Extracting Angular Momentum From Electromagnetic Fields

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Abstract

It is not widely recognized that a circularly polarized electromagnetic wave impinging upon a sail from below can *spin* as well as propel. Our experiments show the effect is efficient and occurs at practical microwave powers. The wave angular momentum acts to produce a torque through an effective moment arm of a wavelength, so longer wavelengths are more efficient in producing spin, which *rules out lasers*.

A variety of conducting sail shapes can be spun if they are *not* figures of revolution. Spin can stabilize the sail against the drift and yaw, which can cause loss of *beam-riding*. So, if the sail gets off-center of the beam, it can be stabilized against lateral movement by a concave shape on the beam side. This effect can be used to stabilize sails in flight and to unfurl such sails in space.

Introduction

- Circularly polarized finite electromagnetic fields carry angular momentum
- Transfer of angular momentum to macroscopic objects via ***absorption*** or ***reflection***

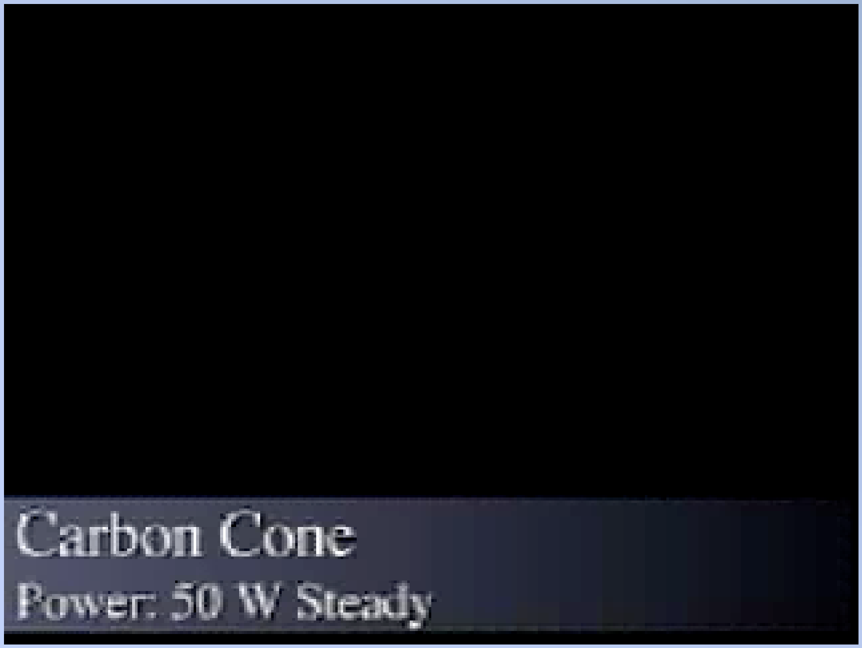
- What are the conditions for the transfer of angular momentum ?
- Physical differences between absorption and reflection ?
- Role of symmetries ?
- Radiation of angular momentum by reflection ?
- Consequences and applications ?

Experiments

Performed at JPL, Pasadena, CA in spring 2000

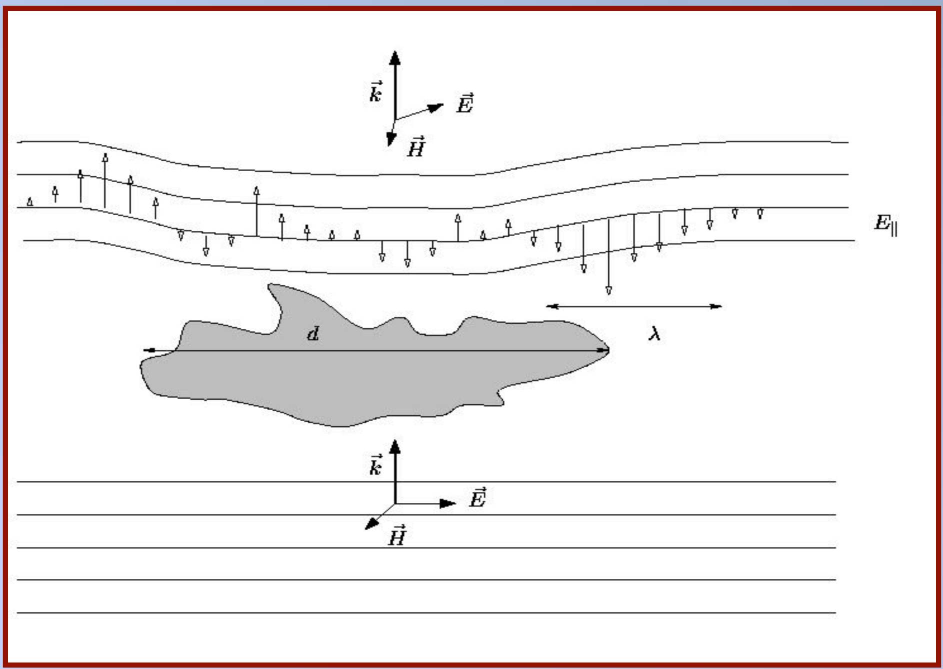
Carbon Cone

Aluminum Roof



Diffraction of Plane Waves

obstacle of size comparable to the wavelength λ



deformation of plane waves by diffraction

parallel component $E_{\parallel} \sim \dfrac{\lambda}{d} E$

perpendicular component of the energy flow $S_{\perp}$

angular momentum conveyed to the obstacle

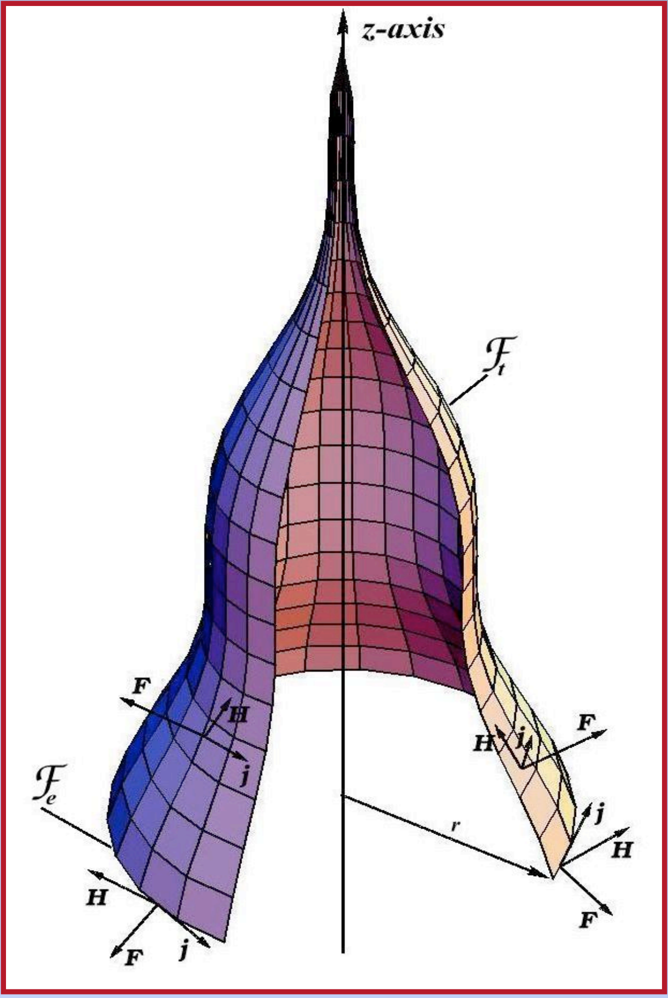
Symmetries

Theorem:

A perfectly conducting body of revolution extracts no angular momentum from an axisymmetric wave field.

No angular momentum transfer from an axisymmetric wave field to a body of revolution via diffraction or reflection

Proof of the Theorem



skin depth $\delta = \sqrt{c/2\pi\sigma\omega}$

conductivity σ

electromagnetic force density F_t

normal on smooth surfaces

surfaces without tangential planes
are one-dimensional circles

=> $\vec{F}$ does not have an azimuthal component

torque $\vec{T} = \vec{r} \times \vec{F}$

How to calculate α^* ?

<u>Solve the scattering or diffraction problem for the given object:</u>

given incident fields $\overrightarrow{E^i}, \overrightarrow{H^i}$

determine scattered fields $\overrightarrow{E^s}, \overrightarrow{H^s}$

such that the complete fields $\vec{E} = \overrightarrow{E^i} + \overrightarrow{E^s}, \quad \vec{H} = \overrightarrow{H^i} + \overrightarrow{H^s}$

fulfill the *boundary conditions* for a perfect conductor:

1.) tangential component of electric field vanishes at object's surface $E_t = 0$

2.) normal component of magnetic field is zero at object's surface $H_n = 0$

Approximate Methods

Kirchhoff, Fresnel, Fraunhofer:

discard the angular momentum by assuming a perfectly 'black' obstacle (setting wave field to zero on the obstacle's surface, equating incident and complete wave field on transparent parts of the screen containing the obstacle)

