Lab #12: The Mass of Jupiter

Galileo Galilei discovered the four largest moons of Jupiter in 1609, and they were named Io, Europa, Ganymede and Callisto. These moons provide an excellent way to illustrate Kepler's III law about the speed of revolution, and they will be used in this exercise to determine the mass of the planet and to calculate its average density. This average density is the most important indicator of the internal composition of a planet, about which we cannot gain information in any other way. The basic idea is that the gravity of Jupiter determines the orbit of the moons, and gravity is caused by mass; so out of the motion of the moons you can tell the total mass of the planet.

Almost all work in astronomy starts with a long series of observations. A one-time look rarely tells us enough to understand much, but the two hours of a student laboratory are quite too short to do any sensible observation series. For this reason, we will use a computer simulator, part of the CLEA program developed by Gettysburg College, which shows Jupiter and its moons as you see them in a telescope at any time of your choice. You'll be able to take a simulated "observation" and record the data. Another feature of astronomy is that the raw data means little in itself; new data is added to previous knowledge to produce one bit of additional knowledge at a time. You'll follow this process; previous data will be given to you and you proceed to conclude on the internal composition of Jupiter.

Kepler's third law in its modern form relates the time period T of the revolution to the radius a of the orbit (more precisely the semi-major axis) through $M \times T^2 = a^3$. This gives us the only way to determine the mass M of celestial objects – provided they have a satellite whose orbit we can use. Jupiter has four large moons and Earth has one; we'll measure each moon's a and T, and calculate the mass of the planet. The units in this form of Kepler's law must be solar masses (M_{Sun}) for M, *years* for T, and *astronomical units* (AU) for a; you can convince yourself of the correctness of that by substituting in Earth's orbit.

You will use the Julian Date (JD) for counting the number of days between two events. This JD is actually the number of days since 12:00 noon Universal Time on January 1, 4713 BC – an arbitrarily chosen time in the past, old enough to make sure that in all practical situations JD is a positive number. The usefulness of JD is that you can calculate the time that passes between two JD's very simply: subtract the two dates to get the elapsed time in days. For comparison, think how hard it is to count how many days pass between, say, February 27 and August 12.

The incremental nature of astronomical knowledge requires previously determined data. Here are a few that you will need in this lab:

Astronomical unit	Radius of Jupiter	Radius of Earth	Mass of the Sun	Length of year
1AU=149.6×10^6 km	71,492 km	6,371 km	2.00×10^{33} g	365.2425 days

Recall also that *1 km = 1000 m* and *1 AU* is the distance from the Sun to Earth.

Note: The purpose is to use your own (simulated) "measurements" to find the mass and density of Jupiter, so **do not use** any data other than that given in this worksheet!

PROCEDURE AND LAB REPORT

Date:_______ Your name: ___________Partner's name: ___________ Section: ___/___

Maximum two students may work together as partners!

1. Start up the computer in Windows. If it is running MAC, you will have to restart it while holding down the OPTION key. Use the *Student* account to log in.

2. Read the introduction, then answer these questions:

(i) What numbers do you find on the two sides of Kepler's III law, when you substitute in Earth's orbital data? Which object's mass is *M* then? ______________________________

(ii) What is the only way to determine the mass of a planet? ______________________________

(iii) Why is JD used in astronomy instead of date and time? ______________________________

3. Start up the simulation (click *Desktop→AstroLabs→MassOfJupiter→CLEA*). Use *File→Log In* to insert your names (one or two students in a group), and select the *Revolution of the Moons of Jupiter* option. Click *File→Run*.

Use these settings (under *File*):

Magnification	100 ×		
Features	Animation, Use ID Color, Show Top View		
Timing	Obs. Steps: 0.1 h	Anim. Steps: 0.1 h	Anim. Time: 10 ms.
Observation Date	Today's date. Set time to 00:00:00 by deleting the time altogether.		

Place the *Top View* next to the main panel, so they do not overlap. Set the *View* on the *Top View* panel at: *Large*, *Show Orbits*, *Show eclipse zone*, and *Eclipses → Show Eclipsed Moon As "+"*, and do not later change these top view settings at all.

4. Start the animation (press *Cont.*) and observe what is happening. Notice that the outer moons move more slowly than the inner ones. Change the magnification. Notice that the moons pass in front of or behind Jupiter, and that they cast a shadow on Jupiter sometimes. Stop the animation when a moon approaches Jupiter, and proceed step-by-step. (Keep clicking *Next*.) Using the display of the time (UT), determine how long it takes for the moon to pass in front of Jupiter. Click on your moon to reveal its name. Your answer is:

On / / , the moon __________ took $\_\_^{h}\_\_^{m}$ of time to pass in front of Jupiter.

