GREATER FOOL BUBBLES
A SURVEY

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ABSTRACT: Recent models of greater fool bubbles are surveyed and compared.

KEYWORDS: Greater Fool Bubbles, Rational Bubbles, Overconfidence, Behavioral Traders

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1. INTRODUCTION

Economists like a good question. Consider the topic of asset price bubbles.[1] Economists like to ask "why would a smart person hold an asset they know is overpriced?" When the person-on-the-street says "they're hoping to sell it to another person just before the bubble bursts," the economist responds with the follow-up question "why would that other smart person buy an asset that's about to collapse?"

On some level, that follow-up question is what really makes us economists. If the people we're studying are smart, then taking their problem seriously forces us to be smarter ourselves.

For some economists, that follow-up question is intended as a conversation-stopper. Obviously that second smart person wouldn't buy the asset in that situation, the first smart person would recognize this, and no one would ever hold an overpriced asset. Thus bubbles are impossible, and markets are efficient.

For others, who like dynamic models, an alternative response has traditionally been that the second smart person buys the overpriced asset because they expect the overpricing to grow forever, so they'll be able to sell it to a third smart person, and so on forever. Unfortunately, these models do not capture market participants' intuition

[1] Of course, asset price fluctuations are among the most controversial subjects in economics. Some (Kindleberger, 2000, Shiller, 2000) argue that these fluctuations are caused by asset price bubbles, while others (e.g., Garber, 2000) argue that bubbles are either impossible or empirically irrelevant. In addition, there is no consensus on appropriate policies towards bubbles, even among those who believe that bubbles exist (Bernanke and Gertler, 1999, Cecchetti et al., 2002).

that, e.g., timing is important for investors hoping to get out of bubbles before they burst. They also do not create any policy dilemmas, since these infinite-horizon bubbles generally improve welfare.

Recently, several models of "greater fool" bubbles have been developed, which take this follow-up question more seriously. These models capture the intuition of market participants that, if they want to ride a bubble, they must carefully time the point at which they sell to a "greater fool," and so, get out of the bubble. However, they also take more seriously the question of why a smart person would volunteer for the role of greater fool.

The goal of this paper is to provide an accessible introduction to these greater fool bubble models and present them in a form which makes it easier to compare their strengths and weaknesses. Section 2 begins with the standard infinite horizon model, both in its simple difference equation form, and in a simplified version of its more complete general equilibrium overlapping generations form. Section 3 turns to greater fool bubbles, beginning with the earliest such model, due to Harrison and Kreps (1978), in which the volume of trade is driven by overconfidence. Section 4 then considers the most neoclassical of the greater fool bubble models, i.e., a simplified version of the Allen et al. (1993) finite horizon model, based on asymmetric information. The virtues of the asymmetric information model are that (a) it permits analysis of greater-fool bubbles in a very standard neoclassical model which allows ordinary welfare analysis, etc., and (b) it shows that the richer models in later sections, which build on various kinds of irrationality, do not actually depend on this irrationality to generate their bubbles.

Since it is possible to generate greater fool bubbles with or without irrationality, the irrationality issue simply becomes a question of empirical plausibility. For example, the volume of trade turns out to play an important role in these bubble models. One could therefore ask which more realistically explains the volume of trade, fluctuating marginal rates of substitution between different states of the world, or investor overconfidence? If the later drives most trade, then bubble models assuming perfect rationality might be misleading in a way which should be investigated by economic theory.

Section 5 returns to an overconfidence model, i.e., the one due to Scheinkman and Xiong (2003). Section 6, finally, considers a model where rational traders ride a bubble to exploit behavioral traders. This model, due to Abreu and Brunnermeier, combines features of the asymmetric information model of Section 4 and the models with irrationality, of Sections 3 and 5.

Hopefully models like those in Sections 3 through 6 represent only the infancy of the field. However, I believe that the development, analysis and testing of these types of models will move the profession beyond the repetitive debates of the past.

2. OLDER INFINITE-HORIZON BUBBLE MODELS

A. A Simple Difference Equation Model

The standard model of an asset price bubble arises from simply solving the usual difference equation for a rational asset price:

$$p_t = d_t + \beta E_t p_{t+1}, \tag{1}$$

where p_t is the price of the asset in period t, d_t is the period t payoff (e.g., dividend) from the asset, E_t denotes expectations with respect to period-t information, and β is a discount factor, e.g., $1/(1+r)$ if the agent is risk neutral and the risk-free interest rate is r. This equation has a "fundamental" solution

$$F_t = d_t + \beta E_t d_{t+1} + \beta^2 E_t d_{t+2} + \cdots \tag{2}$$

In addition, the model also has a large number of other solutions. Specifically, assume that B_t is any random or nonrandom process such that $B_t = \beta E_t B_{t+1}$. Then the price process

$$p_t = F_t + B_t \tag{3}$$

is also a solution to the pricing equation. The term B_t, if it is nonzero, is often referred to as a bubble. For example, the simplest type of bubble process would be

$$B_t = B_0 \beta^{-t}, \tag{4}$$

with B_0 some constant.

However, richer models, with e.g., stochastically bursting bubbles, are also possible (see Blanchard and Watson, 1982). For example, one possibility is for B_t to grow by a constant factor $1/(\pi\beta)$ until the bubble bursts, with the probability of the bubble bursting in any period constant at π. In this case the bubble almost certainly bursts in finite time. Nevertheless, the bubble must grow forever in expected value terms, in order for investors to be willing to hold the asset.

