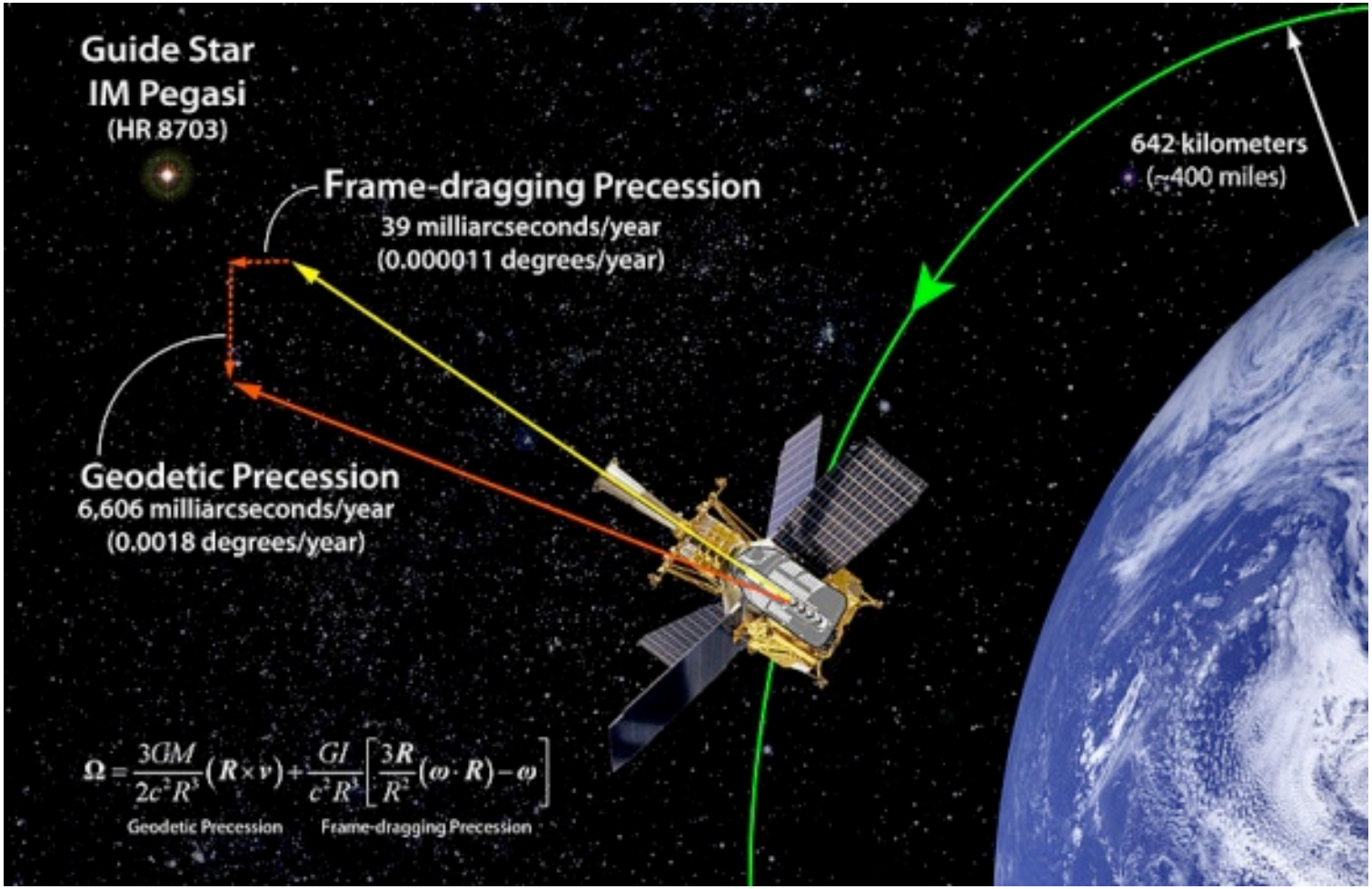
Gravity Probe B Constraints on Kaluza Klein Gravity and in the SME

