Exam Notes

- Multiple Choice
- No Calculators → Quick estimate
- Use estimates to eliminate wrong answers
- Use part of an answer to narrow the field (i.e. direction of a vector, sign of a charge, ...)

- Kinematics: use energy rather than forces

* These notes will be posted on my web site
  www.phy.olemiss.edu/~jgladdenu/
Topics

- Math Review
- Kinematics - Projectile Motion
- Newton's Laws
- Vectors
- Rotational Motion
- Friction
- Gravity
- Energy & Work
- Momentum
- Properties of Fluids & Solids
- Electrodynamics
- Circuits
- Magnetism
- Waves
- Simple Harmonic Motion
- Light and Optics
Math Review

- Trig Functions

Sine, Cosine, Tangent

SoH CAH TOA

\[
\sin \theta = \frac{O}{H} \\
\cos \theta = \frac{A}{H} \\
\tan \theta = \frac{O}{A}
\]

\( \Theta \) in degrees

0 - 360°

in radians

0 - 2\pi

360° = 2\pi radians

Pythagorean Theorem

\[ H^2 = A^2 + O^2 \]
Radians

\[ S = \theta \cdot r \]

if \( \theta \) is in radians

\[ C = 2\pi r \quad (\theta = 2\pi) \]

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**Graphs**

\[ y = mx + b \]

\[ m = \text{slope} \]

\[ = \frac{\text{rise}}{\text{run}} \]

\[ = \frac{\Delta y}{\Delta x} \]

at \( x_1 \):

\[ m = \frac{\Delta y}{\Delta x} \]

of tangent line
Scalars: Magnitude Only (Number + Unit)

Vectors: Magnitude and Direction

Scalar
Mass
Time
Current
Energy/Work

Vector
Acceleration
Force
Electric Field

Distance
Speed

Vector Addition

\[ \vec{V} = \vec{V}_1 + \vec{V}_2 \]
Adding Vectors

Components

\[ \vec{F} = \vec{F}_x + \vec{F}_y \]

\[ \sin \theta = \frac{|F_y|}{|F|} \]

\[ F_y = F \sin \theta \]

\[ \cos \theta = \frac{F_x}{F} \]

\[ F_x = F \cos \theta \]

Kinematics

Displacement
- Vector
- Units: Meters

Velocity: Time rate of change of Displacement

\[ \vec{v} = \frac{\Delta \vec{x}}{\Delta t} \]
Units: \[ \frac{m}{sec} \]

Direction is same as \[ \Delta \vec{x} \]
Acceleration: time rate of change of velocity
\[ \vec{a} = \frac{\Delta \vec{v}}{\Delta t} \]

Units: \( \left[ \frac{m}{s^2} \right] \)

\( \Delta \vec{v} \) can be a change in magnitude or direction.

Equations of Motion

For \( \vec{a} = \) constant:

1. \( v_f = v_o + a \, t \) \( \rightarrow \) line

2. \( v_f^2 = v_o^2 + 2a(x-x_o) \)

3. \( \vec{v} = \frac{v_o + v_f}{2} \)

4. \( x(t) = x_o + v_o \, t + \frac{1}{2} \, a \, t^2 \)

Para bola
Projectile Motion

\[ a = g = -9.8 \, \text{m/s}^2 \approx 10 \, \text{m/s}^2 \]

Acceleration due to gravity

\[ y(t) = y_0 + v_0 t + \frac{1}{2} g t^2 \]

\[ v = \frac{\Delta y}{\Delta t} \]

\[ v = v_0 + g t \]
What is $h$?

$a = -9.8 \text{ m/s}^2$

$v_0 = 10 \text{ m/s}$

$v = -10 \text{ m/s}$

\[
y(t) = y_0 + v_0 t + \frac{1}{2} g t^2
\]

\[
v^2 = v_0^2 + 2 g \left( y - y_0 \right)
\]

\[
o = 100 \frac{m^2}{s^2} + 2 \left( -10 \frac{m}{s^2} \right) (h)
\]

\[
h = \frac{100 \frac{m^2}{s^2}}{20 \frac{m^2}{s^2}} = 5 \text{ m}
\]
Z-D Projectile Motion

$\vec{v}_0 = \vec{v}_0 \sin \theta$
$\vec{v}_0 = \vec{v}_0 \cos \theta$

Treat vertical and horizontal components separately.

in vertical: $a_y = g$

in horizontal: $a_x = 0$

**Vertical**

$\vec{v}_y(t) = \vec{v}_{0y} + gt$

$\vec{v}_y^2 = \vec{v}_{0y}^2 + 2g(y - y_0)$

$y(t) = y_0 + v_{0y}t + \frac{1}{2}gt^2$

**Horizontal**

1. $\vec{v}_x(t) = \vec{v}_{0x} + ax \to 0$
2. $x(t) = x_0 + v_{0x}t + \frac{1}{2}at^2 \to 0$
Newton's Laws of Mechanics

1. Law of Inertia

Inertia: resistance to a change in the motion of a mass [unit: kg]

Force is required to change velocity.

