Quantum gravitational Kasner transitions in Bianchi-I spacetime

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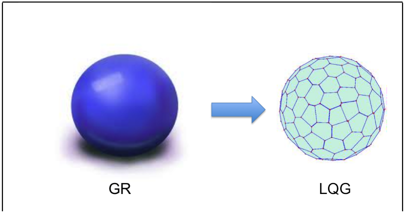
Loop quantum gravity(LQG)

Loop quantum gravity is a quantum theory of gravity based on the canonical quantization of spacetime geometry- [Ashtekar, Lewandowski, Rovelli, Smolin, Thiemann, Gambini, Pullin ...]

- Discrete structure of spacetime opposed to continuum picture in GR



Continuum geometry	Discrete geometry
Differential equations	Difference equations

Loop quantum cosmology

[Bojowald; Ashtekar, Pawlowski, Singh ...]

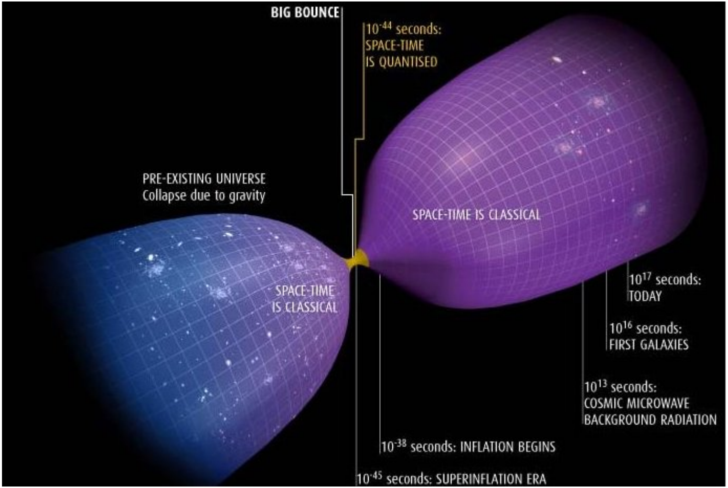


Figure: Big bounce in LQC(source: *New Scientist*)

Why study Bianchi models?

- Anisotropic spacetime introduces more degrees of freedom compared to isotropic spacetime
- Much richer physics due to non-vanishing Weyl scalar
- Classically the anisotropic shear scalar in Bianchi-I model varies as $\sigma^2 \propto a^{-6}$. Singularity can also take place due to diverging anisotropic shear as $a \to 0$
- According to Belinskii-Khalatnikov-Lifshitz (BKL) behavior, during a generic approach to a spacelike singularity, each point transits from one Bianchi-I type universe to another Bianchi-I type (Kasner transition), giving rise to Mixmaster behavior

Loop quantum cosmology of Bianchi-I spacetime

$$ds^2 = -dt^2 + a_1^2\, dx^2 + a_2^2\, dy^2 + a_3^2\, dz^2$$

In the classical theory, approach to singularity can be classified as (Doroshkevich, Ellis, Jacobs, MacCallum, Thorne ...)

- **Point or Isotropic singularity:** $a_1,\, a_2,\, a_3 \to 0$.
- **Barrel:** $a_1 \to \mathrm{const},\, a_2,\, a_3 \to 0$
- **Pancake:** $a_1 \to 0,\, a_2,\, a_3 \to \mathrm{const}$
- **Cigar:** $a_1 \to \infty,\, a_2,\, a_3 \to 0$

Quantization performed by (Ashtekar, Wilson-Ewing(09)). Earlier approaches to quantization developed by Bojowald, Chiou, Date, Martin-Benito, Mena Marugan, Pawlowski, Szulc, Vandersloot

- Classical singularity resolved
- Resolution of all physical singularities studied in the effective dynamics (Singh (11))
- Physics of effective dynamics studied: big bang is replaced by bounce (Artymowski, Cailleteau, Chiou, Lalak, Maartens, Singh, Vandersloot)

This Talk:

Questions:

- What is the relation between the geometrical nature of spacetime in pre and post bounce regime?
- Are there transitions from one type to other?
- Are some transitions favored over others? If yes, depending on what?

Answers:

- Kasner transitions are seen in Bianchi-I spacetime for the first time, a feature not present in classical theory
- Transitions are not random, there turns out to be a "selection rule"
- Depending on the anisotropy and matter content some transitions are favored over others

Kasner solution: classical theory

Vacuum:

$$a_i \propto t^{k_i} \quad \text{such that} \quad k_1 + k_2 + k_3 = 1; \quad k_1^2 + k_2^2 + k_3^2 = 1 \tag{1}$$

Stiff matter, $w = P/\rho = 1$:

$$a_i \propto t^{k_i} \quad \text{such that} \quad k_1 + k_2 + k_3 = 1; \quad k_1^2 + k_2^2 + k_3^2 = 1 - k^2 \tag{2}$$

where k_i are Kasner exponents and k is a constant.

