Sound Propagation in the Nocturnal Boundary Layer

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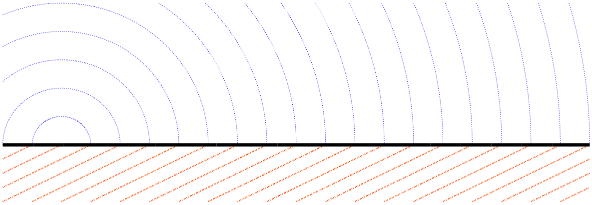
The Propagation of Sound Outdoors

(over flat ground)



The atmosphere is a gas under the influence of

- gravity
- thermal stresses
- the Earth's rotation

The ground is a porous elastic solid

- has large (compared with air) thermal conductivity
- reflects and attenuates sound
- properties can differ dramatically from place to place

The Diurnal Cycle

(fair weather meteorology)

The ground is thermally coupled to space

- heats up during the day
- cools off at night

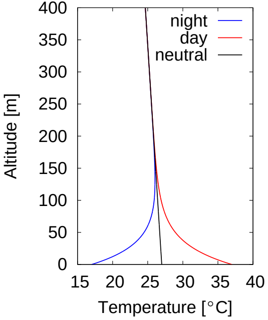
The air is thermally coupled to the ground

Daytime

- temperature decreases with altitude
- turbulent ~ 1 km
- winds slowed by friction

Nighttime

- temperature increases with altitude
- stable with buoyancy waves
- stable layer acts as a lubricant for the wind



Refraction of Sound by the Atmosphere

The speed of sound in air is $c \approx 20\sqrt{T}$

Temperature gradients

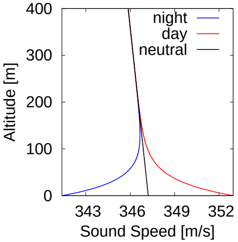
$\Rightarrow$ sound speed gradients

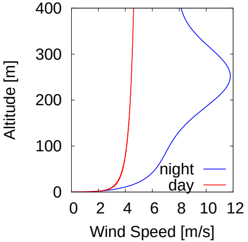
$\Rightarrow$ refraction

wavefronts propagate towards colder air
clear days are upward refracting
clear nights are downward refracting

Similarly for wind shear

wavefronts propagate towards slower air
upwind is upward refracting
downwind is downward refracting





Equations of Atmospheric Mechanics

The atmosphere is described by fluid mechanics

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \qquad \text{mass conservation}$$

$$\rho\Big(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v}\Big) + \nabla P = -\rho g \hat{\mathbf{z}} \qquad \text{Newton's law}$$

$$\frac{\partial S}{\partial t} + \mathbf{v} \cdot \nabla S = 0 \qquad \text{heat equation}$$

$$P = \rho R T \qquad \text{ideal gas law}$$

$$\frac{dS}{R} = \frac{c_p}{R}\frac{dT}{T} - \frac{dP}{P} \qquad \text{second law}$$

The ground is described by poro-elasto-dynamics

The two are coupled by interface conditions

Meteorology versus Acoustics

Variables split into slow (meteorological) and fast (acoustic) terms:

$$\begin{pmatrix} \mathbf{v} \\ \rho \\ P \\ T \\ S \end{pmatrix} = \begin{pmatrix} \mathbf{v}_{\rm met} \\ \rho_{\rm met} \\ P_{\rm met} \\ T_{\rm met} \\ S_{\rm met} \end{pmatrix} + \begin{pmatrix} \mathbf{v}_{\rm ac} \\ \rho_{\rm ac} \\ P_{\rm ac} \\ T_{\rm ac} \\ S_{\rm ac} \end{pmatrix}$$

The meteorological terms split into mean and fluctuating parts:

$$\begin{pmatrix} \mathbf{v}_{\rm met} \\ \rho_{\rm met} \\ P_{\rm met} \\ T_{\rm met} \\ S_{\rm met} \end{pmatrix} = \begin{pmatrix} \mathbf{v}_M \\ \rho_M \\ P_M \\ T_M \\ S_M \end{pmatrix} + \begin{pmatrix} \mathbf{v}_{\rm turb} \\ \rho_{\rm turb} \\ P_{\rm turb} \\ T_{\rm turb} \\ S_{\rm turb} \end{pmatrix}$$

