PHYS 622 – HOMEWORK # 3 – DUE WEDNESDAY, 3/16/2010

Problem 1. Alice moves with uniform velocity $\mathbf{v}$ relative to Bob. Alice sees a particle with velocity $\mathbf{v}_A$ and acceleration $\mathbf{a}_A$. Show that Bob measures the particle acceleration

$$\mathbf{a}_{B,\parallel} = \frac{(1-\mathbf{v}^2/c^2)^{3/2}}{(1+\mathbf{v}\cdot\mathbf{v}_A/c^2)^3}\mathbf{a}_{A,\parallel}\,,$$

$$\mathbf{a}_{B,\perp} = \frac{(1-\mathbf{v}^2/c^2)}{(1+\mathbf{v}\cdot\mathbf{v}_A/c^2)^3}[\mathbf{a}_{A,\perp} + \mathbf{v}\times(\mathbf{a}_A\times\mathbf{v}_A)/c^2]\,,$$

where $\mathbf{a}_\parallel$ and $\mathbf{a}_\perp$ are the accelerations parallel and perpendicular to $\mathbf{v}$, respectively.

Problem 2. Show that the following transformation forms a group with parameter α:

$$T(\alpha) = \begin{cases} \tilde{x}^1 &= \dfrac{x^1}{1-\alpha x^1}\,, \\ \tilde{x}^2 &= \dfrac{x^2}{1-\alpha x^1}\,. \end{cases}$$

Problem 3. (a) Write the transformation laws for the following tensors under the Lorentz group: $A^{\sigma}{}_{\nu\mu}$, $A^{\mu\lambda}{}_{\rho}$, $A^{\rho\nu}{}_{\sigma\mu}$. (b) If $a_{\mu\nu}$ is a skew-symmetric rank-two tensor, show that *i)* $b_{\mu\nu\sigma} = \partial_\sigma a_{\mu\nu} + \partial_\mu a_{\nu\sigma} + \partial_\nu a_{\sigma\mu}$ is a rank-three tensor, *ii)* $b_{\mu\nu\sigma}$ is skew-symmetric in all pairs of indices and *iii)* determine the number of independent components this tensor has.

Problem 4. Consider a symmetric rank-two contravariant tensor $T^{\mu\nu}$ in the coordinate system x^μ ($\mu,\nu = 0,1,2,3$). (a) Show that T is symmetric in all coordinate systems. (b) If T satisfies the condition $\partial_\mu T^{\mu\nu} = 0$, show that

$$\frac{d^2}{dt^2}\int d^3x\, T^{00}x^i x^j = 2\int d^3x\, T^{ij}\,,$$

where $i,j = 1,2,3$ and $T^{\mu\nu}$ is assumed to be localized (boundary terms can be neglected).