Self-consistent evolution of a scalar point charge around a Schwarzschild black hole

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Extreme Mass Ratio Inspirals (EMRI's).

- EMRI's are expected to be important sources for future space based gravitational wave detectors.
- EMRI's are intractable with standard numerical relativity methods.
- Smallness of mass ratio suggests using perturbation techniques instead (point particle moving in the backgorund field).
- To zeroth order the motion of a point particle is geodesic.
- To first order the point particle experiences an acceleration caused by it's interaction with it's own field.
- The field at particle location can be decomposed into a singular piece and a regular piece $\psi = \psi^{\mathsf{S}} + \psi^{\mathsf{R}}$.
- The singular piece ψ^{S} does not contribute to the self-force.
- The regular piece ψ^{R} contains a tail contribution.
- The ultimate goal is a fully self-consistent evolution.

Traditional approaches to self-force calculations.

Most calculations of the self-force uses a δ-function source.

- Using a δ-function source requires subtraction of the singular part of the field in a mode sum regularization scheme.
- This can be fairly expensive especially at high eccentricity where many modes has to be taken into account.

This can be done in both the frequency and time domain.

Frequency domain Requires a prescribed geodesic orbit. Very accurate.

Time domain Requires evolving many modes simulteneously. Less accurate.

Traditional approaches to EMRI evolutions.

- Adiabatic approaches without explicit self-force calculation.
  - Energy and angular momentum fluxes through horizon and infinity are integrated over a complete geodesic orbit.
  - The geodesic orbit is then "evolved" by changing the orbital parameters according to the energy and angular momentum losses.
  - This approach ignores the conservative part of the self-force.
- Geodesic self-force evolution.
  - Build up a large table of self-force calculations in geodesic orbit parameter space.
  - Evolve the orbit according to the "geodesic self-force" (interpolated from the table) as if the particle had been moving on that geodesic forever.
  - This does take into account the conservative part of the self-force but ignores part of the tail contribution coming from the evolving orbit.

Effective source approach.

... is a general approach to self-force and self-consistent orbital evolution that doesn't use any delta functions.

Key ideas

- Compute a regular field, ψ^{R}, such that

$$(\text{self-force}) \propto \nabla\psi^{\mathsf{R}}$$

  where $\psi^{\mathsf{R}} = \psi - \psi^{\mathsf{S}}$, and ψ^{S} can be approximated via local expansions.

- The effective source, S, for the field equation for ψ^{R} is regular at the particle location.

$$\Box\psi^{\mathsf{R}} = \Box\psi - \Box\psi^{\mathsf{S}} = -4\pi q \int_\gamma \frac{\delta^{(4)}(x - z(\tau))}{\sqrt{-g}} d\tau - \Box\psi^{\mathsf{S}} = S(x|z, u)$$

  where $S(x|z, u)$ is regular at the particle location.

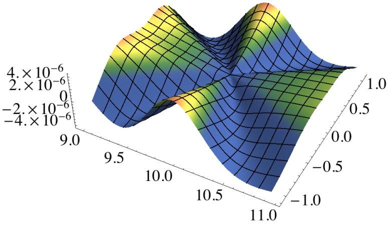
Effective source approach.

Evolve the coupled particle-field dynamics:

$$\Box\psi^{\mathsf{R}} = S(x|z(\tau), u(\tau))$$

$$\frac{Du^\alpha}{d\tau} = \frac{q}{m(\tau)}(g^{\alpha\beta} + u^\alpha u^\beta)\nabla_\beta\psi^{\mathsf{R}}$$

$$\frac{dm}{d\tau} = -qu^\beta\nabla_\beta\psi^{\mathsf{R}}$$



A bound orbit can be specified by its eccentricity (e) and semi-latus rectum (p):

$$r_1 = \frac{pM}{1+e}, \quad r_2 = \frac{pM}{1-e}$$

where r_1 and r_2 are the turning points of the radial motion.

$e = 0$, stable circular orbits

$p = 6 + 2e$, (separatrix), unstable circular orbits

$0 \leq e < 1, \;\; p > 6 + 2e$, bound orbit

Evolution code.

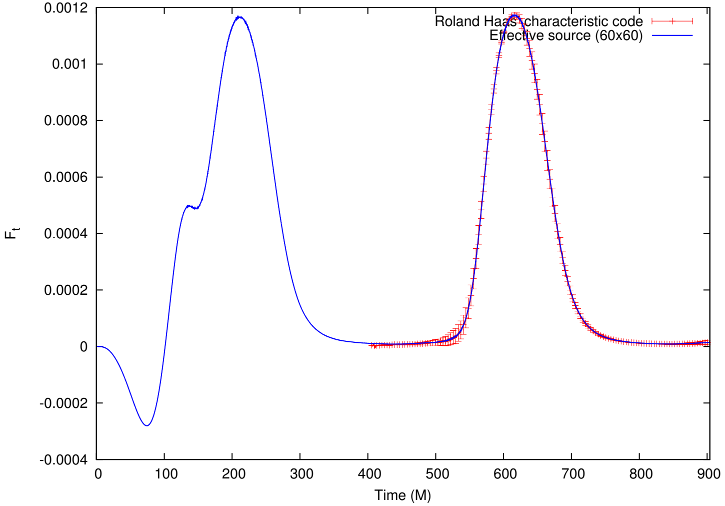
- A 3D multi-block scalar wave equation code.
- Kerr background spacetime in Kerr-Schild coordinates.
- Spherical inner boundary placed inside the black hole.

