

PHYS 221 Measurement

Uncertainty example using simple propagation of uncertainty rules

The following error propagation (sample calculations) consists of the ‘simple’ methods outlined in lab appendix (pages A7-A9). This method yields uncertainties **which are slightly high, but still gives ‘reasonably good values’**.

For **added/subtracted quantities**, the uncertainties are obtained (propagated) by simply **adding the absolute uncertainties** (*i.e., they are not added in quadrature*).

- Write correct significant figures based on the final uncertainty.

For **multiplied/divided** quantities, the uncertainties are obtained by 1) **converted** to percent uncertainties (*i.e., fractional uncertainties*), and 2) the percent uncertainties are **simply added** (*i.e., they are not added in quadrature*).

- Convert from percent to absolute uncertainties (to get correct significant figures for final answer).

Important note for uncertainty calculations –Keep extra significant figures in uncertainties when doing computations. Convert to one significant figure in the final number (*i.e., final answer*)!!!

Sample Calculations for uncertainty of a volume (using simple method estimation of uncertainty propagation)

Volume of block (a cuboid) from lengths measured using vernier caliper:

$$V_{metal} = lwh = (2.540 \pm 0.005)cm \times (5.080 \pm 0.005)cm \times (7.620 \pm 0.005)cm \Rightarrow (98.32238 \pm \delta V_{metal})cm^3$$

Convert to percent (fractional uncertainties)

$$V_{metal} = lwh = (2.540cm \pm 0.197\%) \times (5.080cm \pm 0.0984\%) \times (7.620 \pm cm \pm 0.0656\%)$$

*Approximate percent uncertainty of volume $\delta_{\%}V_{metal}$ is obtained by simple addition of uncertainties, *i.e.*,*

$$\delta_{\%}V_{metal} \approx 0.197\% + 0.0984\% + 0.0656\% = 0.361 = 0.4\%$$

Thus we see $V_{metal} = lwh = (98.32238cm^3 \pm 0.4\%)$

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Convert to absolute uncertainties to get correct number of significant figures.

We see 0.4% of 98.32238cm^3 is

$$98.32238\text{cm}^3 \times 0.4 / 100 = 98.32238\text{cm}^3 \times 0.004 = 0.393\text{cm}^3 = 0.4\text{cm}^3$$

Thus we see $V_{\text{metal}} = lwh = (98.3 \pm 0.4)\text{cm}^3$

Mass of metal block obtained from triple beam balance (given in absolute and fractional uncertainties):

$$M_{\text{metal}} \pm \delta M_{\text{metal}} = (450.90 \pm 0.05)\text{g} \quad \Rightarrow \quad M_{\text{metal}} \pm \delta_{\%} M_{\text{metal}} = 450.90\text{g} \pm 0.01\%$$

Now we need to calculate the density and its uncertainty

$$\rho_{\text{metal}} = \text{mass} \div \text{volume} = (450.90\text{g} \pm 0.01\%) \div (98.3\text{cm}^3 \pm 0.4\%) = 4.5869\text{g} / \text{cm}^3 \pm \delta\rho_{\% \text{metal}}$$

Now we see the (approximate) fractional uncertainty in density of metal block, $\delta\rho_{\text{metal}}$ is

$$\delta\rho_{\% \text{metal}} \approx (0.01\% + 0.4\%) = 0.411\% = 0.4\%$$

The density of the block is

$$\rho_{\text{metal}} = 4.5869\text{g} / \text{cm}^3 \pm 0.4\%$$

Lastly, convert to absolute uncertainty to get correct number of significant figures, i.e., 0.4% of 4.5869 is

$$\rho_{\text{metal}} = 4.5869\text{g} / \text{cm}^3 \times 0.004 = 0.018\text{g} / \text{cm}^3 = 0.02\text{g} / \text{cm}^3$$

Finally, the density (with the correct number of significant figures is)

$$\rho_{\text{metal}} = 4.59 \pm 0.02\text{g} / \text{cm}^3$$