CHAPTER 9- CONSERVATION of MOMENTUM

DEFINITION of MOMENTUM

In physics we define force as the time rate of change of momentum. Momentum as force is a vector quantity.

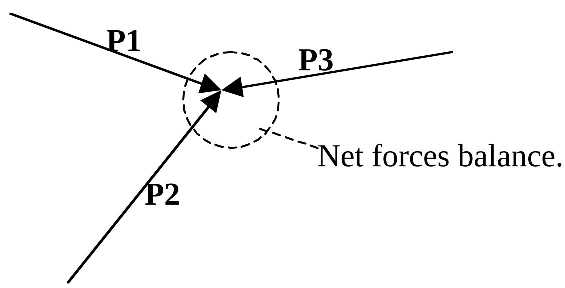
$$\mathbf{F} = d\mathbf{P}/dt \quad \text{where } \mathbf{P} = M\,\mathbf{V}$$

COLLISONS

In a collision between objects the internal forces are equal and opposite thus balance. The net work performed is zero! Since F = M dV/dt = d/dt (MV) = 0 we say that the net Momentum = MV in the collision is conserved.

$$\mathbf{P}_i = M\mathbf{V}_I \quad \text{for the ith particle}$$

$$d/dt\,(\mathbf{P1} + \mathbf{P2} + \mathbf{P3} + ..) = 0 \text{ then} \qquad \mathbf{P1} + \mathbf{P2} + \mathbf{P3} = \text{Constant}$$



Momentum Conservation always hold whether the collision is *elastic* or *inelastic*!
In *elastic* collision energy is conserved, in *inelastic* collisions energy is not conserved.

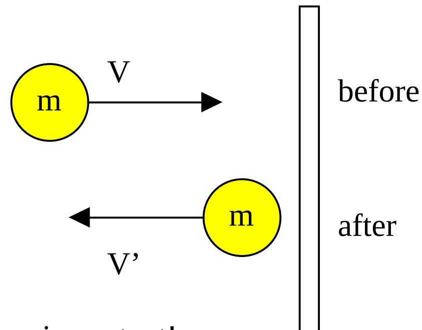
IMPULSE $\mathbf{I} = \Delta\mathbf{P}$

We define the impulse on an object felt in a collision as the integral of the force exerted upon it over the time of impact. Below we see that $\mathbf{I} = \Delta\mathbf{P}$.

$$\mathbf{I} = \int \mathbf{F}\,dt = \int (d\mathbf{P}/dt)\,dt = \Delta\mathbf{P} = \mathbf{Pf - Pi}$$

In the elastic collision of a ball with a solid wall where V = V' the Impulse is calculated

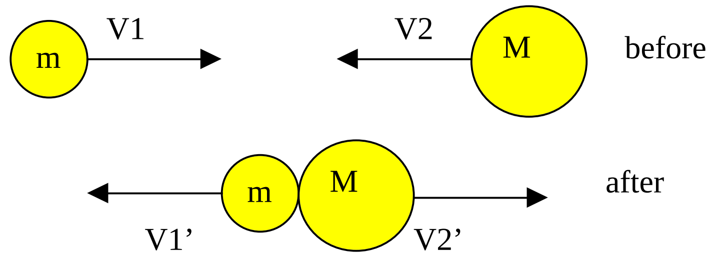
$\mathbf{I}$ = (Pf – Pi) $\mathbf{i}$ = (- m V') - (m V) = <u>-2mV **i**</u>



Note that the signs (directions) of the velocity vectors are important!