Ryan Everett

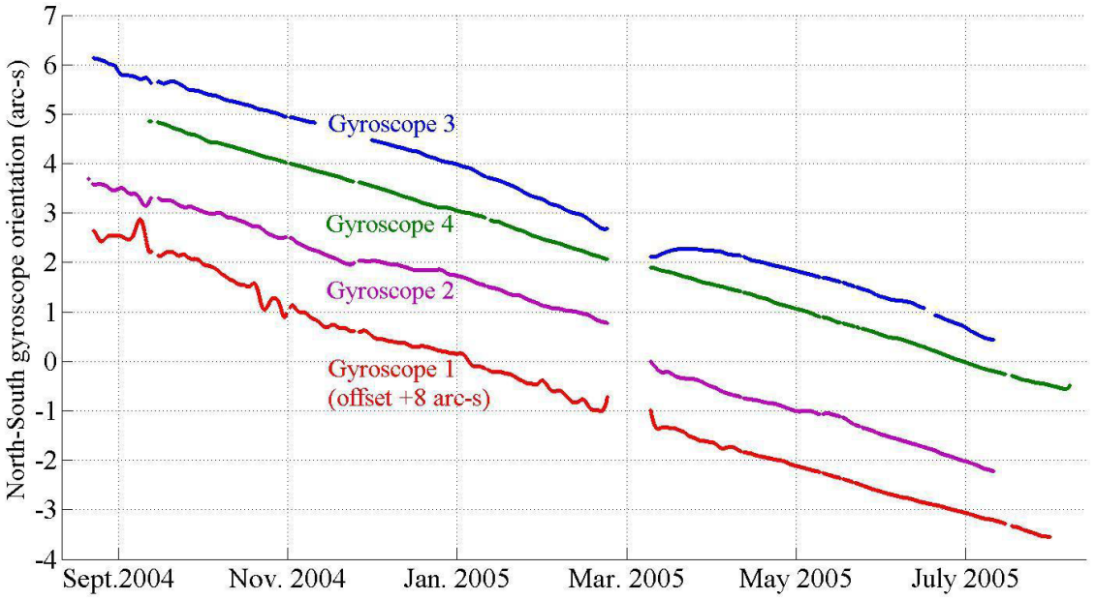
Outline

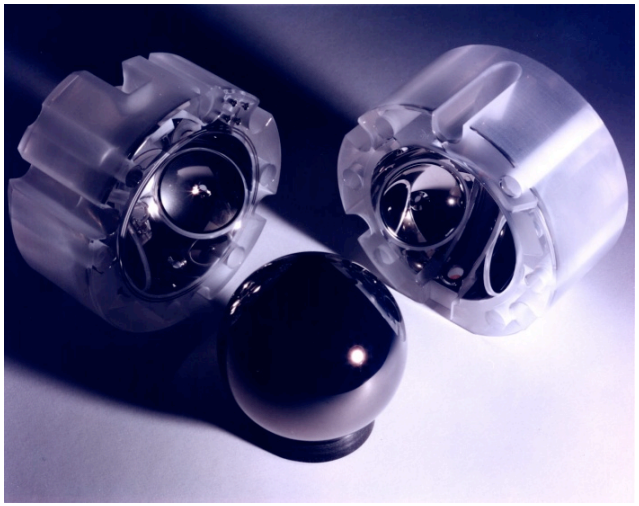
- ***Brief Background***
  - GPB
  - KKG
  - SME
- Solutions in Kaluza Klein Gravity
  - Soliton
  - Canonical
  - Parameter Constraints
- Standard Model Extension
  - $s^{\mu\nu}$ Constraint Equation Derivation
  - 4 New Independent Limits
- Short Discussion

Brief Background - GPB



Brief Background - GPB





Brief Background - KKG

- Theodore Kaluza – 1921
  - GR extension to 5D
  - Unify Gravity and Electromagnetism
- Induced Matter (Paul Wesson)
  - $R_{AB} = 0$
  - $G_{\mu\nu} = 8\pi T_{\mu\nu}$
- Birkhoff's Theorem Breakdown

Brief Background - SME

- Alan Kostelecky and Don Colladay – 1997
  - Minimal SME in flat space-time
- Allows Lorentz Violation
  - CPT violations and preservation
- Vacuum Expectation Value
  - Pure Gravity Sector - $s^{\mu\nu}$
- Experiments Constraining $s^{\mu\nu}$
  - Laser Lunar Ranging
  - Atom Interferometric Gravimeter

Outline

- Brief Background
  - GPB
  - KKG
  - SME
- ***Solutions in Kaluza Klein Gravity***
  - Soliton
  - Canonical
  - Parameter Constraints
- Standard Model Extension
  - $s^{\mu\nu}$ Constraint Equation Derivation
  - 4 New Independent Limits
- Short Discussion

Solutions in Kaluza Klein Gravity - Soliton

Solutions in Kaluza Klein Gravity - Soliton

- $dS^2 = A^a dt^2 - A^{-a-b} dr^2 - A^{1-a-b} r^2 (d\theta^2 + \sin^2\theta \, d\phi^2) - A^b d\ell^2$
  - $A(r) \equiv 1 - \frac{2GM}{c^2 r}$
- $1 = a^2 + ab + b^2$
- $\frac{d^2 x^C}{dS^2} + \Gamma^C_{AB} u^A u^B = 0$ $\qquad$ $\frac{d^2 S^C}{dS^2} + \Gamma^C_{AB} S^A u^B = 0$
- S^A is the 5D spin vector and u^B is the orbital velocity
  - $u^C S_C = 0$

