PHYS 622 – FINAL EXAM (TAKE-HOME PART)

DUE FRIDAY, MAY 7th, 2010 AT 9:00 A.M.

Solve all four problems. Each problem counts 5 points (5%) towards your overall final grade. **PLEASE write clearly and in a legible way.** Write your name on all pages, staple them and return this sheet with them. Please feel free to email me (cavaglia@phy.olemiss.edu) if anything is unclear.

Problem 1: Consider a charged particle moving with constant velocity v_0. At time $t = 0$, the particle uniformly accelerate for a finite time interval ΔT. Compute: (a) The total energy radiated in EM waves; (b) The spectral distribution of the radiation.

Problem 2: A variation of Maxwell electrodynamics is described by the Lagrangian density

$$\mathcal{L} = -\frac{b^2}{4\pi}\left(1 - \sqrt{1 + \frac{2S}{b^2} - \frac{P^2}{b^4}}\right) ,$$

where b is a real constant, $S = F^{\mu\nu}F_{\mu\nu}/4$ and $P = \mathcal{F}_{\mu\nu}F^{\mu\nu}/4$ with $\mathcal{F}_{\mu\nu}$ being the dual field tensor of $F^{\mu\nu}$. (a) Compute the field equations; (b) Show that this theory can be defined in terms of a four potential A_μ; (c) Show that when $b \to \infty$ Maxwell equations are recovered.

Problem 3: An electromagnetic wave traveling in the z-direction strikes a point particle at rest with charge q at $t = 0$. The E-field is taken in the x-direction and the B-field is taken in the y-direction. The magnitudes of the electric and magnetic fields of the light wave are given by $E = E_0 \sin(\omega t + \phi)$ and $B = B_0 \sin(\omega t + \phi)$, where ϕ is an arbitrary phase angle. (a) Compute the velocity of the particle at $t > 0$; (b) Compute the limit of the velocity for large frequencies; (c) Repeat the calculations of (a) and (b) using the Abraham-Lorentz equation and show that the time-averaged total force on the particle is equal to the Thomson cross section times the Poynting vector of the wave.

(turn over)

Problem 4: Consider a medium with permittivity and permeability varying synchronously, i.e. $\epsilon(t) = \epsilon_0 a(t)$, $\mu(t) = \mu_0 a(t)$. (a) Derive an equation which gives the displacement field in terms of the given source current density J; (b) Taking a Fourier and a Laplace transform, find the solution of the above equation in the $\mathbf{x}, t$ domain in terms of the Green's function of the Helmoltz equation.