ELASTIC COLLISIONS $\sum$ **P** = Constant & $\sum$ E =Constant

Elastic collisions are impacts where energy as well as momentum is conserved. Note that the signs (directions) of the velocity vectors are important.



$$m V1 - MV2 = -m V1' + M V2'$$
$$1/2\ m V1^2 + 1/2\ MV2^2 = 1/2\ m V1'^2 + 1/2\ M V2'^2$$

Solving this set of two equations for V1' and V2' the final velocities

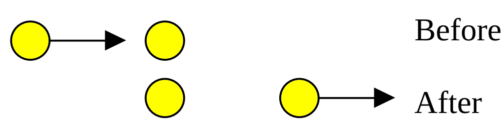
$$V1' = [(m-M)/(m+M)]\ V1 + (2M)/(m+M)\ V2 \qquad \text{eq 9-20}$$
$$V2' = (2m)/(m+M)]\ V1 + [(M-m)/(m+M)]\ V2 \qquad \text{eq 9-21}$$

Case I : m=M V2 =0

V1' = 0

V2' = V1

For equal masses they exchange velocities. This we observe when the cue ball hits another straight on.

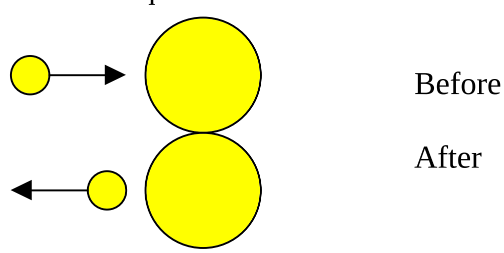


Case II M>>m V2 =0

V1' = -V1

V2' ≅ 0

We expect M to barely move and m to recoil letting M>>m in eq 9-20

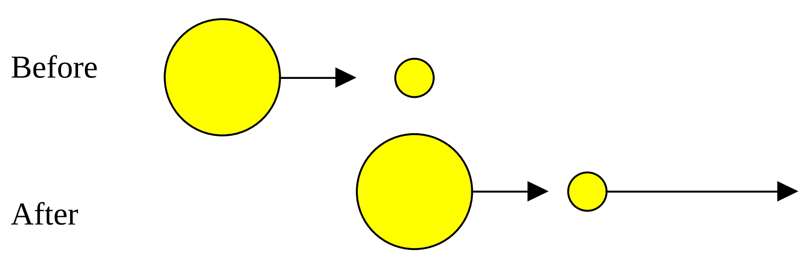


Case III m>>M V2=0

V1' = V1

V2' = 2V1

We expect both to move to he right



NEUTRON MODERATOR - Maximum Kinetic Energy Transfre- Case 1

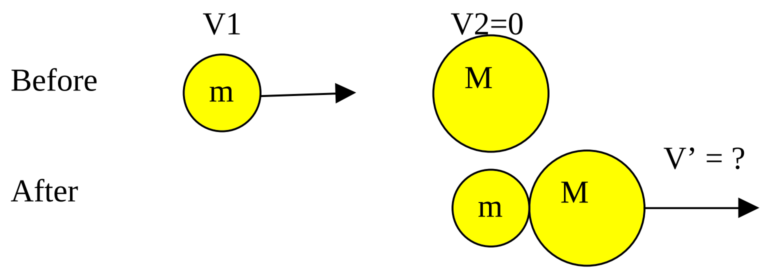
In a collision between 2 atoms the most energy is transferred when the atoms are of equal mass. This is Case 1. To slow down the neutrons (moderator) hydrocarbons (plastics) are often used. The neutrons and hydrogen atoms have about the same mass, and the neutron will knock out a hydrogen atom in the plastic and come to a stop, transfering all of its energy.

INELASTIC COLLISIONS

In a n inelastic collision nergy is NOT conserved. Consider a collision in which the objects stick together.

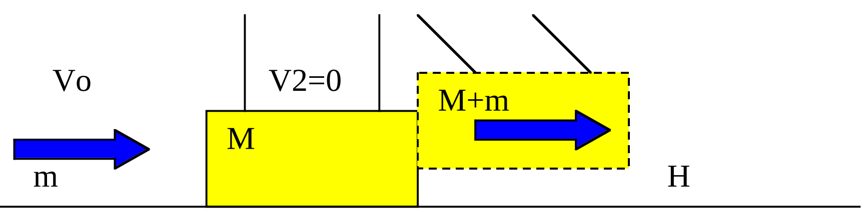
MV1 = (M+m)V2'

V' = M/(m+M) V1



BALLISTIC PENDULUM (2 step problem)

Consider projectile-1 fired in to second object-2 and they stick (above). Now the connected pair swing up to a height H. In this way we can measure the initial velocity Vo of the projectile –1.



