On spherically symmetric solutions in Nonlinear Massive Gravity

Caixia Gao
In collaboration with Bombelli, Cai, Easson and Saridakis

University of Mississippi

April 18, 2013

Outline of the talk

- Introduction to massive gravity
- Linear FP action
- Non-linear dRGT action
- On spherically symmetric charged background
- Non-linear mass varying massive gravity
- Summary and future work

Motivation for massive gravity

- Can the graviton have a mass?
- Cosmic self-acceleration
- Modify General Relativity

The FP action: Fierz and Pauli 1939

$$
\begin{aligned}
L = & \quad L_{EH}(h) + m^2(h_{\mu\nu}h^{\mu\nu} - h^2) \\
= & \quad -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h \\
& \quad +\frac{1}{2}\partial_\lambda h\partial^\lambda h - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) \qquad (1)
\end{aligned}
$$

- $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- Massive spin 2 with 5 d.o.f (2 tensor,2 vector,1 scalar)
- Unique linear massive gravity theory without ghost
- Deffeomorphism invariance is broken

Nonlinear extension

- Problem of the FP action
  - vDVZ discontinuity (van Dam, Veltman,Zakharov 1970)
    - GR is not recovered in the massless limit
    - Post Newtonian parameter is $\gamma = 1/2$ instead of $\gamma = 1$
    - Light bending is off 25 percent from the GR prediction
- Non-linear extension
  - Vainshtein mechanism (Vainshtein 1972)
    - GR is recovered within Vainshtein radius: $r_v = (\frac{GM}{m^2})^{1/3}$
    - M: mass of the Sun, $m = 10^{-33} eV$, $r_v = 10^{18}$km
  - BD ghost:Boulware-Deser 1972
    - 6 d.o.f, extra scalar mode

The dRGT action: de Rham, Gabadadze, Tolley 2010

$$S = \int d^4x\sqrt{-g}\frac{M_p^2}{2}\left[R + m^2\mathcal{U}(g,\phi^a)\right] , \tag{2}$$

where $\mathcal{U}$ is the potential of graviton

$$\mathcal{U}(g,\phi^a) = \mathcal{U}_2 + \alpha_3\mathcal{U}_3 + \alpha_4\mathcal{U}_4 \ . \tag{3}$$

$$\mathcal{U}_2 \equiv [\mathcal{K}]^2 - [\mathcal{K}^2] \ , \tag{4}$$
$$\mathcal{U}_3 \equiv [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3] \ , \tag{5}$$
$$\mathcal{U}_4 \equiv [\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 8[\mathcal{K}][\mathcal{K}^3] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4] \ . \tag{6}$$

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \sqrt{g^{\mu\sigma}\eta_{ab}\partial_\sigma\phi^a\partial_\nu\phi^b} \ . \tag{7}$$

where $M_p = 1/\sqrt{8\pi G}$ and ϕ^a are the Stückelberg scalars.
Choosing unitary gauge: $\phi^a = x^\mu\delta^a_\mu$

The dRGT action

- Vainshtein mechanism
- Free of the BD ghost
- Cosmic self-acceleration

Spherically symmetric charged background

$$ds^2 = -N^2(r)dt^2 + \frac{dr^2}{F^2(r)} + \frac{r^2 d\Omega_2^2}{H^2(r)} \ , \tag{8}$$

Assume static electric charge in the system:

$$E_r = F_{0r} = E(r) \ , \quad E_\theta = E_\varphi = 0 \ , \quad \vec{B} = 0 \ . \tag{9}$$

Inhomogeneous Maxwell equation:

$$D_\mu F^{\mu\nu} = -J^\nu . \tag{10}$$

Source free $J^\nu = 0$, we obtain

$$E(r) = \frac{QNH}{4\pi F r^2} \tag{11}$$

Weak field limit: Koyama,Niz,Tasinato 2011

$$\begin{aligned} N(r) &= 1 + n(r) \\ F(r) &= 1 + f(r) \\ H(r) &= 1 + h(r) \end{aligned} \quad (12)$$

Rescale the radial component

$$\rho = \frac{r}{H} \Rightarrow 1 + \tilde{f} = \frac{1+f}{1+h+\rho h'}$$

Linearized metric

$$ds^2 = -(1+2n(\rho))dt^2 + (1-2\tilde{f}(\rho))d\rho^2 + \rho^2 d\Omega^2 \quad (13)$$

Parameter space: arXiv:1211.0563

- $\alpha = 0$, $\beta = 0$ ruled out because of the vDVZ discontinuity
- $\alpha \neq 0$, $\beta = 0$ shows the Vainshtein mechanism
- $\alpha \neq 0$, $\beta < 0$ ruled out due to dramatic change to GR
- $\alpha \neq 0$, $\beta > 0$ shows the Vainshtein mechanism
- Exactly analytic solution: $\beta = \frac{\alpha^2}{6}$

Case II: $\alpha \neq 0$ and $\beta = 0$

$$\rho_Q \equiv (\frac{GQ^2}{4\pi m^2})^{1/4} \ , \ \ \rho_V \equiv (\frac{GM}{m^2})^{1/3} \ .$$

When $2GM < \rho < \rho_Q$,

$$n \simeq \frac{GQ^2}{8\pi\rho^2} - \frac{GM}{\rho} + \frac{m^2\rho_Q^2}{2\alpha^{1/2}} \ln(m\rho + c) \ , \tag{14}$$

$$\tilde{f} \simeq \frac{GQ^2}{8\pi\rho^2} - \frac{GM}{\rho} - \frac{m^2\rho_Q^2}{\alpha^{1/2}} + \frac{GM\rho}{2\alpha^{1/2}\rho_Q^2} \ . \tag{15}$$