Note that the random aspect of this second bubble process introduces a "sunspot" element into the bubble. Nevertheless, a bubble can exist without these sunspot fluctuations. Similarly, sunspots are possible in models without bubbles.

B. A General Equilibrium Overlapping Generations Model of a Bubble

Concern is sometimes expressed that the above infinite-horizon models are inconsistent in the long run. As Mussa (2003), p. 42, has recently written, "Did the Good Lord teach people how to solve, intuitively, difference and differential equations but forget to imbue them with the insight to impose the relevant boundary conditions?" While several interpretations of this quote are possible (see below), one interpretation is that, if one considers the long run outcome of the above bubble process, then bubble models are simply dynamically inconsistent. However, as shown in this subsection, bubbles which grow forever are consistent in the long run if the economy is growing faster than the bubble. However, this also requires something like an overlapping-generations structure, as in Samuelson (1958) and Tirole (1985), even in this infinite horizon framework. This is because the only reason why one would hold an asset with a price above the fundamental value of the dividend stream is if one hoped to sell it to someone else in the future. On the other hand, in an infinite horizon model, that future buyer need not be a "greater fool." Also, if, say, the bubble is nonstochastic, then the seller need not worry about the timing of the sale, in terms of possible bursting of the bubble.

We will consider an extremely simple overlapping generations model. Time is discrete and extends infinitely into the future. There is a single consumable good. A fixed number, N, of people are born each period and live two periods. People receive an endowment when young, which depreciates, so, if a young person saves an amount s of her endowment, she gets δs when old, with $0 < \delta < 1$. Their utility function is assumed to be Leontief, and given by

$$u(c_t^y,\ c_t^o) = \min(c_t^y,\ c_t^o/\alpha), \tag{5}$$

where c_t^y and c_t^o are the consumption, when young and old, respectively, of the typical member of the generation born in period t. Thus, if the price of the good is positive in both periods, the optimal consumption would satisfy

$$c_t^o = \alpha c_t^y. \tag{6}$$

In the absence of a bubble asset, the only way that the young can save for old age is by physically storing some of their endowment. This yields a budget constraint

$$c_t^o = \delta(e - c_t^y). \tag{7}$$

This, together with (6) gives

$$c_t^y = \frac{\delta}{\alpha+\delta}e \quad \text{and} \quad c_t^o = \frac{\alpha\delta}{\alpha+\delta}e, \quad \text{so} \quad u_t = \frac{\delta}{\alpha+\delta}e. \tag{8}$$

This is "dynamically inefficient" since everyone could do better if each generation, when young, shared some of their endowment with the previous generation's old. Specifically, if each young person gave $(\alpha/[\alpha+1])e$ to that period's old people, everyones' consumption would be

$$c_t^y = \frac{1}{\alpha+1}e \quad \text{and} \quad c_t^o = \frac{\alpha}{\alpha+1}e, \quad \text{so} \quad u_t = \frac{1}{\alpha+1}e. \tag{9}$$

Since $\delta < 1$, everyone is better off sharing as in (9), so the free market outcome in (8) is inefficient.

However, in the presence of a bubble asset, efficiency can be restored. For suppose there is a bubble asset with price in period t of p_t relative to the consumption good in period t. Assume that the price process is nonstochastic, so agents can plan easily. The budget constraint for people who are young in period t is

$$p_{t+1}c_t^o = p_t(e - c_t^y), \tag{10}$$

where we are assuming that

$$p_{t+1}/p_t > \delta \tag{11}$$

so buying the bubble asset is a better investment than storing the good. In addition, (6) still holds. Let $R_t = p_{t+1}/p_t$ be the gross return (capital gain) on the bubble asset. Then solving (6) and (10) gives

$$c_t^y = \frac{R_t}{\alpha + R_t}e \quad \text{and} \quad c_t^o = \frac{\alpha R_t}{\alpha + R_t}e. \tag{12}$$

In addition, markets must clear, so

$$c_t^y + c_{t-1}^o = e. \tag{13}$$

That is, the consumption of the period t young and the period $t-1$ old must equal today's endowment. Combining the demand functions in (12) with the equilibrium condition in (13) and simplifying gives

$$R_t = (1-\alpha) + \frac{\alpha}{R_{t-1}}. \tag{14}$$

In addition, in this equilibrium, the old will trade all of their bubble asset for goods. Thus, if the total amount of the bubble asset per old person is m, then the price of the bubble asset will be

$$1/p_t = c^o_{t-1}/m = \frac{\alpha R_{t-1}}{\alpha + R_{t-1}} \frac{e}{m}. \tag{15}$$

One solution of equilibrium condition (14) is $R_t = 1$ for all t, in which case the price of the asset is

$$1/p_t = \frac{\alpha}{1+\alpha} \frac{e}{m} \tag{16}$$

for all t, so it is possible for the bubble asset to have a positive price.

Also, in this case, the consumption and welfare are as in the sharing case, (9) above, so the bubble asset has also improved welfare. This is like the original Samuelson (1958) model, where a bubble in an asset called "money" provides a vehicle by which the young can save for old age. Thus, in this case a bubble, far from representing a market failure, is a remarkable example of the invisible hand at work!

It should also be noted that one could model sunspots or other types of endogenous fluctuations in this framework. For example, in the above model, let $\alpha = 0.5$, and suppose that $\delta < 0.75$. Then, in addition to the equilibrium path $R_0 = R_1 = R_2 = \cdots = 1$, equation (14) yields a continuum of other equilibrium paths, for example, $R_0 = 2$, $R_1 = 0.75$, $R_2 = 1.1667$, and so on. Note, however, that these endogenous fluctuations are not bubbles. The overpricing is the bubble. Thus, while sunspots and endogenous fluctuations are related to bubbles, the two types of phenomena should be kept separate.