=> no k-parallel components of the fields

neglect of edge effects

Born:

amplitudes of scattered fields comparable to those of incident fields

=> no perturbative methods

Exact Solutions

Solve the three-dimensional *wave equation*

$$\Delta V - \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} = 0$$

rigorously for the boundary conditions for the perfect conductor

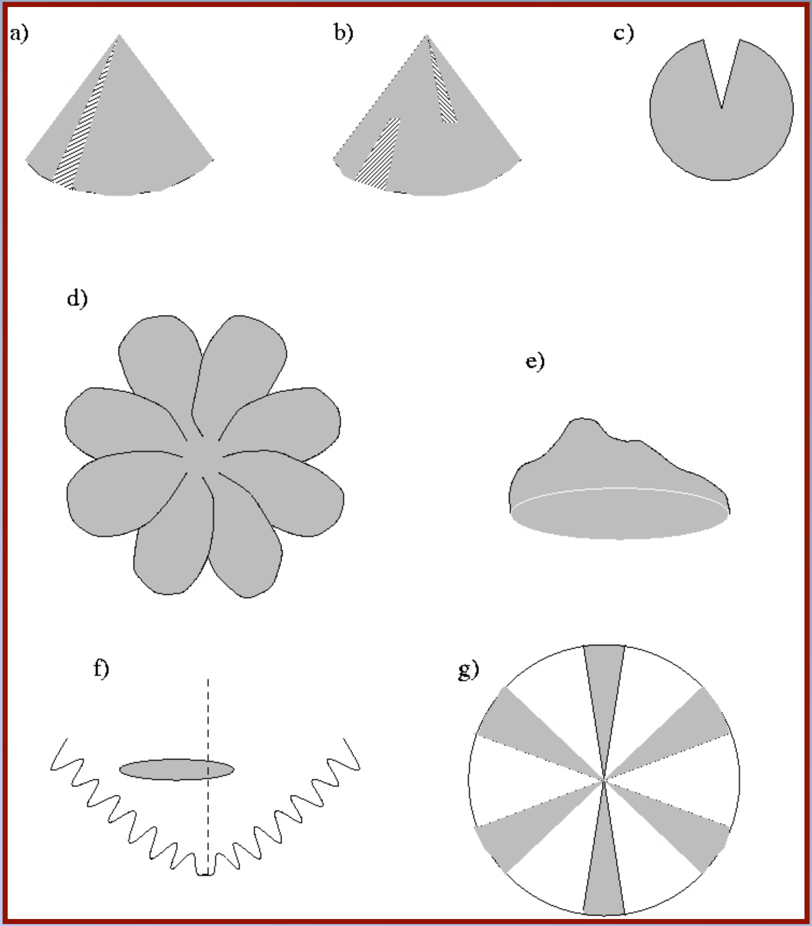
few exact solutions exist:

half plane (Sommerfeld), wedge (Macdonald),
cylinders, discs, and spheres

Shapes

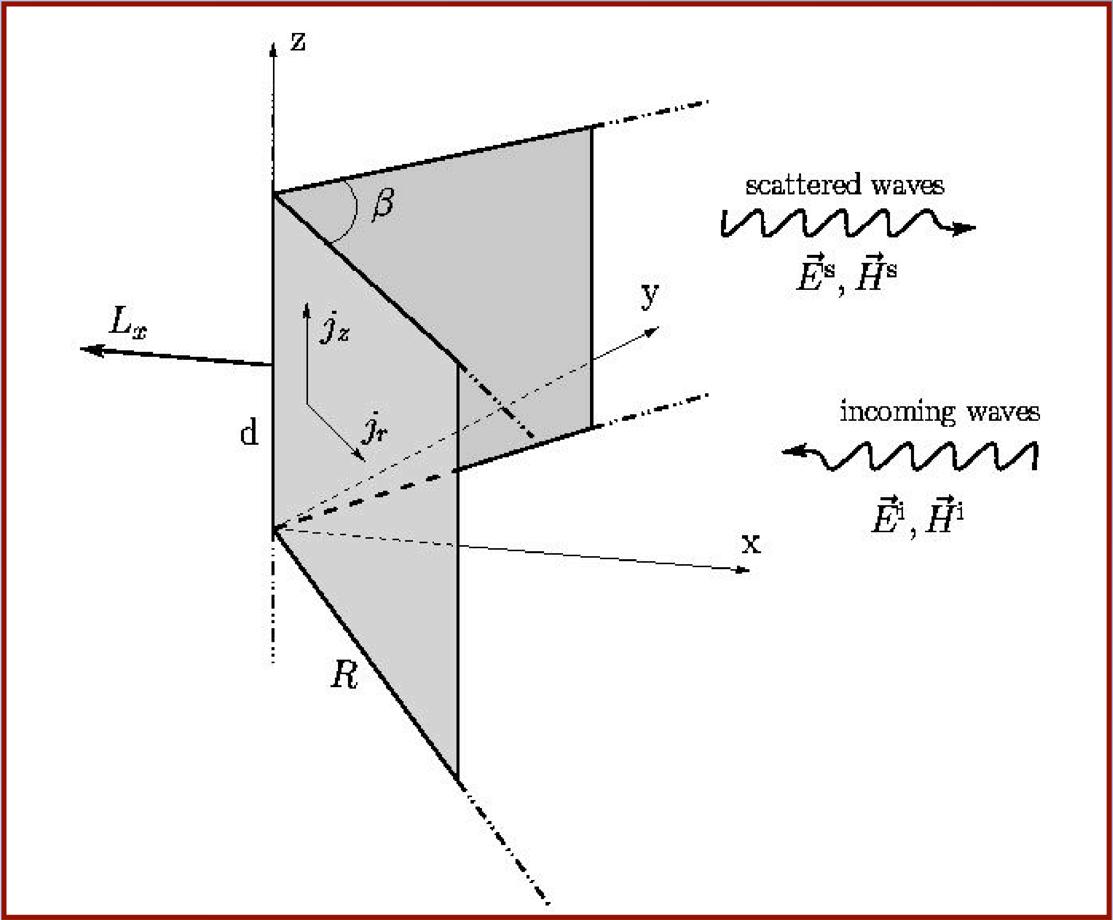
perfectly conducting objects that spin:

Field or geometric asymmetries destroy the theorem !



object displaced from the wave symmetry axis generally spins around a transverse axis

The Wedge: An Exact Solution



<u>infinitely extended, perfectly conducting wedge</u>

separation of the time-independent *Helmholtz* equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \kappa^2 u = 0$$

into E-polarization and H-polarization (two independent solutions)

wave vector $\kappa = \frac{2\pi}{\lambda}$

Exact Solutions:

for E- and H-polarization: series of Bessel functions with argument $\chi = \kappa r$
and trigonometric functions depending on θ

set of two independent solutions each of which corresponding to a *linearly polarized* plane wave (phase shifted) incident along the x-axis