2. Net force acting on a body is proportional to the acceleration produced

\[ \Sigma \vec{F} = M \vec{a} \]

\[ \Sigma \vec{F} = \vec{F}_s + \vec{F}_g \]

Net = 0

Unit of Force: kg \( \cdot \) \( \frac{m}{s^2} \) = 1 Newton

\[ = 1 \text{ N} \]
3) Action / Reaction

- For every force acting on an object, there is an equal and opposite force acting on a different object.

- Forces come in Pairs

\[ mg \uparrow \quad F_e \]

+ Newton's 3rd Law: Pair forces can never cancel each other!
Uniform Circular Motion

- Radius is constant
- Speed constant
- $|\mathbf{v}|$ is constant, but direction is changing

Centripetal Acceleration
\[
\mathbf{a}_c = \frac{\mathbf{v}^2}{\mathbf{r}}
\]
\[
F_c = M \mathbf{a}_c = M \frac{\mathbf{v}^2}{\mathbf{r}}
\]

\[
\mathbf{v} = \frac{2\pi \mathbf{r}}{T_{oz}} \quad \text{"period"}
\]

\[
S = \Theta \mathbf{r}
\]

\[
\mathbf{v} = \omega \mathbf{r}, \quad \omega = \frac{\Delta \Theta}{\Delta t}
\]

\[
\mathbf{a}_T = \alpha \mathbf{r}, \quad \alpha = \frac{\Delta \omega}{\Delta t}
\]

\[\rightarrow \text{tangential acceleration} = 0 \quad \text{for uniform circular motion}\]
Equilibrium

Conditions:

1. \[ \sum F = 0 \implies \ddot{a} = 0 \]
2. \[ \sum \vec{F} = 0 \]

\[ \implies \text{torque - rotational counterpart to force} \]

Torque

\[ \tau = F \cdot \sin \theta \]

Newton's 2nd:

\[ \sum \vec{F} = m \ddot{x} \]
\[ \sum \vec{F} = I \ddot{\theta} \]

"Moment of inertia" depends on the mass and distribution of mass
Friction

\[ F_f = \mu_s \vec{F}_N \]  \hspace{1cm} \text{normal force}

\[ F_f = \mu_k \vec{F}_N \]

\[ \mu_s = \text{Static} \]
\[ \mu_k = \text{Kinetic} \]

\[ \mu_s > \mu_k \]

\[ \sum \vec{F}_x = m \ddot{a}_x \]
\[ \sum \vec{F}_y = m \ddot{a}_y \]

\[ F_g = mg \]
\[ F_{gx} = mg \sin \theta \]
\[ F_{gy} = mg \cos \theta \]

\[ \sum \vec{F}_x = -\mu_k \vec{F}_N + mg \sin \theta = M \dot{a}_x \]
\[ \sum \vec{F}_y = \vec{F}_N - mg \cos \theta = 0 \]
Gravity

- Universal Gravity

\[ F_g = G \frac{M_1 M_2}{r^2} \]

\[ \Rightarrow \text{Universal gravitational Constant} \]

\[ F_g = G \frac{M_{\text{Earth}} M_E}{R_E^2} = \frac{M_{\text{Earth}} g}{R_E} \]

\[ g = G \frac{M_E}{R_E^2} \]
Fundamental Forces
"action at a distance"

1. Gravity
2. Electro magnetic
3. Weak Nuclear Force
4. Strong Nuclear Force

Energy + Work

Scalars

\[
W = F \cdot d \cdot \cos \theta
\]

Centripetal Force does no work!
\[
\theta = 90^\circ \Rightarrow \cos \theta = 0
\]
Work - Energy Theory

\[ Work = \Delta E \]

Kinetic Energy = \( \frac{1}{2} mv^2 \)

Potential Energy
- gravitational
  \[ PE_g = mg \ell \]
- elastic
  \[ PE_e = \frac{1}{2} k (\Delta x)^2 \]