While the above infinite-horizon bubble element might play a role in the pricing of certain types of collectable items such as rare coins or paintings, (LeRoy, 2004), it seems implausible to me that such infinitely lived bubbles could play a major role in the classic historical examples of dramatic asset price movements such as Tulip mania or the dot-com boom. In the above models people hold assets they know are overpriced because they believe that this overpricing will grow for ever in expected value terms. It seems unlikely that any investors who saw themselves as buying overpriced tulips or dot-com stocks expected this overpricing to last forever, even in expected value terms.

It also seems doubtful that these price movements, if they were bubbles, were allowing investors to save for old age. These models require the economy to be dynamically inefficient. Since actual economies appear to be dynamically *efficient* (Abel et al., 1989), this suggests that these overlapping-generations bubble models are not possible in the economies in which we actually live, since there is no dynamic inefficiency of this type for these bubbles to correct (though see Cochrane, 2003, who argues that actual bubble-like assets provide a kind of "convenience yield").

Returning to the Mussa comment, perhaps Mussa was thinking that the economy was dynamically efficient, so that infinite horizon bubbles were impossible. However, another possibility is that he believed that any actual bubbles, if they exist, will eventually burst, even in expected-value terms. The question then becomes, why would anyone ever hold an asset which is expected to burst? One natural answer is that people are just stupid. However, another possibility is that these investors are planning to sell the bubble asset to a "greater fool," who will make the mistake of buying the asset. But then, how can we represent such a greater fool in an economic model without doing too much damage to the usual economic assumption that all agents are rational? The remaining sections of this paper will survey some recent papers which try to accomplish this task.

3. TRADE DUE TO OVERCONFIDENCE–HARRISON AND KREPS

The first real greater fool bubble model was Harrison and Kreps (1978). This is also perhaps the simplest greater fool model. Harrison and Kreps show, essentially, that differences of opinion plus short sales constraints lead to asset overpricing, in the sense that an agent will pay more for an asset if she knows others disagree with her (for a related point, see Miller, 1977).

To understand the Harrison and Kreps story, assume that there are two categories of risk neutral traders, "Momentum Expecting" (ME) traders and "Bounce-back Expecting" (BE) traders. There is an asset which, in any given period, pays a dividend of either zero or one. Thus, in any given period there are two possible states, and each type of trader expects these states to evolve as a Markov process, with transition matrices

$$\begin{pmatrix} 0.5 & 0.5 \\ 0.2 & 0.8 \end{pmatrix} \text{ and } \begin{pmatrix} 0.2 & 0.8 \\ 0.5 & 0.5 \end{pmatrix},$$

for ME and BE traders, respectively. Here the first row/column represents the state with a dividend of zero and the second represents the state where the dividend is one. Thus, an ME trader expects a zero state to be followed by a zero or one state with equal probabilities of 0.5 each, but expects a one state to be followed by a one state with probability of 0.8 (momentum). Similarly, a BE trader expects a one state to be followed by a zero or one state with equal probabilities of 0.5 each, but expects a zero state to be followed by a one state with probability of 0.8 (i.e., bouncing back from a bad period). Assume the discount factor is $\delta = 0.75$.

Now let's calculate the "buy and hold" price for the asset in states zero and one, and compare these to the prices that result if investors can trade the asset with one another. First consider investor ME. Prices p_0 and p_1 satisfy the difference equations

$$p_0^{ME} = 0.75[0.5p_0^{ME} + 0.5(1 + p_1^{ME})]$$

$$p_1^{ME} = 0.75[0.2p_0^{ME} + 0.8(1 + p_1^{ME})].$$

Here p_1^{ME} is the ex-dividend price, i.e., the price an ME-investor would be willing to pay in a period, *after* the dividend had already been paid out to its previous owner. This is a system of two equations in two unknowns. Its solution is

$$p_0^{ME} = 60/31 = 1.9355,\ p_1^{ME} = 69/31 = 2.2258.$$

Similarly, the buy-and-hold prices for Investor BE are

$$p_0^{BE} = 96/49 = 1.9592,\ p_1^{BE} = 87/49 = 1.7755$$

Of course, if investors can trade with each other, then each will take advantage of what they perceive to be the others' mistaken model of the dividend process. Thus, consider state zero. In this state the BE traders expect the asset to "bounce back." They also know that if it *does* bounce back, the ME traders will expect the stock to have momentum, so these ME traders will pay the BE traders a lot for the stock. Thus, the BE traders will hold the asset, planning to sell it to the ME investors when state one arrives. This yields trade prices that satisfy

$$p_0^T = 0.75[0.2p_0^T + 0.8(1 + p_1^T)]$$

$$p_1^T = 0.75[0.2p_0^T + 0.8(1 + p_1^T)].$$

This system has solution

$$p_0^T = 12/5 = 2.4, \; p_1^T = 12/5 = 2.4.$$

Thus, since each believes that she can sell the asset to a "greater fool" in what she sees as a bad state, each is more willing to hold the asset. This leads all investors bid up the price beyond what they see as the asset's buy-and-hold value.