Equations:

$$
\begin{aligned}
\Box\psi^{\mathrm{R}} &= S(x|z^\alpha(\tau), u^\alpha(\tau)), \\
\frac{Du^\alpha}{d\tau} &= \frac{q}{m(\tau)}\left(g^{\alpha\beta} + u^\alpha u^\beta\right)\nabla_\beta\psi^{\mathrm{R}}, \\
\frac{dm}{d\tau} &= -qu^\beta\nabla_\beta\psi^{\mathrm{R}}.
\end{aligned}
$$

- Spherical outer boundary placed at $\mathscr{I}^+$ using Hyperboloidal slicings.
- The field and the particle are evolved together.
  - The particle location $z^\alpha(\tau)$ and four-velocity $u^\alpha(\tau)$ gives the effective source that determines ψ^{R}.
  - $\nabla_\beta\psi^{\mathrm{R}}$ at the location of the particle in turn affects the orbit.
- We use 8th order summation by parts finite differencing and use penalty boundary conditions at patch boundaries.
- We can evolve the orbit using the geodesic equations directly as well as using the osculating orbits framework.

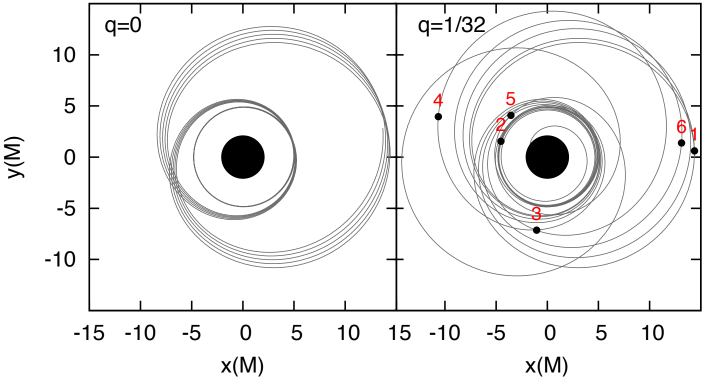
Comparison with (1+1) results.



$$e = 0.5, p = 7.2$$

Self-forced orbit.

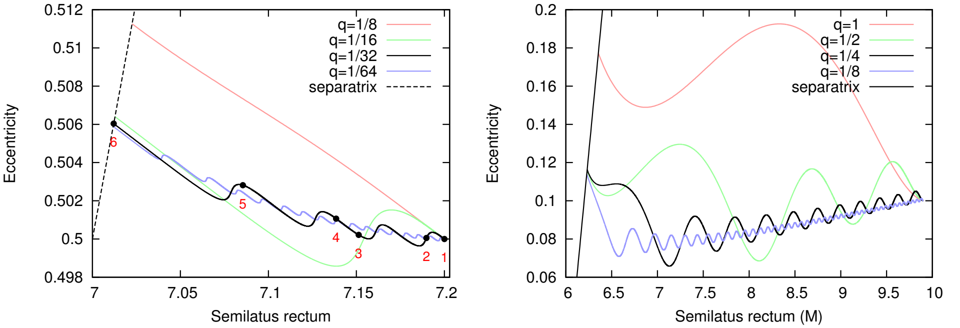
See: Phys.Rev.Lett. 108 (2012) 191102.



$$p = 7.2, e = 0.5,\ r_1 = 4.8M,\ r_2 = 14.4M$$

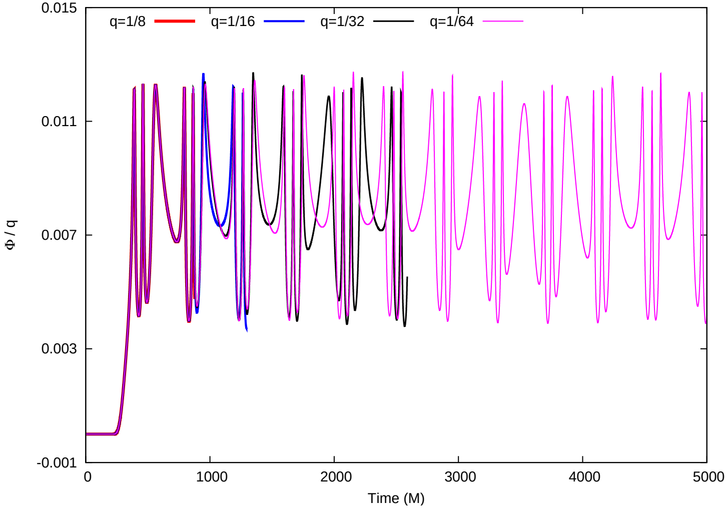
Self-forced orbit: e-p space.

