The Propagation of Sound Outdoors
(over flat ground)

The atmosphere is a gas under the influence of:
- gravity
- thermal stresses
- the Earth’s rotation

The ground is a porous elastic solid:
- has large (compared with air) thermal conductivity
- reflects and attenuates sound
- properties can differ dramatically from place to place
The Diurnal Cycle
(fair weather meteorology)

The ground is thermally coupled to space
- heats up during the day
- cools off at night

The air is thermally coupled to the ground

Daytime
- temperature decreases with altitude
- turbulent $\sim 1$ km
- winds slowed by friction

Nighttime
- temperature increases with altitude
- stable with buoyancy waves
- stable layer acts as a lubricant for the wind
Refraction of Sound by the Atmosphere

The speed of sound in air is \( c \approx 20\sqrt{T} \)

Temperature gradients
\( \Rightarrow \) sound speed gradients
\( \Rightarrow \) refraction

wavefronts propagate towards colder air
clear days are upward refracting
clear nights are downward refracting

Similarly for wind shear

wavefronts propagate towards slower air
upwind is upward refracting
downwind is downward refracting
Equations of Atmospheric Mechanics

The atmosphere is described by fluid mechanics

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad \text{mass conservation}
\]

\[
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) + \nabla P = -\rho g \hat{z} \quad \text{Newton’s law}
\]

\[
\frac{\partial S}{\partial t} + \mathbf{v} \cdot \nabla S = 0 \quad \text{heat equation}
\]

\[
P = \rho RT \quad \text{ideal gas law}
\]

\[
\frac{dS}{R} = \frac{c_p}{R} \frac{dT}{T} - \frac{dP}{P} \quad \text{second law}
\]

The ground is described by poro-elasto-dynamics

The two are coupled by interface conditions
Meteorology versus Acoustics

Variables split into slow (meteorological) and fast (acoustic) terms:

\[
\begin{pmatrix}
\mathbf{v} \\
\rho \\
P \\
T \\
S
\end{pmatrix}
= \begin{pmatrix}
\mathbf{v}_{\text{met}} \\
\rho_{\text{met}} \\
P_{\text{met}} \\
T_{\text{met}} \\
S_{\text{met}}
\end{pmatrix}
+ \begin{pmatrix}
\mathbf{v}_{\text{ac}} \\
\rho_{\text{ac}} \\
P_{\text{ac}} \\
T_{\text{ac}} \\
S_{\text{ac}}
\end{pmatrix}
\]

The meteorological terms split into mean and fluctuating parts:

\[
\begin{pmatrix}
\mathbf{v}_{\text{met}} \\
\rho_{\text{met}} \\
P_{\text{met}} \\
T_{\text{met}} \\
S_{\text{met}}
\end{pmatrix}
= \begin{pmatrix}
\mathbf{v}_{\text{M}} \\
\rho_{\text{M}} \\
P_{\text{M}} \\
T_{\text{M}} \\
S_{\text{M}}
\end{pmatrix}
+ \begin{pmatrix}
\mathbf{v}_{\text{turb}} \\
\rho_{\text{turb}} \\
P_{\text{turb}} \\
T_{\text{turb}} \\
S_{\text{turb}}
\end{pmatrix}
\]
Local Meteorology

For distances \( \sim \) km’s, times \( \sim \) tens of minutes
mean quantities depend only on altitude
mean vertical wind speed is zero

\[ z \text{ is altitude and } H \text{ indicates horizontal} \]

\[
\begin{pmatrix}
v_H(z) \\
\rho(z) \\
P_M(z) \\
T_M(z) \\
S_M(z)
\end{pmatrix}
\]

Then
\[
\frac{dP_M}{dz} = -g \rho_M \\
P_M = \rho_M RT_M
\]

\[ S_M - S_0 = c_p \ln \frac{T_M}{T_M(0)} - R \ln \frac{P_M}{P_M(0)} \]

and \( v_H \) is arbitrary.

example: \( S_M = S_0 \Rightarrow \)

\[ T_M(z) = T_0(0) - \frac{g}{c_p} z \]
The North American Nocturnal Jets
(global generation of local winds)

Air flows from warmer regions to cooler regions.