It's not clear whether we should consider this a bubble or not. The asset is always overpriced relative to its buy-and-hold value, but the overpricing never leads to a crash. It is tempting to consider the higher price as simply reflecting the value of liquidity. However, it is more than that, since it has a greater fool flavor. The price is bid up precisely because each side is attempting to exploit what it perceives as pricing mistakes by the other side.

4. A SIMPLE RATIONAL GREATER FOOL BUBBLE

This section presents the most neoclassical of the greater fool bubble models, with all agents rational, and the bubble driven by asymmetric information and short sales constraints. We assume a finite horizon, not because the world actually has a finite horizon, but because this is a simple way to tie our hands and rule out the infinite-horizon bubbles of the previous section.

A. Asset Markets with Asymmetric Information

This section presents the basic model of an asset market incorporating asymmetric information and short-sale constraints. The next section then presents an example of a rational bubble in such asset markets.

There are two risk-neutral agents in this market, Ellen and Frank, and a finite set of states of the world, Ω. The typical state of the world is $\omega \in \Omega$. Below we also use b, e^B, f_2^G, etc., to denote typical states of the world. Ellen's and Frank's prior probability distributions over Ω are denoted by $\pi_E(\omega)$ and $\pi_F(\omega)$, respectively. We allow $\pi_E(\omega)$ and $\pi_F(\omega)$ to differ, in order to give Ellen and Frank an incentive to trade. However, the incentive to trade could also be generated by giving people different marginal utilities of wealth in different states. For example, if Frank's marginal utility of wealth is higher than Ellen's in states with high dividends, then there will be a potential gain from trade from Ellen selling the asset to Frank.

The market lasts for T periods, so $1 \leq t \leq T$, but there is no discounting. There is a riskless asset (money), and a risky asset. A unit of the risky asset will ultimately pay a single dividend of $d(\omega)$ in state ω. Ellen and Frank are initially endowed with state-dependent amounts of the risky asset, $a_0^E(\omega)$ and $a_0^F(\omega)$, respectively, as well as state-dependent endowments of money, $m_0^E(\omega)$ and $m_0^F(\omega)$.

Denote Ellen's and Frank's net sales of the risky asset in period t by $x_t^E(\omega)$ and $x_t^F(\omega)$, respectively. Thus, if $a_t^E(\omega)$ and $a_t^F(\omega)$ are their holdings of the risky asset at the end of period t, then $a_t^E(\omega) = a_{t-1}^E(\omega) - x_t^E(\omega)$ for Ellen, and similarly for Frank. In the same way, if $m_t^E(\omega)$ and $m_t^F(\omega)$ are Ellen and Frank's money holdings at the end of period t, and $p_t(\omega)$ is the price of the risky asset in period t, then $m_t^E(\omega) = m_{t-1}^E(\omega) + p_t(\omega)x_t^E(\omega)$ for Ellen, and similarly for Frank.

Assume that there are no short sales of the risky asset, so $a_t^E(\omega) \geq 0$ and $a_t^F(\omega) \geq 0$ for all ω and t. Assume also that the price of the consumption good, in terms of money, is fixed at one. Since Ellen is risk neutral, her overall expected payoff is then $E^E[m_T^E(\omega)+a_T^E(\omega)d(\omega)]$, where E^E is the expectation with respect to Ellen's prior π_E. Similarly, Frank's overall expected payoff is $E^F[m_T^F(\omega)+a_T^F(\omega)d(\omega)]$.

Ellen's and Frank's underlying information partitions in period t are given by $\mathbf{E}_t = \{E_t^i\}$ and $\mathbf{F}_t = \{F_t^i\}$, respectively. These partitions become finer over time, so Ellen and Frank do not forget. In general, Ellen and Frank can also learn from the market price in period t, $p_t(\omega)$, but in the example below, the underlying partitions will be chosen so that prices reveal no additional information. That is, the example will be set up so that p_t is "measurable" with respect to each of $\mathbf{E}_t$ and $\mathbf{F}_t$.

A competitive equilibrium in this market consists of a state-dependent price, $p_t(\omega)$, each period, and a pair of state-dependent net trades, $x_t^E(\omega)$ and $x_t^F(\omega)$, each period, for Ellen and Frank, respectively, such that:

(i) p_t is measurable with respect to the join (coarsest common refinement) of $\mathbf{E}_t$ and $\mathbf{F}_t$, so p_t depends only on information actually possessed by market participants,

(ii) an agent's net trades in period t are measurable with respect to the join of his own partition and the partition generated by p_t, so each agent's net trades depend only information he/she actually possesses at the time of trade,

(iii) the market clears, so $x_t^E(\omega) + x_t^F(\omega) = 0$, and

(iv) each agent's net trades are optimal, given his/her information, the set of state-dependent prices, the short-sale constraints, and his/her (correct) beliefs about the other's strategy rule.

Finally, we follow Allen et al. (1993) by saying that a *strong bubble* exists at a state of the world, ω, if all agents know that the risky asset is overpriced for sure. Specifically, a strong bubble exists at the state ω and time t if $\omega \in E_t^i$ implies that, for all $\omega' \in E_t^i$, $p_t(\omega') > d(\omega')$, and similarly when $\omega \in F_t^i$.

B. A Simple Example of a Rational Bubble

This section presents a simple bubble. This is a slight simplification of the example towards the end of Conlon (2004), and is simpler than the Allen et al. example.