We consider the problem in two parts-

Part –1 (Inelastic Collision)   V2' = m/(m+M) Vo

Part –2 (Energy Conservaion)   (M+m) g H = 1/2 (M+m) V2' ^{2}

Vo = (m+M)/m V2' = [(m+M)/m] $\sqrt{2gH}$

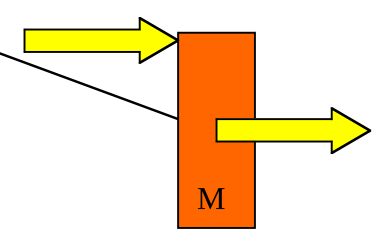
CENTER OF MASS

Any force acting on a perfectly rigid body acts through the bodies center of mass

Mathematically we define the <u>center of mass:</u>

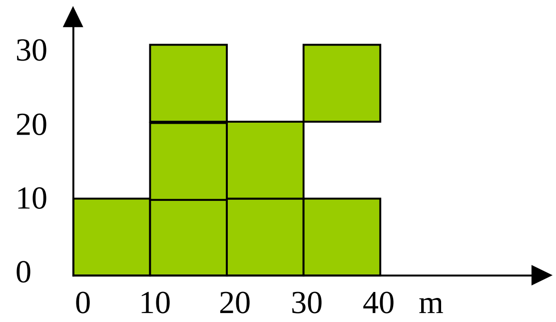
Xcm = 1/M $\sum$ Mi Xi = 1/M $\int x \;\; dm$

Ycm = 1/M $\sum$ Mi Yi = 1/M $\int y \;\; dm$



SUM FORMULA

The center of mass of this cluster of blocks, all of mass M=1kg is:



$Xcm = (1/8)\,[\,1\times5 + 3\times15 + 2\times25 + 2\times35]\ m$
$= 19.5\ m$

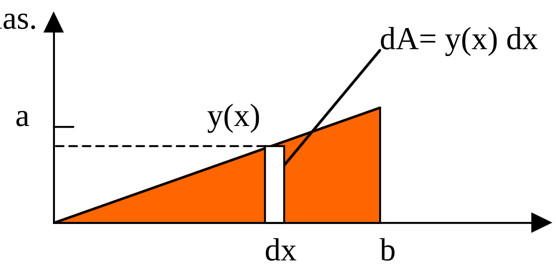
$Ycm = (1/8)\,[\,4\times5 + 2\times15 + 2\times25\,]\ m$
$= 12.5\ m$

Rcm =(19.5 **i** + 12.5 **j**) m

For solid objects we can approximate the center of mass by using the location of their centers and using the $\sum$ formulas.

INTEGRAL FORMULA

In some cases we must use the integral formulas. Consider a solid wedge of uniform density.



ρ = Mass / Area. = $1 kg/m^2$

$dm = \rho\, dA$ where $dA = y(x)\, dx$

$y(x) = (a/b)\, x$ the equation of the line

$dm = \rho\, (a/b)\, x\, dx$

First find the total mass M

$$M = \int_0^b dm = \int_0^b \rho\,(a/b)\, x\, dx$$

$$= \rho\,(a/b) \int_0^b x\, dx = \rho\,(a/b)\, b^2/2 = 1/2\, \rho\, a\, b$$ (the area of a triangle times the density!)

Next the Xcm and Ycm

$$Xcm = \int_0^b \rho\,(a/b)\, x\, dx = (1/M) \int_0^b x\, dm = \int_0^b x\, \rho\,(a/b)\, x\, dx = (1/M)\, \rho\,(a/b) \int_0^b x^2\, dx$$

$$= (2/\rho\, a\, b)\, \rho\,(a/b)\, b^3/3 = (2/3)\, b$$

$Ycm = (1/3)\, a$