## CHAPTER 6 CIRCULAR MOTION

## CENTRIPETAL FORCE

The force $\mathbf{F}$ necessary to keep a body in uniform circular motion is defined as the centripetal force. The magnitude of the force is $\mathrm{F}=\mathrm{m} \mathrm{v}^{2} / \mathrm{r}$ and it is directed to the center of rotation. If F were not present object $m$ would move along it's velocity vector $\mathbf{v}$.

F can be produced by gravitational attraction, a string, or a roadbed pushing on a tire.


EXIT RAMP (see examples 6-4 and 6-5)
A curved exit ramp is normally inclined to facilitate a higher speed of exit with no slippage. Consider a car of mass $m$ moving with velocity $v$ along a curved path of radius R . There is static friction $\mu_{\mathrm{k}}$ between the tire and road. This frictional force $f=\mu_{\mathrm{k}} \mathrm{N}$ must oppose the outward $\mathrm{m} \mathrm{v}^{2} / \mathrm{r}$ force. Then the maximum velocity before slippage is

$$
\mathrm{mv}^{2} / \mathrm{r}=\mathrm{f}=\mu_{\mathrm{k}} \mathrm{~N}=\mu_{\mathrm{k}} \mathrm{mg}
$$

$$
\mathrm{v}_{\max }=\left(\begin{array}{rll}
\mathrm{r} & \left.\mathrm{~g} \mu_{\mathrm{k}}\right)^{1 / 2}
\end{array}\right.
$$

If the exit ramp is inclined the forces opposing slippage can be increased.

$$
\begin{aligned}
& \mathrm{mv}^{2} / \mathrm{r}=\mathrm{N} \sin \theta+f \cos \theta \\
& \mathrm{mg}=\mathrm{N} \cos \theta
\end{aligned}
$$



We can determine the banking angle under extreme conditions $f=0$ (ice!!), thus not relying on frictional forces.

$$
\begin{aligned}
& \mathrm{m} \mathrm{v}^{2} / \mathrm{r}=\mathrm{N} \sin \theta \\
& \mathrm{mg}=\mathrm{N} \cos \theta
\end{aligned}
$$

$$
\begin{aligned}
\tan \theta & =v^{2} / r g \\
& \text { or } \\
\mathrm{v}_{\max } & =(\mathrm{rg} \tan \theta)^{1 / 2}
\end{aligned}
$$



## LOOP-THE-LOOP (see example 6-7)

Consider a pilot making a vertical loop in an airplane at speed constant speed v. What forces does he aircraft experience? How does he avoid a stall at position A?
Let $P$ be the force of lift on the aircraft wing ( the same as the normal force on the pilot in example 6-7). Zero net force insures uniform motion at each location. P changes direction to balance the weight and centripetal force.

A: $\quad \mathrm{P}+\mathrm{mg}=\mathrm{mv}^{2} / \mathrm{r}$
$\mathrm{P}=\mathrm{mv}^{2} / \mathrm{r}-\mathrm{mg}$ (lightest)
B: $P=\left(\left(\mathrm{mv}^{2} / \mathrm{r}\right)^{2}+(\mathrm{mg})^{2}\right)^{1 / 2}$
C: $P=\mathrm{mv}^{2} / \mathrm{r}+\mathrm{mg}$
$\mathrm{P}=\mathrm{mv}^{2} / \mathrm{r}+\mathrm{mg}$ (heaviest)
D: $P=\left(\left(\mathrm{mv}^{2} / \mathrm{r}\right)^{2}+(\mathrm{mg})^{2}\right)^{1 / 2}$
A stall condition occurs If $\mathrm{P}=0$ (freefall) at the top (A). Then $\mathrm{mv}^{2} / \mathrm{r}=\mathrm{mg}$

$$
\mathrm{v}>(\mathrm{rg})^{1 / 2} \text { or stall }
$$