Before presenting the formal example, it may help to explain the intuition. There are two traders, Ellen and Frank. In some states of the world (i.e., b and e^B), Ellen is a "bad seller" who knows that the asset is worthless, but wants to sell it to Frank. In certain other states of the world (e_1^G, e_2^G and e_3^G) Ellen is a "good seller" who believes that the asset may be valuable, but is willing to sell it to Frank because Frank places more value on the asset (in expected value terms) than Ellen. Frank is willing to buy the asset from Ellen in some of these states because there are potential gains from trade in the good states, and Frank can't always tell the difference between the good states and the bad states. That is, Frank is willing to run the risk of buying in the bad states, to get the asset in the good states.

In the same way, there are certain states (b and f^B) in which Frank is a bad seller who knows that the asset is worthless and wants to sell, and certain states (f_1^G, f_2^G and f_3^G) in which Frank is a good seller who believes that the asset may be valuable, but is willing to sell, and Ellen is willing to buy in some of these states, since she cannot tell them apart. Finally, in one of the bad states, b, both know that the asset is worthless, but each is willing to hold it in the (mistaken) belief that he/she will be able to sell it later. A strong bubble therefore exists in state b.

Assume, then, that there are three periods, $t = 1,\ 2,\ 3$, so $T = 3$, and nine states of nature, b, e^B, f^B, e_1^G, e_2^G, e_3^G, f_1^G, f_2^G, and f_3^G. Ellen and Frank each initially owns one unit of the asset in all states, so $a_0^E(\omega) = a_0^F(\omega) = 1$ for all $\omega \in \Omega$. The asset only pays a nonzero dividend in states e_3^G and f_3^G, and this dividend is 4.

Following Allen et al., assume Ellen and Frank have different prior probabilities over the states of the world, as given in the two rows of Table 1, where $A = 1/14$ so the probabilities add to one. These different prior probabilities are used to give the agents an incentive to trade, though other approaches also work.

Table 1: Prior Probabilities

State	b	e^B	f^B	e_1^G	e_2^G	e_3^G	f_1^G	f_2^G	f_3^G
Ellen	A	A	A	$2A$	A	A	$4A$	A	$2A$
Frank	A	A	A	$4A$	A	$2A$	$2A$	A	A

Next, the information partitions, $\mathbf{E}_t = \{E_{it}\}$ and $\mathbf{F}_t = \{F_{it}\}$, for Ellen and Frank, respectively, are given as follows. Ellen's initial information partition has three cells:

$$E_1^{BS} = \{b,\ e^B\},\ E_1^{BUY} = \{f^B,\ f_1^G,\ f_2^G,\ f_3^G\},\ E_1^{GS} = \{e_1^G,\ e_2^G,\ e_3^G\} \tag{17}$$

while Frank's initial information partition has the three cells

$$F_1^{BS} = \{b,\ f^B\},\ F_1^{BUY} = \{e^B,\ e_1^G,\ e_2^G,\ e_3^G\},\ F_1^{GS} = \{f_1^G,\ f_2^G,\ f_3^G\} \tag{18}$$

(see Figure 1). Note that E_1^{BS} represents states where Ellen is a bad seller, who knows that the asset is worthless, and E_1^{GS} represents states where Ellen is a good seller, who thinks that the asset may be valuable, i.e., may pay a positive dividend. Note also that Frank cannot distinguish between the good states from E_1^{GS}, and the bad state, e^B, from E_1^{BS}. This is why Frank is willing to buy from Ellen in some of the states, in F_1^{BUY}, where Ellen is bad. For similar reasons, Ellen is willing to buy from Frank in some of the states where Frank is bad.

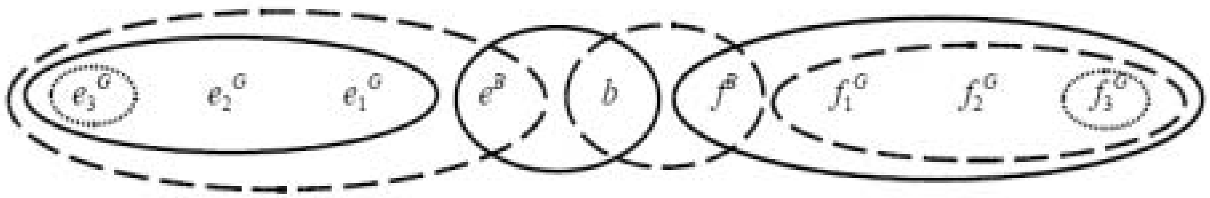

FIGURE 1: The players' information sets. **Solid curves:** Ellen's information sets. **Dashed curves:** Frank's information sets. **Dotted curves:** Dividend paying states.

In period 2, both players learn the true state, ω, if it is b, e_1^G, or f_1^G. Ellen's underlying information partition in period 2 is therefore:

$$\begin{gathered} E_2^1 = \{b\},\ E_2^{BS} = \{e^B\},\ E_2^2 = \{f_1^G\}, \\ E_2^{BUY} = \{f^B,\ f_2^G,\ f_3^G\},\ E_2^3 = \{e_1^G\},\ E_2^{GS} = \{e_2^G,\ e_3^G\} \end{gathered} \tag{19}$$

and Frank's underlying partition in period 2 is

$$\begin{gathered} F_2^1 = \{b\},\ F_2^{BS} = \{f^B\},\ F_2^2 = \{e_1^G\}, \\ F_2^{BUY} = \{e^B,\ e_2^G,\ e_3^G\},\ F_2^3 = \{f_1^G\},\ F_2^{GS} = \{f_2^G,\ f_3^G\}. \end{gathered} \tag{20}$$

In period 3 Ellen and Frank learn the true state if it is e^B e_2^G, f^B or f_2^G, so Ellen's underlying partition in period 3 is

$$E_3^1 = \{b\},\ E_3^2 = \{e^B\},\ E_3^3 = \{f_1^G\},\ E_3^4 = \{f^B\},$$
$$E_3^5 = \{f_2^G\},\ E_3^6 = \{f_3^G\},\ E_3^7 = \{e_1^G\},\ E_3^8 = \{e_2^G\},\ E_3^9 = \{e_3^G\} \tag{21}$$

and symmetrically for Frank. Thus, no one ever forgets, and by the end of period 3, both agents have perfect information.