During the day flow is impeded by ground friction mediated through the turbulent layer.

At night flow is decoupled from the ground. A stiff wind develops above the stable layer. Can be “super-geostrophic”.
Meteorological Measurements

meteorological equipment: 10 m tower, tethersonde, sodar
Meteorological Data collected in the Delta
Locke Station MS 11/09/06 at 18:15

Data is averaged over 15 minute intervals
and 15 meter vertical slices
The Acoustic Terms

Reduces to determining $P_{ac}(x_H, z, t)$ from

$$\left(\nabla^2_H + \frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c_{eff}(\hat{n}, z)^2}\right) \hat{P}_{ac}(x_H, z, \omega) = 0$$

$\hat{n}$ is the unit horizontal from source to receiver and

$$c_{eff}(\hat{n}, z) = 20\sqrt{T(z)} + \hat{n} \cdot v_M(z).$$
Interaction of Sound with the Ground

local impedance approximation

\[ \frac{\partial \hat{P}_{ac}}{\partial z} \bigg|_{z=0} = -\frac{i\omega \rho_M}{\mathcal{Z}} \hat{P}_{ac} \bigg|_{z=0} \]

Well understood above 200 Hz

ground is porous and essentially rigid
reaction is essentially local
modeled by an impedance condition

Local impedances are often used below 200 Hz
Deviations from locality have been reported \( \sim \) a few Hz
Methods of Solution

\[
\left( \nabla_H^2 + \frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c_{\text{eff}}(\hat{n}, z)^2} \right) \hat{P}_{ac}(x_H, z, \omega) = 0
\]

\[
\left. \frac{\partial \hat{P}_{ac}}{\partial z} \right|_{z=0} = -i\omega \rho M \left. \hat{P}_{ac} \right|_{z=0}
\]

generic: \( \hat{P}_{ac} \approx A e^{iS} \); \( \nabla S \) are wavefront normals

PE: solve 1-way equ. \( \left( \frac{\partial}{\partial x} \pm i \sqrt{\frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c_{\text{eff}}(\hat{n}, z)^2}} \right) \hat{P}_{ac} = 0 \)

FFP: Fourier transform w.r.t. \( x_H \)

modes: Expand w.r.t. eigenfunctions of \( \frac{d^2}{dz^2} + \frac{\omega^2}{c_{\text{eff}}(\hat{n}, z)^2} \)
Contrasting Day and Night

Daytime, 100 Hz source at 10 m

Nighttime, 100 Hz source at 10 m
Modal Expansion for the Acoustic Pressure

Eigenvalue problem:

of Schrödinger type non-self-adjoint

Ducted modes:

eigenvectors $\psi_j(\omega, z)$
eigenvalues $k_j^2$

$$
\left( \frac{d^2}{dz^2} + \frac{\omega^2}{c_{\text{eff}}(\hat{n}, z)^2} - k^2 \right) \psi = 0
$$

$$
\frac{d\psi}{dz} \bigg|_{z=0} = - \frac{i\omega \rho_M}{Z} \psi(0)
$$

$$
k_j = \frac{\omega}{c_j} + i\alpha_j
$$

$$
\hat{P}(x_H, z, \omega) \approx \sum_{j=0}^{N(\omega)} p(k_j, \omega, x_H) \psi_j(\omega, z)
$$

$$
\left( \nabla_H^2 + k^2 \right) p(k, \omega, x_H) = 0 \quad \text{plus source terms.}
$$

Validity: ducted propagation ranges $> 500$ m
altitudes $< 50$ m
Finding the Modes
(the atomic analogy)

