2. A particle of mass \( m \) moves freely in one dimension between impenetrable walls located at \( x = 0 \) and \( a \). Its initial wavefunction is

\[
\psi(x, 0) = \sqrt{2/a} \sin(3\pi x/a).
\]

What is the subsequent time evolution of the wavefunction? Suppose that the initial wavefunction is

\[
\psi(x, 0) = \sqrt{1/a} \sin(\pi x/a) [1 + 2 \cos(\pi x/a)].
\]

What now is the subsequent time evolution? Calculate the probability of finding the particle between 0 and \( a/2 \) as a function of time in each case.

Given \( |\psi_n\rangle = \sqrt{\frac{2}{a}} \sin \left( \frac{n\pi x}{a} \right) \quad E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{\hbar^2}{2m} \left( \frac{n\pi}{a} \right)^2 \)

**Superposition Principle**

(a) \( \Psi(x, 0) = \sqrt{\frac{2}{a}} \sin \left( \frac{3\pi x}{a} \right) = \sum_n c_n |\psi_n\rangle \quad \rightarrow \quad E_1 = \frac{\hbar^2}{2m} \left( \frac{3\pi}{a} \right)^2 \quad c_1 = 1 \quad c_{ns} = 0 \)

\[
\Psi(x, t) = \sum_n c_n \psi_n e^{-iE_n t/a} = \sqrt{\frac{2}{a}} \sin \left( \frac{3\pi x}{a} \right) e^{-iE_1 t/a} \quad \text{Superposition Principle}
\]

\[
P_{[0-a/2]} = \int_0^{a/2} |\Psi|^2 \, dx = 1/2 \quad \text{because} \quad \int_0^a |\Psi|^2 \, dx = 1
\]

(b) \( \Psi(x, 0) = \sqrt{\frac{1}{a}} \sin \left( \frac{\pi x}{a} \right) + 2 \sqrt{\frac{1}{a}} \sin \left( \frac{2\pi x}{a} \right) \cos \left( \frac{\pi x}{a} \right) \)

\[\sin(\alpha) \cos(\alpha) = 1/2 \sin(2\alpha)\]

\[
\Psi(x, 0) = \sqrt{\frac{1}{a}} \sin \left( \frac{\pi x}{a} \right) + \frac{1}{\sqrt{2}} |\psi_1\rangle + \frac{1}{\sqrt{2}} |\psi_2\rangle
\]

\[
\Psi(x, t) = \frac{1}{\sqrt{2}} |\psi_1\rangle e^{-iE_1 t/a} + \frac{1}{\sqrt{2}} |\psi_2\rangle e^{-iE_2 t/a} \quad \text{and} \quad P_{[0-a/2]} = \int_0^{a/2} |\Psi|^2 \, dx
\]

4. A stream of particles of mass \( m \) and energy \( E > 0 \) encounter a potential step of height \( W < E \): i.e.,

\( V(x) = 0 \) for \( x < 0 \) and \( V(x) = W \) for \( x > 0 \) with the particles incident from \(-\infty\). Show that the fraction reflected is

\[
R = \left( \frac{k - q}{k + q} \right)^2,
\]

where \( k^2 = (2m/\hbar^2) E \) and \( q^2 = (2m/\hbar^2) (E - W) \). [from Squires]

\[
\psi_I = I e^{ikx} + R e^{-ikx} \quad \psi_H = T e^{ik'x}
\]

\[
k = \sqrt{2m/E} \quad k' = \sqrt{2m/|E - W|}
\]

\[
\psi_I(0) = \psi_H(0) \quad \psi_I(0)' = \psi_H(0)'
\]