Recall that the asset only pays a nonzero dividend in states e_3^G and f_3^G. Thus, when Ellen observes the event E_1^{BS}, she knows the asset is actually worthless, and when Frank observes the event F_1^{BS}, *he* knows the asset is worthless. This implies that, in state b, both Ellen and Frank know the asset is worthless (though neither knows that the other knows). Thus, if the price of the asset is nevertheless positive in this state, this will represent a strong bubble. State b will therefore be our "bubble state."

We now construct an equilibrium with a positive price in state b, so a strong bubble really does exist in this state. First, Table 2 gives an equilibrium set of prices.

Table 2: Equilibrium Prices

State	b	e^B	f^B	e_1^G	e_2^G	e_3^G	f_1^G	f_2^G	f_3^G
$t = 1$	1	1	1	1	1	1	1	1	1
$t = 2$	0	2	2	0	2	2	0	2	2
$t = 3$	0	0	0	0	0	4	0	0	4

Thus, in most states, prices rise for various periods before crashing. In the dividend paying states e_3^G and f_3^G the price rises to the true dividend, $d(e_3^G) = d(f_3^G) = 4$.

Next, Table 3 gives the net sales, from Ellen to Frank, in the equilibrium. Thus, Ellen sells to Frank in information sets $E_2^{BS} = \{e^B\}$ and $E_2^{GS} = \{e_2^G,\ e_3^G\}$, and Frank sells to Ellen in information sets $F_2^{BS} = \{f^B\}$ and $F_2^{GS} = \{f_2^G,\ f_3^G\}$.

Table 3: Net Sales from Ellen to Frank

State	b	e^B	f^B	e_1^G	e_2^G	e_3^G	f_1^G	f_2^G	f_3^G
$t=1$	0	0	0	0	0	0	0	0	0
$t=2$	0	1	-1	0	1	1	0	-1	-1
$t=3$	0	0	0	0	0	0	0	0	0

To see that this is an equilibrium, first note that prices and net trades in each period are measurable with respect to each player's partition for that period. Thus, conditions (i) and (ii) for an equilibrium are met. In addition, markets clear (condition (iii)) and the short-sale constraints are satisfied.

Next, consider whether Ellen's behavior is optimal (Frank's problem is clearly symmetrical). In each of Ellen's initial three information sets, Ellen's expectation of the second period price, p_2, is 1. For example, at cell $E_1^{BS} = \{b,\ e^B\}$,

$$E^E[p_2|E_1^{BS}] = \frac{(0)(A) + (2)(A)}{A + A} = 1 = p_1(\omega) \text{ for } \omega \in E_1^{BS}. \tag{22}$$

Ellen's expected period 2 price is therefore equal to the period 1 price, given $\omega \in E_1^{BS}$. Thus, since Ellen is risk neutral, she is willing to hold the asset in period 1, given E_1^{BS}. Similar results hold for E_1^{BUY} and E_1^{GS}.

Turn, then, to period 2. At Ellen's information sets $E_2^{BUY} = \{f^B,\ f_2^G,\ f_3^G\}$ and $E_2^{GS} = \{e_2^G,\ e_3^G\}$, her expected value for p_3 is 2, by calculations like that in (22). This, in turn, equals p_2 at these information sets. Thus, the period 2 price again equals her expected period 3 price at these information sets. Therefore, since she is risk neutral, she is indifferent between buying, selling, and doing nothing at these information sets. Thus, she is willing to buy at E_2^{BUY} and sell at E_2^{GS}, as called for by the equilibrium.

At information set $E_2^{BS} = \{e^B\}$, the second period price $p_2 = 2$ as well, but Ellen knows that the state is e^B, so p_3 will be zero for sure. Ellen therefore strictly prefers selling her unit of the asset at this cell. However, she is short-sale constrained, and so, can only sell the one unit she owns. Finally, at Ellen's other singleton information sets, E_2^1, E_2^2 and E_2^3, the period 2 price is zero, as is the expected period 3 price. Thus, Ellen is willing to *not* trade, as called for by the equilibrium.

In period 3, price equals dividend ($p_3(\omega) = d(\omega)$), and all information is revealed. Therefore, Ellen is willing to not trade in period 3, as called for by the equilibrium. Similarly, Frank is willing to do everything that the equilibrium calls on him to do. Thus, the above prices and net trades actually do constitute an equilibrium.

Also, $p_1(b) = 1$, so p_1 is positive in state b, even though both Ellen and Frank know that the asset is actually worthless. There is therefore a strong bubble in state b. Intuitively, in state b, Ellen believes that the state might be e^B. But in state e^B, Ellen will be able to sell the asset to Frank. This is because, in state e^B, Frank believes that the state might be e_3^G, where the asset pays a dividend of 4. Similarly, Frank, in state b, thinks he might be able to sell the asset to Ellen. Of course, in state b, neither of them wants to *buy* the asset, so the bubble will burst by the second period.

