Lorentz violations and gravity

The Newtonian limit

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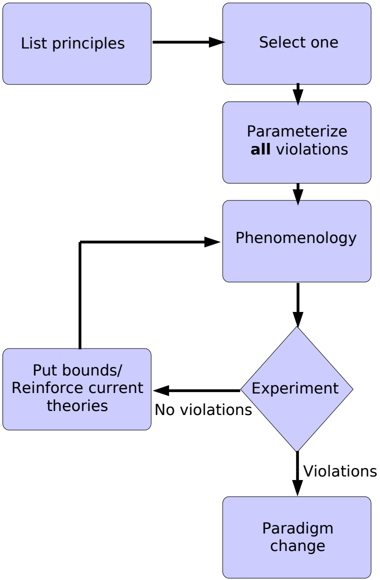
Marriage impossible?

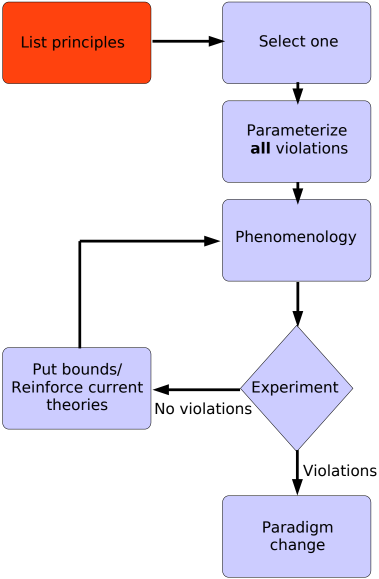
- Current status of Physics:

- No QG theory reconciling QM and GR.
- Two strategies: Theoretical and Phenomenological.

Phenomenologists' workflow

Phenomenology = Search empirical evidence of paradigm change.



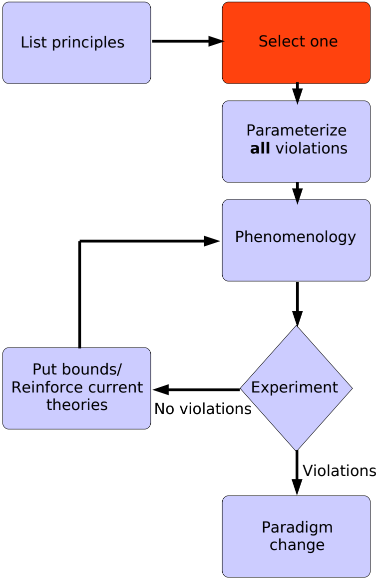


Principles underlying GR

- Causal structure.
- (Local) Lorentz invariance.
- Equivalence principles.
- Diffeomorphism invariance.
- Einstein-Hilbert action.
- Minimal coupling.
- Torsion-free.

$\vdots$

These principles are **not** independent.



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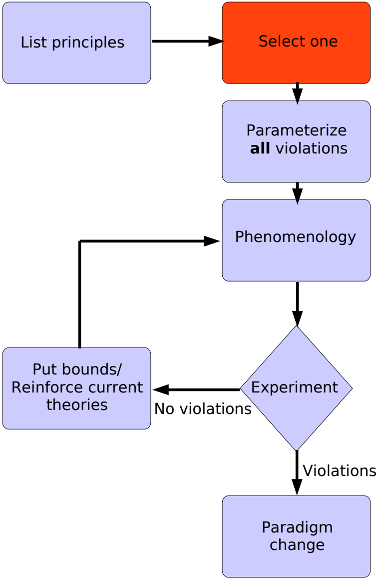
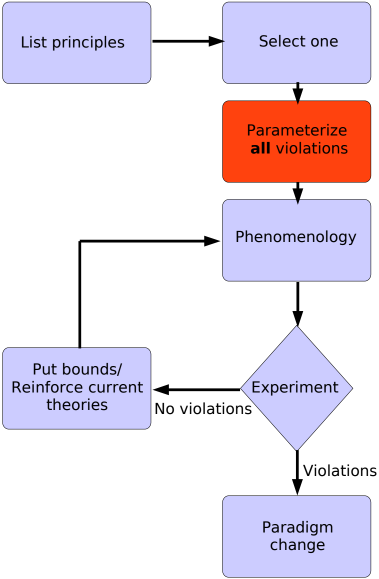


LV $=$ $\exists$ preferred spacetime directions (tensors).

