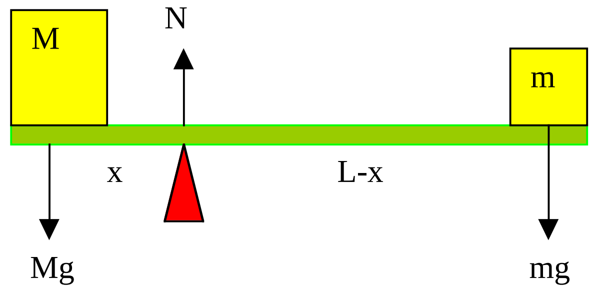
CHAPTER 12: STATIC EQUILIBRIUM & ELASTICITY

A body is in static equilibrium if the vector sum of all forces acting on it and the vector sum of all torques acting on it are zero.

$$\Sigma \mathbf{F}i = 0 \text{ and } \Sigma \boldsymbol{\tau}\mathbf{i} = 0$$

The torques can be summed about any point!

LEVER IN STATIC EQUILIBRIUM



$\Sigma \mathbf{F}i = 0$ N –Mg –mg = 0

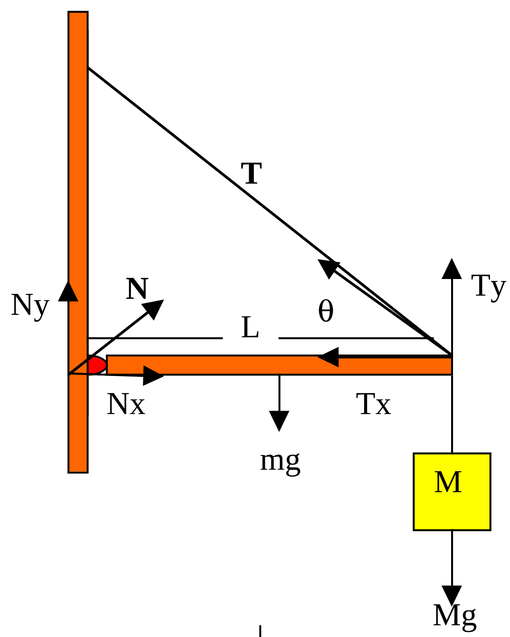
N = (M+m) g

$\Sigma \boldsymbol{\tau}\mathbf{i} = 0$ Mg x - mg (L-x) = 0

(M+m) g x = mg L

x = [m/(m+M)] L

LOADED BEAM



$\Sigma \mathbf{F}i = 0$ Ny + Ty = mg + Mg

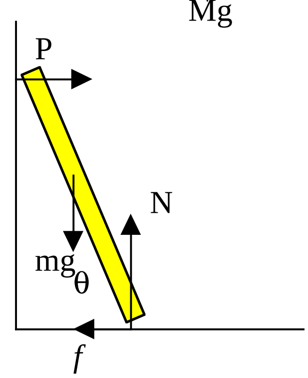
Nx = Tx

Ty / Tx = tan θ

$\Sigma \boldsymbol{\tau}\mathbf{i} = 0$ Mg L + mg L/2 =Ty L

4 equations and 2 unknowns Nx Ny Tx ty !

SLIPPING LADDER see example 12-8



$\Sigma \mathbf{F}i = 0$ P = f

N = mg

$\Sigma \boldsymbol{\tau}\mathbf{i} = 0$ P L sinθ = mg (L/2) cosθ

$f = \mu_S N$ when ladder about to slip!

Three equations and 3 unknowns f, N, P

CENTER OF GRAVITY

The pont about which an object balances in a gravitational field is its center of gravity. In a uniform gravitational field much like on the earth's surface the center of gravity is the same as the center of mass! But there are cases where gravity may not be the same over the length of the bodies in question, near a black hole for example.

$$X_{CG} = \Sigma\ W_i X_i \ / \ \Sigma W_i \quad \text{and} \quad Y_{CG} = \Sigma\ W_i Y_i \ / \ \Sigma W_i$$

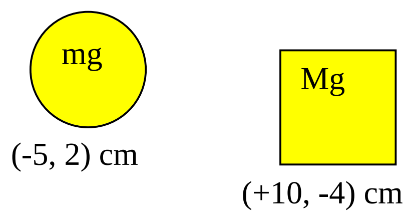
Here $W_i = (Mg)_i$ is the weight of the ith object.

Example

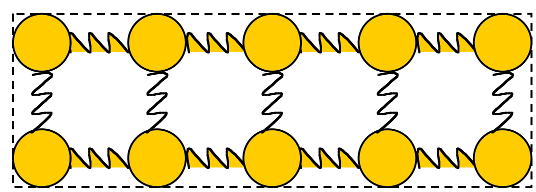
If we know the position of the centers of the objects then

$X_{CG} = 1 / (mg + Mg)$ [-5 mg +10 mg]

$Y_{CG} = 1 / (mg + Mg)$ [+2 mg -4 mg]



DEFORMATION of SOLIDS



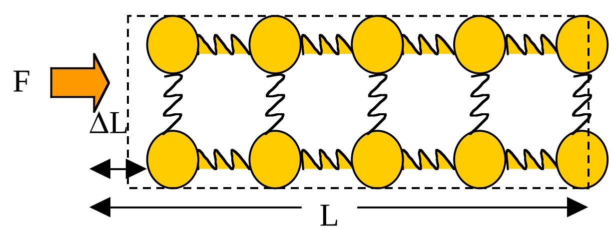
STRESS = Force of deformation per unit area
= F / A

STRAIN = relative degree of deformation
= $\Delta L/L$

ELASTIC MODULUS = STRESS / STRAIN

By Hooke's Law $F = -\kappa\, \Delta X$,

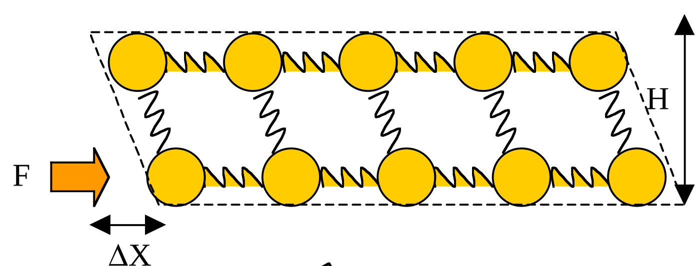
Strain is proportional to Stress!



YOUNG'S MODULUS (elongation)

$(F/A) = Y\ (\Delta L/L)$

$Y = (F/A) / (\Delta L/L)$



SHEAR MODULUS (sliding force)

$S = (F/A) / (\Delta x/h)$

BULK MODULUS (volume change)

$B = (\Delta F/A) / (\Delta V/V)$