A useful way to understand bubbles of this sort is that, since some agent types know the asset is worthless, this creates a two-sided *lemons problem* in this market (Akerlof, 1970). That is, agents are more willing to sell if they have negative information about the asset, so buyers tend to distrust sellers. In the bubble equilibrium, some gains from trade are realized in spite of this lemons problem. This suggests that *asset price bubbles* *tend to occur in markets where there is a lemons problem of this sort, but agents trade in spite of the lemons problem. A bubble is then a situation in which all agents expect to be on the selling side of a lemons market.*

5. TRADE DUE TO OVERCONFIDENCE–THE SCHEINKMAN-XIONG MODEL

The model of Scheinkman and Xiong (2003) is economically very similar to the Harrison-Kreps (1978) model. However, in their model time is continuous, so tools from stochastic calculus must be used. In addition, the differences of opinion that lead to trade are motivated by having different traders placing confidence in different signals of asset value.

We will consider an extremely simple two-period discrete-time version of their model. Assume an asset has a random fundamental value X, which can ultimately take on two values, zero or one. All investors place prior probability 1/2 on each of these two values, so it is reasonable to consider the fundamental value of the asset to be one half. In addition, assume that there is *one* signal S, which can take on two values, S_H and S_L (Scheinkman and Xiong consider a model with *two* signals). Finally, there are two kinds of traders, C-Traders (i.e., confident traders), who believe that the signal is informative, specifically that $\text{Prob}_{\text{C}}(X_H|S_H) = \text{Prob}_{\text{C}}(X_L|S_L) = q$, with $q > 1/2$, and N-Traders (nonconfident, or neutral traders) who believe that S is completely uninformative.

The sequence of play is that, at the beginning of period 0 agents trade. Then they observe the signal S. Then at the beginning of period 1 they trade again, then they observe the true value of X, which is paid out to whoever owns the asset.

This model, of course, is very easy to solve by backwards induction. If the C-Traders observe S_H, then their posterior probability is

$$\text{Prob}_{\text{C}}(X = 1|S_H) = 0.5q/[0.5q + 0.5(1-q)] = q,$$

by Bayes' rule. Thus, the C-Traders buy all of the asset from the N-Traders at price $p_1^H = q$. On the other hand, if the C-Traders observe S_L, then they sell all of their asset to the N-Traders, who are willing to pay $p_1^L = 1/2$, since they think the signal is uninformative. This gives the period 1 prices.

As of Period 0, all traders think that S_H and S_L are equally likely, so all traders are willing to hold the asset at price

$$p_0 = \frac{1}{2}q + \frac{1}{2}\frac{1}{2} = \frac{1}{4} + \frac{1}{2}q > \frac{1}{2},$$

since $q > 1/2$. Thus, there is a bubble in the sense that the market price exceeds the "fundamental" value at which any agent would have been willing to buy and hold the asset.

Of course, this simple example does not do justice to Scheinkman and Xiong's rich model. In addition, one of the values of their model is that it has provided a very convenient work-horse with which Scheinkman, Xiong, and their coauthors have been able to analyze a number of interesting financial phenomena. For example, Hong, Scheinkman and Xiong (2006) have used the model to explain the dramatic drop in price, documented by Ofek and Richardson (2003), which followed the expiration of lock ups of dot com stocks. Essentially, they argue that, if agents are over confident, then they assume that well-informed insiders share their opinion. Thus, since the optimistic traders will be holding the asset on the eve of lock up expiration, their overpricing will be aggravated because they believe that insiders share their optimism.

6. RIDING BUBBLES

Intuitively, it makes sense that, as a bubble develops, more and more people begin to suspect that the asset is overpriced, and that this puts pressure on the bubble to eventually burst. However, the above models do not conform to this intuition, since, in those models, all agents believe that prices are above fundamentals from the beginning. In addition, in the Harrison and Kreps and Scheinkman and Xiong models, the bubble never really bursts. The model which best captures the picture of bubbles eventually bursting under the pressure of increasing skepticism is Abreu and Brunnermeier (2003). This section will present a simple special case of the Abreu-Brunnermeier model.

Consider an asset market with some rational traders and some irrational traders. Assume that the normal rate of return on assets is $r = 0$. Let the market have an asset with a fundamental value which grows at a constant rate $g > 0$ until some random time $\tilde{t}_0$, after which the fundamental value ceases to grow. Thus, normalizing the initial value to one, the value at time t is $\min(e^{gt},\ e^{g\tilde{t}_0})$. Abreu and Brunnermeier assume that the time $\tilde{t}_0$ is exponentially distributed with PDF $\phi(t_0) = \lambda e^{-\lambda t_0}$. However, it simplifies the algebra to take the limit as $\lambda \to 0$, so $\tilde{t}_0$ has an improper prior distribution which is uniform on $[0, \infty)$.

Though the fundamental value stops growing at time $\tilde{t}_0$, the market price continues to grow for a while at the old rate g. This is partly because the rational traders do not immediately learn that the fundamental value has ceased growing, partly because rational traders do not initially sell out when they learn the asset has ceased growing, and partly because there are some "behavioral" traders, who do not sell out until after the price crashes.

Thus, assume that there is a mass of rational investors, indexed by i. Initially, at time $\tilde{t}_0$, the rational traders are not informed that the fundamental value has stopped growing, though they do know the distribution of $\tilde{t}_0$. However, once the fundamental value has ceased growing, the rational investors begin to learn this. Specifically, at time t_i, investor i learns that $\tilde{t}_0 \leq t_i$. Assume that the t_i are uniformly distributed between $\tilde{t}_0$ and $\tilde{t}_0 + \eta$.

