Numerical Evolution of Soliton Stars

Dr. Jayashree Balakrishna
(HSSU Saint Louis, Missouri)

Collaborators: M. Bondarescu (Ole Miss.), R. Bondarescu (Penn. State), G. Daues (N.C.S.A), F.S. Guzman (Mexico), E. Seidel (LSU, NSF)
Overview

• What is Dark Matter?
• Possible Dark Matter Candidates.
• What are soliton stars
  ° role in dark matter cosmology: halos, compact objects.
  ° formation of soliton stars.
  ° properties of compact soliton stars.
    In spherical Symmetry.
    In Full 3D GR- gravitational radiation.
Dark Matter

Dark matter: matter whose presence is inferred by its gravitational effects even though it cannot actually be seen.
Dark Matter: Galactic Rotation Curves
Direct Proof of Dark Matter
Hubble Expansion: Relevance of scalar fields in cosmology

• Redshift in light (distant galaxies) proportional to distance.
• Expansion of the universe: co-moving distance between points (difference in their coordinates) remains constant but physical distance between
• points increases.
Dark Matter and Dark Energy

• Hubble Telescope observations 1998 of distant supernovae showed that the expansion of the universe was accelerating rather than slowing down.

• Postulated dark energy (non zero cosmological constant working like a repulsive force).

• Present estimates are about 70% of the energy density is provided by this dark energy. Only about 5% of the density is luminous matter leaving about 25% being dark matter.
Nature of Dark Matter

• Primordial Nucleosynthesis:
  Current estimates of light element abundances fit predictions of BBN to a high degree. This is an estimate of baryon (protons+neutrons) abundances.

• These estimates show that dark matter is essentially non-baryonic.
Dark Energy

73%

Cold Dark Matter

23%

Atoms

4%
Axions as Dark Matter

Why they are preferred?

i) They are non-baryonic

ii) They are predicted by particle physics as a solution to the lack of CP violation in strong interactions.

iii) They are light and behave as cold-dark matter and can explain galaxy formation and large scale structure.
Axion Mass Range

• Lower bound $10^{-6} \text{eV}/c$
  to keep the density $\approx$ critical density
• Upper bound $10^{-3} \text{eV}/c$
  to prevent excessive energy loss in stars and supernovae.

The mass of a compact star $\sim \frac{M_{Pl}^2}{m}$ where $m$ is the mass of the particle it is made of.
Detection?

- Extremely light scalar particles ($10^{-22} \text{eV}$) could in principle form dark matter halos.
- Heavier particles can form compact objects.
- Can there be stars made of scalar particles that have signatures that could be detected?

Gravitational Radiation
What is an axion-star?

- Axions are scalar particles that can be described by real fields. These particles could clump together by a Jeans instability mechanism to form stars called soliton stars.
- There are also scalar particles that can be described by complex scalar fields (also possible dark matter candidates) that could form stars by the same mechanism. Such hypothetical stars are called boson-stars.
- These stars held together by a balance between the attractive force of gravity and the dissipative nature of the uncertainty principle (field).
Boson and Soliton Stars: Equations, Configurations and Evolutions.

• Spherically Symmetric Boson Stars:
• Soliton Stars: Ground State and Formation:
• Soliton Stars Ground State:
  M. Alcubierre, R. Becerril, F. S. Guzman
• Boson Stars: Spherically Symmetric and 3D evolutions:
• Boson Stars on a 3D Grid:
Recent Work

• Evolution of 3D Boson Stars with Waveform Extraction:


• Numerical Simulations of Oscillating SolitonStars:
  Excited States in Spherical Symmetry and Ground State Evolutions in 3D:

  J. Balakrishna, R. Bondarescu, G. Daues, M. Bondarescu
The equations

• Coupled Einstein-Klein Gordon system:

Action:

\[ I = \int d^4x \sqrt{-g} \left( \frac{1}{16\pi G} R - \frac{1}{2} [g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + V(\Phi)] \right) \quad V(\phi) = m^2\phi \]