- Assumptions:
  - Circular Orbit
    - $\theta = \frac{\pi}{2}$
    - $\dot{\theta} = \dot{r} = 0$
  - S^{ℓ} is unrestricted

- Precession Angular Speed of S^A
  - $\Omega = \sqrt{\frac{aGM}{cr_0^3}}\left[1 + \frac{3GM}{2c^2 r_0}(1 - a - b)\right]$
- Orbital Angular Speed
  - $\omega = d\phi/dS$

- Geodetic Precession Angle per Orbit
  - $\delta\phi = 2\pi\left(\frac{\omega-\Omega}{\omega}\right) = \frac{3\pi GM}{c^2 r_0}(1+\Delta)$
- Δ is the Deviation from GR
  - $\Delta = a + \frac{2}{3}b \approx \frac{b}{6}$
- Where b < 0 for physical reasons
  - Positivity of Mass Density
  - Less Observed Precession
- b "measures" the flatness of the fifth dimension

Solutions in Kaluza Klein Gravity - Canonical

Solutions in Kaluza Klein Gravity - Canonical

- $ds^2 = \frac{\ell^2}{L^2}[Bdt^2 - B^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2)] - d\ell^2$
  - $B(r) \equiv 1 - \frac{2GM}{c^2 r} - \frac{r^2}{L^2}$
  - $\Lambda = 3/L^2$

- $\frac{d^2x^C}{dS^2} + \Gamma^C_{AB}u^A u^B = 0$ $\qquad$ $\frac{d^2S^C}{dS^2} + \Gamma^C_{AB}S^A u^B = 0$

$$u^C S_C = 0$$

- Additional Assumptions:
  - Spin Vector in the Orbital Plane
    - $S^{\theta} = 0$
  - $rS^{\phi} \ll S^{r}$

- Precession Angular Speed of the Spin Vector
  - $\Omega = \frac{c}{r_0}\sqrt{\frac{GM}{c^2 r_0} - \frac{r_0^2}{L^2}}$

- Orbital Angular Speed
  - $\omega = d\phi/dS$

- Geodetic Precession Angle per Orbit
  - $\delta\phi = 2\pi\left(\frac{\omega-\Omega}{\omega}\right) = \frac{3\pi GM}{c^2 r_0}(1+\Delta)$
- Δ is the Deviation from GR
  - $\Delta = -\frac{2c^2 r_0^2}{3GM}\frac{\mathcal{H}}{L}$
- Consider $\mathcal{H}$ to be of Order Unity and:
  - $L = -\frac{2c^2 r_0^2}{3GM}\frac{1}{\Delta}$
- Relate L to Normalized Cosmological Constant, Ω_Λ
  - $\Omega_\Lambda = \frac{\Lambda c^2}{3H_0} \rightarrow L = \frac{c}{H_0\sqrt{\Omega_\Lambda}}$
  - $\therefore \mathcal{H} = -\frac{3GM}{2cr_0^2 H_0\sqrt{\Omega_\Lambda}}\Delta$

Solutions in Kaluza Klein Gravity – Parameter Constraints

Table 1. GP-B Limits on 5D Metric Parameters

Gyro	r_{NS} (mas/yr)	$\Delta(\times 10^{-3})$	$\lvert b\rvert_{\max}$	$\mathcal{H}_{\max}$	$L_{\min}$ (pc)
1	-6588.6 ± 31.7	(-7.4,+2.2)	0.045	1.5×10^{8}	32
2	-6707.0 ± 64.1	(+5.6,+25.0)	N/A	N/A	N/A
3	-6610.5 ± 43.2	(-5.9,+7.2)	0.043	1.2×10^{8}	41
4	-6588.7 ± 33.2	(-7.7,+2.4)	0.046	1.5×10^{8}	31
Joint	-6601.8 ± 18.3	(-3.4,+2.1)	0.020	6.8×10^{7}	70