Assume also that, if a fraction γ of rational investors sell their holdings of the asset, then the price will crash. Now, by time $\tilde{t}_0 + \eta$, all rational traders will know that the asset is overpriced. One can ask, then, whether this will cause the asset price to collapse by that date, or indeed, whether *any* rational trader will sell their holdings by that date.

To investigate this question, assume that investor i follows the strategy "sell the asset at date $t_i + s$." Assume also that investor i expects *other* rational investors, e.g., investor j to sell out at time $t_j + s^e$, where s^e is common to the other rational investors. One can ask whether there is a symmetric Nash equilibrium where all rational investors choose the same holding period s^*, and how big this s^* can be.

It turns out to be fairly easy to find such a Nash equilibrium. To see this, look at the problem from the perspective of investor i. She knows that $\tilde{t}_0$ is somewhere between $t_i - \eta$ and t_i. Also, since i's prior was that $\tilde{t}_0$ was uniform on $[0,\ \infty)$, her posterior is uniform on $[t_i - \eta,\ t_i]$.

Now, investor i believes that the market will crash at time $\tilde{t}_0 + \gamma\eta + s^e$. Thus, if she plans to sell out at time $t_i + s$, her expected payoff is

$$\begin{aligned}
&\text{Prob}(\tilde{t}_0 + \gamma\eta + s^e \geq t_i + s)e^{g(t_i+s)} \\
&\quad +\text{Prob}(\tilde{t}_0 + \gamma\eta + s^e < t_i + s)E[e^{g\tilde{t}_0}|t_i - \eta < \tilde{t}_0 < t_i + s - s^e - \gamma\eta] \\
&\quad = \tfrac{1}{\eta}(\gamma\eta + s^e - s)e^{g(t_i+s)} + \int_{t_i-\eta}^{t_i+s-s^e-\gamma\eta} \tfrac{1}{\eta}e^{gt_0}dt_0 \\
&\quad = \tfrac{1}{\eta}(\gamma\eta + s^e - s)e^{g(t_i+s)} + \tfrac{1}{\eta g}\left[e^{g(t_i+s-s^e-\gamma\eta)} - e^{g(t_i-\eta)}\right]
\end{aligned}$$

The first order conditions for Investor i's choice of s then give

$$\frac{g}{\eta}(\gamma\eta + s^e - s)e^{g(t_i+s)} + \frac{1}{\eta}\left[e^{g(t_i+s-s^e-\gamma\eta)} - e^{g(t_i+s)}\right] = 0.$$

Setting $s^e = s$ for a symmetric equilibrium gives

$$(g\gamma - \frac{1}{\eta})e^{g(t_i+s)} + \frac{1}{\eta}e^{g(t_i-\gamma\eta)} = 0,$$

or

$$e^{gs} = \frac{1}{1-g\gamma\eta}e^{-g\gamma\eta},$$

or

$$s^* = \frac{1}{g}ln\left(\frac{1}{1-g\gamma\eta}e^{-g\gamma\eta}\right).$$

Since $e^{-g\gamma\eta} > 1 - g\gamma\eta$, it follows that $s^* > 0$, so agents do ride the bubble in this model. Also, if η gets bigger, s^* gets bigger, so if there is more uncertainty about when rational traders receive their information, then they ride the bubble longer. Similarly, if the fraction, γ, of rational traders needed to burst a bubble increases, then rational traders ride the bubble longer, as expected. Finally, rewriting s^* as

$$s^* = ln\left(\frac{1}{(1 - g\gamma\eta)^{1/g}} e^{-\gamma\eta}\right)$$

shows that, if the growth rate, g rises, then rational traders ride the bubble longer.

7. COMPARING THE MODELS

This survey has presented a number of recent greater fool bubble models. There is, of course, insufficient grounds, at this point, to choose between them on empirical grounds. Nevertheless, it is possible to list some of the strengths and weaknesses of each model, with a view towards deciding how they can best be combined and applied to understand various asset-market phenomena.

The most obvious advantage of the pure asymmetric-information model of Section 4 is that it is methodologically very conservative. Agents are perfectly rational, they can be given a common prior, and they are in equilibrium at all times. This also means that welfare analysis can be performed in a methodologically very conservative way – the policy maker's standard of Pareto optimality is given by the utility functions which agents themselves maximize.

That said, to me, one of the major benefits of this methodological conservatism is that, once we realize that one can capture bubbles in a very traditional model, it becomes much easier to set philosophical issues aside, and look at less conservative models. For example, if one wants to model the volume of trade on the typical asset market, then models with perfectly rational investors seem much less realistic than models in which investors are overconfident. This is important since, in bubble models, the volume of trade is necessary to answer the follow-up question with which we began this survey – "why would a rational person buy an overpriced asset?"

In the end, however, I believe that it is hybrid models, which combine overconfidence with nontrivial information structures, which hold the most promise for capturing the dynamics of greater-fool bubbles. In simple overconfidence models, people learn nothing about fundamentals from the behavior of other people. On the other hand, in actual market crashes, for example, an important factor in the crash must be that investors see falling prices as reflecting negative information that other investors have learned earlier.

Thus, asymmetric information should be important in capturing the *fragility* in the prices of bubble assets. If investors are not confident that prices equal fundamentals, then their expectations about asset prices presumably become very sensitive to what they think others believe about asset prices, and this sensitivity may be important in explaining the volatility of bubble assets.

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