Local Meteorology

For distances $\sim$ km's, times $\sim$ tens of minutes

mean quantities depend only on altitude
mean vertical wind speed is zero

z is altitude and H indicates horizontal

$$\begin{pmatrix} \mathbf{v}_{HM}(z) \\ \rho_M(z) \\ P_M(z) \\ T_M(z) \\ S_M(z) \end{pmatrix}$$

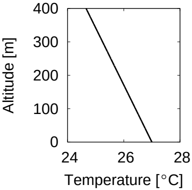
Then
$$\frac{dP_M}{dz} = -g\rho_M \qquad P_M = \rho_M R T_M$$

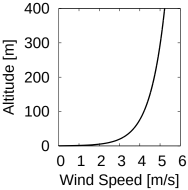
$$S_M - S_0 = c_p \ln \frac{T_M}{T_M(0)} - R \ln \frac{P_M}{P_M(0)}$$

and $\mathbf{v}_H$ is arbitrary.

example: $S_M = S_0 \Rightarrow$

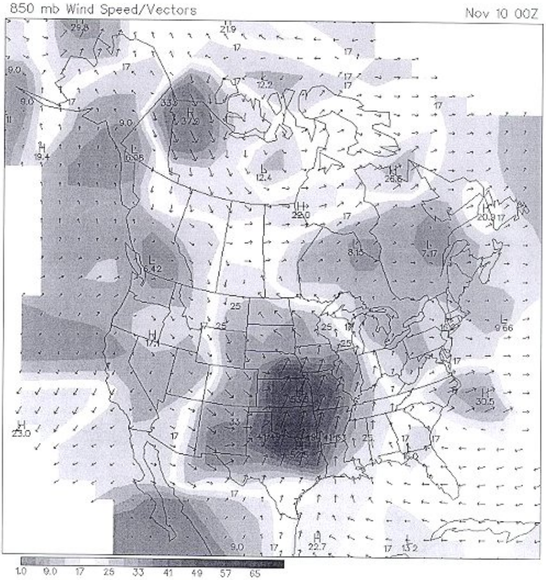
$$T_M(z) = T_0(0) - \frac{g}{c_p} z$$





The North American Nocturnal Jets

(global generation of local winds)



Air flows from warmer regions to cooler regions.

During the day flow is impeded by ground friction mediated through the turbulent layer.

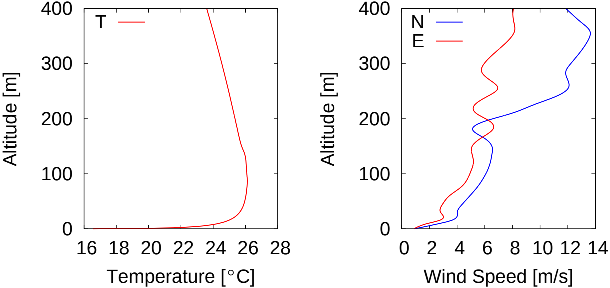
At night flow is decoupled from the ground. A stiff wind develops above the stable layer. Can be “super-geostrophic”.

Meteorological Measurements

meteorological equipment: 10 m tower, tethersonde, sodar

Meteorological Data collected in the Delta

Locke Station MS 11/09/06 at 18:15



Data is averaged over 15 minute intervals

and 15 meter vertical slices

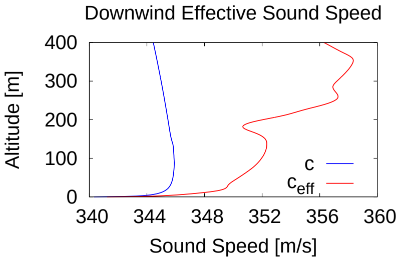
The Acoustic Terms

Reduces to determining $P_{\mathrm{ac}}(\mathbf{x}_H, z, t)$ from

$$\left(\nabla_H^2 + \frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c_{\mathrm{eff}}(\hat{\mathbf{n}}, z)^2}\right)\widehat{P}_{\mathrm{ac}}(\mathbf{x}_H, z, \omega) = 0$$

$\hat{\mathbf{n}}$ is the unit horizontal from source to receiver and

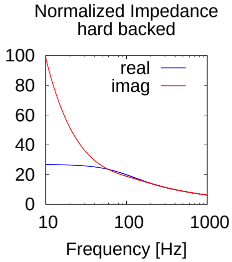
$$c_{\mathrm{eff}}(\hat{\mathbf{n}}, z) = 20\sqrt{T(z)} + \hat{\mathbf{n}} \cdot \mathbf{v}_M(z).$$