- Friction
  - no PE
  - non-conservative force
Conservation of Energy

\[ \Sigma E_{\text{before}} = \Sigma E_{\text{after}} \]

\[ \left( KE + PE \right)_{\text{before}} + W_{\text{NC}} = \left( KE + PE \right)_{\text{after}} \]

\[ W_{\text{NC}}: \text{Work done by non conservative forces (friction)} \]

\[ u_0 = 0 \]

\[ \text{Frictionless} \]

\[ \Sigma E_{\text{before}} = \left[ KE + PE \right] = Mgh \]

\[ \Sigma E_{\text{after}} = \left[ KE + PE^{\theta} \right] = \frac{1}{2} m u^2 \]

\[ Mgh = \frac{1}{2} m u^2 \Rightarrow u = \sqrt{2gh} \]
Work by conservative force does not depend on path.

Power

- time rate at which work is done

\[ P = \frac{\text{Work}}{\text{time}} \]

\[ \left[ \frac{N \cdot m}{s} = \frac{J}{s} = 1 \text{ Watt} \right] \]
Momentum
\[ \vec{P} = m \vec{V} \]

Conservation of Momentum
\[ \Sigma \vec{P}_{\text{before}} = \Sigma \vec{P}_{\text{after}} \]

A \hspace{1cm} B

After
\[ \begin{align*}
\Sigma P_{\text{after}} &= m_A \vec{V}_A' + m_B \vec{V}_B' \\
0 &= m_A \vec{V}_A' + m_B \vec{V}_B' \\
m_A \vec{V}_A' &= -m_B \vec{V}_B'
\end{align*} \]
Collisions

1. Elastic - KE is conserved

\[ \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A v_A'^2 + \frac{1}{2} m_B v_B'^2 \]

2. Inelastic Collision

Objects Stick together

\[ v_A' = v_B' \]
Properties of Fluids and Solids

Fluid: anything that will flow (liquid, gas)
Solid: rigid (hold its shape)

Mass density: \( \rho = \frac{\text{Mass}}{\text{Volume}} \)

Specific gravity: \( \frac{\rho}{\rho_{\text{water}}} \)

\( \rho_{\text{water}} = 1 \ \frac{\text{gram}}{\text{cm}^3} = 1000 \ \frac{\text{kg}}{\text{m}^3} \)

Stress = \( \frac{\text{Force}}{\text{area}} \) [Units: \( \frac{N}{m^2} = \text{Pascal} \)]

\[ \text{Strain} = \frac{\Delta L}{L_0} \]

Young's modulus \( \frac{F}{A} = \frac{E}{L_0} \frac{\Delta L}{L_0} \)
Fluids

Pressure: Force per unit area

\[ P = \rho g h + P_{atm} \]

Atmospheric Pressure

Buoyancy

\[ F_b = S_f Vg \]

Volume of fluid displaced

Will it float?

\[ F_g \leq F_b \]
Fluid Flow

Laminar flow - no turbulence

1. Equ. of Continuity

\[ S_1 A_1 u_1 = S_2 A_2 u_2 \]

If \( S_1 = S_2 \)

\[ A_1 u_1 = A_2 u_2 \Rightarrow \frac{A_1}{A_2} = \frac{u_2}{u_1} \]
Bernoulli Equ

\[ p_1 + \frac{1}{2} \rho v_i^2 + \rho g y_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 \]
**Electric Statics**

- 2 types of electric charge
  
  (+) - Source is Protons  
  
  (-) - Source is electrons

- Charge is quantized
  
  \[ q_e = -1.6 \times 10^{-19} \text{ C} \]

  \[ q_p = +1.6 \times 10^{-19} \text{ C} \]

- Charge is conserved

- If equal # of (+) and (-)
  
  \[ \rightarrow \text{ electrically neutral} \]

\[ + - + - \\
- + - + \]
Charge Transfer

① Contact - Direct transfer of charge (Conduction)

② Induction
- Like charges repel, opposite charges attract
- Conductor: Charge flows very easily
- Insulators: All e's are strongly bound to atoms
- Conducting Sphere

- $-Q$  $+Q$
**Coulomb's Law**

\[ F = \frac{k q_1 q_2}{r^2} = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2} \]

\[ k = \frac{1}{4\pi \varepsilon_0} \]

\[ \text{Coulomb's Constant} \]

\[ k = 9 \times 10^9 \text{ N m}^2/\text{C}^2 \]

\[ \varepsilon_0 \]

\[ \text{permittivity of free space (vacuum)} \]

**Electric Field**

\[ \vec{E} = \frac{\vec{F}}{q} \]