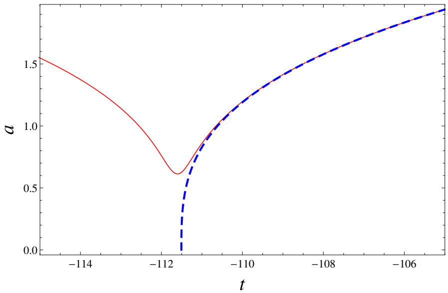
Point	$k_1,\ k_2,\ k_3 > 0$
Barrel	$k_1 = 0,\ k_2, k_3 > 0$
Pancake	$k_1, k_2 = 0,\ k_3 > 0$
Cigar	$k_1 < 0,\ k_2, k_3 > 0$

$0 \leq w < 1$:

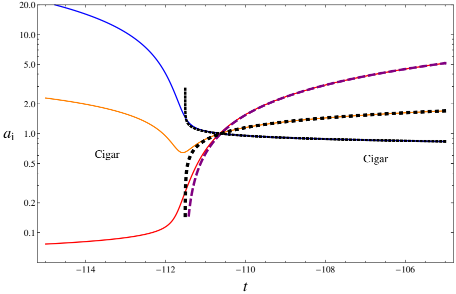
- Close to singularity, behaves like vacuum for all $0 \leq w < 1$
- In the future asymptotic limit
  - $a_i \propto t^{2/3}$ for Dust $(w = 0)$
  - $a_i \propto t^{1/2}$ for Radiation $(w = 1/3)$

Kasner transition: Stiff matter, $P = \rho$

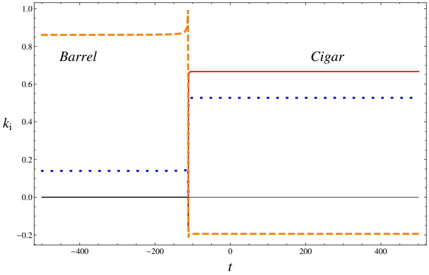
- Classical trajectory undergoes singularity
- The mean scale factor $a = (a_1 a_2 a_3)^{1/3}$ in LQC bounces.

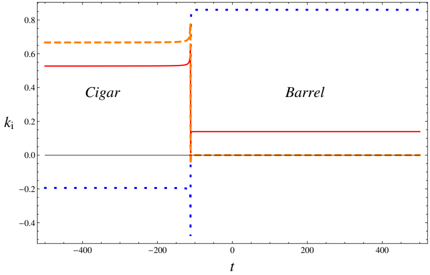


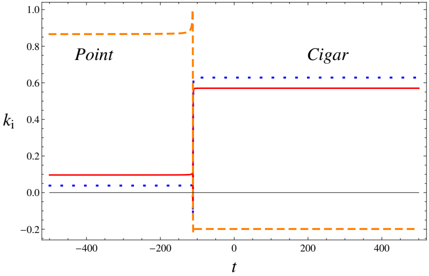
- The directional scale factors undergo Kasner transition across the bounce
- Transitions depend on anisotropy present in the spacetime

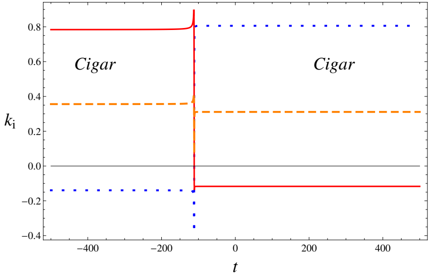


Stiff matter, $w = 1$









Kasner transition for $w = 1$

$0 < \lvert\delta\rvert < \frac{1}{2}$	$\lvert\delta\rvert = \frac{1}{2}$	$\frac{1}{2} < \lvert\delta\rvert < \frac{1}{\sqrt{3}}$	$\lvert\delta\rvert = \frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}} < \lvert\delta\rvert < 1$
P $\leftrightarrow$ P	P $\leftrightarrow$ P	P $\leftrightarrow$ P	P $\not\leftrightarrow$ P	P $\not\leftrightarrow$ P
B $\not\leftrightarrow$ P	B $\leftrightarrow$ P	B $\leftrightarrow$ P	B $\not\leftrightarrow$ P	B $\not\leftrightarrow$ P
C $\not\leftrightarrow$ P	C $\not\leftrightarrow$ P	C $\leftrightarrow$ P	C $\leftrightarrow$ P	C $\leftrightarrow$ P
B $\not\leftrightarrow$ B	B $\not\leftrightarrow$ B	B $\not\leftrightarrow$ B	B $\leftrightarrow$ B	B $\not\leftrightarrow$ B
B $\not\leftrightarrow$ C	B $\not\leftrightarrow$ C	B $\not\leftrightarrow$ C	B $\not\leftrightarrow$ C	B $\leftrightarrow$ C
C $\not\leftrightarrow$ C	C $\not\leftrightarrow$ C	C $\not\leftrightarrow$ C	C $\not\leftrightarrow$ C	C $\leftrightarrow$ C

where $\lvert\delta\rvert = \sqrt{\frac{3\sigma^2}{2\theta^2}}$.

- Depending on the value of δ some transitions are favored over others.
- In the low anisotropy regime only Point-Point transition takes place
- Cigar-Cigar transition only happens in the large anisotropy regime.

Summary

- There are Kasner transitions across the bounce in Bianchi-I spacetime, giving rise to "mixmaster-like" behavior with some differences
- Kasner transitions are seen in Bianchi-I spacetime for the first time, a feature not present in classical theory
- These transitions follow a pattern and depending on anisotropy and matter content some of them are favored- "*selection rule*"