Interaction of Sound with the Ground

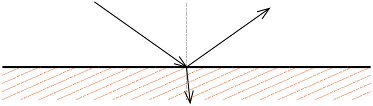
local impedance approximation

$$\frac{\partial \widehat{P}_{\mathrm{ac}}}{\partial z}\Big|_{z=0} = -\frac{i\omega\rho_M}{\mathcal{Z}}\widehat{P}_{\mathrm{ac}}\Big|_{z=0}$$



Well understood above 200 Hz

- ground is porous and essentially rigid
- reaction is essentially local
- modeled by an impedance condition



Local impedances are often used below 200 Hz

- Deviations from locality have been reported $\sim$ a few Hz

Methods of Solution

$$\left(\nabla_H^2 + \frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c_{\text{eff}}(\hat{\mathbf{n}}, z)^2}\right)\widehat{P}_{\text{ac}}(\mathbf{x}_H, z, \omega) = 0$$

$$\frac{\partial \widehat{P}_{\text{ac}}}{\partial z}\Big|_{z=0} = -\frac{i\omega\rho_M}{\mathcal{Z}}\widehat{P}_{\text{ac}}\Big|_{z=0}$$

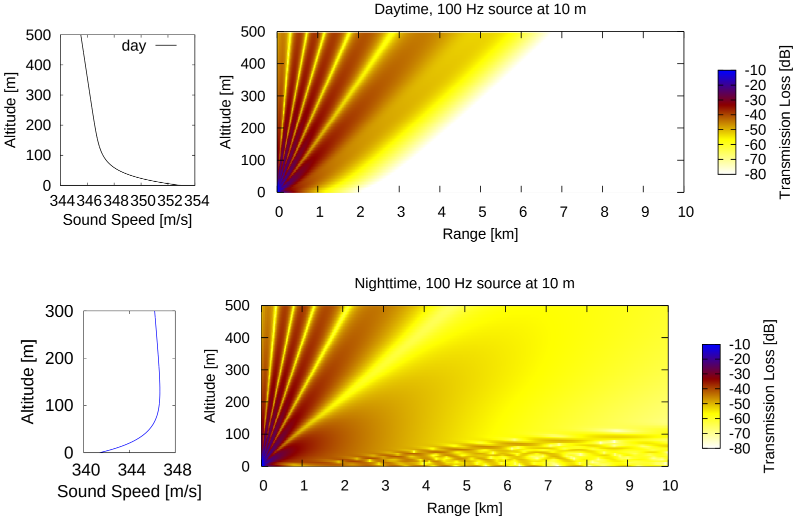
geometric: $\widehat{P}_{\text{ac}} \approx Ae^{iS}$; $\quad \nabla S$ are wavefront normals

PE: solve 1-way equ. $\left(\frac{\partial}{\partial x} \pm i\sqrt{\frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c_{\text{eff}}(\hat{\mathbf{n}}, z)^2}}\right)\widehat{P}_{\text{ac}} = 0$

FFP: Fourier transform w.r.t. $\mathbf{x}_H$

modes: Expand w.r.t. eigenfunctions of $\frac{d^2}{dz^2} + \frac{\omega^2}{c_{\text{eff}}(\hat{\mathbf{n}}, z)^2}$

Contrasting Day and Night



Modal Expansion for the Acoustic Pressure

Eigenvalue problem:

of Schrödinger type
non-self-adjoint

$$\left(\frac{d^2}{dz^2} + \frac{\omega^2}{c_{\text{eff}}(\hat{\mathbf{n}}, z)^2} - k^2\right)\psi = 0$$

$$\frac{d\psi}{dz}\Big|_{z=0} = -\frac{i\omega\rho_M}{\mathcal{Z}}\psi(0)$$

Ducted modes:

eigenvectors $\psi_j(\omega, z)$
eigenvalues k_j^2

$$k_j = \frac{\omega}{c_j} + i\alpha_j$$

$$\widehat{P}(\mathbf{x}_H, z, \omega) \approx \sum_{j=0}^{N(\omega)} p(k_j, \omega, \mathbf{x}_H)\psi_j(\omega, z)$$

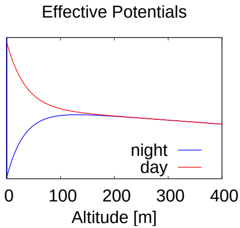
$$\left(\nabla_H^2 + k^2\right)p(k, \omega, \mathbf{x}_H) = 0$$

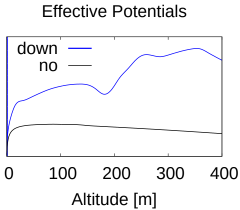
plus source terms.