consider a ***circularly polarized*** incoming wave

=> superposition of E- and H-polarization

+ = right-handed

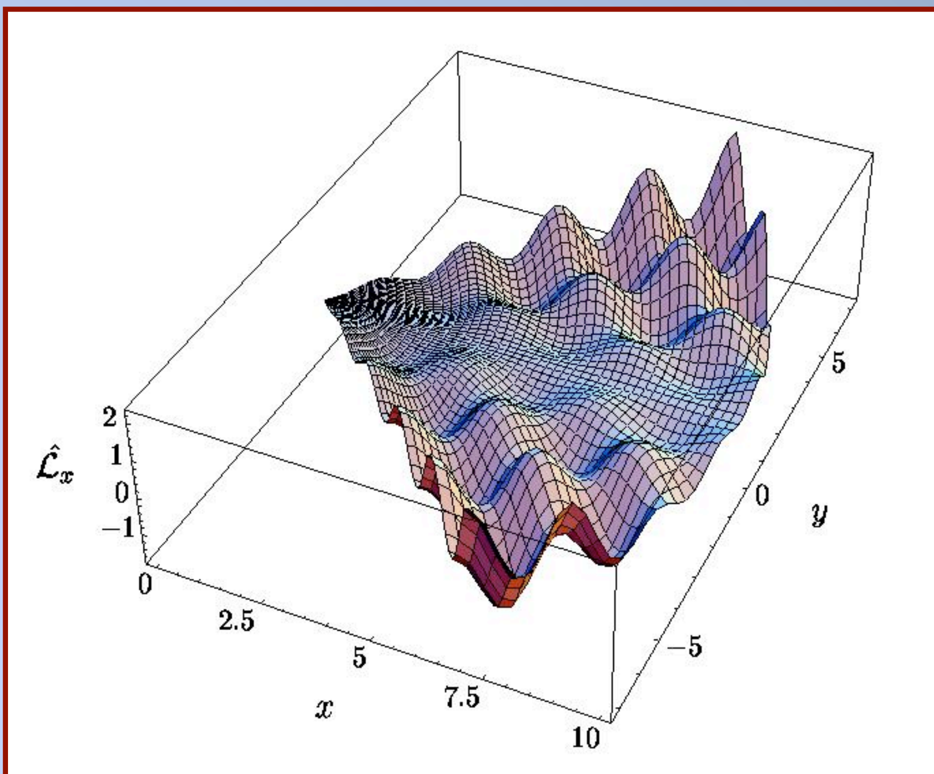
- = left-handed

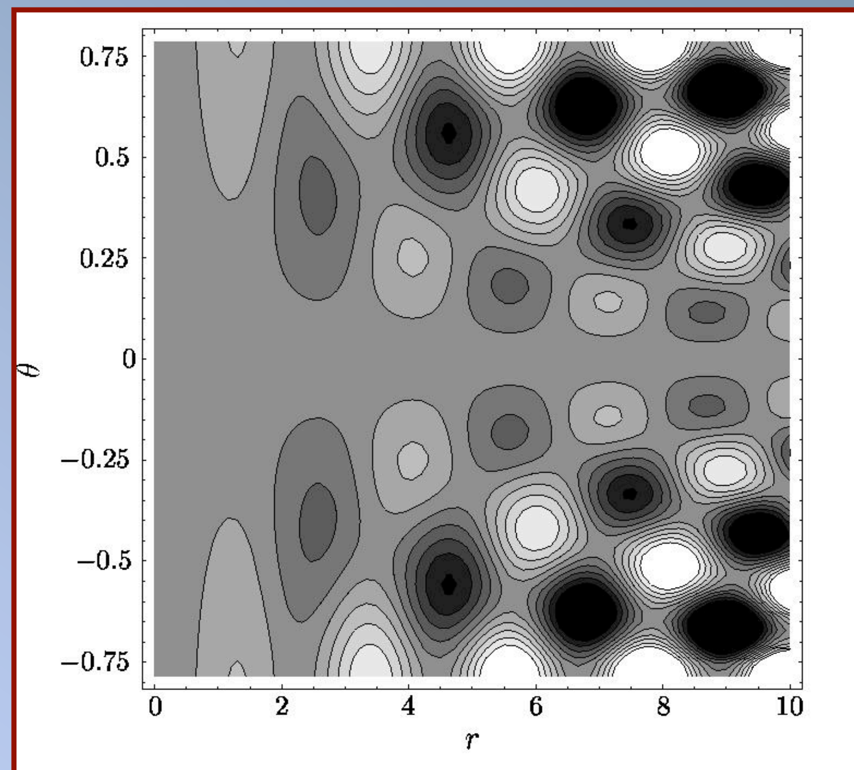
inverting the polarization changes the sign of the

angular momentum density $\vec{\mathcal{L}}(\chi,\theta) = \frac{1}{c^2}\vec{r} \times \vec{S}$

Angular Momentum Density Current Density

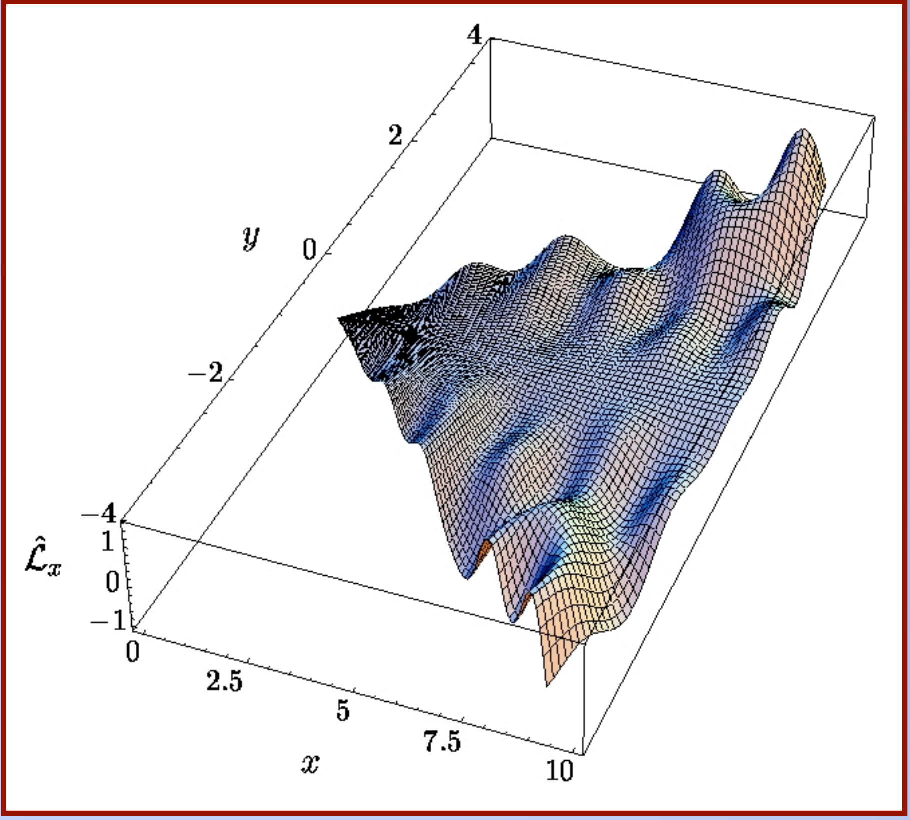
wedge with $\beta = \frac{\pi}{2}$ $\lambda = \pi\,\mathrm{cm}$

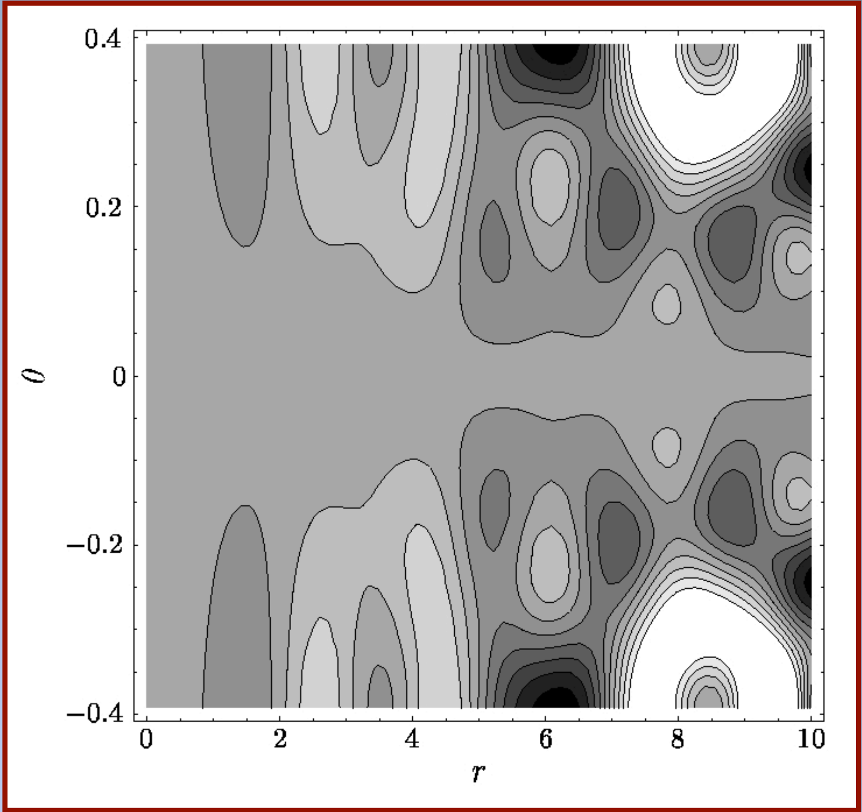




Angular Momentum Density 2

wedge with $\beta = \frac{\pi}{4}$ $\lambda = \pi\,\text{cm}$





Properties of the angular momentum density:

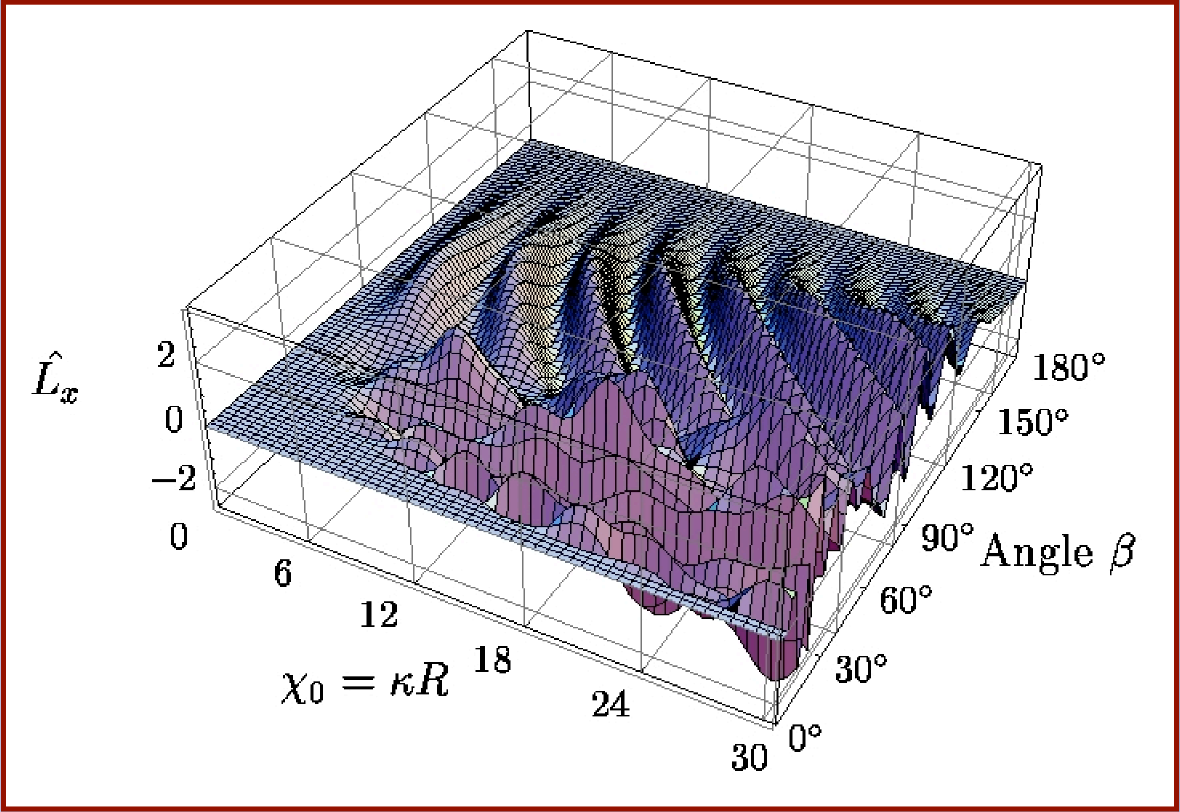
- structure by **wave interference**
- maxima and minima $\approx \lambda/2$ apart from each other
- symmetric around $\theta = 0$
- regions with positive and negative angular momentum density, but don't cancel out
- $\hat{\mathcal{L}}_x$ asymptotically goes to zero for $r \longrightarrow \infty$
- angular momentum mostly concentrated at the faces of the wedge !