Metric (spherical symmetry + polar slicing):

\[ ds^2 = -N^2 dt^2 + g^2 dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \]

\[ A' = 4\pi Gr A (C' \dot{\Phi}^2 + \dot{\Phi}^2 + Am^2 \Phi^2) + \frac{A}{r} (1 - A), \]

\[ C' = \frac{2C}{r} \left[ 1 + A \left( 4\pi Gr^2 \Phi^2 - 1 \right) \right], \]

\[ C' \Phi = -\frac{1}{2} \dot{C} \dot{\Phi} + \Phi'' + \dot{\Phi} \left( \frac{2}{r} - \frac{C'}{2C} \right) - Am^2 \Phi, \]

\[ \dot{A} = 8\pi Gr A \dot{\Phi} \Phi', \]

\[ A(r, t) = g^2, \quad C(r, t) = [g(r, t)/N(t, r)]^2 \]
Solutions

- Boson Stars (complex field) - time dependent field, time independent metric with energy density being time independent.
- Soliton Stars (real field) - no equilibrium configuration. Fields and metrics have time dependence.

Asymptotically Flat:

\[ \Phi(t, r) = \sum_{j=1}^{j_{\text{max}}} \phi_{2j-1} \cos((2j-1)\omega t), \]
\[ A(t, r) = \sum_{j=0}^{j_{\text{max}}} A_{2j}(r) \cos(2j\omega t), \]
\[ C(t, r) = \sum_{j=0}^{j_{\text{max}}} C_{2j}(r) \cos(2j\omega t), \]

\[ \phi(r = \infty) = 0 \Rightarrow \phi_{1,3,\ldots}(r = \infty) = 0. \]
\[ A_0(\infty) = 1, A_{2,4,\ldots} = 0. \]
\[ C_{2j} = \text{eigenvalue} \]
Soliton Star Profile (Alcubierre et. Al)
Boson Star Profile (Seidel-Suen Boson)
Mass Profile: Boson Star
Strong Perturbation of Stable Configuration

$A, B, C, D$

$t = 226, 339, 458, 571$

Unperturbed line

$g^2$

$R_B, R_D, R_C, R_A$
Ground State S-Branch Boson Star Evolution (Seidel-Suen boson unpert. M=0.33, \( \phi(0)=0.1 \))
Star Radius: Perturbed Stable Star

Oscillating Radius of Stable Perturbed Boson Star

Radius of \( \text{max}_{t} g^{2} \)

\[ t \]

Points A, C, B, D
Mass Loss

Mass Loss for Stable Perturbed Boson Star

![Graph showing the mass loss over time for a stable perturbed boson star. The graph plots total mass on the y-axis and time (t) on the x-axis. There are points labeled A and C on the graph.](image)
Mass Profile: Ground State Soliton Star
Soliton Stars

• Expected to be unstable (No equilibrium configurations).

• Truncated solution as a small perturbation:
S-branch Soliton Star: $M=0.5726$, $\phi_1(0)=0.2828$, $j_{\text{max}}=3$ (Alcubierre et al)

Mass loss < 0.003% of the original mass by $t=5000$. 

$\Delta x = 0.01, \Delta t/\Delta x = 0.5$

Slightly perturbed S-oscillation: $\phi_1(0)=0.2828$
Excited States (j.b et al.): Oscillatons

• Role of Excited States in Cosmology: could be intermediate states in the formation process of these stars.

• Are they stable?
Mass Profile: First Excited State

![Graph showing mass profile with first excited state identified by $M_c$.]
S-branch Excited State star collapsing to a black hole

In the polar slicing condition we use the radial metric rises sharply as an apparent horizon forms signaling the onset of black hole formation. \( \phi(0)=0.2828, \ dr=-0.1 \) perturbation is due to discretization of the grid.
S-branch star going to the ground state: Initial configuration

\[ \phi_1(0) = 0.041 \]

mass \( 0.655M_{P1}^2/m \)

\[ \Phi \]

\[ t = 0 \]

\[ t = 688.002.5 \]
Mass Profile Evolution of S-branch $n=1$ star going to ground state.
Metric at the end of the run compared to the metric of a ground state star close to the one it settles down to.

- The mass of the star at the end of the run is \( M = .44M_{Pl}^2/m \)

The profile shown in comparison has a mass of \( M = .43M_{Pl}^2/m \)
Decay Times

If an excited state $S$-branch star can lose enough mass it goes to the ground state otherwise it collapses to a black hole. A comparison of time scale of collapse of $n=1$ $S$-branch stars with the same numbers of grid points ($\sim 187$) covering the star is shown.

<table>
<thead>
<tr>
<th>$\phi_1(0)$</th>
<th>$M(M_{\odot}^2/m)$</th>
<th>$R_{95}(1/m)$</th>
<th>Collapse time</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.41</td>
<td>1.32</td>
<td>14.9</td>
<td>250</td>
</tr>
<tr>
<td>0.2828</td>
<td>1.26</td>
<td>18.7</td>
<td>445</td>
</tr>
<tr>
<td>0.1414</td>
<td>1.07</td>
<td>27.0</td>
<td>1280</td>
</tr>
<tr>
<td>0.10</td>
<td>0.92</td>
<td>33.7</td>
<td>2250</td>
</tr>
<tr>
<td>0.08</td>
<td>0.84</td>
<td>38.0</td>
<td>4160</td>
</tr>
<tr>
<td>0.075</td>
<td>0.82</td>
<td>39.5</td>
<td>goes to gr. state</td>
</tr>
</tbody>
</table>
Cascading Stars: 5 node star

$\phi_1(0) = 0.006; \ M = 0.79M_P^2/m$
5 node star: Density profile with intermediate 4 node state
FULL 3D EVOLUTIONS

• 3D evolutions take longer (more equations): resolution, convergence issues.
• instabilities and assymetries $d/dx \ d/dy$ versus $d/dy \ d/dx$
• Gauge issues- how to step through spacetime (more degrees of freedom).
• SO WHY? More realistic
Cactus Code: www.cactuscode.org
developed by AEI LSU

• It is a modular code. The modules are called thorns. (We changed the initial data and added gauge condition. We also had an initial value solver for the Boson Star.)

• It uses a BSSN formalism for the evolution equations with and without matter.

• Special credit to F. Siddhartha Guzman for his scalar field evolver.
Boson Stars versus Soliton Stars

• Challenges for Soliton Star.

Gravitational Waves have a high damping rate allowing full extraction on a short time scale (compared to Neutron Stars).

Gravitational Waveform

Zerilli function (l=2 m=0)

Stable Boson Star – Perturbation ~ Y_{20}

(a)
Gravitational Wave: Newman-Penrose Scalar

Stable Boson Star – Perturbation $\sim Y_{20}$
Soliton Star 3D (J.B. et al. PRD 2008)
Code Test
Newman-Penrose scalar: Metric Y20 perturbation of stable soliton star.
Zerilli Function Stable Star

$L=2$, $m=0$ Zerilli Function [$x \times 10^{-4}$]

- Low Resolution
- Medium Resolution
- High Resolution
- No Perturbation

$t$
Energy Output

![Graph showing the energy output over time.](image-url)
Conclusion

• We have seen that, in principle, scalar particles can form stable stars.
• They have ground states and excited states. The excited states although inherently unstable can cascade to a stable ground state configuration losing mass via emission of scalar radiation.
• Excited states can be intermediate states during the formation of these stars.
• They have gravitational wave signatures that damp on a short time scale.
Critical Density

- The smooth universe has a ‘geometry’ initially parallel free trajectories
- Flat: stay parallel,
- Closed: converge,
- Open: diverge.

\[ H^2 = \left( \frac{a}{a} \right)^2 = \frac{8\pi G \rho}{3} - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3} \]

\[ \rho_{cr} = \frac{3H^2}{8\pi G} \]
How do we know there is dark matter

• Theoretical: The critical density is needed for inflation and its predictions to work. Luminous matter is 5% of this. There must be non-luminous stuff. If 70% is dark energy then 25% must be dark matter.

• Rotation curves of stars shows the speeds do not match what they would be if the observed mass was all there was.
Friedmann-Robertson Walker Metric.

$$ds^2 = a(t)^2 ds_3^2 - dt^2$$

$$a(t) = \text{scale factor.}$$

$$H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\Pi G \rho}{3} - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

from oo component of Einstein’s equation

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{4\Pi G}{3} \left( \rho + \frac{3p}{c^2} \right) + \frac{\Lambda c^2}{3}$$

from trace of Einstein Field Eqns.

$$\Lambda = \text{Cosm. const.}$$

$$k / a^2 = \text{curvature}$$

$$G = \text{grav. const.}$$
dominated: $w=0 \Rightarrow$
Radiation dominated $w=1/3 \Rightarrow$

\[ p = w \rho c^2 \Rightarrow a(t) = a_0 t^{\frac{2}{3(w+1)}} \]

\[ a(t) \propto t^{2/3} \]
\[ a(t) \propto t^{1/2} \]