Validity: ducted propagation ranges > 500 m
altitudes < 50 m

Finding the Modes

(the atomic analogy)

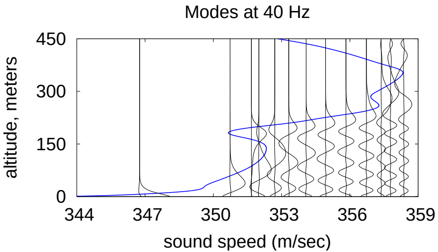




potential: $-\frac{\omega^2}{c_{\text{eff}}^2}$

bound states: ducted

continuum states: upward propagating

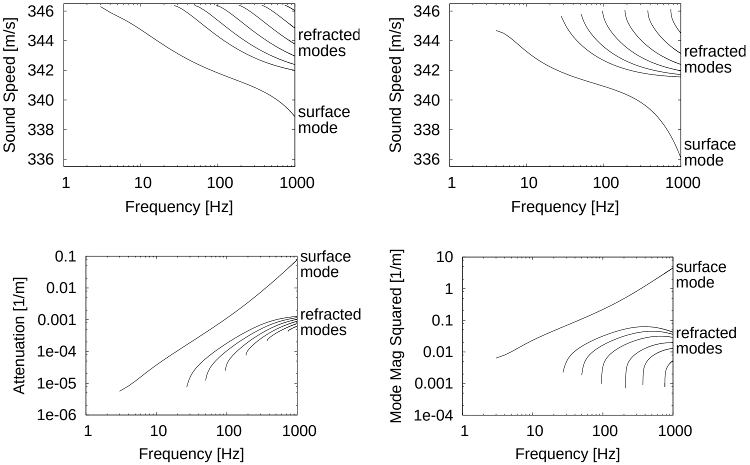


a non-self-adjoint tunneling problem

Modal Dispersion for a Simple Duct

(temperature inversion, no wind)

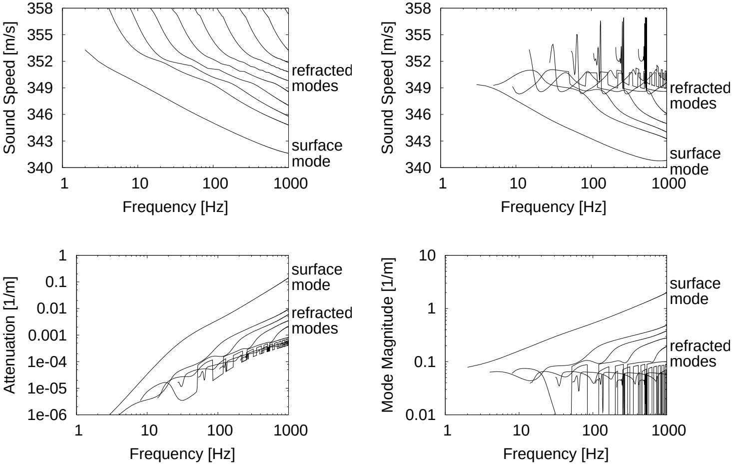
Dispersion for the surface and every $2^{n\text{th}}$ refracted mode.



Modal Dispersion for a Complex Duct

(temperature inversion and down wind)

Dispersion for the surface and every $2^{n\text{th}}$ refracted mode.

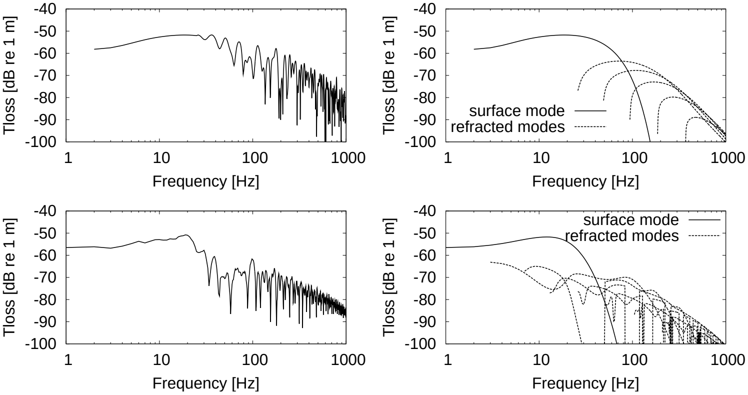


Propagation from a Point Source

$$\widehat{P} \approx e^{-i\frac{\pi}{4}} \sum_{j=0}^{N(\omega)} \frac{e^{ik_j r}}{\sqrt{8\pi k_j r}} \psi_j(\omega, z_S)\psi_j(\omega, z)$$

Ground to Ground Transmission Losses at 3 km

Total | Modal



Surface Mode vs Refracted Modes

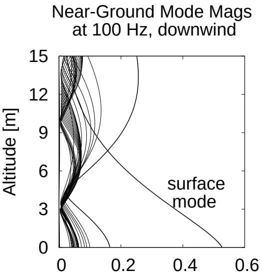
At long ranges the surface mode

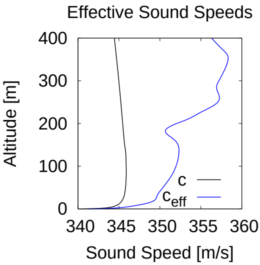
- is very sensitive to the ground surface
- is attenuated primarily by the ground
- has no low frequency cutoff
- carries energy up to about 100 Hz
- propagates at roughly the ground sound speed
- propagates horizontally

At long ranges the refracted modes

- are not very sensitive to the ground surface
- are attenuated primarily by the atmosphere
- have cutoff frequencies
- carry energy up to about 1000 Hz
- propagate with speeds greater than the ground sound speed
- propagate at shallow angles to the horizontal

The Narrow Band Near-ground Structure





The Quiet Height

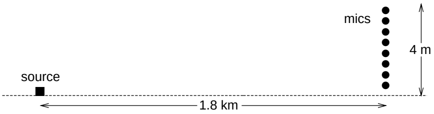
The surface mode has no “nodes”.

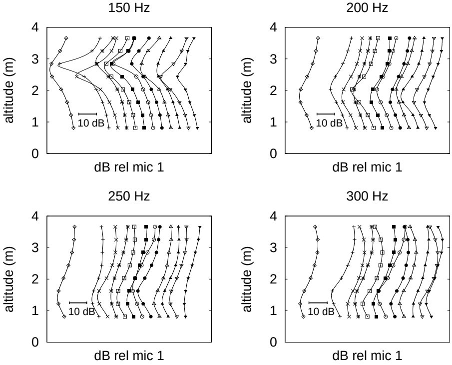
The lowest “node” of the refracted modes are at ≈ the same altitude.

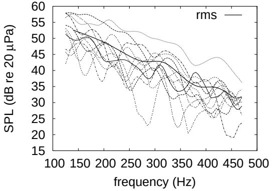
$$\widehat{P} \approx \frac{e^{ik_0 r - i\frac{\pi}{4}}}{\sqrt{8\pi k_0 r}}\psi_0(\omega, z_S)\psi_0(\omega, z) + \sum_{j=1}^{N(\omega)} \frac{e^{ik_j r - i\frac{\pi}{4}}}{\sqrt{8\pi k_j r}}\psi_j(\omega, z_S)\psi_j(\omega, z)$$

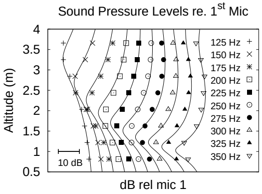
As the surface mode attenuates a “quiet height” develops.

The Quiet Height Experiment





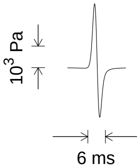


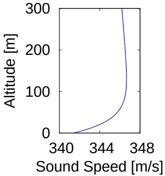


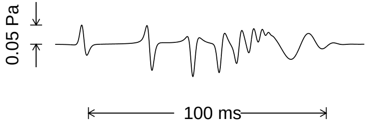
The Broad Band Pulse Tail

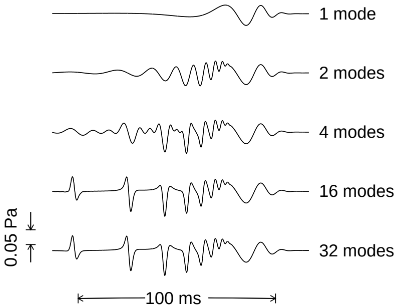
Point source with spectrum $Q(\omega)$ at 3 km range

$$P(r,z,t) = \int Q(\omega) \sum_{j=0}^{N(\omega)} \frac{e^{ik_j(\omega)r - i\omega t - i\frac{\pi}{4}}}{\sqrt{8\pi k_j(\omega) r}} \psi_j(\omega, z_S)\psi_j(\omega, z)\, d\omega$$