Transfer of angular momentum to the wedge clearly is a *boundary effect* !

Energy Density Polarization

Angular momentum transfer from a wave field to a wedge-like object is *maximized* for opening angles around 90 degrees.

Total Angular Momentum



normalized total angular momentum inside a wedge of opening angle β and 'quasi-radius'

$$\chi_o = \kappa R$$

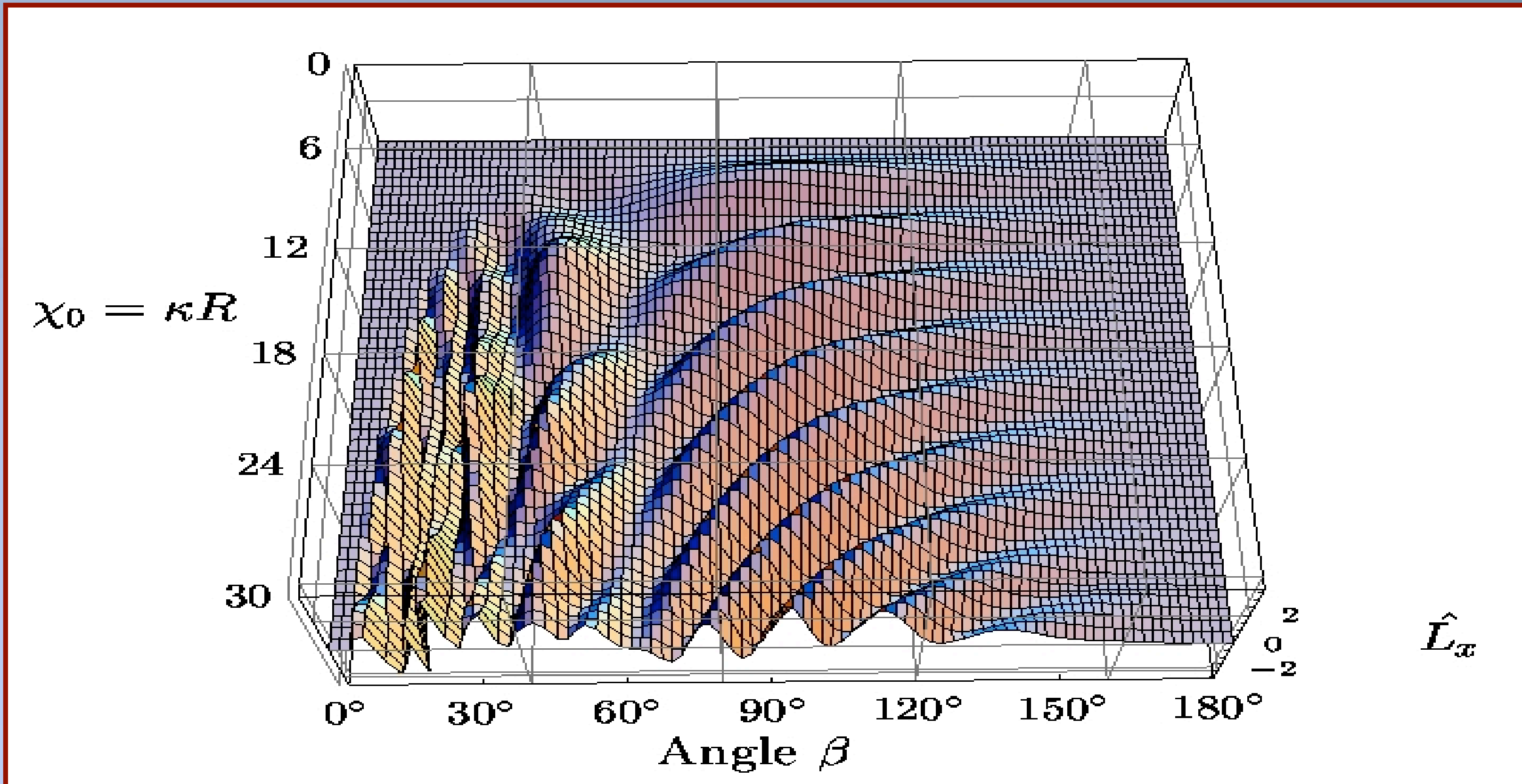
positive and negative values (positive values expected for right-handed circularly polarized waves)

Energy Density

$$W(r,\theta)=\frac{1}{16\pi}\left(\vec{E}(r,\theta)\cdot\vec{E}(r,\theta)^{*}+\vec{H}(r,\theta)\cdot\vec{H}(r,\theta)^{*}\right)$$

Angular momentum density and energy density are not correlated.

all three components of the Poynting flux $\vec{S}$ contribute to the energy flow, but only the component S_z perpendicular to the wave vector accounts for the angular momentum density

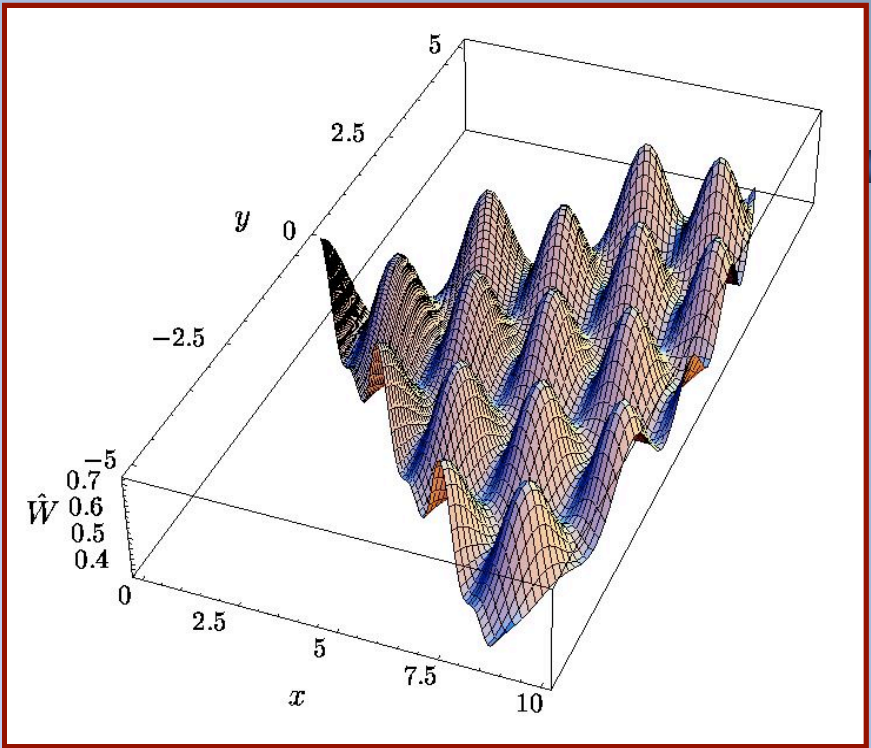


Two qualitatively different regions:

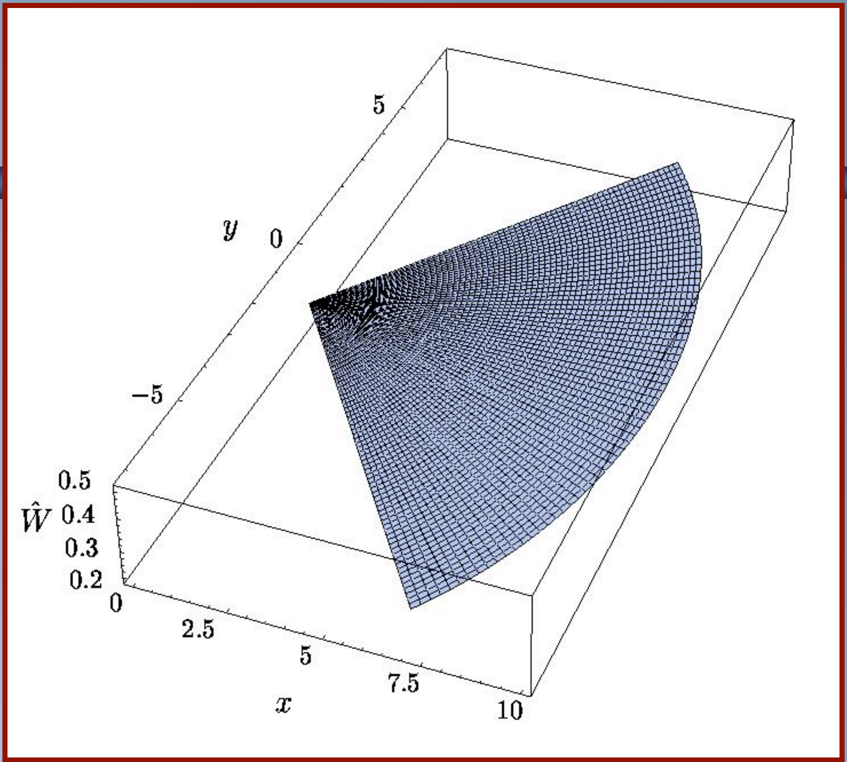
$\beta < \frac{\pi}{3}$ saddles, shoulders and parallel valleys

$\beta > \frac{\pi}{3}$ simple wave-like interference structure with maxima separated by $\sim \lambda/2$

$$\beta = \pi/3$$



$$\beta = \pi/2$$



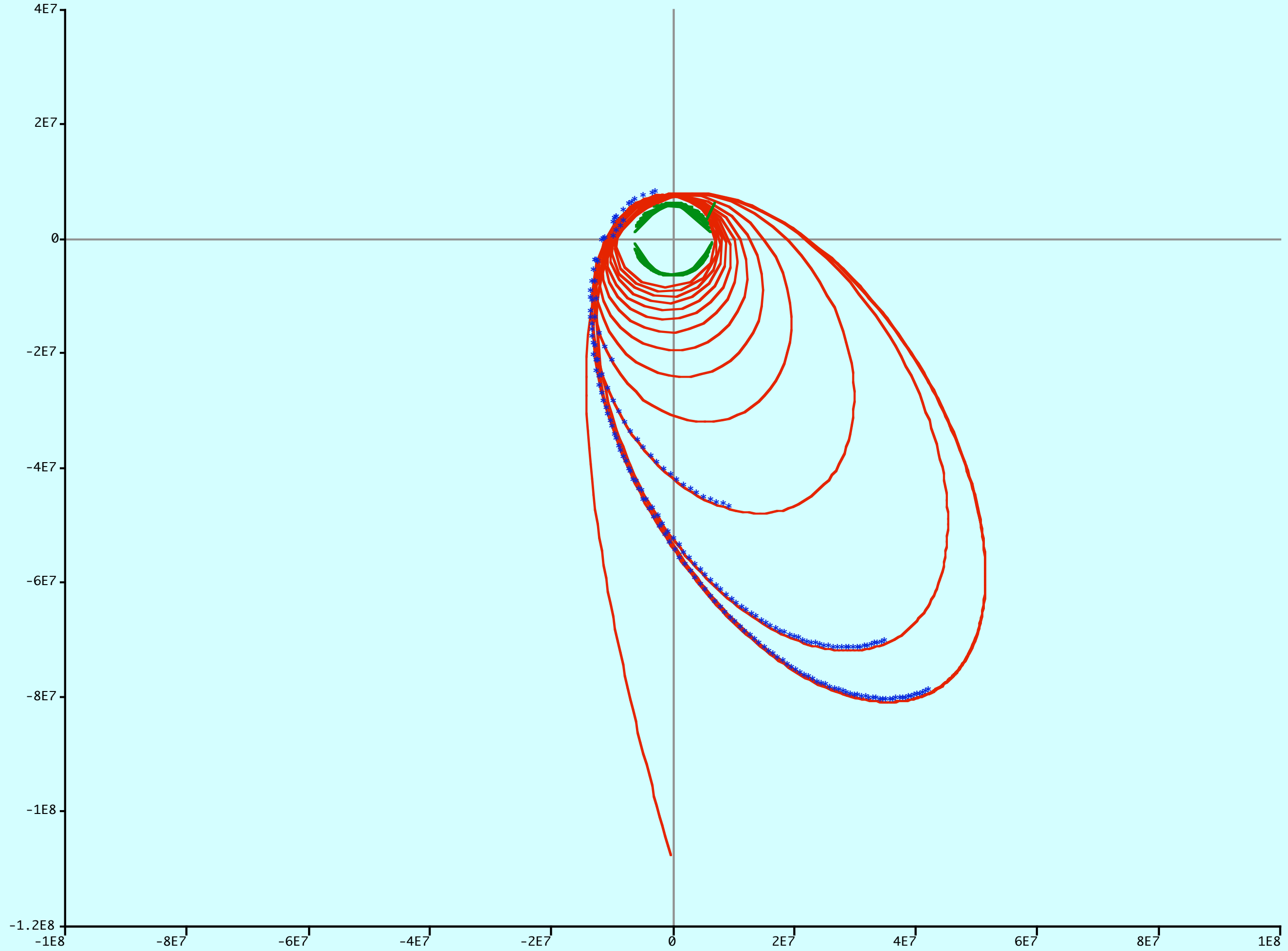
- oscillatory energy density
- no angular momentum

Wave field transports energy without transporting angular momentum

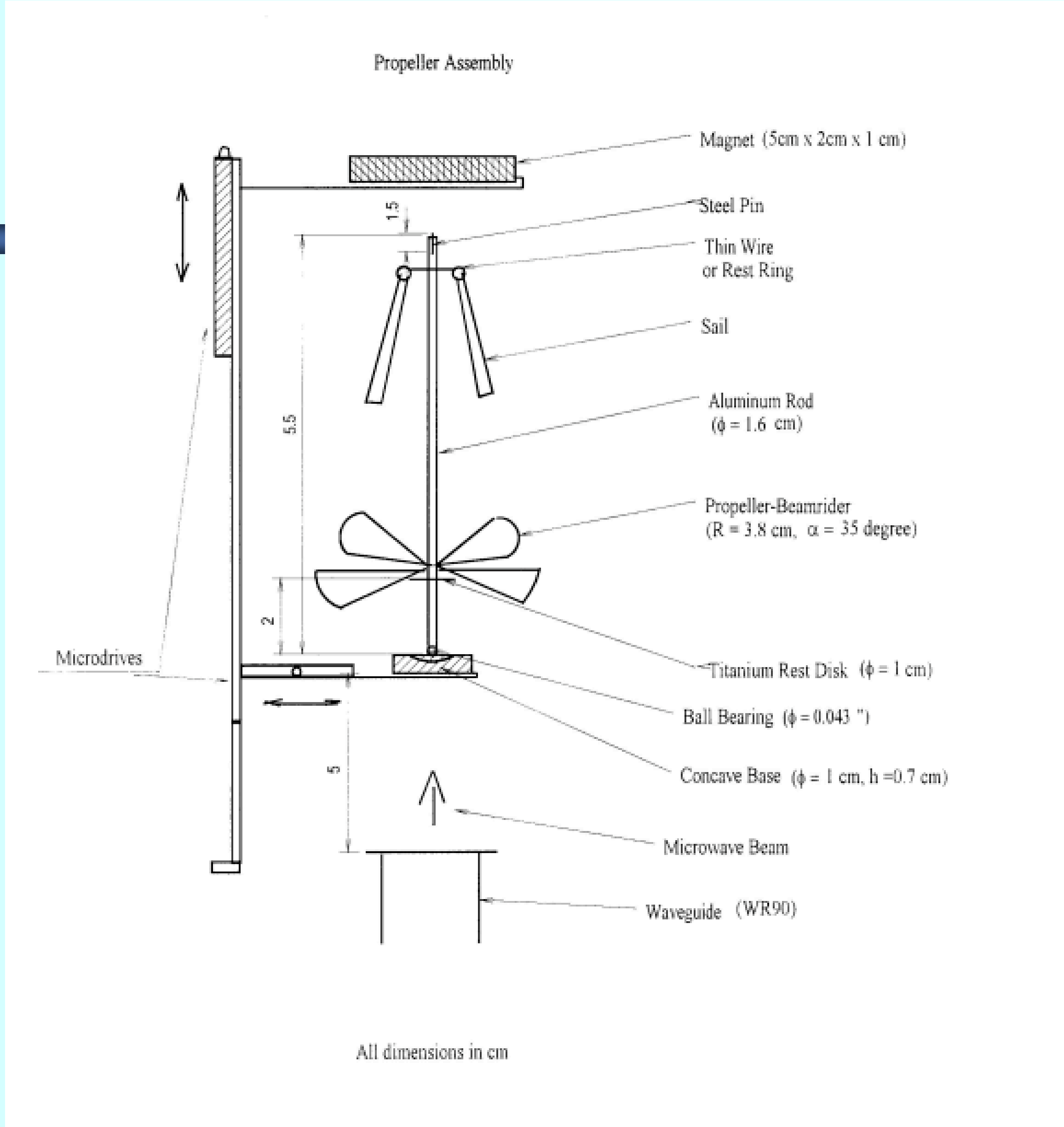
- homogeneous energy density
- oscillatory angular momentum density

Energy and angular momentum density not coupled

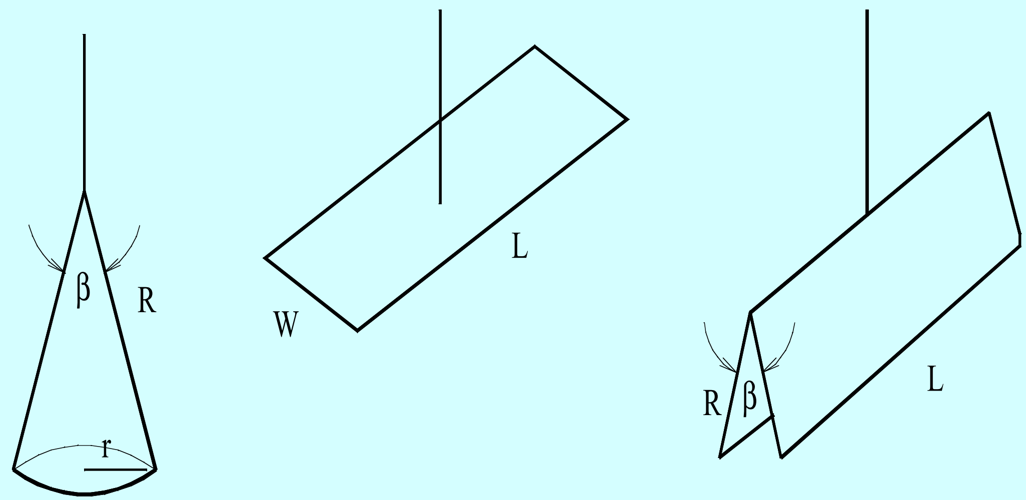
Theory and experiment demonstrate clearly the possibility of extracting angular momentum from electromagnetic fields purely by reflection in specific geometries.