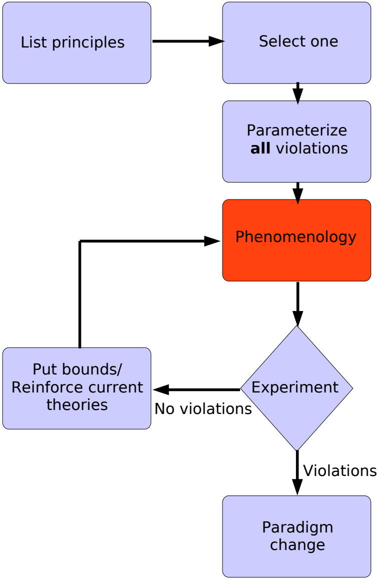
Why Lorentz violation?

- Lorentz invariance underlies SM and GR.
- Associated with violations of other principles/symmetries (*e.g.*, CPT).
- Can be accommodated by QG candidate theories.
- New interactions could manifest as LV.



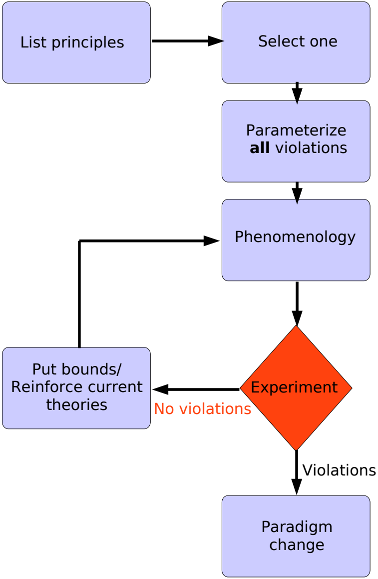
SME

- Standard-Model Extension.
- Colladay+Kostelecký (PRD 97, 98), Kostelecký (PRD 04), *etc.*
- Effective field theory formalism.
- $\mathcal{L}_{SME} = \mathcal{L}_{GR} + \mathcal{L}_{SM} + \mathcal{L}_{LV}$.
- $\mathcal{L}_{LV} \supset k \cdot J$
  - k "coefficients" for LV.
  - J standard field operators.
  - Scalar under coordinate transformations.



Experiments (partial list)

- Accelerator/collider.
- Astrophysical observations.
- Birefringence/dispersion.
- Clock-comparison.
- CMB polarization.
- Laboratory gravity tests.
- Matter interferometry.
- Neutrino oscillations.
- Particle vs. antiparticle.
- Resonant cavities and lasers.
- Sidereal/annual time variations.
- Spin-polarized matter.


List principles
Select one
Parameterize
all violations
Phenomenology
Experiment
No violations
Put bounds/
Reinforce current
theories
Violations
Paradigm
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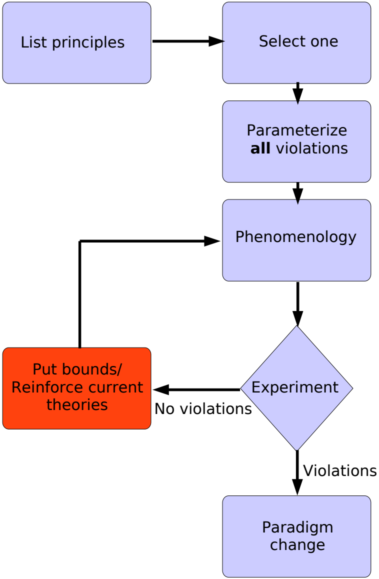


Bounds

- Bounds in many SME sectors.

"Data Tables for Lorentz and CPT Violation"
Kostelecký+Russell,
Rev. Mod. Phys. (2011),
last version arXiv:0801.0287v6.

- Quotes > 150 papers.
- Best bounds (matter sector):

$$|k| \leq 10^{-33}\ \mathrm{GeV}.$$

- Coef. remain unmeasured.

`Matter-gravity couplings offer sensitivity to SME coefficients that are otherwise unobservable.`

Goal of this work

Generate the most general nonrelativistic Hamiltonian for LV matter fields in a uniform Newtonian potential.

Before describing the method:

1. Free Dirac-spinor minimal SME in flat spacetime.
2. Tetrads.
3. Generalization to curved spacetime.
4. Previous work: First-principles calculations.

Free Dirac-spinor minimal SME sector in flat spacetime

- One free Dirac-spinor ψ.
- Minimal $=$ J of renormalizable dimension.
- The Lagrangian density is

$$\mathcal{L}_{\psi,\eta} = \frac{i}{2}\bar{\psi}\Gamma^{\mu}\overleftrightarrow{\partial}_{\mu}\psi - \bar{\psi}M\psi,$$