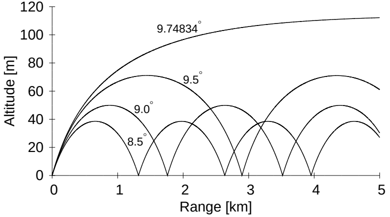


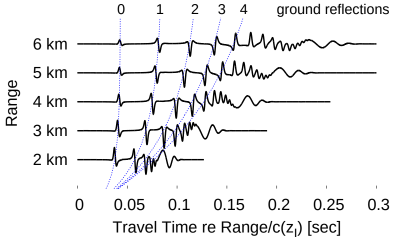




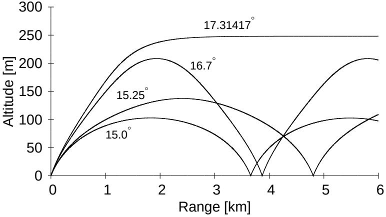
The Geometric Acoustics of the Distinct Early Arrivals

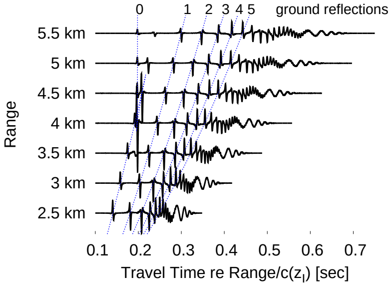
No Wind



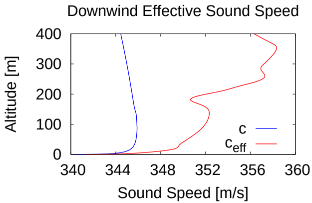


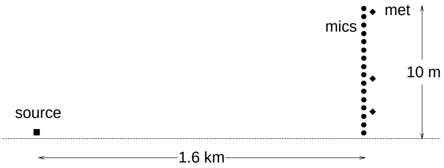
Down Wind

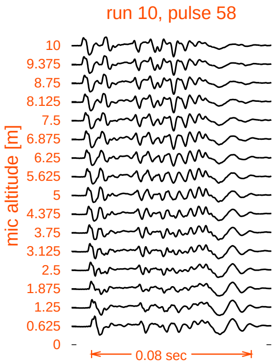




Recent Pulse Propagation Experiments



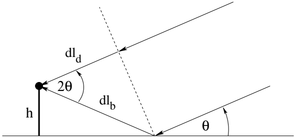




A visible surface mode tail

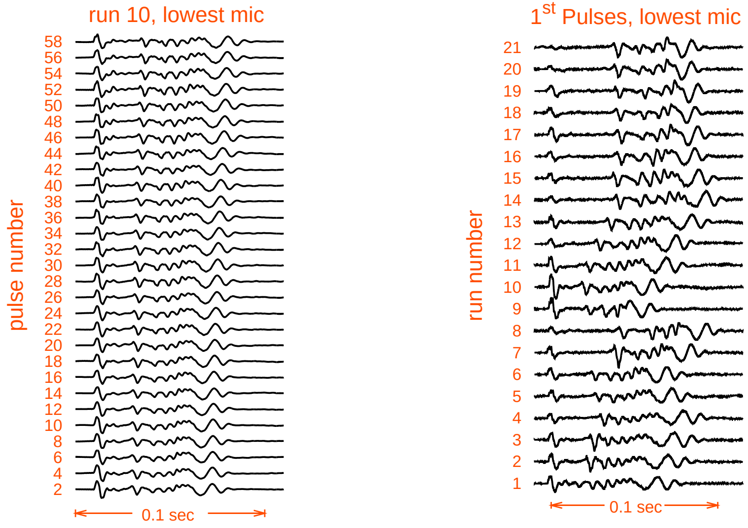
- propagates horizontally
- amplitude decreases with altitude

Front arrives at a shallow angle



Signal Variability

(short term stability   long term variability)



Yet to be Done

(we've got ourselves a probe)

- Study data inversions
  - for source location
  - for elevated wind jets
  - for ground properties
- Investigate the convergence zone
  - theoretically
  - experimentally
- Study the variation in pulse duration
  - study arrival time variability
  - identify the source
  - experiment planned for the spring
- Study upwind propagation
- Study poor weather propagation
  - propagation in fog
  - under cloud cover
  - during rain