Solutions in Kaluza Klein Gravity – Parameter Constraints

Table 1. GP-B Limits on 5D Metric Parameters

Gyro	r_{NS} (mas/yr)	$\Delta(\times 10^{-3})$	$\lvert b\rvert_{\max}$	$\mathcal{H}_{\max}$	$L_{\min}$ (pc)
1	-6588.6 ± 31.7	(-7.4,+2.2)	0.045	1.5×10^{8}	32
2	-6707.0 ± 64.1	(+5.6,+25.0)	N/A	N/A	N/A
3	-6610.5 ± 43.2	(-5.9,+7.2)	0.043	1.2×10^{8}	41
4	-6588.7 ± 33.2	(-7.7,+2.4)	0.046	1.5×10^{8}	31
Joint	-6601.8 ± 18.3	(-3.4,+2.1)	0.020	6.8×10^{7}	70

Solutions in Kaluza Klein Gravity – Parameter Constraints

GR Predicts 6606.1

Table 1. GP-B Limits on 5D Metric Parameters

Gyro	r_{NS} (mas/yr)	$\Delta(\times 10^{-3})$	$\lvert b\rvert_{\max}$	$\mathcal{H}_{\max}$	$L_{\min}$ (pc)
1	-6588.6 ± 31.7	(-7.4,+2.2)	0.045	1.5×10^{8}	32
2	-6707.0 ± 64.1	(+5.6,+25.0)	N/A	N/A	N/A
3	-6610.5 ± 43.2	(-5.9,+7.2)	0.043	1.2×10^{8}	41
4	-6588.7 ± 33.2	(-7.7,+2.4)	0.046	1.5×10^{8}	31
Joint	-6601.8 ± 18.3	(-3.4,+2.1)	0.020	6.8×10^{7}	70

Solutions in Kaluza Klein Gravity – Parameter Constraints

Table 1. GP-B Limits on 5D Metric Parameters

Gyro	r_{NS} (mas/yr)	$\Delta(\times 10^{-3})$	$\lvert b\rvert_{\max}$	$\mathcal{H}_{\max}$	$L_{\min}$ (pc)
1	-6588.6 ± 31.7	(-7.4,+2.2)	0.045	1.5×10^{8}	32
2	-6707.0 ± 64.1	(+5.6,+25.0)	N/A	N/A	N/A
3	-6610.5 ± 43.2	(-5.9,+7.2)	0.043	1.2×10^{8}	41
4	-6588.7 ± 33.2	(-7.7,+2.4)	0.046	1.5×10^{8}	31
Joint	-6601.8 ± 18.3	(-3.4,+2.1)	0.020	6.8×10^{7}	70

Solutions in Kaluza Klein Gravity – Parameter Constraints

Table 1. GP-B Limits on 5D Metric Parameters

Gyro	r_{NS} (mas/yr)	$\Delta(\times 10^{-3})$	$\lvert b\rvert_{\max}$	$\mathcal{H}_{\max}$	$L_{\min}$ (pc)
1	-6588.6 ± 31.7	(-7.4,+2.2)	0.045	1.5×10^{8}	32
2	-6707.0 ± 64.1	(+5.6,+25.0)	N/A	N/A	N/A
3	-6610.5 ± 43.2	(-5.9,+7.2)	0.043	1.2×10^{8}	41
4	-6588.7 ± 33.2	(-7.7,+2.4)	0.046	1.5×10^{8}	31
Joint	-6601.8 ± 18.3	(-3.4,+2.1)	0.020	6.8×10^{7}	70