Notice that the moons either do not show immediately after they pass behind Jupiter, but appear suddenly later or they suddenly vanish before hiding behind the planet. They in fact are eclipsed: that is, they go into Jupiter's shadow. This shadow is the region between the two green lines. Follow one of the moons through an eclipse. By proceeding step-by-step again, determine the time of the vanishing/reappearance of a moon from the shadow. You found that:

On / / , the moon __________ vanished/reappeared (*circle which*) at __^{h}__^{m}.
When a moon goes farthest on one side from Jupiter, it is called an elongation event. The moon's distance from Jupiter will be equal to the radius of the moon's orbit. Run the animation and stop it close to maximum elongation of a moon on the right side (West), and click on the moon to tell how far to the right it is from Jupiter, in units of the diameter of Jupiter's disk.
On / / , at __^{h}__^{m} UT, the moon __________ was ____ D_{Jup} in western elongation.

5. Use these settings (this will slow the animation):

Magnification	100 ×		
Features	Animation, Use ID Color, Show Top View		
Timing	Obs. Steps: 0.01667 h	Anim. Steps: 0.05 h	Anim. Time 10 ms.
Observation Date	Today's date. Set time to 00:00:00 by deleting the time altogether.		

Measure time of two adjacent eastern elongations of each of the four moons, and record them in the table. (You'll need to go step-by-step when the elongation approaches, because the animation is too fast and you cannot run backwards. If you skip the elongation, set the "Observation Date/Time" back. You may change the observation steps if you find it more convenient.) Read off the elongation distances, too, and record them in the table. At this stage you'll have filled out the shaded portion of the table. Change the magnification as convenient.

In all your calculations, keep 2 decimal digits only!

	1st elong.		**2nd elong.**		**Elongation distance (a)**			**Revolution time (T)**	
	Day & Time	***JD*** *24. …*	***Day & Time***	***JD*** *24. …*	D_{Jup}	***km***	***AU***	***day***	***year***
Io	/ / h m		/ / h m				× 10^{-3}		× 10^{-3}
Europa	/ / h m		/ / h m				× 10^{-3}		× 10^{-3}
Ganymede	/ / h m		/ / h m				× 10^{-3}		× 10^{-3}
Callisto	/ / h m		/ / h m				× 10^{-3}		× 10^{-3}
Moon	- - -	- - -	- - -	- - -	- - -	384,748	× 10^{-3}	27.32	× 10^{-3}

6. Calculate the remaining entries in the table below. You will need the data from the explanation above. Use scientific notation for numbers that are too small or too large.

	a^3	T^2	M_{planet}
Unit	*AU^3*	*yr^2*	*M_{solar}*
Io	$\times 10^{-9}$	$\times 10^{-6}$	$\times 10^{-4}$
Europa	$\times 10^{-9}$	$\times 10^{-6}$	$\times 10^{-4}$
Ganymede	$\times 10^{-9}$	$\times 10^{-6}$	$\times 10^{-4}$
Callisto	$\times 10^{-9}$	$\times 10^{-6}$	$\times 10^{-4}$
Moon	$\times 10^{-9}$	$\times 10^{-6}$	$\times 10^{-4}$

7. Kepler's III law, $M \times T^2 = a^3$, predicts that the mass of Jupiter calculated from $M = a^3/T^2$ should be the same, no matter which moon you are using for the calculation. They won't be, because any measurement involves an error, though preferably a small one. (This is not the same thing as *human error*, which is simply doing it wrong!) To minimize this measurement error, calculate the average of the masses you found. After converting solar masses to kilograms, you conclude that the mass of Jupiter is: M_{Jup}= __________ M_{Solar}= ___________ *g*. The same calculation, for comparison, tells the mass of Earth: M_{Earth}= __________ M_{Solar}= ___________ *g*.

8. Out of your measured values of Jupiter's and Earth's masses you can now calculate the density of each. First, use the volume of a sphere, $V = 4\pi R^3/3$, to calculate the volume of both planets. (Make sure you have converted *R* to centimeters!) Show your results: V_{Earth} = ________ cm^3; V_{Jup} = ________ cm^3. Then, calculate the density of both planets, $\rho = M / V$. (Make sure you have converted *M* to grams!) Show your results: ρ_{Earth} = _______ g/cm^3; ρ_{Jup} = _______ g/cm^3.

9. Given that the density of water is $\rho_{water} = 1$ g/cm^3, and the density of rock is about $\rho_{rock} \approx 3$ g/cm^3, what can you tell about the composition of Earth and of Jupiter? (Do not forget that the center of each planet must be much denser than the outside!) Explain:

_______________ _______________ _______________ _______________ _______________

10. ***Review:*** Looking at your table, how many different nights did you have to 'observe' the moons of Jupiter to collect the data? How many hours of observation do you think you would have needed each of these nights? Explain why.

_______________ _______________ _______________ _______________ _______________

11. ***Extra credit:*** The masses and densities that result from this experiment are somewhat too low. What do you think has been overlooked that causes such error?

_______________ _______________ _______________ _______________ _______________