Assignment # 4 Due 02/23/11

1. Let each of the following matrices M describe a deformation of the (x, y) plane. For each given M find: the eigenvalues and eigenvectors of the transformation, the matrix C which diagonalizes M and specifies the rotation to new axes (x', y') along the eigenvectors, and the matrix D which gives the deformation relative to the new axes. Describe the deformation relative to the new axes.

$$\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \qquad \begin{pmatrix} 5 & 2 \\ 2 & 2 \end{pmatrix}$$

2. Show that the trace of a rotation matrix equals $2\cos\theta + 1$ where θ is the rotation angle, and the trace of a reflection matrix equals $2\cos\theta - 1$.

3. Show that if D is a diagonal matrix, then D^n is the diagonal matrix with elements equal to the n^{th} power of the elements of D. Also show that if $D = C^{-1}MC$, then $D^n = C^{-1}M^nC$, so $M^n = CD^nC^{-1}$. *Hint*: For $n = 2$, $(C^{-1}MC)^2 = C^{-1}MCC^{-1}MC$; what is CC^{-1}?

4. Show that each of the following matrices is orthogonal and find the rotation and/or reflection it produces as an operator acting on vectors. If a rotation, find the axis and angle; if a reflection, find the reflecting plane and the rotation, if any, about the normal to that plane.

$$\frac{1}{11}\begin{pmatrix} 2 & 6 & 9 \\ 6 & 7 & -6 \\ 9 & -6 & 2 \end{pmatrix} \qquad \frac{1}{2}\begin{pmatrix} -1 & -1 & \sqrt{2} \\ 1 & 1 & \sqrt{2} \\ \sqrt{2} & -\sqrt{2} & 0 \end{pmatrix}$$

5. Find the eigenvalues and eigenvectors of the following matrices. Do some problems by hand to be sure you understand what the process means. Then check your results by computer.

$$\begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix} \qquad \begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}$$

6. Find the eigenvalues and eigenvectors of the real symmetric matrix

$$M = \begin{pmatrix} A & H \\ H & B \end{pmatrix}.$$

Show that the eigenvalues are real and the eigenvectors are perpendicular.