Solutions in Kaluza Klein Gravity – Parameter Constraints

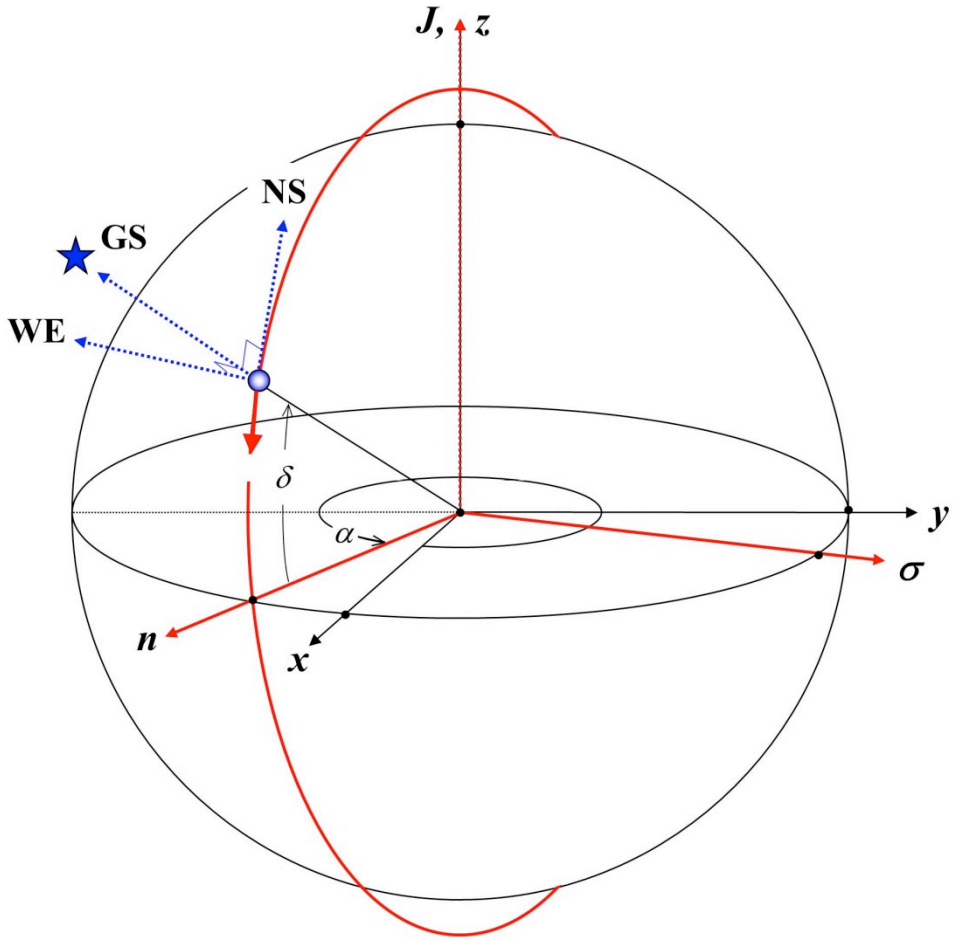
Table 1. GP-B Limits on 5D Metric Parameters

Gyro	r_{NS} (mas/yr)	$\Delta(\times 10^{-3})$	$\lvert b\rvert_{\max}$	$\mathcal{H}_{\max}$	$L_{\min}$ (pc)
1	-6588.6 ± 31.7	(-7.4,+2.2)	0.045	1.5×10^{8}	32
2	-6707.0 ± 64.1	(+5.6,+25.0)	N/A	N/A	N/A
3	-6610.5 ± 43.2	(-5.9,+7.2)	0.043	1.2×10^{8}	41
4	-6588.7 ± 33.2	(-7.7,+2.4)	0.046	1.5×10^{8}	31
Joint	-6601.8 ± 18.3	(-3.4,+2.1)	0.020	6.8×10^{7}	70

Outline

- Brief Background
  - GPB
  - KKG
  - SME
- Solutions in Kaluza Klein Gravity
  - Soliton
  - Canonical
  - Parameter Constraints
- ***Standard Model Extension***
  - $s^{\mu\nu}$ Constraint Equation Derivation
  - 4 New Independent Limits
- Short Discussion

Standard Model Extension - $s^{\mu\nu}$ Constraint Equation Derivation



- Three Coordinate Systems
- Inertial Frame
  - $\hat{x}$ points towards vernal equinox
  - $\hat{z}$ aligns with spin axis of Earth
  - $\hat{y} = \hat{z} \times \hat{x}$
- GPB Frame
  - $\hat{e}_{GS}$ points towards IM Pegasi
  - $\hat{e}_{NS}$ aligns with spin axis of Earth
  - $\hat{e}_{EW} = \hat{e}_{GS} \times \hat{e}_{NS}$
- Hybrid Frame
  - $\hat{\sigma}$ is normal to the orbit
  - $\hat{J}$ aligns with spin axis of Earth
  - $\hat{n} = \hat{\sigma} \times \hat{J}$

- Anomalous Gyroscope Precession in "Hybrid" Frame:

$$\Delta_{\mathrm{BK}} = \begin{pmatrix} \frac{1}{2}\omega_{WE}\sin 2\alpha_{BK}(s^{YY} - s^{XX}) + s^{XY}\omega_{WE}(2\sin^2\alpha_{BK} - 1) \\ s^{TT}\omega_T + s^{XX}\omega_{NS}\sin^2\alpha_{BK} - s^{XY}\omega_{NS}\sin 2\alpha_{BK} + s^{YY}\omega_Y\cos^2\alpha_{BK} \\ s^{XZ}\omega_{WE}\sin\alpha_{BK} - s^{YZ}\omega_{WE}\cos\alpha_{BK} \end{pmatrix}$$

$$\omega(\beta) = \left(1 + \frac{\beta I}{Mr^2}\right)\left(\frac{GM}{c^2 r}\right)^{3/2}$$

Rotation Transformation From Hyrbid to GPB Frame:

$$\mathbb{R} = \begin{pmatrix} \cos\delta & 0 & \sin\delta \\ 0 & -1 & 0 \\ -\sin\delta & 0 & \cos\delta \end{pmatrix}$$