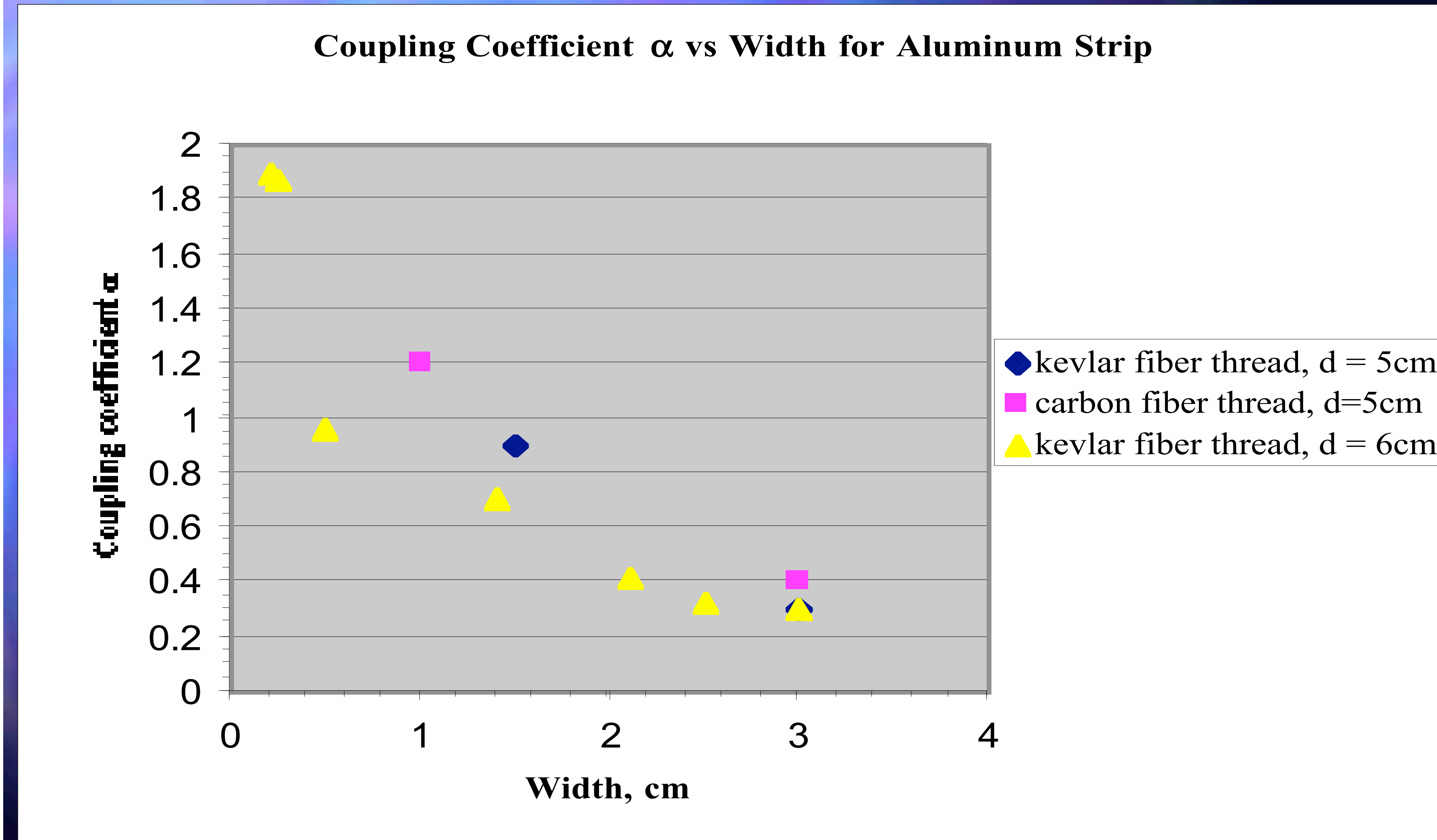


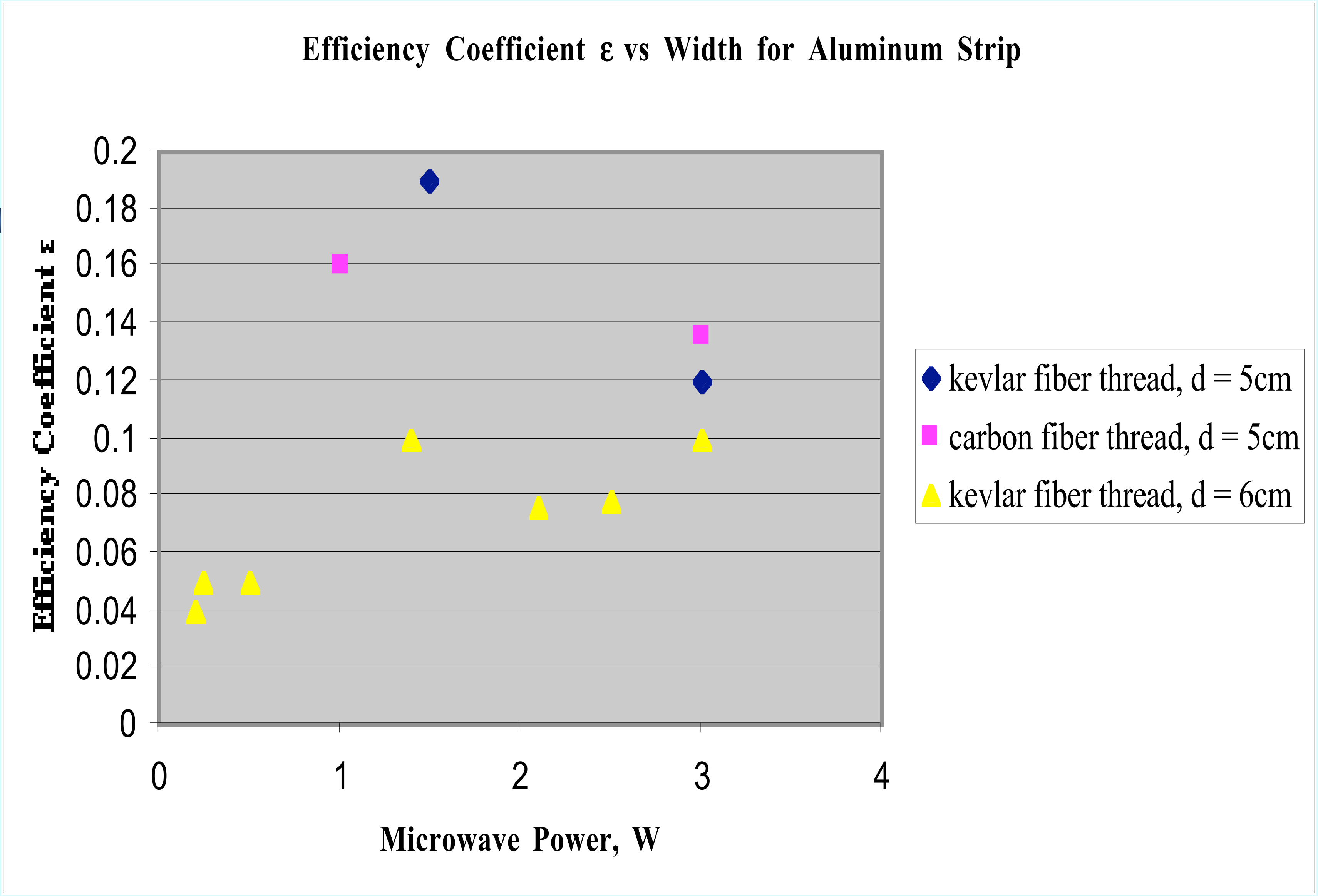
4E7
2E7
0
-2E7
-4E7
-6E7
-8E7
-1E8
-1.2E8
-1E8
-8E7
-6E7
-4E7
-2E7
0
2E7
4E7
6E7
8E7
1E8

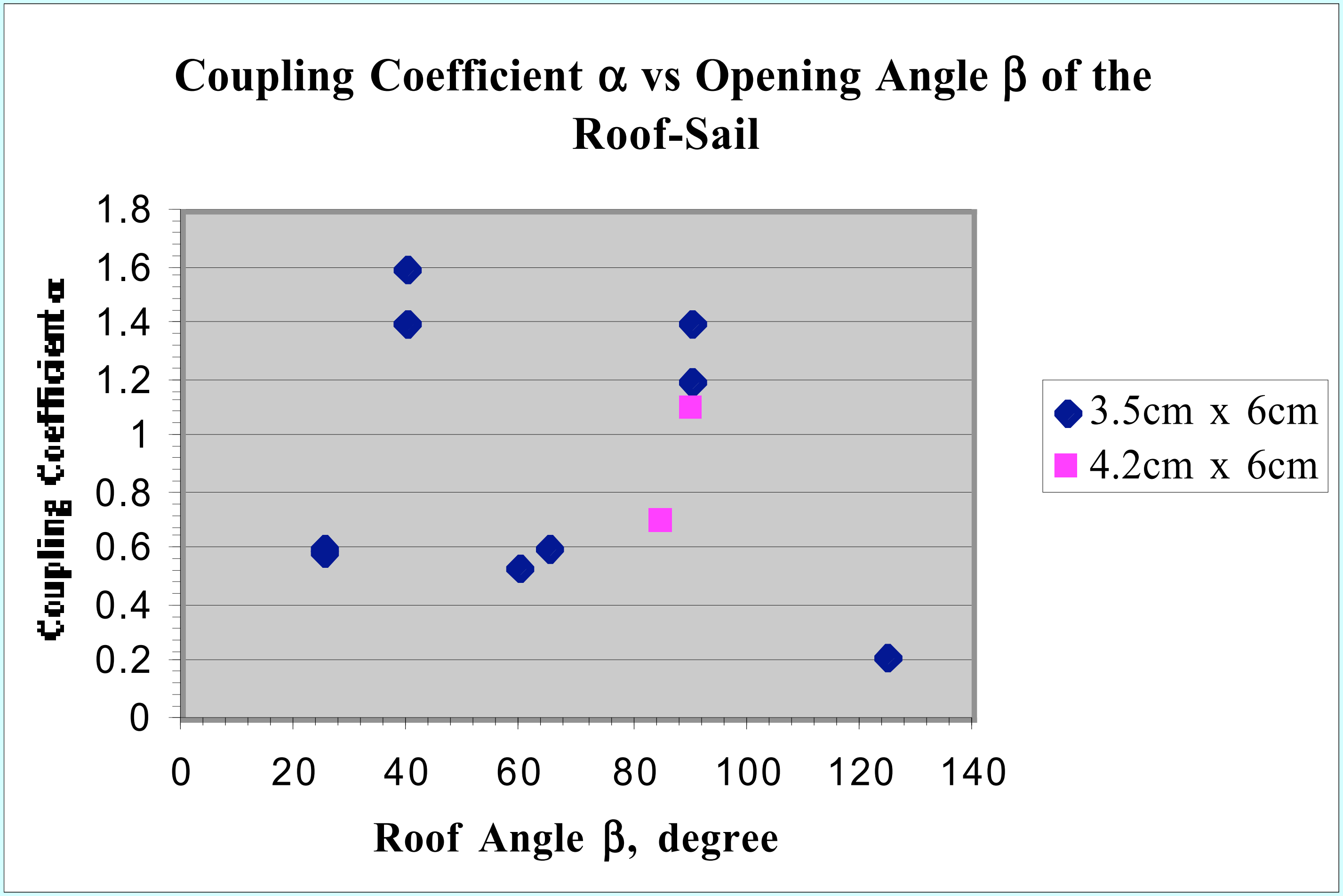


Propeller Assembly
Magnet (5cm x 2cm x 1 cm)
Steel Pin
Thin Wire
or Rest Ring
Sail
Aluminum Rod
(ϕ = 1.6 cm)
Propeller-Beamrider
(R = 3.8 cm, α = 35 degree)
Titanium Rest Disk (ϕ = 1 cm)
Ball Bearing (ϕ = 0.043 ")
Concave Base (ϕ = 1 cm, h =0.7 cm)
Microwave Beam
Waveguide (WR90)
Microdrives
1.5
5.5
2
5
All dimensions in cm

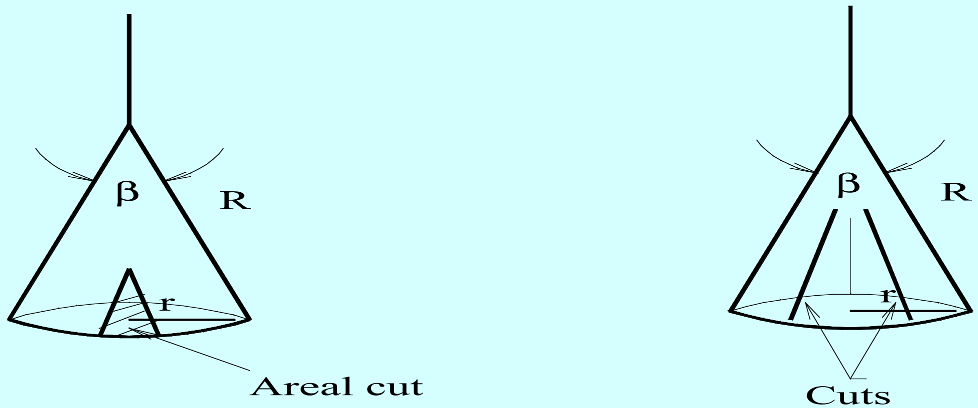

β
R
r
L
W
R
β
L


Coupling Coefficient α vs Width for Aluminum Strip
Coupling coefficient α
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
Width, cm
kevlar fiber thread, d = 5cm
carbon fiber thread, d=5cm
kevlar fiber thread, d = 6cm


Efficiency Coefficient ε vs Width for Aluminum Strip
Efficiency Coefficient ε
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
1
2
3
4
Microwave Power, W
kevlar fiber thread, d = 5cm
carbon fiber thread, d = 5cm
kevlar fiber thread, d = 6cm


Coupling Coefficient α vs Opening Angle β of the Roof-Sail
Coupling Coefficient α
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
20
40
60
80
100
120
140
Roof Angle β, degree
3.5cm x 6cm
4.2cm x 6cm

Types of cuts: Conical sails with an areal cut (left) and thin cuts (right).
We find that the two cut types give very different couplings to circularized microwaves



Comparison with Experiment

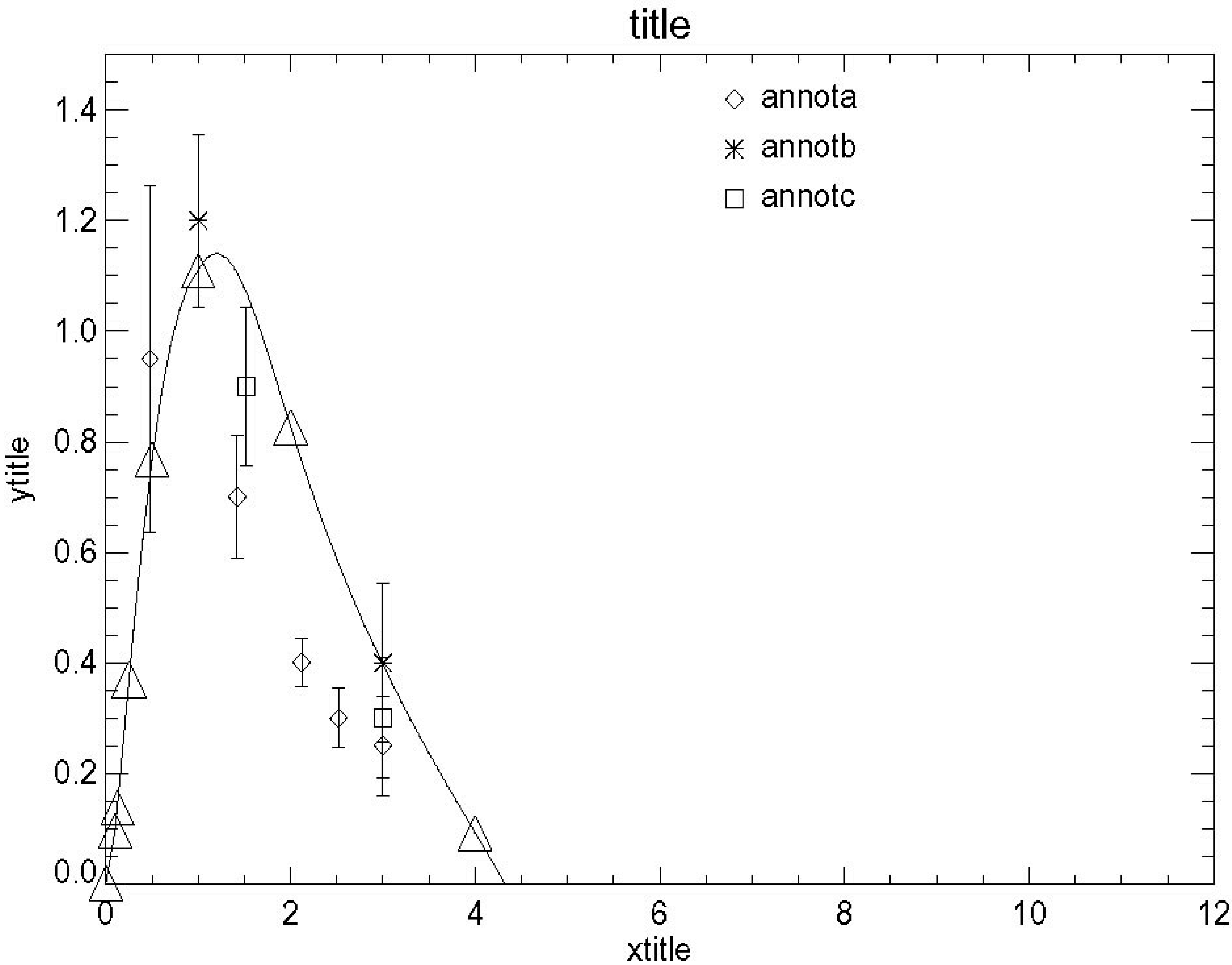
Thin aluminum roofs suspended by 6-micron wide carbon filaments

Circularly polarized microwaves beamed from below at $\approx 100\ W$ and $7.17\ GHz$ spin up the roof

System can be described as a linear torsional spring with an applied, constant electrodynamic torque and a damping torque.

