

The University of Mississippi
Department of Physics and Astronomy
Spring 2011

Mathematical Physics - Phys 308

Preliminary exam

THIS EXAM IS FOR PRELIMINARY TESTING ONLY AND WILL NOT COUNT TOWARDS THE FINAL GRADE OF THE COURSE.

INSTRUCTIONS - Read carefully before turning the page.

- Work all problems.
- The use of books, printed or handwritten notes, tables, and any electronic devices is not allowed.
- Please write your answers neatly in black or blue ink or pencil and include details of all calculations.
- Write your name on top of all pages and staple them together.

1. Compute the definite integral

$$I = \int_0^{\pi} dx \sin(\sqrt{x}).$$

2. Compute the commutator of the matrices A and B , $[A, B] = AB - BA$, where

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

3. Find the solution of the following differential equation that satisfies the boundary conditions $y(0) = 0$, $y'(0) = -1$:

$$y''(x) + 3y'(x) = 0.$$

4. Compute the gradient in Cartesian coordinates of the scalar function $f(x, y, z) = xy^2/z$ at the point $\mathbf{a} = (0, 1, 1)$.

5. Compute the inverse of the matrix

$$A = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 1 & 0 \\ 1 & 2 & 3 \end{pmatrix}.$$

5. Given the Cartesian vectors $\mathbf{a} = 3\mathbf{e}_1 + 2\mathbf{e}_2 - \mathbf{e}_3$ and $\mathbf{b} = -\mathbf{e}_1 + \mathbf{e}_3$, compute:

a) $\mathbf{a} \cdot \mathbf{b}$,

b) $\mathbf{a} \times \mathbf{b}$,

c) $\sqrt{|\mathbf{a}|^2 + |\mathbf{b}|^2}$.

6. Expand the function $f(x, y, z) = x \sin(3x + \pi)$ at second non-trivial order around the point $x = 0$.

7. Evaluate the expression $S(a, b) = a \sum_{k=1}^{\infty} b^{-k}$, where $a = 3$ and $b = 1/3$.

8. Compute the real and imaginary parts of the complex function $f(z) = z^3$, where $z = x + iy$.