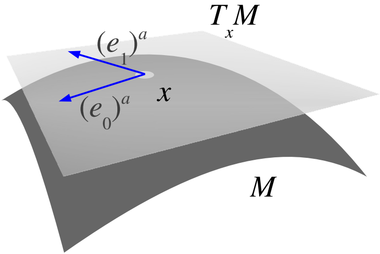
where

$$\begin{aligned} \Gamma^{\mu} &= \gamma^{\mu} - \eta^{\mu\nu}c_{\rho\nu}\gamma^{\rho} - \eta^{\mu\nu}d_{\rho\nu}\gamma_5\gamma^{\rho} - \eta^{\mu\nu}e_{\nu} \\ &\quad -i\eta^{\mu\nu}f_{\nu}\gamma_5 - \frac{1}{2}\eta^{\mu\nu}g_{\rho\sigma\nu}\sigma^{\rho\sigma}, \\ M &= m + a_{\mu}\gamma^{\mu} + b_{\mu}\gamma_5\gamma^{\mu} + \frac{1}{2}H_{\mu\nu}\sigma^{\mu\nu}. \end{aligned}$$

- Γ^{μ} and M are the most general matrices.
- γ^{μ} = Dirac matrices, $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^{\mu},\gamma^{\nu}]$.
- Coefficients controlling LV: $a_{\mu}, b_{\mu}, c_{\mu\nu}, d_{\mu\nu}, e_{\mu}, f_{\mu}, g_{\mu\nu\rho}, H_{\mu\nu}$.

Tetrad formalism reminder

- Tetrad = Orthonormal basis of tangent space $(e_\mu)^a$.
- Freedom to "rotate" $\{(e_\mu)^a\} \leftrightarrow$ Lorentz invariance.
- Connection between spacetime and tangent spaces (a vs. μ).
- Fermions can only be treated with tetrads.
- "Spin connection" $\omega_{a\mu\nu} = (e_\mu)^b \nabla_a (e_\nu)_b$.
- Notation: $e = \sqrt{-g}$.



Dirac-spinor minimal SME sector in a general spacetime

Lagrangian density

$$\mathcal{L}_{\psi,g} = \frac{i}{2} e (e_\mu)^a \bar{\psi} \Gamma^\mu \overleftrightarrow{\nabla}_a \psi - e \bar{\psi} M \psi,$$

where

$$\begin{aligned}
\nabla_a \psi &= \partial_a \psi + \frac{i}{4} \omega_{a\rho\sigma} \sigma^{\rho\sigma} \psi, \quad \nabla_a \bar{\psi} = \partial_a \bar{\psi} - \frac{i}{4} \omega_{a\rho\sigma} \bar{\psi} \sigma^{\rho\sigma}, \\
\Gamma^\mu &= \gamma^\mu - \eta^{\mu\nu} c_{ab} (e_\rho)^a (e_\nu)^b \gamma^\rho - \eta^{\mu\nu} d_{ab} (e_\rho)^a (e_\nu)^b \gamma_5 \gamma^\rho \\
&\quad - \eta^{\mu\nu} e_a (e_\nu)^a - i \eta^{\mu\nu} f_a (e_\nu)^a \gamma_5 \\
&\quad - \frac{1}{2} \eta^{\mu\nu} g_{abc} (e_\rho)^a (e_\sigma)^b (e_\nu)^c \sigma^{\rho\sigma}, \\
M &= m + a_a (e_\mu)^a \gamma^\mu + b_a (e_\mu)^a \gamma_5 \gamma^\mu + \frac{1}{2} H_{ab} (e_\mu)^a (e_\nu)^b \sigma^{\mu\nu}.
\end{aligned}$$

Dirac-spinor minimal SME sector in a general spacetime

First-principles study (Kostelecký+Tasson, PRD 2011):

$$\mathcal{L}_{SME} \supset \mathcal{L}_{GR}(e,\omega) + \mathcal{L}_{SM}(e,\omega;\psi,\nabla\psi) + \mathcal{L}_{LV}(e,\omega;\psi,\nabla\psi).$$

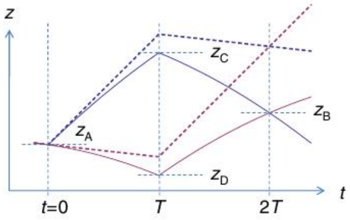
Pros

- General spacetime.
- Backreaction.
- LV mechanism/coef. fluctuations.

Cons

- Complicated.
- Use approximations:
  - Linearized metric.
  - Neglect $k \cdot \partial_a g_{bc}$.
  - No spin.

Bounds on "new" SME coefficients (Hohensee, Chu, Peters and Müller, PRL 2011).



Shortest path to the phenomenological Hamiltonian

Recall: looking for the SME nonrelativistic Hamiltonian in the presence of a uniform Newtonian potential.
Rindler = Minkowski seen by uniformly accelerated observers.

- Write $\mathcal{L}_{\psi,g}$ in Rindler.
- Get the equation of motion.
- Interpret ψ as a wavefunction (go from QFT to RQM).
- Read-off the Hamiltonian to first order in LV coefficients.
- Take the nonrelativistic limit (Foldy-Wouthuysen transf.).
- Identify constant acceleration with a uniform Newtonian gravitational field Φ.

