 1000 seconds. Approximately 50 uncorrelated shells going through.

Wave forms created by uncorrelated solitons passing

![Wave forms created by uncorrelated solitons passing](image)
Variation in energy density can be approximated by

\[ \Delta \rho = A \sin(\omega t), \]

\[ A \simeq 10^{-22.5} \text{ erg/cm}^3, \quad \omega \simeq 2\pi/20 \text{s}^{-1} \]

**Metric**

\[
g = \begin{bmatrix}
- (1 + A_0 (t1)) c^2 & 0 & 0 & 0 \\
0 & (1 + A_1 (t1)) \left( e^{\frac{t}{\alpha_M}} \right)^2 & 0 & 0 \\
0 & 0 & (e^{\frac{t}{\alpha_M}})^2 (1 + A_2 (t1)) & 0 \\
0 & 0 & 0 & (e^{\frac{t}{\alpha_M}})^2 (1 + A_3 (t1))
\end{bmatrix}
\]
Metric Tensor Elements

\[ t_1 = \omega t - k z \]

\[ A_0(t_1) = e^{-2/3 \frac{t_1 \sqrt{3}}{\omega t H}} (A_{0e} + A_{0s} \sin(t_1) + A_{0c} \cos(t_1)) \]

\[ A_1(t_1) = e^{-2/3 \frac{t_1 \sqrt{3}}{\omega t H}} (A_{21e} + A_{1s} \sin(t_1) + A_{1c} \cos(t_1)) \]

\[ A_3(t_1) = e^{-2/3 \frac{t_1 \sqrt{3}}{\omega t H}} (A_{3e} + A_{3s} \sin(t_1) + A_{3c} \cos(t_1)) \]
Divergence relations

\[ \nabla_\mu T^{\mu \nu} = 0 \]

Energy-momentum tensor

\[ T = \begin{bmatrix}
-S_{00} \left(1 + A_0(t)\right) c^2 & 0 & 0 & 0 \\
0 & S_{11} \left(1 + A_1(t)\right) \left(\frac{e^{\frac{t}{\alpha H}}}{\alpha H}\right)^2 & 0 & 0 \\
0 & 0 & S_{22} \left(\frac{e^{\frac{t}{\alpha H}}}{\alpha H}\right)^2 \left(1 + A_2(t)\right) & 0 \\
0 & 0 & 0 & S_{33} \left(\frac{e^{\frac{t}{\alpha H}}}{\alpha H}\right)^2 \left(1 + A_3(t)\right)
\end{bmatrix} \]

where

\[ S_{00} = (P_0 - P_1 \sin(-t)) , \quad S_{11} = (P_0 + P_2 \sin(-t)) \]
\[ S_{22} = (P_0 + P_2 \sin(-t)) , \quad S_{33} = (P_0 - P_1 \sin(-t)) \]
Calculation of Time Shifts

Time variations

Time difference

\[ z_t t_1 t_2 t_0 \]

**FIG. 4:** First wave front is emitted at \( z = 0 \) and \( t = t_1 \), the second one at \( t = t_2 \). They arrive at \( z > 0 \), \( t = t_2' \) and \( t = t_2' \) respectively.
Proper-time difference

\[ c^2 d\tau^2 = -(g_{00} c^2 dt^2 + g_{ij} dx^i dx^j) \]

\[ \Delta \tau_{em} = \int_{t_1}^{t_2} \sqrt{-g_{00}(t, z = 0)} \, dt \]

\[ \Delta \tau_{rec} = \int_{t'_1}^{t'_2} \sqrt{-g_{00}(t, z)} \, dt \]

where \( g_{00} = -(1 + A_0(t1)) \) and \((t1 = \omega t - k z)\)

\[ A_0(t1) = e^{-2/3} \frac{t1}{\omega t H} \left( A_{0e} + A_{0s} \sin(t1) + A_{0c} \cos(t1) \right). \]
Frequencies of emitted and received waves

\[ \nu_{em} = \frac{1}{\Delta \tau_{em}}, \quad \nu_{rec} = \frac{1}{\Delta \tau_{rec}} \]

Fractional time shifts

The fractional time shift is given by forming the ratio

\[ \delta(\Delta \tau) = \frac{\Delta \tau_{em} - \Delta \tau_{rec}}{\Delta \tau_{em}} \]

The relation between \( \Delta \tau \) and \( \nu \) gives the fractional shift in frequency.

Magnitude of fractional time shifts

The fractional time or frequency shifts are determined by the values of the constants \( A_{0e}, A_{0s}, \) and \( A_{0c} \), which are small.
Comparison to Experimental Limits

Current experimental limit on fractional frequency shift

\[ \Delta \nu = \frac{\nu_1 - \nu_2}{\nu_1} \sim 10^{-19} \]

Comparison of model to experiment

Our model requires a precision for the fractional frequency shift of

\[ \frac{\Delta \nu}{\nu} \sim 10^{-22} \]

which is only 3 orders of magnitude below the current limit.
Conclusions

Implications for Cosmology

Detection of time jitter would be a signal of the validity of our model of gravitational waves as the source of dark energy.

Our model replaces the big-bang theory with a scenario in which our universe starts from a Lemaitre ‘atom’ or ‘seed’.

Implications for Quantum Gravity

The validity of our model would imply the existence of a strong-gravity brane and extra dimensions as well as a smallest distance and impenetrable Planck surfaces rather horizons.
Future Extensions of Our Model

Derivation of the creation equation from the Boltzmann equation in an FRW universe.

More formal mathematical description of stimulated emission amplitude for the creation of the solitons.
Pulsar timing experiments

FIG. 5: Gravitational wave background limit from pulsar timing, and our inferred gravitational wave background from stimulated emission of gravitational waves from the background Planck sea constituting dark energy. We use as the ordinate the fraction of closure energy density \( \Omega \), and as abscissa the frequency of the gravitational waves \( f \).
Time interval experiments

FIG. 6: Experimental set-up for measuring frequency stabilization
Fractional instability of frequency measurement

FIG. 7: Frequency instability vs measurement time