

# Physics 221 - Experiment 8

## Conservation of Energy and Linear Momentum



Fig. 8-1 Ballistic Pendulum

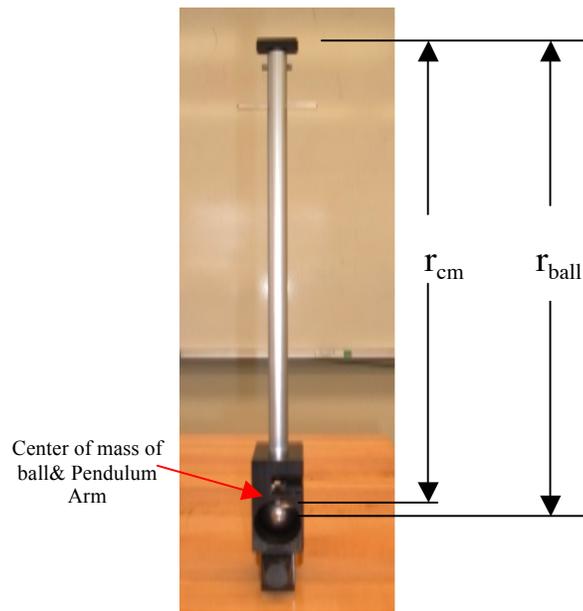


Fig 8-2 Radii of rotation for  $r_{cm}$  &  $r_{ball}$

### Equipment

- Ballistic Pendulum
- 30-cm Ruler
- Triple Beam Balance
- C-clamp
- Safety glasses

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## Conservation of Energy and Momentum

### Advance Reading

**Physics** by Randall Knight, 4<sup>th</sup> edition. Chapter 10, sections 1-4 and Chapter 11, sections, 1-3. Note the difference between elastic and inelastic collisions and the concept of mechanical energy.

### Objective

To determine the speed of the ball as it leaves the ballistic pendulum using conservation of linear momentum and conservation of energy considerations.

### Theory

In this experiment an inelastic collision will occur between the ball and the ballistic pendulum arm. The momentum of the system before and after the collision must be the same (i.e., as always, momentum [both linear and angular] is conserved). *The pendulum arm and ball are assumed to be point masses.* We can then write

$$m_b v_b + M_{pa} v_{pa} = m_b v'_b + M_{pa} v'_{pa} \quad (\text{Eq. 8-1})$$

In this case,  $m_b$  is the mass of the ball and  $M_{pa}$  is the mass of the pendulum arm. Before the collision, the ball has a velocity of  $v_b$  and the pendulum arm has a velocity of  $v_{pa}$ .

The collision is perfectly inelastic, with the ball being caught by the pendulum arm, so that the velocities on the right-hand side of Eq. 5-1 are equal. Label this velocity  $V_{pab}$ . The mass of the pendulum-ball system is  $m_b + M_{pa}$ . The result of this is:

$$m_b v_b = (m_b + M_{pa}) V_{pab} \quad (\text{Eq. 8-2})$$

**Mechanical energy is not conserved during an inelastic collision**, but just after the collision (*again the pendulum arm and ball are assumed to be point masses*) the pendulum-ball system has kinetic energy ( $\text{KE} = 1/2mv^2$ ).

We assume this kinetic energy is conserved and transformed into gravitational potential energy ( $\text{PE} = mgh$ ) as the pendulum arm swings up. In mathematical form  $\text{KE}_i = \text{PE}_f$ , or:

$$\frac{1}{2}(m_b + M_{pa})V_{pab}^2 = (m_b + M_{pa})g\Delta h \quad (\text{Eq. 8-3})$$

By measuring the rise of the center of mass of the system,  $\Delta h$ , the initial velocity of the ball can be determined. The center of mass of the pendulum arm is indicated on fig. 8-2. This is the reference point for initial and final height.

When the ball is fired, it causes the arm to rise and the angle is indicated. Use this angle and the radius of the center of mass,  $r_{cm}$ , to calculate  $\Delta h$ .

### PROCEDURE

1. Measure and record the mass of the ball,  $m_b$ . Carefully remove the arm from the ballistic pendulum. Measure and record the mass of the pendulum arm,  $M_{pa}$
2. Measure the distance from the axis of rotation to the center of mass of pendulum and ball,  $r_{cm}$ . See Fig. 8-2. Next, measure the distance from the axis of rotation to the center of mass of the ball,  $r_{ball}$ . See Fig. 8-2.
3. Reattach the arm on the ballistic pendulum.

### Before firing:

### Do not aim at people or computers!

4. Place the ball on the firing mechanism. Cock the firing mechanism to the 3rd detent position. Fire the ball into the pendulum arm catcher.

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5. Measure and record the new angle; calculate  $\Delta h$ .
6. Calculate the velocity of the ball,  $v_b$  using Eq. 8-2 and Eq. 8-3.
7. Repeat Steps 4-6 twice. Average the  $v_b$  for the three trials.
8. Repeat (3 trials) for the 2nd detent.

#### Questions

1. For each position (2nd & 3rd detent):
  - Calculate the mechanical energy of the system *just before the collision*.
  - Calculate the mechanical energy of the system *just after the collision*.
  - What percent of the initial mechanical energy remains after the collision? What happened to the “lost” mechanical energy?
2. Assume that the ballistic pendulum was not held firmly to the table when it was fired. What effect would this have on the velocity of the ball? Explain.
3. It was assumed that the pendulum arm system was a point mass. This is not true. Rather than conservation of linear momentum, *conservation of angular momentum should be used*, and equation 8-2 can be written as

$$L_{initial} = L_{final} = m_b v_b r_{ball} = I_{pab} \omega_{pab}$$

where  $I_{pab}$  is the moment of inertia of pendulum arm and ball system,  $v_b$  is the velocity of the ball and  $r_{ball}$  is the distance

from the center of mass of the ball to the axis of rotation. See step 2.

Since  $\omega_{pab} = \frac{V_{pab}}{r_{cm}}$ , the equation above can be written as

$$m_b v_b r_{ball} = I_{pab} \frac{V_{pab}}{r_{cm}} \quad \text{Eq 8-4}$$

Equation 8-3 now becomes

$$\frac{1}{2} (I_{pab}) \omega_{pab}^2 = (m_b + M_{PA}) g \Delta h .$$

Again, since  $\omega_{pab} = \frac{V_{pab}}{r_{cm}}$ , the equation above can be written as

$$\frac{1}{2} (I_{pab}) \frac{V_{pab}^2}{r_{cm}^2} = (m_b + M_{PA}) g \Delta h \quad \text{Eq 8-5}$$

Using  $I_{pab} = 0.025 \text{ kg}\cdot\text{m}^2$ , your measured values for masses,  $r_{ball}$ ,  $r_{cm}$  and Eqs- 8-4 & 8-5 above, calculate the velocity of the ball,  $v_b$  for the 2nd and 3rd detent.

**Was the assumption of the system being point sources a reasonable assumption. Show all work.**