The effective potentials are shown as a function of altitude. There are two sets of curves labeled 'night' and 'day' for the night-time and day-time conditions, respectively. The potential is given by the formula:

$$\text{potential: } -\frac{\omega^2}{c_{\text{eff}}^2}$$

The modes are divided into two categories:
- **Bound states**: Ducted
- **Continuum states**: Upward propagating

This is an example of a non-self-adjoint tunneling problem.
Modal Dispersion for a Simple Duct
(temperature inversion, no wind)

Dispersion for the surface and every $2^n$th refracted mode.
Modal Dispersion for a Complex Duct
(temperature inversion and down wind)

Dispersion for the surface and every $2^n$th refracted mode.
Propagation from a Point Source

\[ \hat{P} \approx e^{-i\frac{\pi}{4}} \sum_{j=0}^{N(\omega)} \frac{e^{ik_j r}}{\sqrt{8\pi k_j r}} \psi_j(\omega, z_S) \psi_j(\omega, z) \]

Ground to Ground Transmission Losses at 3 km

**Total**

**Modal**

![Graphs showing transmission loss over frequency for both total and modal cases](image-url)
Surface Mode vs Refracted Modes

At long ranges the surface mode
- is very sensitive to the ground surface
- is attenuated primarily by the ground
- has no low frequency cutoff
- carries energy up to about 100 Hz
- propagates at roughly the ground sound speed
- propagates horizontally

At long ranges the refracted modes
- are not very sensitive to the ground surface
- are attenuated primarily by the atmosphere
- have cutoff frequencies
- carry energy up to about 1000 Hz
- propagate with speeds greater than the ground sound speed
- propagate at shallow angles to the horizontal
The Narrow Band Near-ground Structure

The Quiet Height

The surface mode has no “nodes”.

The lowest “node” of the refracted modes are at \( \approx \) the same altitude.

\[
\hat{P} \approx \frac{e^{i k_0 r - i \frac{\pi}{4}}}{\sqrt{8\pi k_0 r}} \psi_0(\omega, z_S) \psi_0(\omega, z) + \sum_{j=1}^{N(\omega)} \frac{e^{i k_j r - i \frac{\pi}{4}}}{\sqrt{8\pi k_j r}} \psi_j(\omega, z_S) \psi_j(\omega, z)
\]

As the surface mode attenuates a “quiet height” develops.
The Quiet Height Experiment

1.8 km
source
mics
4 m

150 Hz

200 Hz

250 Hz

300 Hz

Sound Pressure Levels re. 1st Mic

SPL (dB re 20 µPa) vs. Frequency (Hz)

Altitude (m) vs. dB rel mic 1

rms
The Broad Band Pulse Tail

Point source with spectrum $Q(\omega)$ at 3 km range

$$P(r, z, t) = \int Q(\omega) \sum_{j=0}^{N(\omega)} \frac{e^{ik_j(\omega)r - i\omega t - i\frac{\pi}{4}}}{\sqrt{8\pi k_j(\omega) r}} \psi_j(\omega, z_S)\psi_j(\omega, z) d\omega$$
The Geometric Acoustics of the Distinct Early Arrivals

No Wind

![Graph showing altitude vs. range for no wind conditions.](image1)

Down Wind

![Graph showing altitude vs. range for down wind conditions.](image2)
Recent Pulse Propagation Experiments

A visible surface mode tail propagates horizontally. Amplitude decreases with altitude.

Front arrives at a shallow angle.
Signal Variability

(short term stability       long term variability)
Yet to be Done
(we’ve got ourselves a probe)

Study data inversions
  for source location
  for elevated wind jets
  for ground properties

Investigate the convergence zone
  theoretically
  experimentally

Study the variation in pulse duration
  study arrival time variability
  identify the source
  experiment planned for the spring

Study upwind propagation

Study poor weather propagation
  propagation in fog
  under cloud cover
  during rain