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Pros

- Relatively simple.
- Phen. relevant.
- All SME coefficients (spin).
- Includes $k \cdot \nabla\Phi$, Φ^n.

Cons

- Particular metric/uniform gravitational potential.
- No backreaction/LV source.
- No coefficient fluctuations.

Shortest path to the phenomenological Hamiltonian

The resulting nonrelativistic Hamiltonian to first order in LV coefficients and Φ is

$$\begin{aligned} H^{\mathrm{NR}}_{\psi,g} &= \frac{\vec{p}^2}{2m}(1+\Phi) + m\Phi - \frac{i}{2m}(\nabla\Phi)\cdot\vec{p} + \frac{1}{4m}\vec{\sigma}\cdot(\nabla\Phi)\times\vec{p} \\ &+ \left(A + B\Phi + C^i(\partial_i\Phi)\right) + \Big(D_i + E_i\Phi + F_i^{\,j}(\partial_j\Phi)\Big)\sigma^i \\ &+ \left(G^i + H^i\Phi + I^{ij}(\partial_i\Phi)\right)p_j + \Big(J_i^{\,j} + K_j^{\,i}\Phi + L_i^{\,jk}(\partial_k\Phi)\Big)\sigma^i p_j \\ &+ \left(M^{ij} + N^{ij}\Phi\right)p_i p_j + \Big(O_i^{\,jk} + P_i^{\,jk}\Phi\Big)\sigma^i p_j p_k + \ldots \end{aligned}$$

where the $A, \ldots, P_i^{\,jk}$ are linear combinations of SME coefficients. For instance,

$$H^i = \frac{1}{m}a^i - e^i, \quad K_j^{\,i} = -d_{00}\delta_j^i + d_j{}^i - \frac{1}{2}\epsilon_j{}^{kl}g_{kl}{}^i + \epsilon_j{}^{ik}g_{0k0}.$$

Shortest path to the phenomenological Hamiltonian

$$
\begin{aligned}
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&+ \left(A + B\Phi + C^i(\partial_i\Phi)\right) + \left(D_i + E_i\Phi + F_i^j(\partial_j\Phi)\right)\sigma^i \\
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\end{aligned}
$$

Correct limits:

- Lorentz invariant part $\leftrightarrow$ Hehl+Ni, PRD 1990.
- $\Phi = 0 \leftrightarrow$ Kostelecký+Lane, J. Math. Phys. 1999.
- Where there is overlap, agreement with Kostelecký+Tasson, PRD 2011.

Conclusions

$$
\begin{aligned}
H^{\mathrm{NR}}_{\psi,g} &= \frac{\vec{p}^2}{2m}(1+\Phi) + m\Phi - \frac{i}{2m}(\nabla\Phi)\cdot\vec{p} + \frac{1}{4m}\vec{\sigma}\cdot(\nabla\Phi)\times\vec{p} \\
&+ \left(A + B\Phi + C^i(\partial_i\Phi)\right) + \left(D_i + E_i\Phi + F_i^j(\partial_j\Phi)\right)\sigma^i \\
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\end{aligned}
$$

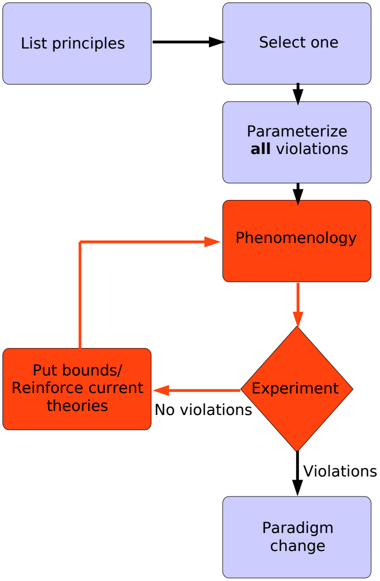
General Hamiltonian for fermions in the lab's gravitational field.

In the near future:

There are bounds on the parenthesis $\Rightarrow$

Reinterpret/disentangle bounds (first or improved bounds?).

Couplings of LV, gravity and spin (σ^i) are interesting as the experiments involving spin have very good sensitivity.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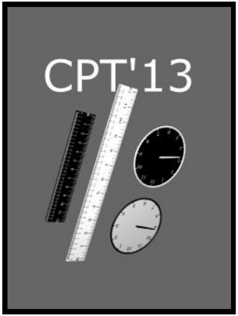
Sixth Meeting on

CPT and Lorentz Symmetry



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Bloomington

www.indiana.edu/~lorentz/cpt13