When $\rho_Q < \rho < \rho_V$,

$$n \simeq \frac{GQ^2}{8\pi\rho^2} - \frac{GM}{\rho} + \frac{GM\rho}{2\alpha^{2/3}\rho_V^2} \ , \tag{16}$$

$$\tilde{f} \simeq \frac{GQ^2}{8\pi\rho^2} - \frac{GM}{\rho} + \frac{GM}{2\alpha^{1/3}\rho_V} + \frac{GM\rho}{2\alpha^{2/3}\rho_V^2} \ . \tag{17}$$

Case II: $\alpha \neq 0$ and $\beta = 0$

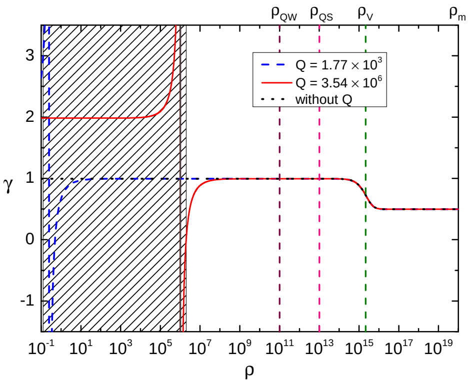

Figure: $\gamma = \frac{\tilde{f}}{n}, \alpha = 1$

Case III: $\alpha \neq 0$ and $\beta > 0$

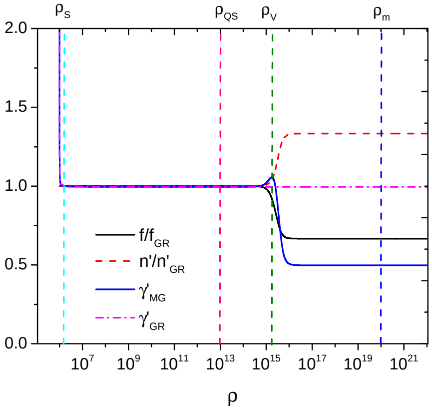

Figure: $\alpha = 1$, $\beta = 3$.

Case III: $\alpha \neq 0$ and $\beta > 0$, Analytic analysis

Neglecting terms proportional to h^3 and h^5, approximate solution

$$h = \frac{-\sqrt{(3\rho^4 - 2\alpha\rho_Q{}^4)^2 + 48\rho\rho_V{}^3(\alpha\rho^4 - 2\beta\rho_Q{}^4 + 2\beta\rho\rho_V{}^3)}}{12(\alpha\rho^4 - 2\beta\rho_Q{}^4 + 2\beta\rho\rho_V{}^3)} + \frac{3\rho^4 - 2\alpha\rho_Q{}^4}{12(\alpha\rho^4 - 2\beta\rho_Q{}^4 + 2\beta\rho\rho_V{}^3)} \quad (18)$$

Special solution: $\beta = \alpha^2/6$

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{A(r)} + r^2 d\Omega^2 \ , \tag{19}$$

where

$$A(r) = 1 + \frac{r_Q^2}{r^2} - \frac{\tilde{r}_S}{r} - \frac{r^2}{r_\Lambda^2} \ , \tag{20}$$

$$\tilde{r}_S \equiv \frac{\alpha^3 r_M}{(1+\alpha)^3} \ , \quad r_\Lambda \equiv \frac{\sqrt{3\alpha}}{m} \ . \tag{21}$$

Mass varying massive gravity: Huang,Piao,Zhou 2012

- A generalization of the dRGT massive gravity
- Free of the BD ghost
- Inflation, bounce and cyclic universe:arXiv:1207.3786

$$S = \int d^4x\sqrt{-g}\frac{M_p^2}{2}\Big[R + V(\psi)(\mathcal{U}_2 + \alpha_3\mathcal{U}_3 + \alpha_4\mathcal{U}_4) -\frac{1}{2}\partial_\mu\psi\partial^\mu\psi - W(\psi)\Big] , \qquad (22)$$

Spherical symmetric solution

- $ds^2 = -N^2(r)dt^2 + \frac{dr^2}{F^2(r)} + \frac{r^2 d\Omega_2^2}{H^2(r)}$
- Ansatz: H(r)=1, F(r)=N(r)
- Numerical solution for F(r)is always singular

Out[117]=

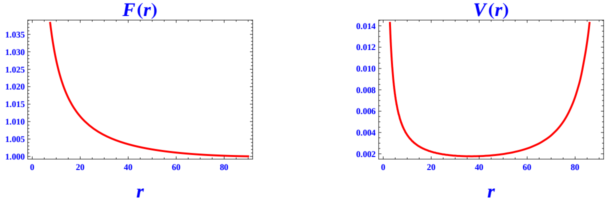

Figure: $\alpha = 1$, $\beta = 0$.

Summary and future work

- Linear massive gravity
  - vDVZ discontinuity
- Non-linear massive gravity
  - dRGT massive gravity
    - Vaishtein mechanism
    - Free of BD ghost
  - Mass varying massive gravity
    - Start with known non-singular metric factor to look for the mass of graviton
    - Start with known non-singular mass of graviton and scalar potential to look for metric factors