For a point charge

\[ E = k \frac{Q}{r^2} \]
\[ \vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots \]
Field Lines

1. Never cross

2. More dense regions of higher density of lines are strong
For Parallel Plates
\[ C = k \varepsilon_0 \frac{A}{d} \]

For dielectric constant \( \varepsilon \) of air
\[ = 1 \]
Circuits

- Closed conductive path through which charge can flow

\[ E = \frac{\Delta Q}{\Delta t} \]

\[ I = \frac{C}{\text{sec}} = 1 \text{ ampere} \]

Series

\[ R_{\text{tot}} = R_1 + R_2 + R_3 \]
Ohm's Law

\[ E = I R_{\text{tot}} \]

or

\[ V = I R \]

Parallel Circuit

\[ I = I_1 + I_2 + I_3 \]

\[ \frac{1}{R_{\text{tot}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \]
Electric Power

\[ P = I \ V, \text{ if } V = IR \]
\[ P = I^2 R \]
\[ = \frac{V^2}{R} \]

Magnetism

All magnetic fields come from moving charges.

\[ B = \frac{\mu_0 I}{2\pi r} \]

\[ \text{(B)} \]
Charge moving in a magnetic field

\[ F = q v B \sin \theta \]
Waves

\[ y(x) = A \sin \left( \frac{2\pi}{\lambda} x \right) \]

\[ y(t) = A \sin \left( \frac{2\pi}{T} t \right) \]

\( T \): "period"

\[ y(x, t) = A \sin \left( \frac{2\pi}{T} t + 2\pi \frac{x}{\lambda} \right) \]

**Frequency:** \( f = \frac{1}{T} \quad [\text{sec}^{-1} = \text{Hertz}] \)

**Speed:** \( v = f \lambda \)
1) Transverse Wave
   - Displacement is \( \perp \) to wave velocity
     - Light
     - Ocean waves
     - Waves on a string

2) Longitudinal Waves
   - Displacement is \( \parallel \) to wave velocity
     - Sound

Sound

Faster in a dense medium
The faster in a stiff medium

Amplitude / Volume

Intensity of Sound

\[ \beta = 10 \log_{10} \left( \frac{I}{I_0} \right) \text{ decibels} \]
$I_o = \text{threshold intensity for hearing} \quad 10^{-12} \text{ W/m}^2$

Pitch : Frequency

20 Hz - 20,000 Hz : human hearing

Infrasound

Ultrasound
Simple Harmonic Motion

- Need a force $F$ to the displacement

Hooke's Law

\[ F = -k \Delta x \]

\[ \frac{\ell}{2 \mu} \]

\[ \frac{\ell}{m} \]

\[ F = ma \]

\[ x(t) = A \sin(\omega t) \]

\[ \frac{2\pi f}{T} = \frac{2\pi}{1} \]

Energy: \[ PE = \frac{1}{2} k \Delta x^2 \]

Mass-Spring: \[ T = 2\pi \sqrt{\frac{m}{k}} \]

Pendulum: \[ T = 2\pi \sqrt{\frac{L}{g}} \]
Light + Optics

Light is an electromagnetic field wave. 

Visible: 

\[ \lambda = 390 \text{ nm} \quad \text{to} \quad 700 \text{ nm} \]

\( \lambda \) = long \( \lambda \) \quad \rightarrow \quad \text{Short } \lambda \\

ROYGBIV

Speed of light

\[ C = 3 \times 10^8 \frac{m}{s} \quad \text{in a vacuum} \]

\[ C = \frac{\lambda}{\nu} \]

Index of Refraction

\[ \nu = \frac{C}{\nu} \]

\( \nu_{\text{air}} \approx 1.0003 \)

\( \nu_{\text{glass}} \approx 1.5 \)

\( \nu_{\text{water}} \approx 1.3 \)
Law of Reflection

$$\theta_i = \theta_r$$

Law of Refraction

Medium 1

Medium 2 (water)

Fast to slow: bends toward the normal, $$\theta_r < \theta_i$$

Slow to fast: bends away from normal, $$\theta_r > \theta_i$$
Snell's Law

\[ n_1 \sin \theta_i = n_2 \sin \theta_r \]

Slow to Fast

Total internal Reflection

When \( \theta_r = 90^\circ \)

then \( \theta_c = \) Critical angle (\( \theta_c \))

\[ \theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right) \]

Mirrors

\[ f = \frac{R}{2} \]

Concave

Convex
\[ \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \]
Lens Makers' Equ.

\[ \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \]

\[ M = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \]