Extracting Angular Momentum From Electromagnetic Fields

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Abstract

It is not widely recognized that a circularly polarized electromagnetic wave impinging upon a sail from below can spin as well as propel. Our experiments show the effect is efficient and occurs at practical microwave powers. The wave angular momentum acts to produce a torque through an effective moment arm of a wavelength, so longer wavelengths are more efficient in producing spin, which rules out lasers.

A variety of conducting sail shapes can be spun if they are not figures of revolution. Spin can stabilize the sail against the drift and yaw, which can cause loss of beam-riding. So, if the sail gets off-center of the beam, it can be stabilized against lateral movement by a concave shape on the beam side. This effect can be used to stabilize sails in flight and to unfurl such sails in space.
Introduction

- Circularly polarized finite electromagnetic fields carry angular momentum
- Transfer of angular momentum to macroscopic objects via absorption or reflection

- What are the conditions for the transfer of angular momentum?
- Physical differences between absorption and reflection?
- Role of symmetries?
- Radiation of angular momentum by reflection?
- Consequences and applications?
Experiments

Performed at JPL, Pasadena, CA in spring 2000

- Carbon Cone
- Aluminum Roof
Diffraction of Plane Waves

obstacle of size comparable to the wavelength

deformation of plane waves by diffraction

parallel component $E_{\parallel} \sim \frac{\lambda}{d} E$

perpendicular component of the energy flow

angular momentum conveyed to the obstacle
Symmetries

**Theorem:**
A perfectly conducting body of revolution extracts no angular momentum from an axisymmetric wave field.

No angular momentum transfer from an axisymmetric wave field to a body of revolution via diffraction or reflection.
Proof of the Theorem

skin depth \( \delta = \sqrt{c/2\pi \sigma \omega} \)

conductivity \( \sigma \)

electromagnetic force density \( F_t \)

normal on smooth surfaces

surfaces without tangential planes

are one-dimensional circles

\( \Rightarrow \) \( \vec{F} \) does not have an azimuthal component

torque \( \vec{T} = \vec{r} \times \vec{F} \)
How to calculate $\alpha^*$?

Solve the scattering or diffraction problem for the given object:

given incident fields $\vec{E}^i, \vec{H}^i$
determine scattered fields $\vec{E}^s, \vec{H}^s$
such that the complete fields $\vec{E} = \vec{E}^i + \vec{E}^s$, $\vec{H} = \vec{H}^i + \vec{H}^s$

fulfill the boundary conditions for a perfect conductor:

1.) tangential component of electric field vanishes at object’s surface $E_t = 0$
2.) normal component of magnetic field is zero at object’s surface $H_n = 0$
Approximate Methods

*Kirchhoff, Fresnel, Fraunhofer:* discard the angular momentum by assuming a perfectly ‘black’ obstacle (setting wave field to zero on the obstacle’s surface, equating incident and complete wave field on transparent parts of the screen containing the obstacle)

=> no k-parallel components of the fields

**neglect of edge effects**

*Born:* amplitudes of scattered fields comparable to those of incident fields

=> no perturbative methods
Exact Solutions

Solve the three-dimensional wave equation

\[ \Delta V - \frac{1}{c^2} \frac{\partial^2 V}{\partial t^2} = 0 \]

rigorously for the boundary conditions for the perfect conductor

few exact solutions exist:

half plane (Sommerfeld), wedge (Macdonald), cylinders, discs, and spheres
Field or geometric asymmetries destroy the theorem!

object displaced from the wave symmetry axis generally spins around a transverse axis

perfectly conducting objects that spin:
The Wedge: An Exact Solution

- infinitely extended, perfectly conducting wedge
- separation of the time-independent Helmholtz equation
  \[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \kappa^2 u = 0 \]
- into E-polarization and H-polarization (two independent solutions)
- wave vector \( \kappa = \frac{2\pi}{\lambda} \)
Exact Solutions:

for E- and H-polarization: series of Bessel functions with argument $\chi = kr$
and trigonometric functions depending on $\theta$

set of two independent solutions each of which corresponding to a *linearly polarized* plane wave (phase shifted) incident along the x-axis

consider a *circularly polarized* incoming wave

=> superposition of E- and H-polarization

+ = right-handed

- = left-handed

inverting the polarization changes the sign of the angular momentum density

$$\vec{L}(\chi, \theta) = \frac{1}{c^2} \vec{r} \times \vec{S}$$
Angular Momentum Density

Current Density

wedge with \( \beta = \frac{\pi}{2} \), \( \lambda = \pi \text{ cm} \)
Angular Momentum Density 2

wedge with $\beta = \frac{\pi}{4}$  $\lambda = \pi \text{ cm}$
Properties of the angular momentum density:

- structure by **wave interference**
- maxima and minima \( \approx \frac{\lambda}{2} \) apart from each other
- symmetric around \( \theta = 0 \)
- regions with positive and negative angular momentum density, but don’t cancel out
- \( \chi \) asymptotically goes to zero for \( r \to \infty \)
- angular momentum mostly concentrated at the faces of the wedge!

Transfer of angular momentum to the wedge clearly is a *boundary effect*!
Angular momentum transfer from a wave field to a wedge-like object is maximized for opening angles around 90 degrees.
normalized total angular momentum inside a wedge of opening angle $\beta$ and ‘quasi-radius’

$$\chi_0 = \kappa R$$

positive and negative values (positive values expected for right-handed circularly polarized waves)
Energy Density

\[ W(r, \theta) = \frac{1}{16\pi} \left( \overrightarrow{E}(r, \theta) \cdot \overrightarrow{E}(r, \theta)^* + \overrightarrow{H}(r, \theta) \cdot \overrightarrow{H}(r, \theta)^* \right) \]

Angular momentum density and energy density are not correlated.

all three components of the Poynting flux \( \overrightarrow{S} \) contribute to the energy flow, but only the component \( S_z \) perpendicular to the wave vector accounts for the angular momentum density.
Two qualitatively different regions:

\[ \beta < \frac{\pi}{3} \]
saddles, shoulders and parallel valleys

\[ \beta > \frac{\pi}{3} \]
simple wave-like interference structure with maxima separated by \( \sim \lambda/2 \)
\[ \beta = \pi/3 \]

- oscillatory energy density
- no angular momentum

Wave field transports energy without transporting angular momentum

\[ \beta = \pi/2 \]

- homogeneous energy density
- oscillatory angular momentum density

Energy and angular momentum density not coupled
Theory and experiment demonstrate clearly the possibility of extracting angular momentum from electromagnetic fields purely by reflection in specific geometries.
Propeller Assembly

- Magnet (5cm x 2cm x 1 cm)
- Steel Pin
- Thin Wire or Resi Ring
- Sail
- Aluminum Rod (d = 1.6 cm)
- Propeller-Beamrider (R = 3.8 cm, α = 35 degree)
- Titanium Rest Disk (d = 1 cm)
- Ball Bearing (d = 0.043")
- Concave Base (d = 1 cm, h = 0.7 cm)
- Microwave Beam
- Waveguide (WR90)

All dimensions in cm
Coupling Coefficient $\alpha$ vs Width for Aluminum Strip

- kevlar fiber thread, $d = 5\text{cm}$
- carbon fiber thread, $d = 5\text{cm}$
- kevlar fiber thread, $d = 6\text{cm}$
Efficiency Coefficient $\varepsilon$ vs Width for Aluminum Strip

- kevlar fiber thread, $d = 5\text{cm}$
- carbon fiber thread, $d = 5\text{cm}$
- kevlar fiber thread, $d = 6\text{cm}$
Coupling Coefficient $\alpha$ vs Opening Angle $\beta$ of the Roof-Sail

Roof Angle $\beta$, degree
Types of cuts: Conical sails with an areal cut (left) and thin cuts (right). We find that the two cut types give very different couplings to circularized microwaves.
Comparison with Experiment

Thin aluminum roofs suspended by 6-micron wide carbon filaments
Circularly polarized microwaves beamed from below at \( \approx 100 \, W \) and \( 7.17 \, GHz \) spin up the roof

System can be described as a linear torsional spring with an applied, constant electrodynamic torque and a damping torque.

Results:
- intact aluminum disks and cones did not rotate
- carbon disks and cones did spin with absorption coefficient \( \alpha \approx 0.1 \)
- aluminum roofs span with coupling coefficient depending on opening angle
maximum value of $\approx 0.13$ at $R \approx \lambda / 2$ and $\beta \approx \pi / 2$

coupling falls fast toward larger $R$ or smaller $\lambda$

negative values down to $\approx -0.05$

$\Rightarrow$ **Geometric absorption coefficient** differs physically from material absorption coefficient which is restricted to values between 0 and 1
A perfectly conducting roof can act as a radiator for angular momentum at small opening angles.
Theory and experiment demonstrate clearly the possibility of extracting angular momentum from electromagnetic fields purely by reflection in specific geometries.
Conclusion

- Electromagnetic waves not only carry energy but transport angular momentum as well if they are circularly polarized and finite.

- While energy can only be absorbed or reflected, angular momentum can also be radiated.

- Objects with specific geometries can act as polarizers.

- Distinction between inherent material absorption coefficient and a geometric absorption coefficient, determined by the shape of the object (may also assume negative values).

- Macroscopic objects can be spun by electromagnetic waves.

- Use angular momentum of a microwave beam to unfold a light sail by spinning it up.

- Axisymmetric, perfectly conducting objects do not electrodynamically spin.

=> break the symmetry by cutting the sail or using roof-like sails.
Carbon Sail Experiments

- Sail masses are 3 cm diameter, ~6 mg,
  - microwave power 2-10 kW
  - pulse duration 0.2 sec
  - Sail flies to altitudes 1 cm to 60 cm

- Flight velocities are 0.3-3 m/s, accelerations are 10-100 m/s² (1-10 gees).

- Sail temperatures are 2000-2500°K. Sails show no erosion or melting.

- Accelerations are several times that predicted for photon pressure.