- Anomalous Gyroscope Precession in "Hybrid" Frame:

$$\Delta_{BK} = \begin{pmatrix} \frac{1}{2}\omega_{WE}\sin 2\alpha_{BK}(s^{YY} - s^{XX}) + s^{XY}\omega_{WE}(2\sin^2\alpha_{BK} - 1) \\ s^{TT}\omega_T + s^{XX}\omega_{NS}\sin^2\alpha_{BK} - s^{XY}\omega_{NS}\sin 2\alpha_{BK} + s^{YY}\omega_Y\cos^2\alpha_{BK} \\ s^{XZ}\omega_{WE}\sin\alpha_{BK} - s^{YZ}\omega_{WE}\cos\alpha_{BK} \end{pmatrix}$$

$$\omega(\beta) = \left(1 + \frac{\beta I}{Mr^2}\right)\left(\frac{GM}{c^2 r}\right)^{3/2}$$

$\hat{n}$ direction

Rotation Transformation From Hyrbid to GPB Frame:

$$\mathbb{R} = \begin{pmatrix} \cos\delta & 0 & \sin\delta \\ 0 & -1 & 0 \\ -\sin\delta & 0 & \cos\delta \end{pmatrix}$$

- Anomalous Gyroscope Precession in "Hybrid" Frame:

$$\Delta_{BK} = \begin{pmatrix} \frac{1}{2}\omega_{WE}\sin 2\alpha_{BK}(s^{YY} - s^{XX}) + s^{XY}\omega_{WE}(2\sin^2\alpha_{BK} - 1) \\ s^{TT}\omega_T + s^{XX}\omega_{NS}\sin^2\alpha_{BK} - s^{XY}\omega_{NS}\sin 2\alpha_{BK} + s^{YY}\omega_Y\cos^2\alpha_{BK} \\ s^{XZ}\omega_{WE}\sin\alpha_{BK} - s^{YZ}\omega_{WE}\cos\alpha_{BK} \end{pmatrix}$$

$$\omega(\beta) = \left(1 + \frac{\beta I}{Mr^2}\right)\left(\frac{GM}{c^2 r}\right)^{3/2}$$

Rotation Transformation From Hyrbid to GPB Frame:

$\hat{\sigma}$ direction

$$\mathbb{R} = \begin{pmatrix} \cos\delta & 0 & \sin\delta \\ 0 & -1 & 0 \\ -\sin\delta & 0 & \cos\delta \end{pmatrix}$$

- Anomalous Gyroscope Precession in "Hybrid" Frame:

$$\Delta_{BK} = \begin{pmatrix} \frac{1}{2}\omega_{WE}\sin 2\alpha_{BK}(s^{YY} - s^{XX}) + s^{XY}\omega_{WE}(2\sin^2\alpha_{BK} - 1) \\ s^{TT}\omega_T + s^{XX}\omega_{NS}\sin^2\alpha_{BK} - s^{XY}\omega_{NS}\sin 2\alpha_{BK} + s^{YY}\omega_Y\cos^2\alpha_{BK} \\ s^{XZ}\omega_{WE}\sin\alpha_{BK} - s^{YZ}\omega_{WE}\cos\alpha_{BK} \end{pmatrix}$$

$$\omega(\beta) = \left(1 + \frac{\beta I}{Mr^2}\right)\left(\frac{GM}{c^2 r}\right)^{3/2}$$

Rotation Transformation From Hyrbid to GPB Frame:

$$\mathbb{R} = \begin{pmatrix} \cos\delta & 0 & \sin\delta \\ 0 & -1 & 0 \\ -\sin\delta & 0 & \cos\delta \end{pmatrix} \quad \hat{J} \text{ direction}$$

- Anomalous Gyroscope Precession in "Hybrid" Frame:

$$\Delta_{\mathrm{BK}} = \begin{pmatrix} \frac{1}{2}\omega_{WE}\sin 2\alpha_{BK}(s^{YY} - s^{XX}) + s^{XY}\omega_{WE}(2\sin^2\alpha_{BK} - 1) \\ s^{TT}\omega_T + s^{XX}\omega_{NS}\sin^2\alpha_{BK} - s^{XY}\omega_{NS}\sin 2\alpha_{BK} + s^{YY}\omega_Y\cos^2\alpha_{BK} \\ s^{XZ}\omega_{WE}\sin\alpha_{BK} - s^{YZ}\omega_{WE}\cos\alpha_{BK} \end{pmatrix}$$

$$\omega(\beta) = \left(1 + \frac{\beta I}{Mr^2}\right)\left(\frac{GM}{c^2 r}\right)^{3/2}$$