Results:

- intact aluminum disks and cones did not rotate
- carbon disks and cones did spin with absorption coefficient $\alpha \approx 0.1$
- aluminum roofs span with coupling coefficient depending on opening angle


title
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annotc
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xtitle
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0.4
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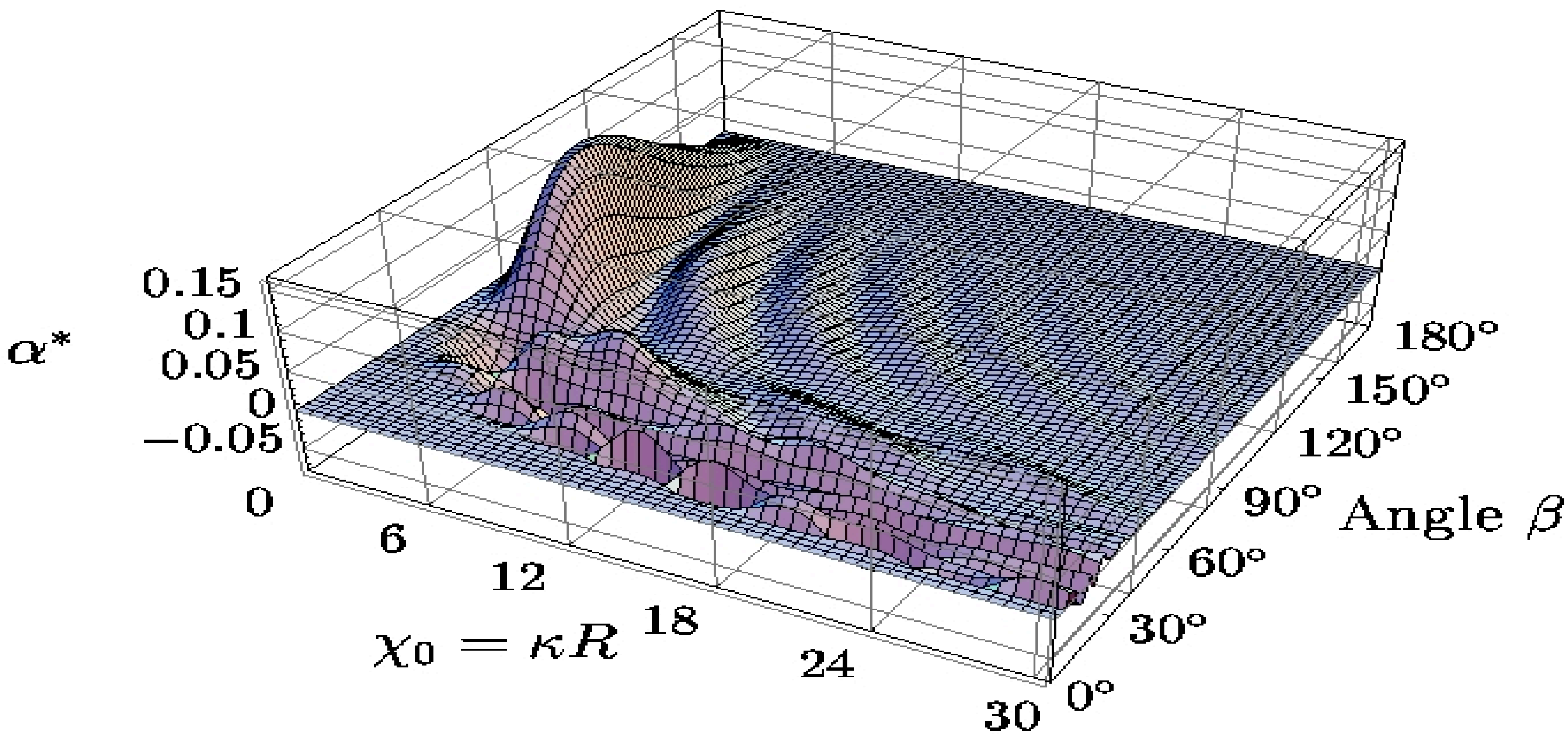
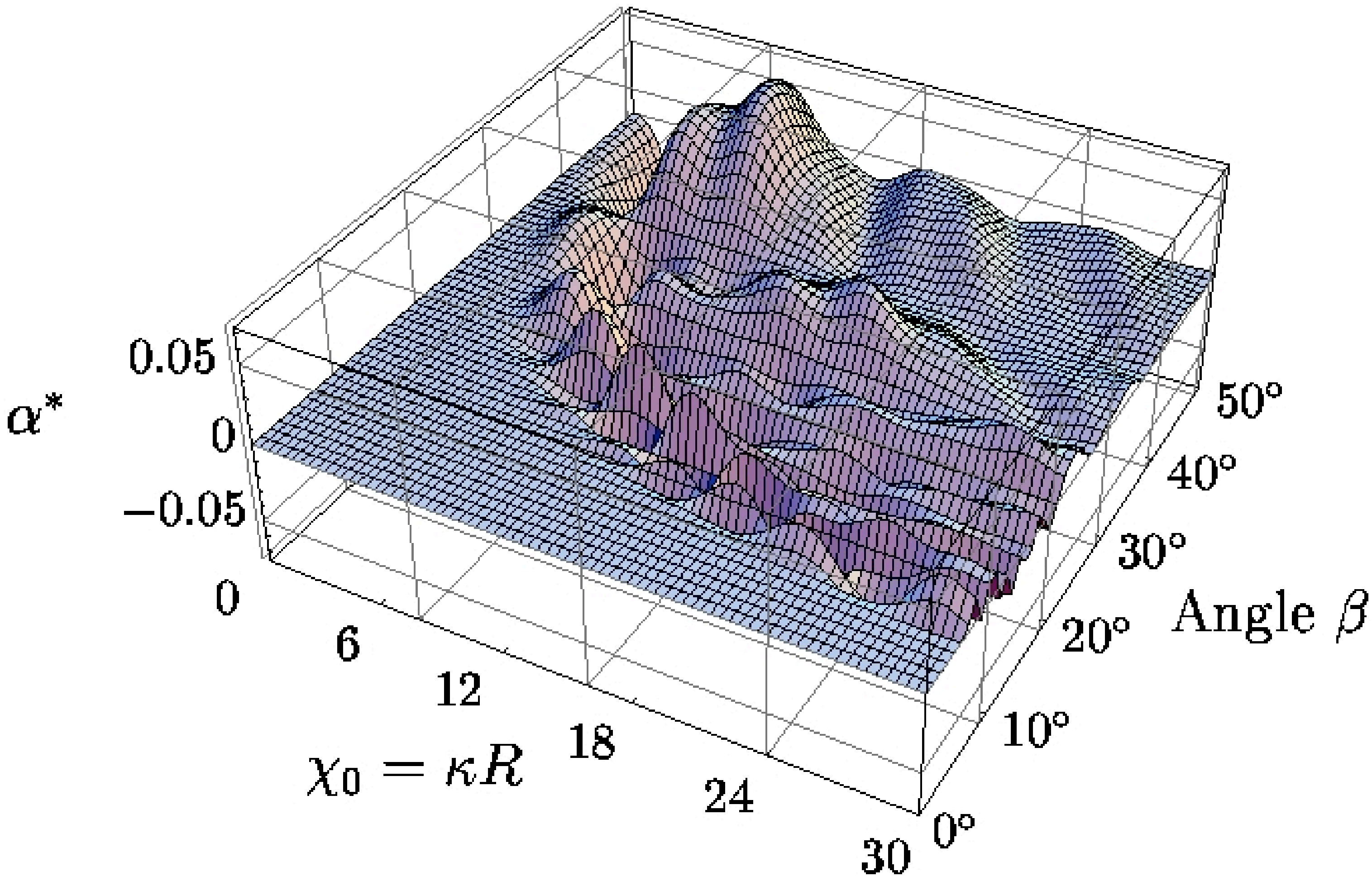
0.8
1.0
1.2
1.4
0
2
4
6
8
10
12



- maximum value of ≈ 0.13 at $R \approx \lambda/2$ and $\beta \approx \pi/2$
- coupling falls fast toward larger R or smaller λ
- negative values down to ≈ -0.05 !

=> **Geometric absorption coefficient** differs physically from material absorption coefficient which is restricted to values between 0 and 1



A perfectly conducting roof can act as a *radiator* for angular momentum at small opening angles.

Theory and experiment demonstrate clearly the possibility of extracting angular momentum from electromagnetic fields purely by reflection in specific geometries.

Conclusion

- Electromagnetic waves not only carry energy but transport angular momentum as well if they are circularly polarized and finite.
- While energy can only be absorbed or reflected, angular momentum can also be radiated.
- Objects with specific geometries can act as polarizers.
- Distinction between inherent material absorption coefficient and a geometric absorption coefficient ,determined by the shape of the object (may also assume negative values)
- Macroscopic objects can be spun by electromagnetic waves.
- Use angular momentum of a microwave beam to unfold a light sail by spinning it up
- Axisymmetric, perfectly conducting objects do not electrodynamically spin

  => break the symmetry by cutting the sail or using roof-like sails

Carbon Sail Experiments

- Sail masses are 3 cm diameter, ~6 mg,
  - microwave power 2-10 kW
  - pulse duration 0.2 sec
  - Sail flies to altitudes 1 cm to 60 cm

- Flight velocities are 0.3-3 m/s, accelerations are10-100 m/s^2 (1-10 gees).
- Sail temperatures are 2000-2500°K. Sails show no erosion or melting.

- <u>Accelerations are several times that predicted for photon pressure</u>.