Some features: p monotonically decreases, while e oscillates. e decreases secularly far from the separatrix (e.g. weak field regime), but then enters an increasing phase as the particle nears plunge.

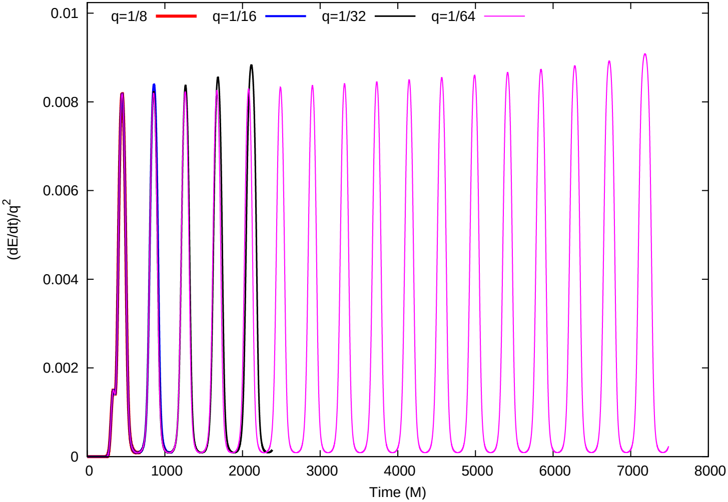


Self-forced evolution in e-p space for an orbit starting at $p = 7.2, e = 0.5$ (left plot) and $p = 9.9, e = 0.1$ (right plot).

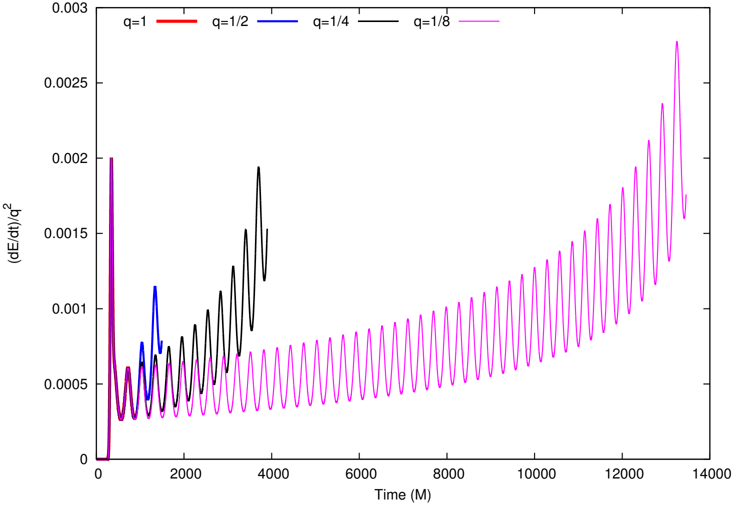
Waveform at $\mathscr{I}^+$ ($e = 0.5$ and $p = 7.2$).



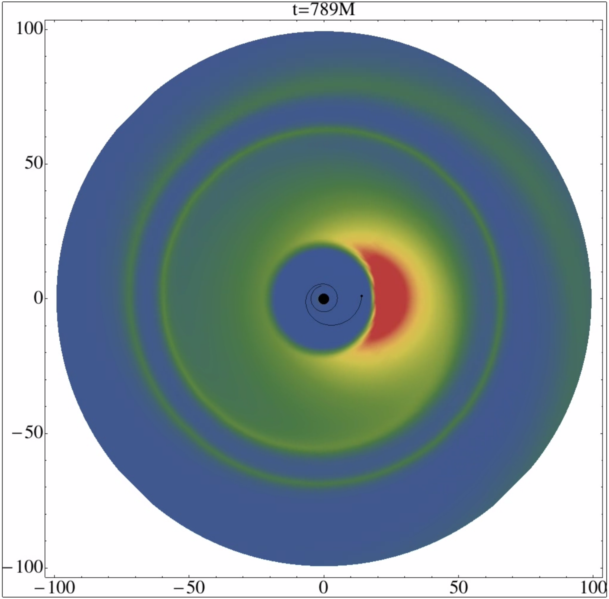
Energy flux through $\mathscr{I}^+$ ($e = 0.5$ and $p = 7.2$).



Energy flux through $\mathscr{I}^+$ ($e = 0.1$ and $p = 10$).



Movie ($e = 0.5$, $p = 7.2$, $q = 1/32$).



Conclusions and future work.

Conclusions

- We have computed the first self-consistent evolutions and waveforms of a scalar charge in orbit around Schwarzschild.
- The code is robust, well parallelized and fully generic.
- The main limitations are the expense of evaluating the effective source and the cost of evolving in 3D.

Future work.

- We plan to do self-consistent orbits in Kerr.
- We would like to compare evolutions based on the geodesic self-force.
- The extension of the method to the gravitational case is underway.