Rotation Transformation From Hyrbid to GPB Frame:

$$\mathbb{R} = \begin{pmatrix} \cos\delta & 0 & \sin\delta \\ 0 & -1 & 0 \\ -\sin\delta & 0 & \cos\delta \end{pmatrix}$$

- $\omega_{WE} = 4607\,^{mas}/_{yr}$
- $\omega_{NS} = 1898\,^{mas}/_{yr}$
- $\omega_{T} = 4505\,^{mas}/_{yr}$
- $\alpha_{BK} = \alpha - \pi = 2.85$ rad
- $\delta = 0.0944$ rad

- Anomalous Gyroscope Precession in GPB Frame:

$$\Delta_{GPB} = \begin{pmatrix} 1216s^{XX} + 3678s^{XY} + 384s^{XZ} - 1216s^{YY} + 1278s^{YZ} \\ -368s^{XX} - 1113s^{XY} + 1270s^{XZ} + 368s^{YY} + 4222s^{YZ} \\ -4506s^{TT} - 157s^{XX} - 1047s^{XY} - 1741s^{YY} \end{pmatrix}$$

- $\omega_{WE} = 4607\,^{mas}/_{yr}$
- $\omega_{NS} = 1898\,^{mas}/_{yr}$
- $\omega_{T} = 4505\,^{mas}/_{yr}$
- $\alpha_{BK} = \alpha - \pi = 2.85$ rad
- $\delta = 0.0944$ rad

$\hat{e}_{GS}$ direction

- Anomalous Gyroscope Precession in GPB Frame:

$$\Delta_{GPB} = \begin{pmatrix} 1216s^{XX} + 3678s^{XY} + 384s^{XZ} - 1216s^{YY} + 1278s^{YZ} \\ -368s^{XX} - 1113s^{XY} + 1270s^{XZ} + 368s^{YY} + 4222s^{YZ} \\ -4506s^{TT} - 157s^{XX} - 1047s^{XY} - 1741s^{YY} \end{pmatrix}$$

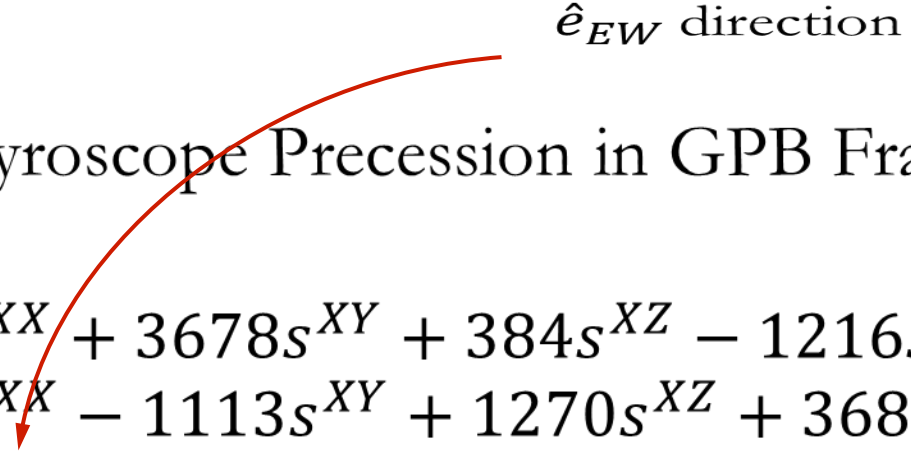
- $\omega_{WE} = 4607\,^{mas}/_{yr}$
- $\omega_{NS} = 1898\,^{mas}/_{yr}$
- $\omega_{T} = 4505\,^{mas}/_{yr}$
- $\alpha_{BK} = \alpha - \pi = 2.85$ rad
- $\delta = 0.0944$ rad

$\hat{e}_{NS}$ direction

- Anomalous Gyroscope Precession in GPB Frame:

$$\Delta_{GPB} = \begin{pmatrix} 1216s^{XX} + 3678s^{XY} + 384s^{XZ} - 1216s^{YY} + 1278s^{YZ} \\ -368s^{XX} - 1113s^{XY} + 1270s^{XZ} + 368s^{YY} + 4222s^{YZ} \\ -4506s^{TT} - 157s^{XX} - 1047s^{XY} - 1741s^{YY} \end{pmatrix}$$

- $\omega_{WE} = 4607\,^{mas}/_{yr}$
- $\omega_{NS} = 1898\,^{mas}/_{yr}$
- $\omega_{T} = 4505\,^{mas}/_{yr}$
- $\alpha_{BK} = \alpha - \pi = 2.85$ rad
- $\delta = 0.0944$ rad



