Self-consistent evolution of a scalar point charge around a Schwarzschild black hole

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Extreme Mass Ratio Inspirals (EMRI’s).

- EMRI’s are expected to be important sources for future space based gravitational wave detectors.
- EMRI’s are intractable with standard numerical relativity methods.
- Smallness of mass ratio suggests using perturbation techniques instead (point particle moving in the background field).
- To zeroth order the motion of a point particle is geodesic.
- To first order the point particle experiences an acceleration caused by it’s interaction with it’s own field.
- The field at particle location can be decomposed into a singular piece and a regular piece $\psi = \psi^S + \psi^R$.
- The singular piece $\psi^S$ does not contribute to the self-force.
- The regular piece $\psi^R$ contains a tail contribution.
- The ultimate goal is a fully self-consistent evolution.
Traditional approaches to self-force calculations.

Most calculations of the self-force uses a $\delta$-function source.

- Using a $\delta$-function source requires subtraction of the singular part of the field in a mode sum regularization scheme.
- This can be fairly expensive especially at high eccentricity where many modes has to be taken into account.

This can be done in both the frequency and time domain.

**Frequency domain** Requires a prescribed geodesic orbit. Very accurate.

**Time domain** Requires evolving many modes simultaneously. Less accurate.
Traditional approaches to EMRI evolutions.

- Adiabatic approaches without explicit self-force calculation.
  - Energy and angular momentum fluxes through horizon and infinity are integrated over a complete geodesic orbit.
  - The geodesic orbit is then “evolved” by changing the orbital parameters according to the energy and angular momentum losses.
  - This approach ignores the conservative part of the self-force.

- Geodesic self-force evolution.
  - Build up a large table of self-force calculations in geodesic orbit parameter space.
  - Evolve the orbit according to the “geodesic self-force” (interpolated from the table) as if the particle had been moving on that geodesic forever.
  - This does take into account the conservative part of the self-force but ignores part of the tail contribution coming from the evolving orbit.
Effective source approach.

... is a general approach to self-force and self-consistent orbital evolution that doesn’t use any delta functions.

Key ideas

▶ Compute a regular field, $\psi^R$, such that

$$(\text{self-force}) \propto \nabla \psi^R$$

where $\psi^R = \psi - \psi^S$, and $\psi^S$ can be approximated via local expansions.

▶ The effective source, $S$, for the field equation for $\psi^R$ is regular at the particle location.

$$\Box \psi^R = \Box \psi - \Box \psi^S = -4\pi q \int_\gamma \frac{\delta^{(4)}(x - z(\tau))}{\sqrt{-g}} d\tau - \Box \psi^S = S(x|z,u)$$

where $S(x|z,u)$ is regular at the particle location.
Effective source approach.

Evolve the coupled particle-field dynamics:

\[ \Box \psi^R = S(x|z(\tau), u(\tau)) \]

\[ \frac{Du^\alpha}{d\tau} = \frac{q}{m(\tau)}(g^{\alpha\beta} + u^\alpha u^\beta) \nabla_\beta \psi^R \]

\[ \frac{dm}{d\tau} = -qu^\beta \nabla_\beta \psi^R \]

A bound orbit can be specified by its eccentricity \((e)\) and semi-latus rectum \((p)\):

\[ r_1 = \frac{pM}{1 + e}, \quad r_2 = \frac{pM}{1 - e} \]

where \(r_1\) and \(r_2\) are the turning points of the radial motion.

\(e = 0\), stable circular orbits
\(p = 6 + 2e\), (separatrix), unstable circular orbits
\(0 \leq e < 1, \ p > 6 + 2e\), bound orbit
Evolution code.

- A 3D multi-block scalar wave equation code.
- Kerr background spacetime in Kerr-Schild coordinates.
- Spherical inner boundary placed inside the black hole.
- Spherical outer boundary placed at $\mathcal{I}^+$ using Hyperboloidal slicings.
- The field and the particle are evolved together.
  - The particle location $z^\alpha(\tau)$ and four-velocity $u^\alpha(\tau)$ gives the effective source that determines $\psi^R$.
  - $\nabla_\beta \psi^R$ at the location of the particle in turn affects the orbit.
- We use 8th order summation by parts finite differencing and use penalty boundary conditions at patch boundaries.
- We can evolve the orbit using the geodesic equations directly as well as using the osculating orbits framework.

Equations:

\[
\Box \psi^R = S(x | z^\alpha(\tau), u^\alpha(\tau)),
\]
\[
\frac{D u^\alpha}{d\tau} = \frac{q}{m(\tau)} \left( g^{\alpha\beta} + u^\alpha u^\beta \right) \nabla_\beta \psi^R,
\]
\[
\frac{dm}{d\tau} = -qu^\beta \nabla_\beta \psi^R.
\]
Comparison with \((1+1)\) results.

\[ e = 0.5, p = 7.2 \]
Self-forced orbit.


\[ p = 7.2, \ e = 0.5, \ r_1 = 4.8M, \ r_2 = 14.4M \]
Self-forced orbit: $e-p$ space.

Some features: $p$ monotonically decreases, while $e$ oscillates. $e$ decreases secularly far from the separatrix (e.g. weak field regime), but then enters an increasing phase as the particle nears plunge.

Self-forced evolution in $e-p$ space for an orbit starting at $p = 7.2, e = 0.5$ (left plot) and $p = 9.9, e = 0.1$ (right plot).
Waveform at $\mathcal{I}^+$ ($e = 0.5$ and $\rho = 7.2$).
Energy flux through $\mathcal{I}^+$ ($e = 0.5$ and $p = 7.2$).
Energy flux through $\mathcal{I}^+$ ($e = 0.1$ and $p = 10$).
Movie \((e = 0.5, \ p = 7.2, \ q = 1/32)\).
Conclusions and future work.

Conclusions

- We have computed the first self-consistent evolutions and waveforms of a scalar charge in orbit around Schwarzschild.
- The code is robust, well parallelized and fully generic.
- The main limitations are the expense of evaluating the effective source and the cost of evolving in 3D.

Future work.

- We plan to do self-consistent orbits in Kerr.
- We would like to compare evolutions based on the geodesic self-force.
- The extension of the method to the gravitational case is underway.