- Anomalous Gyroscope Precession in GPB Frame:

$$\Delta_{GPB} = \begin{pmatrix} 1216s^{XX} + 3678s^{XY} + 384s^{XZ} - 1216s^{YY} + 1278s^{YZ} \\ -368s^{XX} - 1113s^{XY} + 1270s^{XZ} + 368s^{YY} + 4222s^{YZ} \\ -4506s^{TT} - 157s^{XX} - 1047s^{XY} - 1741s^{YY} \end{pmatrix}$$

- $\omega_{WE} = 4607\,^{mas}/_{yr}$
- $\omega_{NS} = 1898\,^{mas}/_{yr}$
- $\omega_{T} = 4505\,^{mas}/_{yr}$
- $\alpha_{BK} = \alpha - \pi = 2.85$ rad
- $\delta = 0.0944$ rad

- Anomalous Gyroscope Precession in GPB Frame:

$$\Delta_{GPB} = \begin{pmatrix} 1216s^{XX} + 3678s^{XY} + 384s^{XZ} - 1216s^{YY} + 1278s^{YZ} \\ -368s^{XX} - 1113s^{XY} + 1270s^{XZ} + 368s^{YY} + 4222s^{YZ} \\ -4506s^{TT} - 157s^{XX} - 1047s^{XY} - 1741s^{YY} \end{pmatrix}$$

The 7 Constraint Equations and Free Parameters

- Previous Experimental Upper Limits on $s^{\mu\nu}$ Coefficients
  - $s^{XY} < 10^{-9}$
  - $s^{XZ} < 10^{-9}$
  - $s^{YZ} < 10^{-9}$
  - $s^{XX} - s^{YY} < 10^{-9}$
  - $s^{XX} + s^{YY} - 2s^{ZZ} < 10^{-7}$
- Traceless Condition on $s^{\mu\nu}$
  - $s^{TT} - s^{XX} - s^{YY} - s^{ZZ} = 0$
- EW GPB
  - $|-4505s^{TT} - 157s^{XX} - 1047s^{XY} - 1741s^{YY} < 22.6$

Standard Model Extension – 4 New Independent Limits

Parameter	Upper Limit
$\lvert s^{TT} \rvert$	4.4×10^{-3}
$\lvert s^{XX} \rvert$	1.5×10^{-3}
$\lvert s^{YY} \rvert$	1.5×10^{-3}
$\lvert s^{ZZ} \rvert$	1.5×10^{-3}

Acknowledgements

- Dr. James Overduin
- Dr. Quentin Bailey
- Towson University Jess and Mildred Fisher College of Science and Mathematics Undergraduate Research Travel Grant

References

- C.M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge University Press, 1993)
- J.M. Overduin and P.S. Wesson, "Kaluza-Klein Gravity," Phys. Rep. 283 (1997) 303-378
- D. Gross, and M. J. Perry, "Magnetic Monopoles in Kaluza Klein Theories," Nuclear Phys B. 226 (1983) 29-48; R.D. Sorkin, "Kaluza-Klein Monopole," Phys. Rev. Lett. 51 (1983) 87-90
- B. Mashhoon *et al*, "Particle Masses and the Cosmological Constant in Kaluza-Klein Theory," Phys. Lett. B 331 (1994) 305-312
- H. Liu and J.M. Overduin, "Solar System Tests of Higher Dimensional Gravity," Astrophys. J. 538 (2000) 386-394
- C.W.F. Everitt *et al*, "Gravity Probe B: Final Results of a Space Experiment to Test General Relativity," Phys. Rev. Lett. 106 (2011) 221101
- H. Liu and P. S. Wesson, "The Motion of a Spinning Object in a Higher-Dimensional Spacetime," Class. Quantum Grav. 13 (1996) 2311-2318
- J.M. Overduin and P.S. Wesson, "Kaluza-Klein Cosmology with Noncompactified Extra Dimensions," in M. Rainer and H.-J. Schmidt, *Current Topics in Mathematical Cosmology* (Singapore: World Scientific, 1998) 293-301
- Q.G. Bailey and V.A. Kostelecky 2006, \Signals for Lorentz violation in post-Newtonian gravity," Phys. Rev. D74, 045001
- J.M. Overduin 2007, Constraints on Lorentz violation from Gravity Probe B," in V.A. Kostelecky (ed.) proc. Fourth Meeting on CP and Lorentz Symmetry (Singapore: World Scientific, 2007), p. 199-205
- V.A. Kostelecky and N. Russell 2011, \Data tables for Lorentz and CPT violation," Rev. Mod. Phys. 83, 11-31; http://arxiv.org/abs/0801.0287